CONSTRAINTS ON GLOBAL $N = 2$ SUPERSYMMETRIC GAUGE THEORIES
FROM PERTURBATIVE UNIFICATION

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ABSTRACT

We consider the implication of perturbation theory on $N = 2$ supersymmetric theories for electroweak and strong interactions. We compute the scale at which colour becomes non-perturbative in the ultra-violet region as a function of the supersymmetry breaking scales and of the number of families. The constraints of these results on schemes with and without unification and the problems with model building are discussed.

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Gauge theories that are invariant under *extended* ($N > 1$) supersymmetry transformations provide an interesting way of constructing *finite* field theories of elementary particle interactions [1]. The best example of this kind is provided by the $N = 4$ supersymmetric gauge theories, which are finite to all orders in perturbation theory [2]. However, they face a serious problem as far as the application to realistic model building is concerned: the matter multiplets belong to the adjoint representation of the gauge group. Confining ourselves to the case of *global* supersymmetry, the class of $N = 2$ supersymmetric gauge theories appears as a better candidate from a phenomenological point of view. Actually, the only dimensionless coupling constants they contain are the gauge couplings, and the corresponding β functions have contributions at one-loop *only* [3] which are given by^{*)} $\beta(g_A) = -g_A^3/8\pi^2[C_2(G_A) - \sum_R T(R_A)]$ for the gauge coupling g_A in the gauge group $\Pi_A G_A$, where $C_2(G_A)$ is the quadratic Casimir for the adjoint representation and $T(R_A)$ (multiplied by the rank of the group) are the second-order Dynkin indices for the representations R_A of G_A . As a consequence of this point, the requirement of finiteness naïvely calls for a grand unification^{**)} . In fact, the ‘observed’ low-energy theory is *not* finite since the matter representations give rise to non-vanishing β functions. Therefore, one is possibly led to understand such a theory as being the low-energy remnant of a *finite* GUT, namely a GUT for which the β function vanishes above the unification point. At lower energies, the heavy particles decouple, and since they have in general non-vanishing contributions to the β functions, they will make the gauge couplings run logarithmically below their mass threshold. Recently, a systematic analysis has been performed by some authors, and the various matter representations of the classical groups which lead to a finite $N = 2$ GUT have been singled out [4].

Remarkably enough, there is yet a totally different reason which makes these theories as appealing as they are challenging: their great predictive power for the fermion mass spectrum. Actually, in the supersymmetry limit no free Yukawa coupling is allowed, in that the fermion-fermion-scalar interactions are governed by gauge couplings *only*. To break the gauge group spontaneously and supersymmetry explicitly, as well as to get generation-dependent fermion masses, the simplest strategy is to add ‘soft’ breaking terms [5], which allow for a very limited freedom in the free parameters^{***)}. The corresponding increase in predictivity with respect to the models considered so far is impressive. Last but not least, a great virtue of the theories under consideration is the ease with which they can be disproved in LEP and supercollider experiments. In fact, they predict that all known quarks and leptons should have ‘mirror’ partners, which enjoy low-energy $V + A$ weak interactions. Clearly, since the mirror matter fermions are chirally protected by $SU(2)_L \otimes U(1)_Y$, their masses are expected to be in the 100 GeV range, at least as long as the theory is realized in the perturbative regime.

The aim of the present letter is to analyse the constraints dictated by perturbation theory on realistic unified $N = 2$ supersymmetric gauge models of elementary particle interactions. We expect such constraints to be *non-trivial* since, as a consequence of the large number of scalars and fermions implied by the multiplet structure, the theories of this class are not asymptotically free.

*) In the case of pseudoreal representations of G , the contribution to the β function coefficients is $+(g_A^3/16\pi^2) \sum_{RA} T(R_A)$ (see J.P. Derendinger et al. in Ref. [4]).

**) This point of view is actually rather controversial since at the unification scale the $O(M_P)$ gravitational effects and new physics may play a crucial role, and the requirement of finiteness in a ‘truncated’ renormalizable theory may be misleading.

***) Notice that since the low-energy theory is not finite, the coefficients of the soft breaking terms are not required to obey the algebraic relations that preserve finiteness.

We first give a very schematic account of the structure of the models under consideration in terms of $N = 1$ superfields generated by the supersymmetry charge Q_1 . The two building blocks are the $N = 2$ vector multiplet and hypermultiplet [6]. The former belongs to the adjoint representation of the gauge group G and contains one $N = 1$ vector multiplet $V_a \equiv (v_a^\mu, \lambda_a)$ and one $N = 1$ chiral multiplet $\Phi_a \equiv (\phi_a, \psi_a)$. The latter contains two $N = 1$ chiral multiplets $X \equiv (x, \psi_x)$, $Y \equiv (y, \psi_y)$, transforming as the representations R and $\bar{R}$ of G , respectively.

Throughout this paper, we assume that the $N = 2$ supersymmetric low-energy model is based on the gauge group $G = SU(3)_C \otimes G_W$, where $SU(3)_C$ is just the conventional colour and $G_W \supseteq SU(2)_L \otimes U(1)_Y$, with N_F standard families of quarks and leptons in the form of hypermultiplets in the fundamental representation of G . The case of $SU(3)_C$ replaced by $SU(4)_{PS}$ Pati-Salam with $B - L$ as the fourth colour is briefly discussed. Of course, the Higgs supermultiplets have also to be introduced to spontaneously break G_W , but they are irrelevant to our analysis. Actually, we will consider only the energy behaviour of the $SU(3)_C$ [$SU(4)_{PS}$] coupling constant $\alpha_C(E) \equiv g_3^2(E)/4\pi$. Before plunging into the renormalization group calculations, we would like to discuss an important point. Conceptually, the most serious reason for applying supersymmetry to particle physics is perhaps to get rid of quadratic divergences associated with the elementary scalar fields. It is by now well-known that this can be achieved if, for example, the supersymmetry generated by Q_1 is broken in the TeV range [7]. What about the second supersymmetry? We do not have any argument against the possibility that Q_2 can be broken at a much higher scale, even though the breaking of Q_2 at a low scale is phenomenologically much more appealing. As we said, we consider only the situation in which supersymmetry is broken explicitly in a ‘soft’ way^{*)}. Now, since there exist soft breaking terms preserving Q_1 but not Q_2 , we will discuss the general case in which Q_2 is broken at a scale M_2 and Q_1 at a scale $M_1 \leq M_2$. With a view to later convenience, we parametrize these scales as $M_1 \equiv 10^\alpha \text{ GeV}$, $M_2 \equiv 10^\beta \text{ GeV}$.

Next, as in any renormalization group calculation, we have to exhibit the various mass thresholds for the coloured particles. The $SU(3)_C$ gluons V_a^μ are massless, and the gauginos λ_a have a tree-level Majorana mass of order M_1 , since it breaks the SUSY generated by Q_1 . On the other hand, we take the masses of the mirror gauginos ψ_a and of the mirror gauge scalars ϕ_a of order M_2 , since the latter may have a soft breaking mass term which preserves Q_1 . The quarks and their mirrors are chirally protected by $SU(2)_L \otimes U(1)_Y$: consequently we assume for the latter a common average mass $k \times 10^2 \text{ GeV}$, keeping k as a free parameter, whereas ordinary quarks have a mass smaller than $M_W \simeq 80 \text{ GeV}$. Although our analysis applies to an arbitrary number of families N_F , we assume that possible unknown families lie beyond the M_W mass. Therefore we have taken, as indicated by UA1 results at the CERN $p\bar{p}$ Collider, $m_{top} < M_W$. How heavy are the squarks and mirror squarks? They become massive at tree-level by soft breaking terms which necessarily violate both Q_1 and Q_2 . Therefore, we assume that their masses are of order M_1 .

We are now in a position to compute the scale at which colour becomes non-perturbative in terms of the free parameters k , α , β and the number N_F of conventional families. To be sure, nobody can tell the exact point at which this phase-transition takes place. The analogy with the infrared behaviour of QCD is useless, for in such a case the phase-transition is triggered by the

^{*)} Actually we are not considering that soft breaking terms leading to realistic $N = 2$ models can be obtained by a hidden sector of $N = 2$ supergravity as is the case for $N = 1$ theories [8]. Constraints occurring from $N = 2$ SUGRA may restrict the soft breaking terms of $N = 2$ global supersymmetry.

gluon interactions, whereas in our case we are interested in the ultraviolet regime where the fermion interactions are responsible for the non-perturbative behaviour. So the best we can do is to determine the energy scale $E = 10^x \text{ GeV}$ at which α_s becomes one. There is a very remarkable point about the one-loop renormalization group analysis which we are going to present. In standard theories our results would be rather unreliable, since for α_s close enough to one, the higher-loop contributions are not negligible. The situation is *different* in $N = 2$ supersymmetric gauge theories: as we said, the only contributions to $\beta(g)$ come from one-loop. Therefore, our results are *exact*, as long as perturbation theory works. The computation of the β function coefficients in the various energy ranges is straightforward and the renormalization group equation for α_s reads:

$$\begin{aligned} \alpha_s (10^x \text{ GeV})^{-1} &= \alpha_s (M_W)^{-1} + \frac{1}{2\pi} \left\{ \left(-11 + \frac{4}{3} N_F \right) \left[\ln(4/5) - \ln k \right] + \right. \\ &+ \left(-11 + \frac{8}{3} N_F \right) \left[\ln k + (2 - \alpha) \ln 10 \right] + \left(-9 + 4 N_F \right) (\alpha - \beta) \ln 10 + \\ &\left. + \left(-6 + 4 N_F \right) (\beta - x) \ln 10 \right\}. \end{aligned} \quad (1)$$

We take $\alpha_s(M_W) \approx 0.12$, which corresponds to $\Lambda_{\overline{\text{MS}}} = 250 \text{ MeV}$ [9]. Requiring $\alpha_s(10^x \text{ GeV}) \leq 1$, Eq. (1) becomes

$$\begin{aligned} &-1.1199 + 0.2122 N_F \ln k + 1.9072 N_F + 0.4886 N_F \alpha + \\ &+ 0.7330 \alpha + 1.0994 \beta + 2.1989 x - 1.4659 N_F x > 0. \end{aligned} \quad (2)$$

Recently, two models for $N = 2$ supersymmetric low-energy gauge interactions have been discussed in detail [10]. The first one is based on $G = \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$: it faces difficulties both in splitting the mirror fermions from ordinary ones and in its vacuum structure. The second one has gauge group $G = \text{SU}(3)_C \otimes \text{SU}(4)_W \otimes \text{U}(1)_{B-L}$ and is free from the above problems. In particular, the mirror fermions get tree-level masses $\approx 100 \text{ GeV}$ from $\langle \Phi \rangle \neq 0$, whilst the standard fermions have realistic radiative masses. Unfortunately, the second point forces the SUSY-breaking scale to be rather high, thereby disfavours the naturalness of the Fermi scale. Specifically, in these models it is assumed that $\alpha = \beta$, and it is required that $M_1 = M_2 \geq 10^5 \text{ GeV}$. Moreover, it has been pointed out [10] that within the second model the K_L - K_S mass difference yields the bound $M_2 \geq 7 \times 10^5 \text{ GeV}$ for natural values of the free parameters. We would like to discuss the implications of Eq. (2) with an eye to these models.

First, we can safely drop the term containing $\ln k$ in Eq. (2), since the mirror fermion masses come from the spontaneous breakdown of $\text{SU}(2)_L \otimes \text{U}(1)_Y$, and so $k \lesssim 2.5$ from perturbation

theory. Consequently such a term is at most $0.1944 \times N_F$, which in turn is smaller by one order of magnitude than the third term in Eq. (2).

Now we consider different situations, but we will anyway assume throughout that M_1 and M_2 should not exceed the Planck mass $M_P \approx 10^9$ GeV, so as to have an effective unbroken supersymmetry when the quantum gravitational effects become dominant.

First, it can be supposed that no new physics shows up between the Fermi scale — or the scale of $SU(4)_W$ breaking — and the Planck scale. Moreover, it can also be supposed that the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ or $SU(3)_C \otimes SU(4)_W \otimes U(1)_{B-L}$ theories are realized in the perturbative phase up to the Planck mass. Consider first the case in which both supersymmetries are broken at the same scale, i.e. $M_1 = M_2 \equiv M_{\text{SUSY}}$. Then we get $M_{\text{SUSY}} \geq 2 \times 10^{11}$ GeV for $N_F = 3$ and $M_{\text{SUSY}} \geq 5 \times 10^{16}$ GeV for $N_F = 4$, whilst for $N_F > 4$ it turns out that $M_{\text{SUSY}} > M_P$. Next, in the case $M_1 \neq M_2$, we get $M_1 \geq 1.5 \times 10^7$ GeV for $N_F = 3$ and $M_1 \geq 6.3 \times 10^{15}$ GeV for $N_F = 4$, whilst no solution exists for $N_F > 4$. Notice that the constraint $M_2 \leq M_P$ has been used here. Therefore, the option considered so far is untenable for more than four conventional families, and even for three families it seems to totally upset the spectrum of standard fermion masses considered in the two models discussed above. This last point requires, however, a much more thorough analysis than the one presented in Ref. [10].

Alternatively, one can take the attitude that a *conventional* grand unification occurs — hopefully, a *finite* one. By this we mean that: i) at some scale M_{GUT} the low-energy gauge groups disappear into a larger one; ii) the nucleon decay is mediated by some gauge bosons without any extra suppression; iii) the theory stays perturbative up to M_{GUT} . Then M_{GUT} has clearly to be at least as large as 10^{15} GeV. Since the theory is not asymptotically free, we expect the gauge coupling of the grand group to be rather close to one at M_{GUT} , which requires a higher value of M_{GUT} in order to ensure consistency with proton decay negative experimental results. In this case it is obvious that M_2 can be at most M_{GUT} since we are describing an $N = 2$ renormalizable GUT. As before, consider first the case $M_1 = M_2 \equiv M_{\text{SUSY}}$. One gets $M_{\text{SUSY}} \geq 4 \times 10^8$ GeV for $N_F = 3$ and $M_{\text{SUSY}} \geq 6.3 \times 10^{12}$ GeV for $N_F = 4$, whereas as before no solution exists for $N_F > 4$.

Next, in the case $M_1 \neq M_2$, one gets $M_1 \geq 2.5 \times 10^5$ GeV for $N_F = 3$ and $M_1 \geq 8 \times 10^{11}$ GeV for $N_F = 4$; again no solution exists for $N_F > 4$. This follows from the above-stated requirement that $M_2 \leq M_{\text{GUT}}$. So the moral of this analysis seems to indicate that the model described in Ref. [10] might have problems with perturbative unification since it assumes $M_1 = M_2$. One should rather look for models with $M_1 < M_2$, with M_2 not far from the Planck scale.

The last option we consider is that, at some scale $\tilde{M} \ll 10^{15}$ GeV, $SU(3)_C$ disappears as a consequence of new physics showing up. It might be that a *low-energy* grand unification occurs with nucleon decay very suppressed or even absent [12], or that at least the colour interactions change. We would like to discuss briefly this last possibility and to show how this can improve the situation considered above. We take the Pati-Salam group $SU(4)_{\text{PS}}$ [13] which breaks down to $SU(3)_C$ at a scale $\tilde{M}$. The breakdown is accomplished by a Higgs hypermultiplet in the 4 of $SU(4)_{\text{PS}}$. The corresponding renormalization group equation analogous to Eq. (1) can be straightforwardly derived. As an illustration, we report only the case $M_1 = M_2 = \tilde{M} \equiv M_{\text{SUSY}}$ only. Then the request that $SU(4)_{\text{PS}}$ stays perturbative up to M_P implies $M_{\text{SUSY}} \geq 3.1 \times 10^7$ GeV for $N_F = 3$, almost four orders of magnitude *lower* than in the $SU(3)_C$ case.

Finally, we would like to summarize the above results. The naturalness of the Fermi scale as well as radiative conventional fermion masses call for a rather low SUSY-breaking scale. However, in the $N = 2$ theories under consideration, the request that perturbation theory works up to superheavy energies (not far away from the Planck scale) pushes the SUSY-breaking scale

to rather high values. A compromise can be found, but at the price of breaking the two supersymmetries at very different scales. Hence, we see that the difficulties in performing the symmetry breaking of a finite $N = 2$ standard GUT [14] come along with the difficulties of keeping the theory perturbative up to M_{GUT} and possibly beyond. This seems to us a good indication that in the context of $N = 2$ supersymmetry one should look for something which differs from the standard GUT philosophy. Certainly, the theories discussed here do not like to stay perturbative all the way up to the Planck mass. An interesting possibility is that around M_{GUT} the non-perturbative regime takes over, leading to the formation of condensates which then accomplish the GUT symmetry breaking [15]. Alternatively, we find that the $N = 2$ SUSY gauge theories may provide the ideal framework for a scenario *à la* Maiani–Parisi–Petronzio [16], thanks to their intrinsic asymptotically divergent nature. We will discuss this latter point in a forthcoming paper.

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