CERN-TH.4074/84
LAPP-TH-129

THE DISTORTIONS OF THE MICROWAVE BACKGROUND RADIATION,
HEAVY NEUTRAL DECAYING LEPTONS AND DECAYING GRAVITINOS

P. Salati *)

CERN, Geneva, Switzerland
LAPP, Annecy-le-Vieux, France

A B S T R A C T

Recent measurements of the spectral distortions of the cosmic microwave background radiation (CBR) indicate that the chemical potential μ of the photon gas filling the early Universe is less than $O(10^{-2})$ and might be as small as 10^{-3} , 10^{-4} . We have reinvestigated the bounds derived from this low value of μ , integrating numerically the evolution in time of the typical length scale factor of the Universe, the entropy per baryon and the chemical potential of photons.

A heavy "Dirac" neutrino is much more constrained than a "Majorana" one, owing to a leptonic asymmetry between the neutrino and its antiparticle which arose at the Grand-Unified epoch. If $\mu < O(10^{-4})$, its lifetime is less than $O(10^5)$ seconds, even for a 1 TeV neutrino.

Finally the present density of an unstable gravitino, produced at the end of the inflation era, during the reheating, must be sufficiently small not to distort the CBR overmuch with its decay products. This results in an upper limit upon the maximal reheating temperature $T^{\max}$ which, roughly, lies between $4 \cdot 10^8$ to $3 \cdot 10^{10}$ GeV if μ is less than $O(10^{-3})$.

*) On leave of absence from Université de Savoie (Chambéry).

Since it was discovered in 1965¹⁾ the Cosmic Background Radiation (CBR) has been extensively studied²⁾. The present CBR is a relic of the photon gas that filled the early Universe. The fact that its energy distribution approaches a planckian shape, and its high angular isotropy are a great support to the so-called Hot Big-Bang Model^{3),4)}. However precise measurements of the CBR have been carried out and may leave room for deviations from this standard case.

- Angular inhomogeneities could exist. (At the present time, only the dipole effect⁵⁾ has been seen).
- The energy/frequency curve could show distortions from a planckian distribution.

The lack of large angular scale inhomogeneities is a strong evidence for the non-local origin of the CBR, arguing for its cosmological nature. The small angular scale inhomogeneities are a probe of the process of galaxy formation. In this paper we shall focus on the possible departures of the spectrum from a pure black-body one. Recent measurements carried out at low frequencies by an Italo-American group at the White-Mountain in California⁶⁾ disagree with previous data obtained in the infrared region by Woody and Richards⁷⁾ (if a planckian distribution is assumed). At the present time the observational situation is not clear. The results of Woody and Richards might be too high by 25% because of an error of calibration. If so, from 0.07 to 12 cm, the observed spectrum of the CBR would be consistent with a pure black-body (chemical potential $\mu = 0$) of a 2.74 K temperature. However if the Woody and Richards' calibration is correct, evidence might be produced for a non-zero μ . Therefore the observational bound on μ is 10^{-2} but could be decreased if the Woody and Richards' calibration proved to be incorrect. Such a low value of μ provides constraints on the behaviour of decaying⁴⁾ particles which are prohibited from distorting the CBR overmuch. In this paper we reinvestigate the limits on their lifetime or density and we vary μ from 10^{-2} (Woody and Richards are correct) to a lower value (we take 10^{-4} for illustration purpose).

In the first part of the paper, we establish differential equations governing the expansion of the Universe, the increase of the photon entropy per baryon and the evolution of the chemical potential μ . We solve them numerically and do not use any approximation.

In the second part, we apply our calculations to neutral leptons (the most popular one is the neutrino) and gravitinos. We integrate numerically the equations set up previously and we require the CBR not to be overmuch distorted. We study the bounds on the neutral lepton lifetime, varying its mass and using different values for the baryon density $\Omega_B = n_B/n_T$. We show that if this particle is of the Dirac type it must decay earlier than if it were of the Majorana type.

The gravitino abundance depends on the maximal reheating temperature $T^{\max}$ just after inflation. A 100 GeV gravitino decays 10^8 seconds after the Big-Bang.

Any release of energy at that time is dangerous for the CBR. We find that $T^{\max}$ ranges from 10^9 GeV to 10^{11} GeV. For a 20 GeV gravitino and $\mu = 10^{-4}$, $T^{\max}$ is small : $3.6 \cdot 10^8$ GeV. (Our limits agree with the results obtained by J. Ellis, J.E. Kim, D.V. Nanopoulos and S. Sarkar⁸⁾).

Equations of the evolution in time of the scale factor of the Universe, the entropy per baryon and the chemical potential μ .

As the mechanism of the distortions of the CBR has been studied in detail in the literature⁴⁾, we directly present the method we followed. The innovation of this paper is the exact numerical solution of the evolution equations. The quantities of interest (Universe temperature, entropy per baryon and μ) are time-dependent and their behaviour is studied.

We deal with a radiatively unstable A particle.

$$A \rightarrow \text{light particle} + mT$$

The A mass is M. Its lifetime is τ and we define ω as the ratio:

$$\omega = \frac{n_A \text{ after decoupling}}{n_A \text{ before decoupling}} \quad (1)$$

Therefore, ω describes the depletion in A during its quenching. (We deal with a temperature T_T less than 0.1 MeV. The ν_e , ν_μ and A are already frozen). The average energy of each photon is FM.

The typical length scale factor R of the Universe is related to the ν temperature by:

$$RT_\nu = \text{Constante} \quad (2)$$

The evolution of T_ν in time is governed by the first differential equation

$$-\frac{\dot{T}_\nu}{T_\nu} = \left(\frac{8\pi G}{3}\rho\right)^{1/2} \quad (A) \quad (3)$$

ρ stands for the Universe energy density. We take into account the ultra-relativistic ν_e 's, ν_μ 's and T gas, the baryons and the A particles. The latter may sometimes dominate ρ until they decay and largely perturb the Universe expansion. Their contribution writes:

$$\rho_A = \frac{3}{2\pi^2} \xi(3) \omega T_\nu^3 M e^{-t/\tau} \quad (4)$$

Period 1 : $t < t_{dc}$

The chemical potential μ of the CBR is always zero, despite the energy released by the decaying A's. The relaxation rate of μ towards 0 ($\Gamma_{\mu \rightarrow 0}$) is basically given by^{9),4)} the double-Compton scattering time τ_{DC} (this process is responsible for the chemical equilibrium of the photon gas).

$$\Gamma_{\mu \rightarrow 0} \sim \frac{1}{\tau_{DC}} = \frac{8\alpha}{\pi} \frac{T_T^2}{m_e^2} \cdot \sigma_{\text{Compton}} \cdot n_e C \quad (5)$$

During the first period, $\Gamma_{\mu \rightarrow 0}$ is larger than the expansion rate H . t_{dc} is computed by the program requiring that

$$\Gamma_{\mu \rightarrow 0} = H \quad \text{at } t = t_{dc} \quad (6)$$

We consider an expanding volume (covolume) with a typical length scale of T_V^{-1} and study the increase of its coentropy S_T

$$S_T = \frac{y}{\pi^2} \left(\frac{4}{3} I_3(\mu) + \mu I_2(\mu) \right) \quad (7)$$

Two parameters describe the photon gas contained in the covolume:

- The chemical potential μ ($\mu = 0$ during this phase).
- The photon entropy per baryon y

$$y = \frac{T_T^3}{T_V^3} \quad (8)$$

We recall that

$$I_n(\mu) = \int_0^{+\infty} \frac{x^n}{e^{x+\mu}-1} dx \quad (9)$$

When A decays and its products thermalize, S_T increases owing to the heat release δQ :

$$dS_T = \frac{\delta Q}{T_T} \quad (10)$$

The time dependence of y during period 1 obeys the second differential equation:

$$\dot{y} = \frac{3}{16} \frac{\xi(3)}{\xi(4)} \frac{m_{FM}}{T_T} \omega \frac{e^{-t/\tau}}{\tau} \quad (B) \quad (11)$$

Period 2 : $t_{dc} < t < t_c$
 -----c

μ may increase. The decay products thermalize and the CBR is in thermal equilibrium whenever the compton scattering rate (this reaction is responsible for the thermalization of the photon gas) is larger than H :

$$\sigma_{\text{compton}} n_e C > H \quad (12)$$

In general, this condition is always fulfilled until the Primeval Plasma recombines¹⁰⁾. (Meanwhile, for highly energetic photons, the compton scattering cross-section falls down as $1/E_T$). Therefore, we compute t_c numerically, requiring that

$$\sigma_{\text{compton}} n_e C = H$$

or

$$T_T = T_{\text{Recombination}} = 4000 \text{ }^\circ\text{K} \quad (13)$$

During period 2, relation (10) remains valid:

$$\frac{\delta Q}{T_T} = \frac{\partial S_T}{\partial \mu} d\mu + \frac{\partial S_T}{\partial y} dy \quad (14)$$

The photon codensity N_T does not adjust itself in order to insure a zero μ , but increases merely because each A 's decay releases m photons

$$dN_T = d\left(\frac{y I_2(\mu)}{\pi^2}\right) = m N_A \tau^{-1} dt \quad (15)$$

The y and μ increase rates are derived:

$$\dot{\mu} = 1.4 \frac{\sigma m}{y \tau} e^{-t/\tau} \left(\frac{F}{3.6} \frac{M}{T_T} \right) \quad (C)$$

and

$$(\text{Log } y) = \frac{\xi(2)}{\xi(3)} \dot{\mu} \quad (16)$$

As μ has to be less than 10^{-2} (or 10^{-4}), the resulting increase of entropy per baryon, during period 2, is very small (less than 1%). We have fixed y at the end of period 1.

We have numerically integrated the coupled differential equations (A) (B) during period 1 starting at a temperature T_T of 10^9 °K ($t \approx 0(100)$ seconds) and the set (A) (C) during period 2. Fig. 1 gives the example of a 2 MeV A particle with a lifetime of 10^5 seconds. T_T , μ and y are plotted against the age of the Universe t . In this illustration, the temperature decreases as $t^{-0.66}$

at the beginning because the energy density is A dominated. Between 10^4 and 10^5 seconds, the released heat increases the entropy per baryon by a factor $O(20)$. After 10^5 sec., μ is allowed to evolve and grows up to 0.5. T_T now behaves as $t^{-0.5}$ because the Universe has become radiation dominated. It is important to remark that T_T keeps decreasing. The rate of decrease is merely slowed down as has also been recently noticed by R.J. Scherrer and M.S. Turner¹¹⁾.

Heavy "Dirac" neutrinos and gravitinos

We now turn to heavy neutrinos N and the limits upon their lifetime derived from the CBR distortions. If they are of the "Majorana type" N and its anti-particle $\bar{N}$ are identical. Therefore no asymmetry between N and $\bar{N}$ is possible and their present density can be computed "à la Lee and Weinberg"¹²⁾. If M is smaller than $O(2)$ MeV, the neutrino gas is still ultra-relativistic when it decouples. On the other hand, if N is heavier, the decoupling occurs when the neutrinos have substantially annihilated among themselves. The resulting density decreases with increasing mass and it has been shown^{12),13)} that when M is larger than a few GeV, the fossilized mass density is sufficiently small not to overreach the currently observed mass density of the Universe. No limits can be put on N if it is a "Majorana" particle with a mass larger than a few GeV, since even its stability is cosmologically allowed.

The situation is different if N is of the "Dirac type". As the electron and the ν_e can be put into a $SU(2) \times U(1)$ doublet, N should be associated with a charged heavy lepton E^- . At the GUT era, when the lepto-baryonic asymmetry was generated, the doublets

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \text{ and } \begin{pmatrix} N \\ E^- \end{pmatrix}$$

were ultra-relativistic and equally considered by the Grand-Unified forces. Therefore a leptonic asymmetry between N and $\bar{N}$ was generated, and it was of the same order of magnitude than for e^- or baryons. We have assumed, for the sake of simplicity, that

$$N_{\text{asymmetry}} = \delta_N = \frac{N - \bar{N}}{n_T} = \frac{n_B - \bar{n}_B}{n_T} \quad (17)$$

The resulting asymmetry δ_N writes:

$$\delta_N = 3 \cdot 10^{-8} h^2 \Omega_B \quad (18)$$

where h is the present Hubble constant expressed in 100 km/s/Mpc units. When M is larger than a few GeV (about 7 GeV), the frozen density is limited by $\delta_N \cdot n_T$ and cannot decrease below. Therefore, the present N mass density dominates the Universe unless N is unstable.

When $M < 2m_e$ the decay modes are purely radiative:

$$N \rightarrow T \text{ or } 2T.$$

As M overreaches the threshold $2m_e$ the predominant decay channel is now

$$N \rightarrow \nu_e e^+ e^-$$

and if $M \gtrsim 300$ MeV, hadronic decays come into play:

$$N \rightarrow \nu_e q \bar{q}.$$

In these two cases, the products are rapidly thermalized. The electron-positron for example⁹⁾ lose their energy by inverse Compton scattering on thermal photons and then annihilate with each other. The hadronic showers strongly interact with baryons and the resulting pions decay into 70 MeV photons. Thermalization always occurs, and we have assumed that 50% of the initial energy reheats the CBR, setting $m_F = 0.5$ (preserving value). Fig. 2 shows a mass to lifetime plot. Evolution equations (A), (B), (C) have been numerically integrated with $\Omega_B = 0.1$ (curves a-b) and $\Omega_B = 1.0$ (curves c-d). The maximal allowed chemical potential μ has been varied from 10^{-2} (Woody and Richards are right - curves b-d) down to 10^{-4} (curves a-c). If no asymmetry is generated for N , no limit can be put on τ when M is larger than $O(10)$ GeV, merely because the resulting N density is very small. Our results are in agreement with previous ones^{4),13)}. If a leptonic asymmetry is generated the constraints are much stricter and a new region in the (M, τ) plane is forbidden. For example, if μ is less than $O(10^{-3})$ τ has to be always less than $O(10^6)$ seconds and when μ is decreased by a factor 10, the lifetime bounds are decreased by an order of magnitude, showing a great sensitivity to the μ value. Thus, it is of high importance to measure the CBR accurately and to determine μ .

Finally we deal with the gravitino, the $3/2$ spin partner of the graviton which appears in many supersymmetric models¹⁴⁾ and could be unstable if heavier than the gluino or the photino. Many phenomenological supersymmetric models use the Super-Higgs mechanism to break supersymmetry spontaneously and consider the possibility of a gravitino with a mass ranging from 20 to 100 GeV. This eventuality has been studied by J. Ellis et al.⁸⁾. They computed the cosmological production of gravitinos due to other particle collisions after the wash out of inflation. They showed that their present density is given by:

$$n_{3/2} \approx 1.6 \cdot 10^{-13} T_9^R (1 - 0.018 \ln T_9^R) T_V^3 \quad (19)$$

where T_9^R is the temperature (expressed in 10^9 GeV units) obtained just after inflation (during the reheating of the Universe when the Higgs-inflaton releases the vacuum energy). Since the lightest supersymmetric particle, in most supersymmetric models, is an absolutely stable photino, the gravitino decays:

$$g_{3/2} \rightarrow T + \tilde{\gamma}$$

with a lifetime given by

$$\tau = 4 \cdot 10^8 \text{ s } (100 \text{ GeV}/m_{3/2})^3 \quad \text{assuming } m_{\tilde{\gamma}} \ll m_{3/2} \quad (20)$$

Moreover if gluinos too are lighter than the gravitino, the gluino channel is open

$$g_{3/2} \rightarrow g + \tilde{g}$$

and the lifetime is now given by

$$\tau = 4.4 \cdot 10^7 \text{ s } (100 \text{ GeV}/m_{3/2})^3 \quad \text{with } m_{\tilde{\gamma}}, m_{\tilde{g}} \ll m_{3/2} \quad (21)$$

We have investigated the CBR bounds in detail and showed that, once again, a smaller limit on μ means a smaller $T_9^{\max}$ ($T_9^{\max}$ is the upper bound upon T_9^R).

Fig. 3a is a plot of the maximal allowed reheating temperature $T_9^{\max}$, obtained at the end of the inflation epoch, compatible with the distortions of the microwave background (curves a $\rightarrow$ e) and nucleosynthesis⁸⁾ (curves f-g).

Before discussing these results, we compare the exact calculation of μ with the standard approximation¹⁵⁾. Equation (C) writes:

$$d\mu = 1.4 \frac{\rho_A(t)}{\rho_T(t)} m_F \tau^{-1} dt = 1.4 \frac{d\rho_T}{\rho_T} \quad (22)$$

The final μ is the result of the integration of eq.(22):

$$\mu_{\text{exact}} = 1.4 \int_{t_{dc}}^{t_c} \frac{d\rho_T}{\rho_T} \quad (23)$$

The approximation lies on the assumption that the A's decay all together at τ .

$$\mu_{\text{approx.}} = 1.4 \frac{\Delta\rho_T}{\rho_T} \quad \text{a time } t = \tau \quad (24)$$

A straightforward calculation gives the ratio between the exact and the approximate solutions:

$$\frac{\mu_{\text{exact}}}{\mu_{\text{approx}}} = \left[\frac{\sqrt{\pi}}{2} \Theta(\sqrt{x}) - \sqrt{x} e^{-x} \right]_{t_{dc}/\tau}^{t_c/\tau} \quad (25)$$

In general this ratio is small compared to 1. Therefore, the approximation overestimates the distortions of the CBR and the derived bounds are stricter than the exact ones.

In the case of the gravitino, it turns out that (see fig. 3b):

$$t_{dc} \ll \tau \ll t_c \quad (26)$$

Taking $t_{dc} = 0$ and $t_c = +\infty$ in eq.(25) gives a ratio of $\sqrt{\pi}/2$. In that case, the approximation is very good since it only overestimates μ by 13% !

The best nucleosynthesis bound has been recently obtained by J. Ellis, D.V. Nanopoulos and S. Sarkar. They found that T_9^{max} was pushed down to .2 for $\Omega_B < 8 \cdot 10^{-2}$ ($\delta_B = n_B/n_T < 6 \cdot 10^{-10}$).

Recent data in the infrared region provided¹⁶⁾ by a study of the CN interstellar gas disagree with Woody and Richards' conclusions, and are consistent with low frequency measures. New studies of the CBR have been¹⁷⁾ undertaken and show that, at least, μ is $\lesssim 10^{-3}$. Therefore, T_9^{max} is $\lesssim 1$ to 10 for $\Omega_B > 8 \cdot 10^{-2}$.

The conclusion is that both analyses (nucleosynthesis and CBR) complement each other and that the upper limit upon T_9^{max} lies in the range 0.1 to 10. The precise knowledge of μ is of some importance because a T_9^{max} as small as 1 or 10 presumably rules out the scenario of a $O(M_W)$ gravitino, produced at the end of inflation and decaying long afterwards. The Grand Unified forces are supposed to generate the baryonic number after inflation and a high temperature is needed (larger than 10^{10} GeV). Therefore, in this scenario cosmology forbids the gravitino to be heavier than the photino or the gluino. No problem arises if, this time, the gravitino is assumed to be the lightest supersymmetric particle (LSP). Gravitinos are always produced in the same way. A 100 GeV photino is not a cosmological embarrassment since it decouples as a neutral lepton, fossilizing to a tenuous residual population.

So in these two illustrations, a more accurate measure of the CBR spectrum proves to be of great importance for particle physics merely because the derived bounds heavily constraint the theory, for example restricting the gravitino to a lighter mass than the photino or gluino.

ACKNOWLEDGEMENTS

I would like to thank J. Ellis, G. Girardi and S. Sarkar for helpful discussions and a critical reading of the manuscript. I wish to thank the CERN theory division for warm hospitality and financial support.

REFERENCES

- 1) A.A. Penzias and R.W. Wilson, Ap. J. 142, 419 (1965);
A.A. Penzias and R.W. Wilson, Astron. J. 72, 315 (1967).
- 2) For a review, see for example:
R. Weiss, Ann. Rev. Astron. Astrophys. 18, p.489-535 (1980).
- 3) S. Weinberg, Gravitation and Cosmology (Wiley, New York).
- 4) A.D. Dolgov and Ya. B. Zeldovich, Rev. Mod. Phys. 53, 1 (1981).
- 5) See for example:
D.T. Wilkinson, invited talk at the first ESO-CERN Symposium, p. 153-166,
"Large-Scale Structure of the Universe, Cosmology and Fundamental Physics",
21-25 November 1983, G. Setti and L. Van Hove eds.
- 6) G.F. Smoot et al., Phys. Rev. Lett. 51, 12, p. 1099-1102 (1983).
- 7) D.P. Woody and P.L. Richards, Astrophys. J. 248, 18 (1981).
- 8) J. Ellis, J.E. Kim and D.V. Nanopoulos, Phys. Lett. 145B, 3,4, p.181-186 (1984);
J. Ellis, D.V. Nanopoulos and S. Sarkar, preprint CERN.TH.4057 (1984).
- 9) S. Sarkar and A.M. Cooper, Phys. Lett. 148B, 347 (1984);
See also A.P. Lightman, Astrophys. J. 244, 392-405 (1981).
- 10) P.J.E. Peebles, Astrophys. J. 153, 1 (1968).
- 11) R.J. Scherrer and M.S. Turner, Fermilab Pub. 84/94 (1984).
- 12) B. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977).
- 13) D.A. Dicus, E.W. Kolb and V.L. Teplitz, Astrophys. J. 221, p.327-341 (1978);
L.M. Krauss, Phys. Lett. 128B, n° 1,2, p.37-43 (1983).
- 14) D.V. Nanopoulos, preprint CERN.TH.3699 (1983);
J. Ellis, preprints CERN.TH.3718 (1983) and CERN.TH.3802 (1984);
R. Arnowitt, A.H. Chamseddine and P. Nath, North-Eastern University preprints
2597 and 2600 (1983);
H.P. Nilles, Phys. Rep. 110, 1 (1984).
- 15) R.A. Sunyaev and Ya. B. Zeldovich, Ann. Rev. Astron. Astrophys. 18, 537 (1980).
- 16) D.M. Meyer and M. Jura, Astrophys. J. 276, L1-L3 (1984).
- 17) S. Sarkar, Private communication.

FIGURE CAPTIONS

- Fig. 1 As an illustration, the behaviour of the photon temperature T_γ , the ratio $n_\gamma(t)/n_\gamma^0(\text{today})$ and the chemical potential μ is plotted against the age of the Universe t for a A mass of 2 MeV, a lifetime of 10^5 seconds, $H^0 = 50$ km/s/Mpc and $\Omega_B = 0.1$. The A particle releases half of its mass to reheat the CBR.
- Fig. 2 A mass to lifetime plot shows the bounds, derived from the maximal chemical potential μ of the microwave background, for "Majorana type" and "Dirac type" heavy neutral leptons. In this last case we have :
- curve a : $\mu < 10^{-4}$, $\Omega_B = 0.1$
 curve b : $\mu < 10^{-2}$, $\Omega_B = 0.1$
 curve c : $\mu < 10^{-4}$, $\Omega_B = 1.0$
 curve d : $\mu < 10^{-2}$, $\Omega_B = 1.0$
 $H_0 = 50$ km/s/Mpc.
 A new region arises for Dirac particles.
- Fig. 3a T_9^{max} is plotted against the present baryon to photon abundances ratio for several sets of parameters :
- curve a : $m_{3/2} = 100$ GeV, gluino channel, $\mu < 10^{-2}$
 curve b : $m_{3/2} = 100$ GeV, no gluino channel, $\mu < 10^{-2}$
 curve c : $m_{3/2} = 100$ GeV, gluino channel, $\mu < 10^{-4}$
 curve d : $m_{3/2} = 100$ GeV, no gluino channel, $\mu < 10^{-4}$
 curve e : $m_{3/2} = 100$ GeV, no gluino channel, $\mu < 10^{-4}$
 curve f : Deuterium limit
 curve g : Helium limit. See ref. 8).
- Fig. 3b The evolutions in time of T_γ and μ are shown for a 100 GeV gravitino, $n_B/n_\gamma = 10^{-9}$ and no gluino channel ($\tau = 4 \cdot 10^8$ sec.). τ lies between t_{dc} and t_c . T_γ behaves as $t^{-0.5}$ (radiation domination).

FIG. 1

FIG. 2

FIG. 3

FIG. 3