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RESOLUTION LIMITS WITH COOLED BEAMS

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ABSTRACT

Storage rings for intense proton and ion beams are under construction which will work in the low and intermediate energy range, for experiments with internal targets. These machines will have a very good vacuum and will be equipped with efficient cooling devices which will give small beam dimensions and a small momentum spread. Under these conditions intrabeam scattering becomes a dominant effect. Calculations were made to estimate the resulting beam dimensions and momentum spread for different ions and energies.

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INTRODUCTION

Storage rings are under construction at Uppsala, Sweden, (CELSIUS)¹ and at IUCF, Bloomington² for experiments with proton and ion beams. For good energy and spatial resolution in experiments with internal targets a circulating beam of small size and small energy spread is desirable. Therefore the phase space density of the injected beam will be increased by an efficient cooling device such as electron cooling with time constants of seconds. The stored beam cannot be cooled to infinitely small dimensions and momentum spread because there are other mechanisms which will "heat" the beam, so that the final dimensions are given by an equilibrium between heating and cooling. These mechanisms can be:

- Heating by machine imperfections:
Resonances, ripple in power supplies, alignment errors etc.
- Heating by scattering:
Residual gas (vacuum), internal targets, intrabeam scattering, electron cooling.

For the following discussions one assumes that the machine is well aligned, the ripple of the power supplies is low and the working point is not too close to a resonance so that heating due to machine imperfections is negligible.

The heating of the beam by scattering on the residual gas and on internal targets and the final equilibrium between heating and cooling can be easily estimated³. For example in CELSIUS one finds for 200 MeV protons and a cooling time of ~ 4 sec the following equilibrium emittances⁴:

Residual gas : vacuum 10^{-12} Torr (NEP), no internal target
 $\epsilon_{EQ} = < 10^{-3} \pi$ mm mrad

Internal target: perfect vacuum, target thickness 10^{-9} g/cm²
 $\epsilon_{EQ} = 0.12 \pi$ mm mrad

It will be shown that the equilibrium emittances due to intrabeam scattering, i.e. the scattering of the beam particles on each other, will be much bigger, so that the scattering on the residual gas and the internal target can be neglected at least for beams with more than 10^9 particles.

To calculate the equilibrium between heating by intrabeam scattering and electron cooling some assumptions are necessary which will be explained hereafter.

INTRABEAM SCATTERING

Scattering of the particles within the beam has the tendency to equalize the "temperature" of the beam in all three planes. Here the temperature is defined for one plane by

$$T = 1/2 mc^2 \beta^2 \gamma^2 \Theta^2$$

where Θ is the beam divergence in one plane. For the longitudinal direction Θ is replaced by $1/\gamma \times \Delta p/p$. As the scattering probability depends on the particle density, intrabeam scattering therefore depends on:

- the beam dimensions, which are given by the focusing properties of the storage ring and the beam emittances
- the number of particles.

An additional problem occurs due to dispersion, the dependence of the radial position of the particle on its momentum. Particles which are changing their momentum by scattering start to make betatron oscillations around the orbit which corresponds to their new momentum. The energy for these oscillations is taken from the longitudinal motion of the particles, which is continuously renewed either by a RF-system or a cooling device. As a consequence the beam is heated continuously.

This heating is counteracted by electron cooling which leads to an equilibrium between heating and cooling. A numerical evaluation of this effect can be made with the help of the theory of A. Piwinsky⁵, from which follows that the behaviour of the beam can be well described by the following invariant:

$$\left(\frac{1}{\gamma^2} - \frac{D^2}{\beta_x^2} \right) \left\langle \frac{\Delta^2 p}{p^2} \right\rangle + \langle \Theta_x^2 \rangle + \langle \Theta_z^2 \rangle = \text{const.}$$

with:

- D = dispersion function
- β_x = horizontal focusing function
- γ = relativistic factor
- $\Theta_{x,z}$ = horizontal resp. vertical divergence of the beam
- $p, \Delta p$ = momentum and momentum deviation, respectively.

The three terms of the invariant correspond to the temperature of the beam in the two transversal and the longitudinal plane. Depending on the local value of D and β_x one can discriminate between the following cases:

$$1. \quad D^2/\beta_x^2 < 1/\gamma^2$$

In this case the temperatures in all 3 planes will equalize which means that the emittances and momentum spread will reach an equilibrium even without cooling (Note that one neglects here other effects such as residual gas scattering etc.). This is in particular always the case for $D = 0$.

$$2. \quad D^2/\beta_x^2 > 1/\gamma^2$$

In this case the first term is negative which means that all three "temperatures" can increase until the beam dimensions are so big that the scattering probability tends to 0.

$$3. \quad D^2/\beta_x^2 = 1/\gamma^2$$

As $\langle D^2/\beta_x^2 \rangle \sim 1/\gamma_{tr}^2$ one finds for a well defined energy (transition energy) the longitudinal temperature equal to 0, which in so far has a meaning that at this energy all particles in the machine have independently of their energy the same revolution velocity. Longitudinal scattering cannot take place !

As the dispersion and the beta-functions change around the storage ring it is therefore evident that intrabeam scattering calculations have to be made in small steps along a superperiod using the local lattice functions. For a precise description it is also necessary to take D' and β'_x into account.

Furthermore intrabeam scattering depends strongly ($\beta^3 \gamma^4$) on the energy and (for different ions) on the square of the classical particle radius $r_i^2 = (Z^2/A)^2 \cdot r_p^2$. To calculate the equilibrium between intrabeam scattering and electron cooling a program written by F. Sacherer and D. Möhl in collaboration with A. Piwinski was modified. It is based on further development of Piwinski's original theory to include the deviations of D and β_x . In Fig. 1 the calculations are compared with the results from ICE⁶. They are in good agreement with the measurements.

Fig. 1: Comparison between the measured equilibrium momentum spread in ICE and the calculations (straight line).

ELECTRON COOLING

Cooling means the reduction of the transverse velocities and the longitudinal velocity spread in the proton or ion beam. This is achieved by repetitive interactions with a "cold" electron beam. The amplitude cooling time is given by:

$$\tau = 0.16 \frac{\beta^4 \gamma^5 e (\theta_e^2 + \theta_i^2)^{3/2}}{r_i r_e j L_c \eta} \quad [\text{sec}]$$

where β, γ are the usual relativistic factors, e [Cb] the electric charge, $\theta_{e,i}$ the divergence of the electron resp. ion beam, j [A/m²] the current density of the electron beam, L_c the Coulomb logarithm and η the ratio between the length of the cooling section and the circumference; r_e [m] is the classical electron radius and $r_i = Z^2/A \cdot r_p$ the "ion radius" given by the charge state Z , the atomic number A and the classical proton radius r_p .

Cooling stops when electrons and ions have the same temperature:

$$T = 1/2 m_e c^2 \beta^2 \gamma^2 \theta_e^2 \sim 1/2 M_i c^2 \beta^2 \gamma^2 \theta_i^2$$

$$\theta_i \sim \theta_e \sqrt{m_e/M_i}$$

which means that in the case of protons, for example, θ_p can become 1/43 of θ_e . Below this value the electron beam will heat the ion beam. However, as the divergence of the ion beam is proportional to $\sqrt{\epsilon/\beta_c}$ with β_c being the local focusing function of the storage ring and ϵ the beam emittance, this can be avoided by a careful selection of the beta-function.

Another problem may occur at very low energies because of the space charge depression in the electron beam. Due to this effect there is a velocity difference between the outer and inner electrons of the beam. (Fig. 2)

velocity profile of e⁻ beam

If this velocity difference becomes too big, the ions are alternatively accelerated or decelerated due to their betatron oscillations in the machine thus making the cooling under certain circumstances impossible. As the velocity difference is proportional to $\sqrt{j}$ with j being the current density, this can be avoided by reducing the beam current. However, this also prolongs the cooling time.

The electron cooling devices foreseen for CELSIUS and for the IUCF cooler ring are similar. They both use a Pierce configuration for the gun region accelerating the beam up to 20-50 keV and afterwards an acceleration tube to bring the beam to the desired energy between 250-300 keV. The beam may even be decelerated down to 10 keV by this tube. As a consequence the current density j , normally showing a $U^{3/2}$ behaviour, is constant.

In the following calculations a current density of 0.5 A/cm^2 , a cathode diameter of 2 cm and a ratio η between the cooling length and the circumference of the ring of 3×10^{-2} (2.5m/80m) was assumed. For low ion energies of 10 MeV/n (corresponding to 5.45 KeV electron energy !) the current density was reduced by a factor 10 to lower the above mentioned space charge depression. Otherwise, the potential difference in the electron beam would have been 325 V $\approx$ 6% !!

As shown before the amplitude cooling time τ of electron cooling depends strongly on the angular divergence of the ion beam θ_i and of the electron beam θ_e :

$$\tau \sim (\theta_e^2 + \theta_i^2)^{3/2}$$

The divergence of the electron beam is given by the construction of the cooling device and the transverse temperature of the cathode, while the divergence of the ion beam depends on the beta-function in the cooling straight section which can be adjusted within some limits.

Therefore the fastest cooling time can be achieved if the divergence of the ion beam in the cooling straight section is smaller than the divergence of the electron beam. If $\theta_i < \theta_e$, the cooling time is independent of the ion beam properties.

For calculating the cooling times for different energies and ions the above condition $\theta_i < \theta_e$ was assumed. For the temperature of the electron beam the value $T \sim 0.2 \text{ eV}$ was taken from the results from Novosibirsk and CERN.

Furthermore as the current density stays constant and the divergence of the electron beam varies with $1/\beta\gamma$, the cooling times scale with $\beta \gamma^2$. With these assumptions the following amplitude cooling times were calculated:

COOLING TIMES IN SECONDS

	p	$^{12}\text{C}^{6+}$	$^{20}\text{Ne}^{10+}$	$^{238}\text{U}^{92+}$
10 MeV/n	3.8	1.26	0.76	0.10
200 MeV/n	2.2	0.74	0.44	0.06
500 MeV/n	4.7	1.56	0.94	0.13

EQUILIBRIUM CALCULATIONS

For the equilibrium calculations one of the possible lattices for CELSIUS⁷ was taken. However, repeating the calculations with a lattice for the IUCF Cooler⁸ gives only differences within a factor of 2.

The calculations were made for p, $^{12}\text{C}^{6+}$ and $^{238}\text{U}^{92+}$. Figs 3 and 4 show the dependence of the equilibrium emittance and momentum spread on energy for 10^{10} particles circulating in the machine; Figs 5 and 6 show the dependence from the number of circulating particles for 10 MeV/n and 500 MeV/n.

BEAM ϕ (mm) EMITTANCE
95% $\beta=0,25$ (mRAD)

Fig. 3

Fig. 4

As it can be seen from Fig. 3 and 4 the energy dependence of the equilibrium beam emittance and momentum spread is relatively small. For the energy range from 100 MeV/n down to 10 MeV/n it should be kept in mind that in this region the current density of the electron beam was reduced by a factor 10. Due to this change in cooling "strength" the energy dependence in this energy region is much stronger. As discussed before the reduction of the current density was assumed to avoid a too big velocity difference in the electron beam due to space charge depression. The factor 10 was chosen to keep

the velocity difference within the same range as for the ICE gun. This choice however is somehow arbitrary and should depend on the individual set-up. Therefore the calculations below 100 MeV/n should be considered only as a guide-line.

Fig. 5 and 6 show that the equilibrium also does not depend strongly on the number of stored particles. Below $\sim 10^9$ particles it might be possible that, depending on the particular set-up the equilibrium emittances and momentum spreads are smaller due to the "magnetization" effect⁹ of the electron beam, which will reduce the cooling times. Above $\sim 5 \times 10^{10}$ particles it may be desirable to have bigger equilibrium emittances and momentum spread to avoid instabilities for example caused by the resistive wall impedance or the Laslett tune shift.

Furthermore the calculations show that the equilibrium depends only weakly on the type of fully stripped ion: one finds a factor 2 between protons and $^{238}\text{U}^{92+}$ for the equilibrium momentum spread and a factor 4 for the equilibrium emittance.

CONCLUSION

Intrabeam scattering is an important effect in the discussed energy and beam intensity range. It may present a serious limit to the available energy resolution for experiments.

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