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COHERENT vs. INCOHERENT QUARK FRAGMENTATION PICTURE

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A B S T R A C T

We study the helicity density matrix of vector mesons produced in e^+e^- interactions at high energies through the two step process $e^+e^- \rightarrow q\bar{q} + VX$. Whereas in the usual incoherent fragmentation picture in which each quark decays independently, $\rho(V)$ is predicted to be diagonal, we find that final state interactions (coherent fragmentation) give rise to a non-vanishing $\rho_{1-1}(V)$. Various corrections to the standard picture such as, e.g., gluon bremsstrahlung, quark masses, transverse momenta, are shown to be small. Therefore, any significant non-zero value of $\rho_{1-1}(V)$ found experimentally can be considered as a clear measurement of coherence effects.

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1. - INTRODUCTION

The QCD based hard scattering picture for the inclusive production of hadrons seems to work quite well in a large class of reactions such as e^+e^- annihilation, deep inelastic lepton-hadron scattering or purely hadronically initiated reactions. The predictions for jet cross-sections at the $p\bar{p}$ collider, for example, have been found to be in agreement with data within, say, a factor of two. This can be regarded as a success, considering the uncertainty in the phenomenological parton distributions and in the value of the strong coupling, the ambiguity due to the definition of Q^2 and, last but not least, the systematic errors of the data.

Despite this, there are still many questions to be answered before the hard scattering picture can be regarded as fully settled. Among them there is the most important one of coherence in the hadronization process. Commonly, hadronization is described as the independent (incoherent) fragmentation of a parton into the observed final particle. This picture implies the assumption that any final state interaction between the various partons - even the minimal requirements of energy-momentum conservation or colour neutralization - can be neglected or, respectively, absorbed into fragmentation functions. It is certainly of utmost importance to study the validity of this assumption. The more detailed Monte Carlo simulations of the hadronization process, although lacking theoretical justification, comprise some effects of final state interactions; at least they guarantee energy momentum conservation. In the Lund model¹⁾, a string is stretched between the coloured partons which is an example of coherence in the fragmentation subject to experimental tests.

Another most challenging issue is the spin structure of the hard scattering picture. The underlying perturbative QCD as a vector interaction between (almost) massless quarks exhibits extremely pronounced spin effects²⁾ which till now are open to experimental verification. Unavoidably, in making predictions for inclusive productions of hadrons one also has to deal with the spin dependence of the fragmentation, again an almost unexplored field.

In this investigation our aim is twofold. We are going to discuss consequences of coherence effects and of the spin structure of QCD. The latter goes parallel with a study of the fragmentation process. The simplest case in which these consequences are revealed in a rather clean way is the study of the spin density matrix of vector mesons produced in e^+e^- annihilations. Most of our results can straightforwardly be carried over to any other reaction in the hard

scattering regime. However, this would be technically more difficult as the fragmenting parton has many other constituents to interact with, and the fragmentation is a much more complex one.

In Section 2, we shall briefly present the well-known results³⁾⁻⁶⁾ for the vector meson density matrix in the standard incoherent fragmentation picture under the neglect of quark masses and transverse momenta in the jet, considering only the lowest order electromagnetic interaction (we have typical PETRA-PEP energies in mind). We proceed in Section 3 by presenting the general formalism for the density matrix in the presence of final state interactions. It will be shown that in this case the density matrix may be non-diagonal, contrary to the lowest-order incoherent picture. Since admittedly we are not able to calculate the non-diagonal elements of the density matrix in a reliable way - this would require us to solve QCD non-perturbatively - we shall restrict ourselves to various corrections to the lowest order incoherent picture such as higher order QCD, electroweak interference effects. It will turn out that none of these corrections leads to a sizeable value for the non-diagonal elements of the density matrix (Section 4). Thus, we are in the position to conclude that the measurement of the vector meson density matrix is a clean way of testing the strength of coherence in the hadronization process. In Section 5 we discuss the density matrix of bottom vector mesons for energies close to the resonances where we have some ideas about the final state interaction. The paper terminates with our conclusions and some final remarks.

Gluon bremsstrahlung is one of the corrections to the incoherent approach estimated in Section 4. Apart from that it is a quantity extremely sensitive to the QCD spin structure and therefore has some interest in itself. In the Appendix, we suggest methods to measure this contribution directly, regarding this as a severe test of the partonic interaction.

2. - INCOHERENT FRAGMENTATION PICTURE FOR THE PROCESS $e^+e^- \rightarrow V X$

In this model the inclusive production of vector mesons is visualized as shown in Fig. 1. The centre-of-mass helicity density matrix of the vector meson is then expressed by

$$\rho_{\lambda\lambda'}(V) \frac{d\sigma}{dz} (e^+e^- \rightarrow V X) = \sigma_0 \sum_q \sum_{\nu\nu'} e_q^2 \rho_{\nu\nu'}(q) D_{q\nu\nu'}^{V\lambda\lambda'}(z) \quad (2.1)$$

after having integrated over angles for simplicity and in order to improve statistics in an experiment. All labels are helicity ones and the sum goes over all types of quarks ($u, \bar{u}, d, \dots$) which can be created at a given energy. As usual e_q is the quark charge and z is the fraction of quark momentum carried by the vector meson. The transverse momentum p_T of the vector meson with respect to the jet direction is assumed to be negligible as well as any mass. σ_0 is defined by

$$\sigma_0 = 4\pi\alpha^2/s \quad (2.2)$$

where s is the centre-of-mass energy squared. $\sigma_0 e_q^2$ is the spin-averaged, colour summed cross-section for e^+e^- annihilation into a $q\bar{q}$ pair via one photon exchange. The elementary density matrix $\rho(q)$ defined in terms of $e^+e^- \rightarrow q\bar{q}$ amplitudes by

$$\rho_{\nu\nu'}(q) = \frac{1}{N} \sum_{\lambda_1\lambda_2} \int d\Omega T_{\nu\mu_1\lambda_1\lambda_2}(e^+e^- \rightarrow q\bar{q}) T_{\nu'\mu_1\lambda_1\lambda_2}^*(e^+e^- \rightarrow q\bar{q}) \quad (2.3)$$

is diagonal

$$\rho(q) = \rho(\bar{q}) = \frac{1}{2} \mathbb{1} \quad (2.4)$$

The normalization N is determined by the requirement $\text{Tr} \rho = 1$.

The D function in Eq. (2.1) is the fragmentation matrix. For negligible p_T , rotational invariance implies that only those elements of $D(z)$ for which

$$\gamma - \gamma' = \lambda - \lambda' \quad (2.5)$$

can be different from zero. The other matrix elements behave as $(p_T/p_V)^n$ where n depends on the net-helicity flip. This property was first noted by Nieves³⁾. Combining it with Eq. (2.4) one recognizes that only the diagonal elements of D appear and these are proper fragmentation functions

$$D_{qv}^{V\lambda}(z) \equiv D_{qv}^{V\lambda\lambda}(z) \quad (2.6)$$

Consequently, inserting Eqs. (2.2)-(2.6) into Eq. (2.1), we have the prediction from the lowest order incoherent fragmentation picture

$$p_{\lambda\lambda'}(v) = \delta_{\lambda\lambda'} \frac{1}{2} \sum_{q,v} e_q^2 D_{qv}^{V\lambda}(z) / \sum_q e_q^2 D_q^V(z) \quad (2.7)$$

whereby D_q^V is the spin-averaged fragmentation function

$$D_q^V(z) = \frac{1}{2} \sum_{\lambda\lambda'} D_{qv}^{V\lambda}(z) \quad (2.8)$$

In order to get a feeling of what size $\rho(V)$ might be, let us assume a simple proportionality between the spin-dependent fragmentation functions and the spin-averaged one ($0 < \delta < 1$ and flavour independent)

$$D_{q+}^{V1}(z) = \delta D_q^V(z) ; D_{q+}^{V0}(z) = (1-\delta) D_q^V(z) ; D_{q-}^{V1}(z) = 0. \quad (2.9)$$

Moreover, making use of parity conservation we obtain

$$p_{11}(v) = p_{-1,-1}(v) = \delta/2 ; p_{00}(v) = 1-\delta . \quad (2.10)$$

The assumption (2.9) can be regarded as a generalized statistical fragmentation⁶⁾. For valence quarks, the use of SU(6) spin wave functions would give for δ a value of 2/3. On the other hand, there is an experimental hint that longitudinally polarized vector mesons may be more copiously produced than transversally polarized ones. In the EMC experiment the decay distribution of inclusively produced ρ^0 in deep inelastic μp scattering has been measured⁷⁾ (mean value of Q^2 is 17 GeV²). The results favour a value of δ close to 1/3. A previous result from electroproduction⁸⁾ gives a value close to the SU(6) one, namely 0.59 ± 0.08 . In this experiment, however, the mean value of Q^2 is only 2.5 GeV².

In summary the standard picture for $e^+e^- \rightarrow VX$ predicts $\rho(V)$ to be diagonal. From measurements of the vector meson decay distributions, providing some elements of $\rho(V)$

$$W(\theta, \varphi) = \frac{3}{4\pi} \left\{ \frac{1}{2} (1 - \rho_{00}(V)) + \frac{1}{2} (3\rho_{00}(V) - 1) \cos^2 \theta - \sqrt{2} \operatorname{Re} \rho_{10}(V) \sin 2\theta \cos \varphi - \rho_{1-1}(V) \sin^2 \theta \cos 2\varphi \right\} \quad (2.11)$$

(θ, φ are the decay angles of V in its helicity rest frame, we have written here the angular distribution for the parity conserving decay of V into two pseudoscalar mesons), one learns about the spin-dependent fragmentation functions.

It is easy to convince oneself that no additional information is obtained from the decay distribution when the experiment is carried out with polarized electrons and/or positrons. This is a consequence of the helicity conserving electromagnetic interaction between (almost) massless particles.

3. - COHERENT FRAGMENTATION

As we mentioned in the Introduction, the incoherent fragmentation is a simplification (perhaps a good one). Some coherence effects must exist, at least to guarantee energy-momentum conservation and colour neutralization. A more general fragmentation picture is depicted in Fig. 2 where the bubble accounts for any type of interaction between quark and antiquark. Of course, the hadrons are produced in such a way that they are essentially grouped into two jets. Examples of such final state interactions are the string model, the basis of the Lund Monte Carlo¹⁾, or a type of preconfinement⁹⁾ where the final state interaction is such that first two colourless, massive clusters of quarks and gluons are produced which subsequently decay into ordinary hadrons, forming two jets.

As in the incoherent fragmentation approach, we consider q and $\bar{q}$ as free quarks on their mass-shells which is the common assumption of the parton model, justified by QCD asymptotic freedom and very successful in describing many experimental data. Moreover, the hadronization process is supposed to be an essentially forward $q\bar{q} \rightarrow VX$ interaction, i.e., the vector meson propagates into

roughly the same direction as the quark^{*)}.

In the coherent fragmentation picture for $e^+e^- \rightarrow VX$, the density matrix of the vector meson is given by

$$\rho_{\lambda\lambda'}(V) \frac{d\sigma}{dz}(e^+e^- \rightarrow VX) = \sigma_0 \sum_q \sum_{\substack{\lambda_X, \lambda_x \\ \mu, \mu', \nu, \nu'}} e_q^2 M_{\lambda\lambda_X \nu \mu} \rho_{\nu\mu\nu'\mu'}^{(q\bar{q})} M_{\lambda'\lambda_X \nu'\mu'}^* \quad (3.1)$$

The Σ_X denotes, as usual, the sum (or integral, respectively) over all internal variables of the composite state X ¹⁰⁾. $\rho(q, \bar{q})$ is the joint density matrix of the $q\bar{q}$ system

$$\rho_{\nu\mu\nu'\mu'}(q\bar{q}) = \frac{1}{N} \sum_{\lambda\sigma} T_{\nu\mu\lambda\sigma}(e^+e^- \rightarrow q\bar{q}) T_{\nu'\mu'\lambda\sigma}^*(e^+e^- \rightarrow q\bar{q}) \quad (3.2)$$

From the one-photon exchange amplitudes, one finds that the only non-zero matrix elements of $\rho(q, \bar{q})$ are (quark masses are neglected)

$$\rho_{+-+-} = \rho_{-+ -+} = \frac{1}{2} \quad ; \quad \rho_{+--+} = \rho_{-++-} = \frac{1}{4} \quad (3.3)$$

The M 's in Eq. (3.1) are the helicity amplitudes for the process $q\bar{q} \rightarrow VX$.

Notice that, when keeping only the diagonal elements $\rho_{\nu\mu, \nu\mu}(q, \bar{q})$ of the $q\bar{q}$ density matrix, Eq. (3.1) simplifies to Eq. (2.1) with the $q\bar{q} \rightarrow VX$ fragmentation amplitudes related to the usual probabilistic fragmentation functions by

$$D_{qV}^{\nu\lambda}(z) = \sum_{\lambda_X} |M_{\lambda\lambda_X \nu \nu}|^2 \quad (3.4)$$

Since we again neglect the transverse momentum of the vector meson with respect to the quark direction, the fragmentation amplitudes are subject to the constraints from rotational invariance, telling us that the only products of M 's contributing to Eq. (3.1) are those satisfying the relation^{**)}

*) Of course, one has to add incoherently the contribution from the process $\bar{q}q \rightarrow VX$, where now V has the same direction as $\bar{q}$. This is included as in the incoherent picture, in the sum over q .

**) This relation as well as Eq. (2.5) are consequences of the well-known behaviour of helicity amplitudes for forward scattering¹⁰⁾.

$$\lambda - \lambda' = (\nu - \nu') - (\mu - \mu') \quad (3.5)$$

Inserting Eq. (3.3) into Eq. (3.1) and making use of Eqs. (3.4) and (3.5), we get for the diagonal elements of $\rho(V)$

$$\rho_{\lambda\lambda}(V) = \frac{1}{2} \sum_{q\nu} e_q^2 D_{q\nu}^{V\lambda}(z) / \sum_q e_q^2 D_q^V(z) \quad (3.6)$$

i.e., we recover the results from the incoherent fragmentation approach. However, we obtain in addition a non-zero non-diagonal element, namely

$$\rho_{1-1}(V) = \frac{1}{4} \sum_q \sum_{\lambda\lambda_x} e_q^2 M_{\lambda\lambda_x+-} M_{-1\lambda_x-+}^* / \sum_q e_q^2 D_q^V(z) \quad (3.7)$$

For a parity-conserving fragmentation process as is assumed to be the case here, the sum $\sum_{\lambda\lambda_x}$ over the bilinear products of fragmentation amplitudes appearing in the above equation, is real. Consequently, $\rho_{1-1}(V)$ is real too, as is required for a density matrix of inclusively produced vector mesons by hermiticity and parity invariance

$$\rho_{\lambda\lambda'}(V) = \rho_{\lambda'\lambda}^*(V) \quad ; \quad \rho_{-\lambda-\lambda'}(V) = (-1)^{\lambda-\lambda'} \rho_{\lambda\lambda'} \quad (3.8)$$

In conclusion, the results from the coherent fragmentation picture differ from those obtained in the incoherent scheme in that $\rho_{1-1}(V)$ can be different from zero. Its measurement, although difficult, would constitute an immediate evaluation of the importance of final state interactions which are taken into account in our coherent approach. A theoretical estimate of the magnitude of $\rho_{1-1}(V)$ is out of our scope, since this would require us to solve QCD non-perturbatively (see, however, Section 5). Resummation techniques used in the jet calculus¹¹⁾ may be useful to handle this problem.

Although we cannot calculate $\rho_{1-1}(V)$, we can at least derive a bound for it. For $p_T = 0$, by parity conservation, we can cast Eq. (3.7) in the form

$$\rho_{1-1}(V) = \frac{1}{4} \sum_q e_q^2 \sum_x \eta_x |M_{10+-}|^2 / \sum_q e_q^2 D_q^V \quad (3.9)$$

where $\eta_x = \pm 1$ depending on the intrinsic parities and spins of the states

involved. Equation (3.4) tells us

$$\left| \sum_x \eta_x |M_{10+-}|^2 \right| \leq D_{q+}^{v1} \quad (3.10)$$

and comparison with Eq. (3.6) yields

$$\left| \rho_{1-1}(V) \right| \leq \frac{1}{2} \rho_{11}(V) \quad (3.11)$$

This is stronger than the usual bound from the Schwarz inequality

$$\left| \rho_{1-1}(V) \right| \leq \rho_{11}(V) \quad (3.12)$$

The bound (3.11) is a consequence of the properties of the elementary interaction, Eq. (3.3).

We note that as in the incoherent scheme, experiments with polarized electrons and/or positrons provide no additional information.

One may also think of considering Λ particles instead of vector mesons. The Λ density matrix is fairly easily obtained from the weak decay of the Λ . However, calculating the Λ density matrix for both the approaches, the incoherent and the coherent fragmentation picture, along the lines described above reveals no difference. Predictions for the $\rho(\Lambda)$ derived in the incoherent approach were presented some time ago by Nieves³⁾.

4. - CORRECTIONS TO THE LOWEST ORDER PREDICTIONS IN THE INCOHERENT APPROACH

In this section we are going to discuss several corrections to the lowest-order incoherent fragmentation picture in order to learn whether it is possible or not to produce a sizeable value of $\rho_{1-1}(V)$ this way. It will turn out, however, as we may say in advance, that none of these corrections leads to a value of $\rho_{1-1}(V)$ larger than ≈ 0.02 . Thus, we can safely consider $\rho_{1-1}(V)$ as a clean measure of coherence effects in the hadronization.

4.1. Electroweak Interference and Quark Mass Effects

For PETRA-PEP energies ($\sqrt{s} \approx 30$ GeV) the electroweak interference terms are not really negligible and it is appropriate to estimate their contributions to $\rho_{1-1}(V)$. Calculating now the quark density matrix with regard to the γ -Z interference and keeping also terms proportional to quark masses (having typically in mind charm and bottom quark contributions), we obtain instead of (2.4)

$$\rho_{+-}(q) = \rho_{-+}(q) = \frac{3\pi}{4} \frac{m_q}{\sqrt{s}} \frac{g_A G_V^q}{e_q} \frac{s}{s - m_Z^2} ;$$

$$\rho_{\lambda\lambda}(q) = \frac{1}{2} ; \quad \rho(\bar{q}) = \rho(q) \quad (4.1)$$

g_A being the axial vector coupling of the electron and G_V^q the vector coupling of the quark. The vector coupling of the electron is taken to be zero, as given by the Weinberg-Salam model of electroweak interactions with $\sin^2 \theta_W = 1/4$. With this value of the Weinberg angle, one has $g_A = 1/\sqrt{3}$, $G_V^q = 1/3\sqrt{3}$ for u and c and $G_V^q = -2/3\sqrt{3}$ for d and s quarks.

The extra non-diagonal elements are proportional to both the quark mass and the Z propagator. Thus, it is only too obvious that these contributions are tiny. Even more, inserting the quark density matrix into Eq. (2.1) and exploiting again the rotational invariance rule (2.5), one still has $\rho_{1-1}(V) = 0$ since there is no fragmentation matrix element satisfying this rule. However, one gets now a small contribution to another non-diagonal element of $\rho(V)$

$$\rho_{10}(V) = \rho_{-10}(V) = \frac{3\pi}{4} g_A \sum_q \frac{m_q}{\sqrt{s}} e_q G_V^q \frac{s}{s - m_Z^2} D_{q+}^{V10} / \sum_q e_q^2 D_q^V \quad (4.2)$$

which is most likely too small to be measured ($|\rho_{10}(V)| \lesssim 0.01$ for $m_b = 5$ GeV and $\sqrt{s} = 30$ GeV).

4.2. Transverse Momentum in the Fragmentation

We allow now for a transverse momentum of the vector meson with respect to the jet axis. In addition to the diagonal elements given in Eq. (2.7), we obtain then for purely electromagnetically produced $q\bar{q}$ pairs

$$\begin{aligned}\rho_{10}(V) \frac{d\sigma}{dz}(e^+e^- \rightarrow VX) &= \frac{\sigma_0}{2} \sum_q e_q^2 [\mathcal{D}_{q++}^{V10}(z) + \mathcal{D}_{q--}^{V10}(z)] \\ \rho_{1-1}(V) \frac{d\sigma}{dz}(e^+e^- \rightarrow VX) &= \sigma_0 \sum_q e_q^2 \operatorname{Re} \mathcal{D}_{q++}^{V1-1}(z) \quad (4.3)\end{aligned}$$

These contributions are also very small for $\sqrt{s}$ of the order of 20-40 GeV and not too small values of z since the mean value of p_T is known from experiment to be about 400 MeV and the behaviour of the relevant fragmentation matrix elements for $p_T/p_V \rightarrow 0$ is a generalization of the rule (2.5)¹⁰⁾

$$\mathcal{D}_{q\pm\pm}^{V10} \sim \frac{p_T}{p_V} \quad ; \quad \mathcal{D}_{q++}^{V1-1} \sim \left(\frac{p_T}{p_V}\right)^2 \quad (4.4)$$

To estimate the actual magnitude of these matrix elements, we adopt the point of view that spin instead of helicity is conserved⁴⁾. The diagonal elements of the fragmentation matrix are then the fragmentation functions used in Section 2 [cf. Eq. (2.9)] generalized to include p_T , i.e.,

$$\mathcal{D}_q^V(z, p_T) d p_T^2 = \frac{1}{q^2} e^{-p_T^2/q^2} \mathcal{D}_q^V(z) d p_T^2 \quad (4.5)$$

The non-diagonal helicity matrix elements are obtained by a suitable rotation around a direction orthogonal to that of the quark, yielding after averaging over p_T^2

$$\mathcal{D}_{q++}^{V10}(z) + \mathcal{D}_{q--}^{V10}(z) = -\frac{\sqrt{\pi}}{2\sqrt{2}} \frac{q}{p_V} (3\delta - 2) \mathcal{D}_q^V(z)$$

$$\operatorname{Re} \mathcal{D}_{q++}^{V1-1}(z) = \frac{1}{4} \frac{q^2}{p_V^2} (3\delta - 2) \mathcal{D}_q^V(z) \quad (4.6)$$

Using the parameters $q = 640$ MeV (as an upper bound) and $\delta = 1/3$ this simple-minded model provides the contributions to $\rho_{1-1}(V)$ and $\rho_{10}(V)$ for ρ^0 production shown in Fig. 3. Indeed, these contributions are tiny.

4.3. Gluon bremsstrahlung

The interesting point with gluon bremsstrahlung (cf. Fig. 4) is that the vector meson observed may be one of the gluon fragments. The density matrix Eq. (2.1) has to be completed such that $(x=2E_p/\sqrt{s})$ where E_p is the parton energy)

$$\rho_{\lambda\lambda'}(v) \frac{d\sigma}{dz} = \sum_{vv'} \int_z^1 \frac{dx}{x} \rho_{vv'}(q) \frac{d\sigma}{dx}(e^+e^- \rightarrow q) \sum_q e_q^2 D_{qv'}^{\lambda\lambda'}\left(\frac{z}{x}\right) + \sum_{vv'} \int_z^1 \frac{dx}{x} \rho_{vv'}(g) \frac{d\sigma}{dx}(e^+e^- \rightarrow g) D_{gv'}^{\lambda\lambda'}\left(\frac{z}{x}\right) \frac{1}{2} \sum_q e_q^2 \quad (4.7)$$

The elementary cross-section and density matrices have now to be calculated consistently to the order α_s . To discriminate between hard gluons, forming three-jet configurations, and infra-red ones or those which are almost collinear to a quark - in these cases a $q\bar{q}g$ state differs only marginally from a $q\bar{q}$ state beyond experimental resolution - we follow the prescription given for example by Kramer¹²⁾, of cutting out in x intervals of the width ϵ^*). The cross-sections may be found in the literature^{12), 18)} and read

$$\frac{d\sigma}{dx}(e^+e^- \rightarrow q) = \sigma_0 \left[\beta \delta(1-x) + \frac{2\alpha_s}{3\pi} \sigma_q(x) \right]$$

$$\frac{d\sigma}{dx}(e^+e^- \rightarrow g) = \frac{4\alpha_s}{3\pi} \sigma_g(x) \quad (4.8)$$

with

$$\beta = 1 + \frac{\alpha_s}{2\pi} \frac{4}{3} \left[-2 \ln^2 \epsilon - 3 \ln \epsilon + 4 \epsilon \ln \epsilon - 1 + \frac{\pi^2}{3} \right]$$

$$\sigma_q(x) = \left[(1+x^2) \ln x/\epsilon + x^2/2 - 2x + 2\epsilon \right] / (1-x)$$

$$\sigma_g(x) = 1/x (2 - 2x + x^2) \ln[(x-\epsilon)/\epsilon] - x + 2\epsilon$$

The inclusive density matrix defined for the quark contribution by

$$\rho_{vv'}(q) \frac{d\sigma}{dx}(e^+e^- \rightarrow q) = \frac{1}{4} \sum_{\mu\lambda\sigma} \int d\Omega T_{v\mu\lambda\sigma}(e^+e^- \rightarrow q\bar{q}) T_{v'\mu\lambda\sigma}^*(e^+e^- \rightarrow q\bar{q}) + \frac{1}{4} \sum_{\substack{\mu\kappa \\ \lambda\sigma}} \int d\text{Lips} T_{v\mu\kappa\lambda\sigma}(e^+e^- \rightarrow q\bar{q}g) T_{v'\mu\kappa\lambda\sigma}^*(e^+e^- \rightarrow q\bar{q}g) \quad (4.9)$$

is still given by Eq. (2.4).

This result would hold even if more gluons were radiated or if more complicated loops would be taken into account (with only two fermions in the

*) The results depend somewhat on the cutting procedure [e.g., the Sterman-Weinberg method¹³⁾ instead of that of Ref. 12)]. Since, as will be shown below, gluon bremsstrahlung provides only a tiny contribution to $\rho_{1-1}(v)$ we have not attempted to study this dependence.

final state); it is generally true for an interaction conserving helicity and parity.

The gluon contribution is defined in a similar way (the two-jet part being zero in that case). At first order in α_s , it reads^{*)}

$$\rho_{11}(g) = \rho_{-1,-1}(g) = \frac{1}{2}$$

$$\rho_{1,-1}(g) = \frac{1}{s_g(x)} \frac{1-x}{x} \ln \frac{x-\varepsilon}{\varepsilon} \xrightarrow{\varepsilon \rightarrow 0} \frac{1-x}{1+(1-x)^2} \quad (4.10)$$

i.e., $\rho_{1,-1}(g)$ is finite for $\varepsilon \rightarrow 0$. We shall come back to this expression in the Appendix.

Inserting these results into Eq. (4.7) and exploiting again rotational invariance for $p_T = 0$, we obtain

$$\begin{aligned} \rho_{11}(v) \frac{d\sigma}{dz} &= \rho_{-1,-1}(v) \frac{d\sigma}{dz} = \frac{\sum e_q^2}{4} \int_z^1 \frac{dx}{x} \frac{d\sigma}{dx}(e^+e^- \rightarrow g) \mathcal{D}_{g1}^{V1}\left(\frac{z}{x}\right) \\ &+ \frac{1}{2} \int_z^1 \frac{dx}{x} \frac{d\sigma}{dx}(e^+e^- \rightarrow q) \sum e_q^2 \mathcal{D}_{q+}^{V1}\left(\frac{z}{x}\right); \\ \rho_{00}(v) \frac{d\sigma}{dz} &= \int_z^1 \frac{dx}{x} \frac{d\sigma}{dx}(q) \sum e_q^2 \mathcal{D}_{q+}^{V0}\left(\frac{z}{x}\right) + \frac{\sum e_q^2}{2} \int_z^1 \frac{dx}{x} \frac{d\sigma}{dx}(g) \mathcal{D}_{g1}^{V0}\left(\frac{z}{x}\right); \\ \rho_{1,-1}(v) \frac{d\sigma}{dz} &= \frac{\sum e_q^2}{2} \int_z^1 \frac{dx}{x} \rho_{1,-1}(g) \frac{d\sigma}{dx}(g) \mathcal{D}_{g1,-1}^{V1,-1}\left(\frac{z}{x}\right); \quad (4.11) \\ \rho_{10}(v) &= 0. \end{aligned}$$

The fragmentation functions appearing in the above equations are, strictly speaking, not identical to those of Section 2. The ones here are defined up to order α_s^{15} . In practice, however, the difference is of marginal importance. It is interesting to note that $\mathcal{D}_{g1,-1}^{V1,-1}$ measures the transfer of linear polarization

^{*)} We consider the gluon as massless so that only two out of its three helicity states are excited. Allowing the gluon to be slightly off-shell (its off-shellness being related to the invariant mass of the jet it initiates) opens the possibility of observing other spin density matrix elements. The theoretical treatment is in that case complicated by gauge invariance requirements and we shall not attempt to pursue it here.

from the gluon to the vector meson⁶⁾. Measuring the linear polarization perpendicular ($\perp$) or parallel ($\parallel$) to the jet production plane, one has

$$D_{g\perp}^{V\perp} - D_{g\parallel}^{V\perp} = D_{g1-1}^{V1-1} \quad (4.12)$$

where the D_{g1-1}^{V1-1} are proper spin-dependent fragmentation functions.

In Fig. 3, we present a numerical estimate of $\rho_{1-1}(V)$, using $\alpha_s = 0.17$, $\epsilon = 0.04$ and assuming $D_{g1-1}^{V1-1}(z) = D_g^V(z) = 0.14(1-z)^2/z$ ⁶⁾. This is the maximal value D_{g1-1}^{V1-1} can acquire. The results shown in Fig. 3 may, therefore, be regarded as an upper bound. Still the three-jet contribution to $\rho_{1-1}(V)$ is tiny.

5. - B* PRODUCTION THROUGH BOTTONIUM RESONANCES: A MODEL FOR COHERENT PRODUCTION

Let us consider now hadron production in e^+e^- annihilation, in the T resonance region. The b jet ($\bar{b}$ respectively) has very special fragmentation features. At first a very hard bottomed particle is produced, taking away on average about 80% of the jet energy and subsequently decaying into other particles¹⁶⁾. Not much energy is left for other primary particles. Thus the jet multiplicity is low. This observation leads us to propose the following model for the production of bottom vector mesons (B^*) at energies where one of the broad bottomonium excited states ($T^{(n)}$) can be produced. We adopt the point of view that the B^* production proceeds via the process

$$e^+e^- \rightarrow T^{(n)} \rightarrow B^* X \quad (5.1)$$

where X is a bottom state such as B or $B\pi$. Taking, for simplicity, X as a pseudoscalar state (e.g., B), the matrix elements of the process (5.1) read in the vector meson dominance model^{17),18)} (cf. Fig. 5)

$$T \sim \frac{1}{s} \bar{v}(p_2) \gamma^\mu u(p_1) \frac{1}{m_{T^{(n)}}^2 - s - i m_{T^{(n)}} \Gamma_{T^{(n)}}} \epsilon^{\mu\lambda\nu\sigma} q_{2\lambda} q_{1\nu} a_\sigma^* \quad (5.2)$$

a being the polarization vector of the B^* . Evaluating the density matrix of the B^* from Eq. (5.2), we find

$$\rho_{1-1}(V) = -\frac{1}{2} \frac{1-\cos^2\theta}{1+\cos^2\theta} ; \rho_{10}(V) = 0 \quad (5.3)$$

where θ is the cms scattering angle of the B^* , or if one prefers, to average over θ we get

$$\rho_{1-1}(V) = -\frac{1}{4} \quad (5.4)$$

This is the minimum value $\rho_{1-1}(V)$ can acquire. We would get the same result from Eq. (3.9), assuming that it is applicable in the exclusive limit.

The vector meson dominance model is a clear case of a coherent mechanism for the production of particles in e^+e^- annihilation. Its successes for describing data at relatively low energies leads to its extension to heavy quarkonium states (charmonium and bottomonium)¹⁷⁾. Although we are not advocating here its validity with respect to the successful description in terms of perturbative QCD, we think of it as an interesting example of a coherent mechanism.

How does one measure $\rho_{1-1}(V)$ in the case considered here? The B^* mass is expected¹⁹⁾ to lie about 50 MeV above the B mass so that its dominant decay mode should be

$$B^* \rightarrow B \gamma \quad (5.5)$$

The monochromatic photon can serve as a trigger for the reaction studied here. The helicity density matrix elements of the vector meson B^* are then studied through the angular distribution of the photon which, in the rest system of the B^* , reads¹⁸⁾:

$$W(\theta, \varphi) = \frac{1}{4\pi} \left\{ 1 - \frac{3\rho_{00}-1}{4} (3\cos^2\theta-1) + \frac{3}{2} \rho_{1-1} \sin^2\theta \cos 2\varphi + \frac{3\sqrt{2}}{2} \text{Re} \rho_{10} \sin 2\theta \cos \varphi \right\} \quad (5.6)$$

The question to be answered by the data can then be clearly formulated: does the angular dependence of the photon, $W(\theta, \varphi)$, display the same features on a $T^{(n)}$ resonance as quite far from it? In particular, are the stringent relations deduced in the incoherent picture (in particular the relations $\rho_{1-1}, \rho_{10} \approx 0$) obeyed by the data for B^* production in the $T^{(n)}$ region? A rather precise

measurement seems feasible in existing e^+e^- machines provided a good photon detector is used in the energy region around the T family.

6. - SUMMARY AND CONCLUSIONS

We have considered the helicity density matrix of a vector particle V such as ρ , K^* and D^* produced in e^+e^- annihilations through the two-step process $\Sigma_q (e^+e^- \rightarrow qq \rightarrow VX)$. Taking into account the final state interaction between the decaying quark and its antiparticle, we can write $\rho(V)$ in terms of the elementary joint density matrix $\rho(q\bar{q})$ of the $q\bar{q}$ system which can be exactly computed in QED, and some fragmentation amplitudes, whose moduli squared are related to the usual probabilistic fragmentation functions. By imposing parity conservation for the fragmentation process and by using rotational invariance with respect to the jet axis, in the limit in which we neglect the transverse momentum of V inside its jet, we can give strict constraints on the fragmentation amplitudes.

We find out that the non-diagonal element $\rho_{1-1}(V)$ may differ from zero in a coherent scheme, contrary to what happens in the customary approach treating the fragmentation of the quark independently of that of the antiquark. We have checked that various corrections to the latter approach, such as electroweak interference, quark masses, transverse momentum in the fragmentation or gluon bremsstrahlung do not give a value for $|\rho_{1-1}(V)|$ larger than 0.02 for not too small a value of z. Therefore, we are in the position to conclude that any significant non-zero value of $\rho_{1-1}(V)$ found experimentally can be considered as a clear measurement of coherence effects. Although this is a difficult experimental task, it seems to us very important in order to learn more both about the hadronization process and the elementary interaction.

One may guess that for z tending to 1, final state interactions become small, so that finally the incoherent picture is restored. A transition from one regime to another may therefore be visible in an experiment.

A theoretical estimate of $\rho_{1-1}(V)$ in the coherent picture requires non-perturbative methods which are beyond our scope. However, in the particular situation of B^* production at energies in the vicinity of the $T^{(n)}$ resonances, the vector meson dominance model, as an example of a coherent scheme, allows an estimate of $\rho_{1-1}(V)$. We find a rather large value for it.

In this paper, we have restricted ourselves to e^+e^- annihilation. The same analysis may of course be applied to large p_T vector meson production in hadronic collisions. Here also the incoherent jet fragmentation approach leads to a diagonal vector meson density matrix at Born order. Whatever scheme we use, the coherent or the incoherent one, the analysis in the hadronic case is very similar to the one in e^+e^- annihilation provided one ignores initial and final state interactions between the hard partons and the spectator partons. This latter assumption makes the hadronic case a less sensitive laboratory for the measure of coherence effects. Moreover, the number of elementary scattering processes and the difficulty of evaluating higher order amplitudes render the analysis quite more intricate.

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APPENDIX - MEASURING THE THREE-JET CONTRIBUTION TO $\rho_{1-1}(V)$

In this Appendix, we examine in some detail, in the incoherent approach to fragmentation, the non-zero contribution to non-diagonal spin density matrix elements coming from three-jet production (Fig. 4). As has often been stressed²⁾, the spin structure of QCD has not been much tested experimentally, in particular due to the difficulty of producing polarized beams with a high luminosity. We are here in an experimentally favoured situation since we deal with unpolarized beams. The good statistics accumulated (now and in the near future) in e^+e^- colliding rings at PETRA and PEP allow us to use some selective criteria to enhance a particular signal. In our case, the first step is to select three-jet events through a by now standard procedure such as the one described in Ref. 12). Suppose now that one was able to identify the gluon jet. Among its fragments, vector mesons may be reconstructed and their non-diagonal helicity density matrix elements evaluated through the study of the angular distribution of their decay products. Let us further assume that the fragmentation function for vector mesons, say ρ° 's, in gluon jets, is approximately that used in Section 4.3. Calculating then $\rho_{1-1}(V)$ for those events where the vector meson is a fragment of the gluon jet, one obtains fairly large values for $\rho_{1-1}(V)$ (see Fig. 5). Remember $\rho_{11} = 0.5$. Of course, such a programme is a bit too idealized, but Monte Carlo techniques can at least help to select events where the observed vector meson comes from a gluon jet. The difference between the selected and the rejected samples should present the distinctive features shown in Fig. 6. It is out of the scope of this paper to describe in any detail the data analysis which must take into account specific acceptance properties of the specific experiments.

Up to now, we have concentrated on quantities integrated over the azimuthal angles. An interesting property of the three-jet diagram of Fig. 4 is that it yields a non-vanishing contribution to the imaginary part of the non-diagonal helicity density matrix element of the gluon. This contribution, however, vanishes after integration on azimuthal angles. Such a contribution does not appear at the two-jet level (at Born order and at any order) because of helicity conservation along the fermion line when its mass is neglected. Since the D fragmentation matrices are to be real, this imaginary part of $\rho_{1-1}(g)$ will be transmitted to $\rho_{1-1}(V)$ and thus appear experimentally in the angular distribution of the vector meson decay products through a $\sin^2\theta\sin 2\phi$ term [cf. Eq. (2.11)].

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FIGURE CAPTIONS

Fig. 1 Hadronization in the incoherent picture.

Fig. 2 The coherent fragmentation picture.

Fig. 3 Contributions from transverse momentum corrections (solid lines) and from gluon bremsstrahlung (dashed lines) to the density matrix elements $\rho_{1-1}(V)$ and $\text{Re}\rho_{10}(V)$ for ρ^0 production. Parameters used: $\alpha_s = 0.17$, $\varepsilon = 0.04$, $q = 640$ MeV, $\delta = 1/3$.

Fig. 4 The gluon bremsstrahlung diagrams.

Fig. 5 A coherent model for B^* production in e^+e^- annihilation.

Fig. 6 $\rho_{1-1}(V)$ for ρ^0 production normalized to the inclusive gluon cross-section ($\varepsilon=0.04$).

Fig.1

Fig.2

Fig. 3

Fig. 4

Fig. 5

Fig. 6