CERN-TH.4103/85

PROPERTIES OF E_6 BREAKING AND SUPERSTRING THEORY

S. Cecotti^{*}), J.-P. Derendinger, S. Ferrara, L. Girardello⁺)

CERN - Geneva

and

M. Roncadelli^{&)}

Sektion Physik, Ludwig-Maximilians-Universität
Munich, West Germany

ABSTRACT

We consider symmetry breaking patterns of E_6 down to $SU(3) \times SU(2) \times U(1)$ which may be related to superstring theories. The Higgs structure suggested by these theories leads to few possibilities compatible with the values of the strong, weak and electromagnetic coupling constants at low energies.

^{*}) On leave from Dipartimento di Fisica, Università di Pisa and INFN (Pisa), Italy.

⁺) On leave from Dipartimento di Fisica, Università di Milano and INFN (Milano), Italy.

^{&)} On leave from Dipartimento di Fisica Nucleare e Teorica and INFN, Università di Pavia, Pavia, Italy.

The possibility that the ultimate theory unifying all fundamental interactions could be a superstring theory ¹⁾ has quite recently received renewed attention. The reason for this interest is the discovery by M. Green and J.H. Schwarz that strings can be free of gravitational and Yang-Mills anomalies ²⁾, as well as finite to one loop and possibly to all orders, provided the gauge group is $SO(32)$ or $E_8 \times E_8$ ³⁾. These results suggest that superstring theories can be consistent quantum mechanical systems including gravity and the other forces of nature. In the point-field limit, these theories essentially describe a ten-dimensional supergravity coupled to a Yang-Mills supermultiplet, with additional interactions dictated by the superstring amplitudes. Some of these interaction terms are precisely the Wess-Zumino terms discovered in Ref. 2, and necessary to cancel the gauge anomalies.

At present, three types of these theories are known to exist, i.e. the $SO(32)$ Green-Schwarz theory and the $SO(32)$ and $E_8 \times E_8$ new "heterotic" superstring theories constructed by Gross et al ⁴⁾ *).

Soon after, Candelas et al ⁵⁾ have shown that an $SO(32)$ and $E_8 \times E_8$ ten-dimensional Yang-Mills theory coupled to supergravity can undergo spontaneous compactification in four-dimensional Minkowski space-time with chiral fermions, provided suitable higher derivative terms are introduced in the bosonic part of the Lagrangian **).

The $E_8 \times E_8$ theory appears presently more realistic. If one requires unbroken $N=1$ supersymmetry in four dimensions, there exists for example a compactification for which one E_8 group is broken to E_6 in four dimensions, with zero modes transforming as $5(\underline{27}) + \overline{27}$ chiral multiplets besides the $\underline{78}$ gauge multiplet ⁵⁾. The other E_8 group remains unbroken, but is essentially not coupled to the chiral massless modes. This superstring theory seems thus to realize in four dimensions the scenario of a SUSY-GUT ⁷⁾.

An additional topological configuration, acting like a Higgs field in the adjoint $\underline{78}$ representation, can induce the first symmetry breaking of E_6 , near the Planck scale ⁸⁾.

*) Two other superstring theories ¹⁾, with closed strings only, correspond to two different versions of $N=2$ extended supergravity in 10 dimensions, and do not seem suitable for describing the presently known fundamental interactions, mainly because of the lack of chiral fermions.

**) These terms are actually needed in order to avoid a no-go theorem of Ref. 6.

It is then natural to examine the complete E_6 symmetry breaking pattern down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, and the evolution of the coupling constants using renormalization group equations. It is worth mentioning that the weak interaction scale in this framework is not yet introduced, since it should be related to the further breaking of local $N=1$ supersymmetry at low energies ^{9,7)}. We will comment at the end on this point.

Since the chiral multiplet zero modes contain four standard E_6 generations ¹⁰⁾ and a $\underline{27} + \overline{\underline{27}}$ pair, it is reasonable to consider the latter as containing the scalar multiplets responsible for the symmetry breaking in the effective supersymmetric theory, below the Planck scale.

We analyze in this letter some possible breaking patterns of E_6 . Our conclusion is that the E_6 field content allows to construct a realistic breaking chain when the $\underline{78}$ configuration is used at the Planck scale. Satisfactory values for the low-energy gauge coupling constants are obtained if one assumes that the theory contains some relatively light coloured multiplets. The intermediate symmetries arising in this scheme are however somewhat unusual in unified models.

An alternative approach would have been to explore the possibility of dynamical symmetry breaking with, for instance, bilinear condensates of fermions in $\underline{27}$ to form an effective $\underline{351}'$ Higgs multiplet ¹⁰⁾. We feel that this possibility is more artificial and problematic in the superstring framework. Other considerations do not favour this idea. $\underline{351}'$ is the highest dimensional representation in the product $\underline{27} \times \underline{27}$. Arguments based on the so-called most attractive channel ¹¹⁾ indicates that the formation of such condensates is energetically disfavoured. Although these arguments are not conclusive, we will not consider dynamical symmetry breaking further in this paper.

Unified models based on the gauge group E_6 are of two classes, depending on the classification of quarks and leptons in the $\underline{27}$ representation. In the first attempts at E_6 unification ¹²⁾, each $\underline{27}$ was accomodating one quark with charge $Q = 2/3$ and two $Q = -1/3$ quarks. All known quarks and leptons are then contained in two $\underline{27}$, without top, but with a fourth quark of charge $-1/3$. However, data on the weak interactions of the bottom quark disprove this approach and are naturally explained if the top and bottom quarks form an $SU(2)_L$ doublet. The second class of E_6 models ¹⁰⁾ follows this line. Each $\underline{27}$ contains one conventional quark-lepton generation (15 Weyl fermions), the other states receiving an $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant mass. The $SU(3)_C \times SU(2)_L \times U(1)_Y$ content of $\underline{27}$ is:

$$\begin{aligned} \underline{27} = & (\underline{3}, \underline{2}, 1/6) + (\overline{\underline{3}}, \underline{1}, 1/3) + (\overline{\underline{3}}, \underline{1}, -2/3) + (\underline{1}, \underline{2}, -1/2) \\ & + (\underline{1}, \underline{1}, 1) + [(\underline{1}, \underline{1}, 0) + (\underline{3}, \underline{1}, -1/3) + (\underline{1}, \underline{2}, -1/2) + c.c.] \end{aligned} \quad (1)$$

The massless modes of the superstring theory, besides the gauge multiplet, transform according to $5(27) + \overline{27}$ of E_6 . They form $N=1$ supersymmetric chiral multiplets. However their scalar content does not permit to break E_6 into $G_S \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. To prove this statement, let us consider the E_6 leptoquark gauge bosons, mediating baryon number violating interactions of quarks and leptons. E_6 , like $SO(10)$, possesses the two possible sorts of leptoquarks, classified in $(3, 2, -5/6) + c.c.$ and $(3, 2, 1/6) + c.c.$ of G_S . According to (1), there is no $(\overline{3}, 2, 5/6)$ in 27 or $\overline{27}$. This means that the Goldstone bosons needed to give masses to leptoquarks with $Y = \pm 5/6$ do not exist in the massless modes. It follows that these leptoquarks should necessarily become massive at the Planck scale. We then have to invoke the effective adjoint Higgs multiplet mentioned hereabove to break E_6 at the Planck scale. The resulting unbroken gauge symmetry must not contain $(\overline{3}, 2, 5/6) + c.c.$ leptoquarks. This already indicates that the Georgi-Glashow $SU(5)$ symmetry¹³⁾, which possesses only these leptoquarks, cannot be an intermediate symmetry of the E_6 breaking pattern down to G_S . The same holds for $SO(10)$.

Unified models with leptoquarks transforming like $(3, 2, 1/6) + c.c.$ have been considered in the literature. They are based on the gauge group $SU(5) \times U(1)_Z$, the so-called "flipped $SU(5)$ " containing $SU(3)_C \times SU(2)_L$, while the weak hypercharge is a combination of generators of $SU(5)$ and $U(1)_Z$ ¹⁴⁾.

The E_6 symmetry breaking, induced near the Planck scale by the real 78 , has to give mass to the unwanted class of leptoquark bosons. The 78 offers several different possible vacua, which will be degenerate as long as supersymmetry is unbroken. This degeneracy should be lifted if the full-fledged superstring theory is able to induce spontaneous supersymmetry breaking. However the direction of the vacuum expectation value (v.e.v.) of the 78 should not be arbitrary in the superstring theory; it will be constrained in principle by the form of the effective ten-dimensional theory. Since this aspect of the problem has not yet been investigated, we will only consider here physically relevant breaking patterns. We will assume that the 78 aligns itself in a physically acceptable direction, with an invariance group containing G_S . We will follow the same approach for the symmetry breakings due to v.e.v.'s of the 27 's and the $\overline{27}$.

We now proceed to enumerate the possible breaking patterns at the Planck scale, using the 78 . The maximal subgroups of E_6 which are physically relevant are $SO(10) \times U(1)$, $SU(6) \times SU(2)$ and $SU(3)^3$. The little groups of possible v.e.v.'s of 78 are of the form $G \times U(1)$, with G of rank 5, since 78 is the adjoint representation. Notice that a v.e.v. of a real adjoint multiplet preserves in general supersymmetry¹⁵⁾. Then, 78 can break E_6 into for instance $SO(10) \times U(1)$, but any combination of 27 and $\overline{27}$ is unable to break $SO(10) \times U(1)$ into G_S , as mentioned hereabove. $SO(10) \times U(1)$ must then be broken by 78 down to at least $SU(5) \times U(1)^2$,

where $SU(5)$ is the flipped $SU(5)$. This last group is however a subgroup of $SU(6) \times SU(2)$. The $SO(10) \times U(1)$ direction can then be discarded. $\underline{78}$ can also break E_6 into the $SU(3)^2 \times SU(2) \times U(1)$ subgroup of $SU(3)^3$, which is also a subgroup of $SU(6) \times SU(2)$. All the physically relevant orbits of $\underline{78}$ have then little groups contained in $SU(6) \times SU(2)$. In particular the critical orbits for which $\underline{78}$ possesses only one invariant direction have little groups $SU(6) \times U(1)$, $SU(5) \times SU(2) \times U(1)$, $SU(4) \times SU(2)^2 \times U(1)$ and $SU(3)^2 \times SU(2) \times U(1)$. All other possible invariance groups of $\underline{78}$ will be subgroups of these four groups, containing $SU(3)_C \times SU(2)_L \times U(1)^3$.

The largest relevant invariance group of the $\underline{78}$ is then $SU(6) \times U(1)$. This is also the only case of the form $G \times U(1)$, with G simple. We will first concentrate on this possibility which we find the most attractive and come back shortly to the other little groups later on.

Below the Planck scale, symmetry breaking down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ is induced by the zero modes $5(\underline{27}) + \overline{\underline{27}}$. In supersymmetric unified theories, this breaking has to preserve supersymmetry. The scalar potential will as usual contain two positive terms. The vanishing of the gauge potential $D^a D^a / 2$ for the vacuum will impose ¹⁵⁾ that one $\underline{27}$, defined by its non zero v.e.v., will be aligned with the $\overline{\underline{27}}$, and that the scales of the v.e.v.'s will be the same. The superpotential relevant in superstring theories is unknown. We will come back to this question later, when we will face the problem of giving mass to parts of the chiral multiplets.

The $\underline{27}$, with the embedding as given by eq. (1), has two G_S -invariants which can be used for symmetry breaking. Associated to the $\underline{78}$ multiplet, they will allow us to break E_6 into G_S .

Assuming that the invariance group of the $\underline{78}$ is $SU(6) \times U(1)$, the $\underline{27}$ decomposes into

$$\underline{27} = \overline{\underline{6}}_{1/2} + \overline{\underline{6}}_{-1/2} + \underline{15}_0. \quad (2)$$

The $U(1)$ quantum numbers correspond in fact to the right-handed weak isospin. We will further on denote this group $U(1)_{T_{3R}}$. The embedding of $SU(3)_C \times SU(2)_L$ in $SU(6)$ is simply

$$\begin{aligned} \overline{\underline{6}} &= (\overline{\underline{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{1}) \\ \underline{15} &= (\underline{3}, \underline{2}) + (\underline{3}, \underline{1}) + (\overline{\underline{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{1}). \end{aligned} \quad (3)$$

Each of the three $(\underline{1}, \underline{1})$ states of $\underline{27}$, taken alone, breaks $SU(6) \times U(1)_{T_{3R}}$ into intermediate symmetries defined in the following way. Consider the $SU(5) \times U(1)_A$ subgroup of $SU(6)$ given by

$$\underline{6} = \underline{5}_1 + \underline{1}_{-5}. \quad (4)$$

Then we have the following breaking patterns:

$$\text{I: } \langle \bar{6}_{1/2} \rangle : SU(6) \times U(1)_{T_{3R}} \longrightarrow SU(5) \times U(1)_{Z_1} \quad (5)$$

with $Z_1 = \frac{1}{2}A - 5T_{3R}$,

$$\text{II: } \langle \bar{6}_{-1/2} \rangle : SU(6) \times U(1)_{T_{3R}} \longrightarrow SU(5) \times U(1)_{Z_2} \quad (6)$$

with $Z_2 = \frac{1}{2}A + 5T_{3R}$

$$\text{III } \langle 15_0 \rangle : SU(6) \times U(1)_{T_{3R}} \longrightarrow SU(4) \times SU(2) \times U(1)_{T_{3R}} \quad (7)$$

where $\langle \bar{6}_{1/2} \rangle$, for instance, means a v.e.v. for the $SU(3)_C \times SU(2)_L$ -invariant direction of $\bar{6}_{1/2}$. For cases I and II, the 27 reads

$$27 = 10_1 + \bar{5}_{-3} + 1_5 + 5_{-2} + \bar{5}_2 + 1_0 \quad (8)$$

under $SU(5) \times U(1)_{Z_1}$. These assignments precisely correspond to the $SU(5) \times U(1)$ subgroup of $SO(10)$. Obviously, cases I and II differ by changing the sign of T_{3R} .

It is easy to convince oneself that adding a v.e.v. for 15_0 to either $\langle \bar{6}_{1/2} \rangle$ or $\langle \bar{6}_{-1/2} \rangle$ breaks $SU(6) \times U(1)_{T_{3R}}$ into $G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$. Notice that under $SU(5) \times U(1)_Z$, 15_0 decomposes into

$$15_0 = 10_1 + 5_{-2}. \quad (9)$$

The $SU(3)_C \times SU(2)_L$ -invariant direction lies in 10_1 . It is precisely this component which receives a v.e.v. in the flipped $SU(5)$ models based on $SO(10)^{14)}$. Considering the usual $G_O = SU(3)_C \times SU(2)_L \times U(1)_X$ subgroup of $SU(5)$, we have, under $G_O \times U(1)_Z$:

$$10_1 = (\underline{1}, \underline{1}, 1, 1) + (\underline{3}, \underline{2}, 1/6, 1) + (\bar{\underline{3}}, \underline{2}, -2/3, 1) \quad (10)$$

The direction $(\underline{1}, \underline{1}, 1, 1)$ is invariant under G_S with $^{14)}$

$$Y = \frac{1}{5}(Z - X) \quad (11)$$

One checks easily that the 27 has then the assignments given by eq. (1).

We have then found that assuming E_6 breaking into $SU(6) \times U(1)_{T_{3R}}$ by the 78

at the Planck scale leads to two symmetry breaking patterns using $\underline{27}$ and $\overline{27}$. They use v.e.v.'s of the $\underline{15}_0$ ($\overline{15}_0$) component of $\underline{27}$ ($\overline{27}$) at scale M_1 and of $\underline{6}_{1/2}$ ($\overline{6}_{-1/2}$) at scale M_2 . If $M_1 > M_2$, the breaking pattern is:

$$\begin{aligned} SU(6) \times U(1)_{T_{3R}} &\xrightarrow{M_1} SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}} \\ &\downarrow M_2 \\ &SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned} \quad (12)$$

The intermediate symmetry is the Pati-Salam $SU(4)_{PS}$ group¹⁶⁾, the left-handed weak isospin $SU(2)_L$ and T_{3R} . On the other hand, if $M_2 > M_1$, we have

$$\begin{aligned} SU(6) \times U(1)_{T_{3R}} &\xrightarrow{M_2} SU(5) \times U(1)_Z \\ &\downarrow M_1 \\ &SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned} \quad (13)$$

the intermediate symmetry containing the flipped $SU(5)$ group.

This group-theoretical discussion shows that E_6 symmetry breaking into G_s can be achieved with the field content suggested by an underlying superstring theory. It is worth mentioning that a $\overline{27}$ is essential to obtain this breaking compatible with unbroken supersymmetry. A combination of $\underline{27}$ multiplets only would lead to vector-like symmetries, like for instance F_4 , as a consequence of the necessary cancellation of the D-terms appearing in the gauge part of the scalar potential¹⁵⁾. The existence of the symmetry breaking pattern must however be supplemented by sensible values for the low-energy coupling constants of strong (α_c), weak (α_w) and electromagnetic (α_{em}) interactions. We now analyze this question using one-loop renormalization group equations.

Our starting point will be to assume that E_6 unification occurs for the $SU(6)$ and $U(1)_{T_{3R}}$ coupling constants at $M \equiv M_P/\sqrt{8\pi} \approx 2.4 \cdot 10^{18}$ GeV, but that E_6 is also broken at this scale by the v.e.v. of $\underline{78}$. We then have three parameters: the E_6 coupling constant α at M and the two symmetry breaking scales M_1 and M_2 . The low-energy parameters $\alpha_c(M_W)$, $\alpha_w(M_W)$ and $\alpha_{em}(M_W)$ at $M_W = 81$ GeV are then given in terms of α , M_1 , M_2 , provided some assumptions are made on the number of multiplets becoming massive due to spontaneous symmetry breaking at M_1 and M_2 . In particular, notice that since $5(\underline{27}) + \overline{27}$ are massless modes of the superstring theory, there should be no explicit mass term in the resulting superpotential. The two scales M_1 and M_2 must then be confronted with phenomenological limits, like for instance nucleon lifetime.

The renormalization group analysis will select one of the two symmetry breaking patterns (12) and (13). We find in fact that only $M_1 > M_2$ (eq. 12) gives sensible low-energy parameters. The intermediate symmetry between M_1 and M_2 is then $SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$. Let us now describe the renormalization group analysis for this breaking pattern with some details and give some numerical estimates.

The energy dependence of a gauge coupling constant $\alpha(Q)$, at one-loop level, is given by

$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha(Q_0)} + \frac{b}{2\pi} \ln\left(\frac{Q}{Q_0}\right), \quad (14)$$

with, for $N=1$ supersymmetric theories,

$$b = 3C(G) - T(R). \quad (15)$$

$C(G)$ is the second order Casimir of the corresponding gauge group G and $T(R)$ is defined by

$$\text{Tr} (T_R^a T_R^b) = T(R) \delta^{ab}, \quad (16)$$

where T_R^a are generators of G for the representation R of the chiral multiplets. E_6 symmetry is broken at $M \simeq 2.4 \cdot 10^{18}$ GeV into $SU(6) \times U(1)_{T_{3R}}$ with the embedding defined in eq. (2). Since $SU(6)$ and T_{3R} generators have the same normalization on the $\underline{27}$ of E_6 , their coupling constants will satisfy

$$\alpha_6(M) = \alpha_{T_{3R}}(M) \equiv \alpha, \quad (17)$$

due to E_6 symmetry at M . ($\alpha_6(Q)$ and $\alpha_{T_{3R}}(Q)$ are respectively the $SU(6)$ and $U(1)_{T_{3R}}$ running coupling constants). The chiral multiplet content of the theory below M , is given by $5(\underline{27}) + \overline{\underline{27}}$ decomposed in $SU(6) \times U(1)_{T_{3R}}$ multiplets. As already mentioned, no direct mass term should appear among the zero modes. Since no $SU(6) \times U(1)_{T_{3R}}$ -invariant mass term can be formed within a $\underline{27}$, it is natural to retain all chiral multiplets as massless at this level. We then find, for $SU(6)$

$$T = 18, \quad b = 0, \quad (18)$$

which means that the $SU(6)$ gauge coupling is not renormalized at one loop, and for $U(1)_{T_{3R}}$

$$T = 18 = -b. \quad (19)$$

$SU(6) \times U(1)_{T_{3R}}$ will be broken into $SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$ at M_1 by the v.e.v. of the $\underline{15}_0 + \overline{\underline{15}}_0$ component of $\underline{27} + \overline{\underline{27}}$. The embedding of $SU(4)_{PS} \times SU(2)_L$ in $SU(6)$ is

obviously

$$\underline{6} = (\underline{4}, \underline{1}) + (\underline{1}, \underline{2}). \quad (20)$$

Then, at M_1 ,

$$\alpha_6(M_1) = \alpha_{PS}(M_1) = \alpha_w(M_1), \quad (21)$$

where $\alpha_{PS}(Q)$ and $\alpha_w(Q)$ are respectively the $SU(4)_{PS}$ and the weak running coupling constants. Using the renormalization group equation (14), we get

$$\alpha_6(M_1) = \alpha_w(M_1) = \alpha, \quad (22)$$

$$\frac{1}{\alpha_{T_{3R}}(M_1)} = \frac{1}{\alpha} - \frac{9}{\pi} \ln \left(\frac{M_1}{M} \right). \quad (23)$$

Under $SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$, the $\underline{27}$ decomposes according to

$$\begin{aligned} \underline{27} = & (\underline{4}, \underline{2}, 0) + (\underline{\bar{4}}, \underline{1}, 1/2) + (\underline{4}, \underline{1}, -1/2) \\ & + (\underline{6}, \underline{1}, 0) + (\underline{1}, \underline{1}, 0) + (\underline{1}, \underline{2}, 1/2) + (\underline{1}, \underline{2}, -1/2). \end{aligned} \quad (24)$$

The chiral multiplets containing the Goldstone bosons will combine with gauge multiplets to form massive vector multiplets. The $\underline{27} + \underline{\bar{27}}$ combination with non zero v.e.v. then loses at M_1 the multiplets

$$(\underline{4}, \underline{2}, 0) + (\underline{\bar{4}}, \underline{2}, 0) + 2(\underline{1}, \underline{1}, 0) \quad (25)$$

absorbed in the massive vector multiplets and in the Higgs multiplet. The rest of the spectrum is determined by the quantum numbers and by the superpotential of the theory. Each $\underline{27}$ contains a chiral part

$$(\underline{4}, \underline{2}, 0) + (\underline{\bar{4}}, \underline{1}, 1/2) + (\underline{\bar{4}}, \underline{1}, -1/2) \quad (26)$$

without $SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$ -invariant mass. These 16 chiral multiplets correspond to one quark-lepton generation with a right-handed neutrino. It also contains a real part

$$(\underline{6}, \underline{1}, 0) + (\underline{1}, \underline{2}, 1/2) + (\underline{1}, \underline{2}, -1/2) + (\underline{1}, \underline{1}, 0)$$

which can become massive at M_1 . This can result from a cubic coupling which, at the level of $SU(6) \times U(1)_{T_{3R}}$, gives two terms

$$(\underline{15}_0 \times \underline{15}_0 \times \underline{15}_0)_{SYM}$$

and

$$(\underline{\bar{6}}_{1/2} \times \underline{\bar{6}}_{-1/2} \times \underline{15}_0).$$

A v.e.v. of $\underline{15}_0$ will generate a mass for $(\underline{6}, \underline{1}, 0)$ in the first term, and for $(\underline{1}, \underline{2}, 1/2) + \text{c.c.}$ in the second one. However, we need as usual some massless $SU(2)_L$ - doublets to be able to break the electroweak symmetry.

This requires in generic supersymmetric models particular forms of the super-potential. We will follow here a general approach, and just assume that k $SU(2)_L$ -doublets and also ℓ $SU(4)_{PS}$ -sextets remain massless at M_1 , ($0 \leq k, \ell \leq 6$), so that the chiral content of the theory is:

$$\begin{aligned} & 4[(\underline{4}, \underline{2}, 0) + (\bar{\underline{4}}, \underline{2}, 1/2) + (\bar{\underline{4}}, \underline{2}, -1/2)] \\ & + k[(\underline{1}, \underline{2}, 1/2) + (\underline{1}, \underline{2}, -1/2)] \\ & + \ell(\underline{6}, \underline{1}, 0) + 6(\underline{1}, \underline{1}, 0) \\ & + (\underline{4}, \underline{1}, 1/2) + (\bar{\underline{4}}, \underline{1}, -1/2) + (\underline{4}, \underline{1}, -1/2) + (\bar{\underline{4}}, \underline{1}, 1/2). \end{aligned} \quad (27)$$

The last four terms come from the $\underline{27} + \bar{\underline{27}}$ combination with nonzero v.e.v.'s. They are needed as Goldstone multiplets for the $SU(4)_{PS} \times U(1)_{T_{3R}}$ breaking at M_2 which will absorb a pair of them. T_{3R} symmetry suggests that the four multiplets will then remain massless at M_1 .

We can now pursue the renormalization group analysis between M_1 and M_2 . According to eq. (27), we find for $SU(4)_{PS}$:

$$T = 10 + \ell \quad ; \quad b = 2 - \ell, \quad (28)$$

for $SU(2)_L$:

$$T = 8 + k \quad ; \quad b = -2 - k, \quad (29)$$

and for $U(1)_{T_{3R}}$:

$$T = -b = 12 + k. \quad (30)$$

Using eqs. (14) and (22), we then obtain

$$\frac{1}{\alpha_{PS}(M_2)} = \frac{1}{\alpha} + \frac{2-\ell}{2\pi} \ln \left(\frac{M_2}{M_1} \right), \quad (31)$$

$$\frac{1}{\alpha_{T_{3R}}(M_2)} = \frac{1}{\alpha_{T_{3R}}(M_1)} - \frac{12+k}{2\pi} \ln \left(\frac{M_2}{M_1} \right). \quad (32)$$

At M_2 , $SU(4)_{PS} \times U(1)_{T_{3R}}$ breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ using the v.e.v. of $(\bar{\underline{4}}, \underline{1}, -1/2) + (\underline{4}, \underline{1}, 1/2)$ (or of $(\bar{\underline{4}}, \underline{1}, 1/2) + (\underline{4}, \underline{1}, -1/2)$, which just reverses the sign of T_{3R} in the subsequent discussion). The definition of the weak hypercharge Y follows from the $SU(3)_C \times U(1)_{B-L}$ subgroup of $SU(4)_{PS}$:

$$\underline{4} = \underline{3}_{1/3} + \underline{1}_{-1}. \quad (33)$$

It is then clear that the weak hypercharge which leaves the v.e.v. invariant is

$$Y = \frac{1}{2}(B-L) + T_{3R} \quad (34)$$

as usual. Then, using eq. (34) and the normalization of B-L in eq. (33), one gets the continuity relation of the coupling constants at M_2 :

$$\frac{1}{\alpha_Y(M_2)} = \frac{1}{\alpha_{T_{3R}}(M_2)} + \frac{2}{3\alpha_{PS}(M_2)} ,$$

$$\alpha_C(M_2) = \alpha_{PS}(M_2). \quad (35)$$

Below M_2 , we assume that the spectrum of massless multiplets contains four quark-lepton generations and k Higgs doublets $(\frac{1}{2}, 2, 1/2) + (\frac{1}{2}, 2, -1/2)$. This leads to the following relations:

$$\frac{1}{\alpha_C(M_W)} = \frac{1}{\alpha_{PS}(M_2)} + \frac{1}{2\pi} \ln \left(\frac{M_W}{M_2} \right), \quad (36)$$

$$\frac{1}{\alpha_W(M_W)} = \frac{1}{\alpha} - \frac{2+k}{2\pi} \ln \left(\frac{M_W}{M_1} \right), \quad (37)$$

$$\frac{1}{\alpha_Y(M_W)} = \frac{1}{\alpha_{T_{3R}}(M_2)} + \frac{2}{3\alpha_{PS}(M_2)} - \frac{40+3k}{6\pi} \ln \left(\frac{M_W}{M_2} \right) \quad (38)$$

The relations (22,23,31,32,36-38) allow us to obtain M_1 , M_2 , α in terms of the low-energy coupling constants, for given values of k and ℓ . For instance, we find:

$$\ln \left(\frac{M_1}{M_W} \right) = \frac{-A + \ell \cdot B}{k + 3\ell} , \quad (39)$$

and

$$\ln \left(\frac{M_2}{M_W} \right) = \frac{-4A + (\ell - k) B}{k + 3\ell} , \quad (40)$$

where

$$A = \frac{8\pi}{3\alpha_C(M_W)} - \frac{\pi}{\alpha_{em}(M_W)} + 9 \ln \left(\frac{M}{M_W} \right), \quad (41)$$

$$B = \frac{2\pi}{3\alpha_C(M_W)} + \frac{\pi}{\alpha_{em}(M_W)} (2 \sin^2 \theta_W(M_W) - 1) + 9 \ln \left(\frac{M}{M_W} \right). \quad (42)$$

For proper values of the low-energy coupling constants, A and B will be positive numbers. Eq. (40) then indicates that the number ℓ of $SU(4)_{PS}$ -sextets has to be larger than the number of $SU(2)_L$ -doublets between M_1 and M_2 to realize the breaking chain.

Leptoquark gauge bosons will become massive at M_1 . Limits on the nucleon lifetime correspond to $M_1 \geq 10^{15}$ GeV. M_1 will in fact always exceed 10^{16} GeV. The number of $SU(2)_L$ -doublets is also limited by the necessary requirement that coupling constants remain in the perturbative domain between M_W and M . This is

expressed by demanding $\alpha < 1$ in eq. (37). For the possible values of M_1 , k will be at most two.

Limits on the scale M_2 are essentially related to two phenomena. First, M_2 is the scale where B-L is spontaneously broken. M_2 is related to the right-handed neutrino masses and also to possible generation of $\Delta B = 2$ interactions (neutron-antineutron oscillations for example). Then M_2 is the scale where $\mathbb{1}$ SU(4)-sextets become massive. The scalar fields in these sextets potentially induce proton decay, when coupled to quarks and leptons. This problem is common in supersymmetric unified models. Quark and lepton multiplets have to couple to light Higgs doublets, but not to light colour triplets. Proton lifetime indicates that the sextets should be heavier than $\sim 10^{10}$ GeV for standard values of Yukawa couplings. The form of the superpotential is however crucial. The presence of a cubic coupling between $\underline{27}$'s containing light fermion generations is also unwanted for the same reason. If the SU(4)-sextets come from these $\underline{27}$'s, then the proton lifetime is long enough, at the price of a rather particular (fine-tuned) form of the superpotential.

To estimate M_1 , M_2 and α , we use the low-energy values $\alpha_c(M_W) = .11$ and $\alpha_{em}(M_W) = 128^{-1}$ at $M_W = 81$ GeV. The weak mixing angle will be

$$\sin^2 \theta_W(M_W) = .215. \quad (43)$$

We give all cases for which $M_1 < M$, $\alpha < 1$ and $M_2 > 10^6$ GeV in Table I. The dependence of the scales on variations of the mixing angle for instance, is clear from eqs. (39), (40) and (42). Choosing $k = 1$, $\ell = 2$ and increasing $\sin^2 \theta_W(M_W)$ by .01 will multiply M_2 by a factor ~ 3.2 and M_1 by ~ 10 . This will however push M_1 beyond the Planck scale.

The $\underline{78}$ scalar multiplet can also be used to break E_6 into a symmetry group smaller than $SU(6) \times U(1)_{T_{3R}}$. Subsequent breakings are then achieved using $\underline{27} + \overline{\underline{27}}$ chiral multiplets. One-loop renormalization group equations lead always to the same features. $SU(3)_c$ and $SU(2)_L$ cannot be unified at the lowest scale of the $\underline{27}$. The value of the weak hypercharge is in this case too close to α_c and α_w and requires complete unification far below the Planck scale. One has to use an intermediate scale where some fields (containing mainly colour-triplets) become massive, and where the strong interaction group is possibly enlarged. In our previous case, M_2 was the scale of $SU(4)_{PS}$ breaking, giving mass to some SU(4)-sextets. The role of these "intermediate" fields is in fact to delay the unification of weak and strong interactions.

If for instance $\underline{78}$ breaks E_6 into $SU(5) \times U(1)_Z \times U(1)_B$, the first step of symmetry breaking, at M_1 , is to break $SU(5) \times U(1)_Z$ into $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_A$, using the flipped SU(5) scheme. Under $SU(5) \times U(1)_Z \times U(1)_B$, $\underline{27}$ reads

$$\underline{27} = (\underline{10}, 1, 1) + (\underline{\bar{5}}, -3, 1) + (\underline{1}, 5, 1) + (\underline{5}, -2, -2) + (\underline{\bar{5}}, 2, -2) + (\underline{1}, 0, 4). \quad (44)$$

The $SU(3)_C \times SU(2)_L$ -neutral component in $(\underline{10}, 1, 1)$ receives the v.e.v. The weak hypercharge is given by eq. (11) and we define

$$A = B - Z \quad (45)$$

The second step of symmetry breaking, at M_2 , breaks $U(1)_A$ using the v.e.v. of $(\underline{1}, 0, 4)$. Between M_1 and M_2 , we assume that four generations, k $SU(2)_L$ -doublets and antidoublets, and

$$\ell [(\underline{3}, \underline{1}, -1/3) + (\underline{\bar{3}}, \underline{1}, 1/3)] \quad (46)$$

of $SU(3)_C \times SU(2)_L \times U(1)_Y$ remain massless, the latter fields having finally a mass of order M_2 . The renormalization group equations give M_1 , α and

$$\ell \cdot \ln(M_2/M_W) \quad (47)$$

as functions of the low-energy coupling constants. We use the same values $\alpha_C(M_W) = .11$, $\alpha_{em}(M_W) = 128^{-1}$ and $\sin^2 \theta_W(M_W) = .215$ at $M_W = 81$ GeV as before, and require $M_2 \geq 10^6$ GeV to get the results presented in Table II. A value of k greater than two leads to gauge coupling constants larger than one.

The same analysis can be performed for other possible little groups of the $\underline{78}$, like $SU(5) \times SU(2) \times U(1)$, $SU(4) \times SU(2)^2 \times U(1)$, $SU(3)^2 \times SU(2) \times U(1)$ or their subgroups.

There are also compactifications of the 10-dimensional point-field limit of superstring theory which do not preserve supersymmetry ⁵⁾. They would for instance give chiral fermions in $\underline{16}$ and $\underline{\bar{16}}$ of the gauge group $SO(10)$. Scalar fields in $\underline{16}$ and $\underline{10}$ representations would also be necessary for symmetry breaking. The adjoint $\underline{45}$ configuration breaks $SO(10)$ into $SU(5) \times U(1)$ at the Planck scale. A $\underline{16}$ scalar multiplet can then induce the second breaking into $SU(3)_C \times SU(2)_L \times U(1)_Y$ using the flipped $SU(5)$ scheme. A study of the scalar potential with $\underline{16} + \underline{45}$ Higgses ¹⁷⁾ shows that this breaking chain is naturally obtained by requiring $\langle \underline{45} \rangle \gg \langle \underline{16} \rangle$. However the two scales of symmetry breaking should not be separated by more than an order of magnitude to reproduce low-energy parameters. Both scales lie in the $10^{14} - 10^{16}$ GeV range. Without supersymmetry, there is no way to avoid that Dirac fermions and scalars have heavy masses. The spectrum of light fields is fixed by chirality and cannot be used to modify the scales. The assumption that the v.e.v. of $\underline{45}$ is of order M leads to unrealistic models at low energies.

Let us finally mention very briefly some low-energy features of the E_6 models considered here. The E_6 superpotential is characterized by the coupling 27^3 . Discrimination between "Higgs multiplets", 27_H , and "generation multiplets", 27_G , without v.e.v. should arise from the form of the superpotential. Couplings of the form 27_G^3 are disastrous for the proton lifetime if they involve the lightest quark-lepton families. On the other hand, terms like $27_H \times 27_G^2$ will generate quark and lepton masses when $SU(2)_L \times U(1)_Y$ breaking is induced by supersymmetry breaking. The superpotential must be such that the supersymmetric potential vanishes for the desired v.e.v.'s. If E_6 breaks E_6 into $SU(6) \times U(1)_{T_{3R}}$, the invariant 27^3 contains terms linear in the v.e.v. of order M_1 and also terms linear in the v.e.v. of order M_2 . The scalar potential will then have no v.e.v. as required by supersymmetry. Both M_1 and M_2 will not be in general determined by the tree-level potential. Since M_1 can be close to the Planck scale, non renormalizable interactions may play an important role. Also, radiative corrections induced by supersymmetry breaking could be important to fix the scales.

The spectrum of light multiplets is also related to the form of the superpotential. The existence of light $SU(2)_L$ -doublet Higgs multiplets often requires some tuning of parameters. The same holds for coloured triplets and $SU(4)_{PS}$ -sextets with masses of order M_2 instead of M_1 . On the contrary, obtaining a large enough mass for the right-handed neutrinos can be a source of difficulties.

Our next remark will be concerned with the m_b/m_τ mass ratio. The usual mass relation $m_b = m_\tau$ will remain exact as long as $SU(4)_{PS}$ is unbroken. It is then renormalized mainly because of the effect of the eight quarks contained in 4 (27). The larger effect due to the fourth generation will be compensated by the relative smallness of the $SU(4)_{PS}$ breaking scale M_2 . An estimation based on the assumption that both the supersymmetry breaking scale and the mass of the fourth generation are $\sim M_W$ gives the following results:

$$\begin{array}{llll} M_2 = 10^6 & \text{GeV} & : & m_b/m_\tau \cong 1.8 , \\ M_2 = 10^8 & \text{GeV} & : & m_b/m_\tau \cong 2.1 , \\ M_2 = 10^{10} & \text{GeV} & : & m_b/m_\tau \cong 2.5 , \\ M_2 = 10^{11} & \text{GeV} & : & m_b/m_\tau \cong 2.8 . \end{array}$$

The best values are then obtained for M_2 of the order 10^{10-11} GeV, which is also safer with respect to nucleon lifetime. This estimation uses one-loop calculations ¹⁸⁾. Higher order ¹⁹⁾ and $SU(2)_L \times U(1)_Y$ effects are about 20%.

Our final remark is on possible sources for supersymmetry breaking. It has been argued ⁵⁾ that the E_8' gauge interaction could be used as hidden sector for $N=1$ supergravity. It is natural to assume that the confining nature of E_8'

will give rise to fermion condensates $\langle \bar{\lambda}^{\alpha} \lambda^{\beta} \rangle$ in which λ^{α} is the E_8 ' gaugino ^{20,21)} This condensate does not generally break supersymmetry. However, in N=1 supergravity theories (as well as in σ -models coupled to vector multiplets) the auxiliary fields of chiral multiplets get an extra term ²⁰⁾

$$h^i = f_{\alpha\beta}^i \bar{\lambda}^{\alpha} \lambda^{\beta}$$

proportional to the first derivative of the $f_{\alpha\beta}^i$ function introduced in Ref. 9. Dynamical supersymmetry breaking can occur provided both $\langle \bar{\lambda}^{\alpha} \lambda^{\beta} \rangle \neq 0$ and $\langle f_{\alpha\beta}^i \rangle \neq 0$. This is the supersymmetric version of technicolour. In ten-dimensional supergravity ²²⁾, there is for instance a quartic coupling which gives rise in four dimensions to the coupling of gauge fermion condensates to a spin 1/2 singlet and to the spin 3/2 massless gravitino. Therefore the formation of the $\lambda\lambda$ condensate satisfies the necessary conditions for supersymmetry breaking, since the N=1 $f_{\alpha\beta}^i$ function is non vanishing in this case.

ACKNOWLEDGEMENTS

M.R. would like to thank Harald Fritzsch for the hospitality at the University of Munich. The work of M.R. is partially supported by "Fondazione Angelo Della Riccia".

REFERENCES

- 1) For recent reviews see i.e.: J.H. Schwarz, Proc. of the Trieste Spring School on Supersymmetry and Supergravity 84, eds. B. de Wit, P. Fayet and P. van Nieuwenhuizen, (World Scientific, Singapore, 1984), p. 426.
L. Brink, CERN preprint TH 4006 (1984) to appear in the Proc. of NATO Advanced Study on Supersymmetry, Bonn, August 1984.
M.B. Green, Caltech preprint CALT-68-1219 (1984).
- 2) M.B. Green and J.H. Schwarz, Phys. Lett. 149B (1984) 117.
- 3) M.B. Green and J.H. Schwarz, Phys. Lett. 151B (1985) 21.
- 4) D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Princeton Univ. preprint, November 1984
- 5) P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Princeton preprint, December 1984.
- 6) D.Z. Freedman, G.W. Gibbons and P.C. West, Phys. Lett. 124B (1983) 491.
- 7) For recent reviews see i.e.: S. Ferrara, Int. Europhysics Conf. on High Energy Physics, Brighton, 1983, eds. J. Guy and C. Costain (Rutherford Appleton Laboratory, Chilton, Didcot, UK), p. 522.
D.V. Nanopoulos, CERN preprint TH 3995 (1984), to appear in the Proc. of the XXII Int. Conf. on High Energy Physics, Leipzig, July 1984.
- 8) Y. Hosotani, Phys. Lett. 126B (1983) 309; 129B (1983) 193.
- 9) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Phys. Lett. 79B (1978) 231; Nucl. Phys. B147 (1979) 105.
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 231; Nucl. Phys. B212 (1983) 413.
- 10) Y. Achiman and B. Stech, Phys. Lett. 77B (1978) 389.
P. Ramond, Sanibel Symposium (Florida 1979) Caltech report CALT-68-709 (1979).
R. Barbieri and D.V. Nanopoulos, Phys. Lett. 91B (1980) 369.
- 11) S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B169 (1980) 373.
- 12) F. Gurse, P. Ramond and P. Sikivie, Phys. Lett. 60B (1976) 177.
Q. Shafi, Phys. Lett. 70B (1977) 187.
F. Gurse and M. Serdaroglu, Lett. Nuovo Cim. 21 (1978) 28.
A. Achiman and B. Stech, ref. 10.

- 13) H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
- 14) A. de Rujula, H. Georgi and S.L. Glashow, Phys. Rev. Lett. 45 (1980) 413.
S.M. Barr, Phys. Lett. 112B (1982) 219.
- 15) F. Buccella, J.P. Derendinger, S. Ferrara and C.A. Savoy, Unification of
the fundamental particle interactions II, eds. J. Ellis and S. Ferrara
(Plenum, New-York, 1983), p. 349; Phys. Lett. 115B (1982) 375.
- 16) J. Pati and A. Salam, Phys. Rev. D10 (1974) 275.
- 17) F. Buccella, H. Ruegg and C.A. Savoy, Phys. Lett. 94B (1980) 491.
G. Anastaze, F. Buccella and J.P. Derendinger, Zeit. Phys. C 20 (1983) 269.
- 18) M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475.
J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B202 (1982) 43.
- 19) D.V. Nanopoulos and D.A. Ross, Phys. Lett. 118B (1982) 99.
- 20) S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. 125B (1983) 457.
- 21) J.P. Derendinger, L.E. Ibanez and H.P. Nilles, CERN preprint TH 4123 (1985).
- 22) E. Bergshoeff, M. De Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys.
B195 (1982) 97.
G.F. Chapline and N.S. Manton, Phys. Lett. 120B (1983) 105.

TABLE I

Scales and coupling constant for the E_6
breaking, through $\tilde{78}$, into $SU(6) \times U(1)_{T_{3R}}$.

k	ℓ	M_2 (GeV)	M_1 (GeV)	α
1	2	$1.6 \cdot 10^6$	$1.6 \cdot 10^{17}$	.09
1	3	$4.0 \cdot 10^{10}$	$2.1 \cdot 10^{18}$	.11
2	4	$1.3 \cdot 10^8$	$5.0 \cdot 10^{17}$	.23
2	5	$2.4 \cdot 10^{10}$	$1.8 \cdot 10^{18}$	.28

TABLE II

Scales and coupling constant for the E_6
breaking, through $\tilde{78}$, into $SU(5) \times U(1)^{26}$.

k	M_1 (GeV)	α	ℓ	M_2 (GeV)
1	$1.1 \cdot 10^{18}$	.11	2	$8.3 \cdot 10^{10}$
			3	$2.0 \cdot 10^{13}$
			4	$3.0 \cdot 10^{14}$
			5	$1.6 \cdot 10^{15}$
			6	$4.6 \cdot 10^{15}$
2	$7.7 \cdot 10^{16}$	.26	3	$4.5 \cdot 10^8$
			4	$5.1 \cdot 10^{10}$
			5	$8.8 \cdot 10^{11}$
			6	$5.9 \cdot 10^{12}$