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GEOMETRICAL IMAGE TRANSFORMATIONS AS A MEANS
TO IMPROVE THE QUALITY OF HIGH-ENERGY PICTURES

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ABSTRACT

The human perception of high-energy pictures may be considerably facilitated if one enhances the interesting features of the image by modifying its geometry. Several examples for such an improvement are shown for pictures from bubble chambers, for a drawing of a fixed target experiment and for a simulated image from a collider experiment.

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1. INTRODUCTION

Charged particles produce tracks in bubble chambers which are photographically recorded. A typical picture is reproduced in fig. 1(a), which shows the section of the original photo where the track of an incoming interacting particle is seen, followed by 16 tracks leaving the interaction. The photos are typically about 100 mm long and 50 mm wide with bubble diameters between 10 and 20 microns on the photo. The mean distance - on the photo - between 2 bubbles along a minimum ionising track lies between 50 and 200 microns depending on the experiment. In recent experiments the momenta of the incoming particles are very high, leading normally to a few sideways going tracks and to a narrow bundle of forward going tracks as shown in fig. 1(a). Most interesting are reactions where a charged particle (visible track) or an uncharged particle (no visible track) decays and gives rise to one or more visible "decay tracks" (see figs 1(a) and 1(b): points 2 and 3). The angle between the decaying secondary particle and the decay tracks is normally very small, which is the main problem when trying to find and analyse reliably the decays. This is done mostly on large projection tables. If the data are stored holographically or on video tapes or in digital form it may be preferable to examine the interaction on a TV monitor (or graphic display). In this case however one encounters three problems:

- (a) The resolution of normal monitors is very bad compared to the ratio of bubble size to the length of the photo (about 1 to 10000 for 10 micron bubbles), so that only a small part of the image can be displayed at one time. Assuming that the monitor screen is composed of 500×500 pixels and that one bubble of 10 microns covers just 1 pixel, an image of $5 \times 5 \text{ mm}^2$ can be displayed out of the photo of typically $100 \times 50 \text{ mm}^2$ as indicated in fig. 1(b). With a mean bubble distance of 100 microns a track is then represented by less than 50 bubbles on the screen which is often not enough to examine it reliably as experience shows.
- (b) There are normally some tracks so near to each other that many TV images must be examined following each other consecutively in space before the tracks separate.

- (c) A human operator examining the image on a monitor is able to recognize a change of direction (kink) of a track from about four degrees upwards. Smaller kinks may escape detection, which is for many experiments intolerable.

Similar problems may arise when displaying data from other detectors, where tracks have small kinks and/or lie near to each other.

However one should try that the examination of these images is not limited by the displaying system but rather by the detector.

It is the aim of this paper to demonstrate a method of geometrical image treatment to overcome the three problems above. The method may also be used to visualize data from various other detectors. It may be extended to experiments where particles leave the interaction point in all directions (which is not the case for fixed target experiments), for instance collider experiments.

Finally different realization techniques will be discussed.

2. THE PERSPECTIVE TRANSFORMATION

Monochromatic images may be described by an intensity function $I(X, Y)$. There exist many methods to improve the image quality by intensity transformations $I'(X, Y) = J(I(X, Y))$ or by more complicated transformations which take the intensity distribution in a neighbourhood into consideration.

In order to improve the clarity of the images described above a geometrical method is used here which leaves the intensity unchanged but alters the positions X, Y , that means that the image is geometrically modified to improve its quality:

$$X' = F(X, Y), \quad Y' = G(X, Y), \quad I'(X', Y') = I(X, Y). \quad (1)$$

In order to improve the analysis of forward tracks the transformations F and G should have the following properties:

- (a) High resolution between forward going tracks (the sideways going tracks may be ignored).
- (b) Long visible distance of forward going tracks.

In order not to worsen the human perception of tracks and possible kinks it is necessary that:

- (c) Straight lines transform into straight lines.

As the interactions are symmetric around the line of flight of the incoming particle, the transformation should be symmetric around the X-axis, where the direction parallel to the incoming particle is defined as X-direction with Y perpendicular to it. Combining this with the requirement before one gets:

$$(d) \quad X' = F(X) \text{ and } Y' = G(X, Y) = -G(X, -Y) \quad (2)$$

These requirements are matched by the anamorphic transformation

$$X' = M_x \cdot X, \quad Y' = M_y \cdot Y \text{ with } M_y \gg M_x, \quad M_x, M_y = \text{constant} \quad (3)$$

This transformation solves the next requirement, that

- (e) "kink-angles" (α) of forward tracks are increased:

$$\alpha' = \alpha \cdot \frac{M_y}{M_x} = \alpha \cdot M_\alpha \quad (4)$$

Fig. 1(c) shows the anamorphic transformed image of the one shown in figs 1(a) and 1(b). The "V" at point 2 and the kink at point 3 are now easily perceived.

If one wants to see most of the secondary decays and a large part of the forward tracks, M_x should be sufficiently small for the displayed area to cover a big part of the original photo. However for short lived particles the decays tend to occur near to the primary interaction and cannot reliably be found if M_x is too small. Therefore the anamorphic transformation is disadvantageous near to the interaction point, so that one needs another transformation where

- (f) the X-magnification $M_x(X)$ should be variable, being sufficiently large near the interaction point and small enough far from it.

In this case $Y' = G(X, Y)$ must be chosen in such a way that the requirement (c) is still valid, that means straight tracks remain straight.

The "perspective transformation" (see fig. 2(a)) is the only one which fulfils these requirements:

$$X' = X \cdot \frac{P_x}{P_o + X} \quad Y' = Y \cdot \frac{P_y}{P_o + X} \quad (5)$$

or if one replaces P_x , P_y and P_o by the parameters H , T , B , H' as defined in fig. 2(a) one gets:

$$X' = X \cdot \frac{H' \cdot T}{H \cdot B + X \cdot (T - B)} \quad (6)$$

$$Y' = Y \cdot \frac{H' \cdot H}{H \cdot B + X \cdot (T - B)}$$

This formula transforms a trapezoid of height H , bottom line B and top line T into a square with side of length H' (see fig 2(a)). It is clear that with such a transformation parallelism is not conserved, except for the case where B is equal to T , which leads back to the anamorphic transformation (formula (3)). Fig. 2(b) shows how the original X, Y coordinate system is transformed.

The effect of the perspective transformation is best described by the magnifications M_x , M_y and $M_\alpha = M_y/M_x$ at the bottom ($X = 0$, $Y = 0$) and the top ($X = H$, $Y = 0$) of the trapezoid.

$$\begin{aligned} M_x(0) &= \frac{T \cdot H'}{B \cdot H} & M_y(0) &= \frac{H'}{B} & M_\alpha(0) &= \frac{H}{T} \\ M_x(H) &= \frac{B \cdot H'}{T \cdot H} & M_y(H) &= \frac{H'}{T} & M_\alpha(H) &= \frac{H}{B} \end{aligned} \quad (7)$$

One sees that for a large angular magnification M_α the height H of the trapezoid should be large compared with B and T . With small values of B and T relative to H' the magnification M_y gets large, improving in this way the separation between neighbouring tracks. In order to get a high value of M_x at the bottom, T should be much bigger than B , which decreases M_x at the top. The formulae given above are discussed in more detail in Appendix A.

If one changes X , Y into polar coordinates (see fig. 2(a)):

$$X = R \cdot \cos\phi - R_1 \quad Y = R \cdot \sin\phi \quad (8)$$

one finds an approximation to formula (6):

$$X' = P_r \cdot \left(1 - \frac{R_1}{R}\right) \quad Y' = P_\phi \cdot \phi \quad (9)$$

If one replaces P_r and P_ϕ by the parameters defined in fig. 2(a) one gets:

$$X' = H' \cdot \frac{1 - \frac{R_1}{R}}{1 - \frac{R_1}{R_2}} \quad Y' = H' \cdot \frac{\phi}{\beta} \quad (10)$$

See Appendix A.

3. APPLICATION OF THE PERSPECTIVE TRANSFORMATION

Fig. 1(d) shows the application of the perspective transformation to the picture in fig. 1(a). Due to the variation of magnification from bottom to top the bubble size decreases with increasing distance from the vertex. The untransformed images (figs 1(a) and 1(b)) cover a somewhat smaller length than the transformed ones. The size of the image in fig. 1(d) (also figs 3(a) and 4(b)) is described by fig. 2(b). With the help of the two parallel beam tracks to the right and the left (points 4 and 5) of the interaction one may judge which width of the original is displayed. The "V" (point 2) and the kink (point 3) of a track which runs out of focus are easily distinguishable contrary to the original in fig. 1(a). Due to the small magnification at the top of the image the tracks are well defined there so that a straight line can be superimposed with high accuracy onto the track image to detect or confirm kinks. The distance between the "V" and the interaction point is increased by a factor of 2.5 when comparing the anamorphically transformed image in fig. 1(c) with the perspectively transformed image in fig. 1(d).

Fig. 3(a) shows the perspectively transformed picture of an interaction, which was recorded by holography [1]. If one uses a TV system to visualize the reconstructed image as is the case for the holographic measuring system HOLMES [2], only small sections of the

reconstructed image may be visualized on a television screen for the reasons discussed in the introduction. From many such television pictures the image of fig. 3(a) was constructed. This procedure will be described in more detail below. The drawing in fig. 3(b) shows the untransformed image of the kinked track (kink at point 6) and the track which leads to the three pronged decay (decay at point 2). Only by use of the transformed image one is able to decide if the three-prong decay is a real one or if it consists of a straight, through-going track plus a "V".

In order to study the properties of the transformation when tracks are curved, fig. 4(a) shows an interaction whose tracks are curved by the magnetic field in the bubble chamber BEBC. The transformed image covering the same length is shown in fig. 4(b) where the forward tracks of the event may easily be disentangled and separated from the many background tracks. It is clearly seen here that one of the three most forward going tracks does not belong to the interaction; this is hardly evident on the original in fig. 4(a).

The method may also be applied to display large experiments, which often include small vertex detectors and are equipped with large electronic detectors for different purposes downstream. Fig. 5(a) shows the EHS apparatus [3] with slight simplifications. The size of the vertex detector (fig. 5(a) : A) had to be increased to make it visible in fig. 5(a). The two vertical lines represent the simulated straight trajectories of two particles with an angle of about half a degree between them. The transformed image is shown in fig. 5(b), where the vertex detector even in the original size (fig. 5(a): A') is clearly visible and where the two tracks may easily be followed through all the equipment. The various detectors are labelled A to P to correlate the unmodified and the modified image.

4. TRANSFORMATION OF "COLLIDER" IMAGES

In collider experiments two particles with opposite momenta interact with each other. The interaction produces particles or jets of particles which may go in any direction rather than mainly in a forward direction as in fixed target experiments. The coordinates of points forming the tracks are normally known in all three dimensions. To show these interactions on

a screen it is necessary to project all tracks onto one plane, so that one gets a two dimensional image. These images are quite different from those of fixed target experiments as described in the foregoing chapter.

However the problem remains that:

- (a) Kink angles may be small.
- (b) Some decays are very near to the interaction point.
- (c) It might be necessary to have a better resolution at the centre than further out.

A simulated image with straight tracks is shown in fig. 6(a). Most of the tracks are generated with a kink, the value of which is indicated in this figure as in the following ones. The image was invented so as to bring out the features of the transformation to be described below but not to best resemble the real interactions.

If one wants to transform the whole image into another one, the transformation should have the following properties:

- Alpha: rotational symmetry,
- Beta: straight tracks through the centre of the image should remain straight.

Because of the required rotational symmetry it is useful to express the transformation in polar coordinates:

$$X = \cos\phi \qquad Y = \sin\phi \qquad (11)$$

Three examples for such transformations will be discussed using the image shown in fig. 6(a).

- T1: The polar coordinate transformation:

$$X' = R \qquad Y' = \phi \qquad (12)$$

- T2: The approximate perspective transformation, as given above by eq. (10):

$$X' = R_2 \cdot \frac{1 - \frac{R_1}{R}}{1 - \frac{R_1}{R_2}} \qquad Y' = \phi \qquad (13)$$

The parameters R_1 and R_2 are defined in fig. 2(a).

- T3: The rotational symmetric (approximate) perspective transformation is derived from formula (5) by changing X in the denominator to R and by setting $P_x = P_y = P_s$ in order to obtain a rotational symmetric transformation.

$$X' = X \cdot \frac{P_s}{P_o + R} \quad Y' = Y \cdot \frac{P_s}{P_o + R} \quad (14)$$

Replacing X , Y and X' , Y' by polar coordinates one gets:

$$R' = R \cdot \frac{P_s}{P_o + R} = R \cdot \frac{1 + E \cdot R_2}{1 + E \cdot R} \quad (15)$$

$$\phi' = \phi \quad \text{with } E = \frac{R'_1 - R_1}{R_1 \cdot (R_2 - R'_1)}$$

This formula transforms $R = 0$ into $R' = 0$, $R = R_2$ into $R' = R_2$ and $R = R_1$ into $R' = R'_1$. From R' and ϕ' the coordinates X' and Y' are calculated^(*).

All three transformations given above should be followed by a linear scaling to fit the image onto the monitor screen.

Figs 6(b-d) show the image of fig. 6(a) transformed by the three methods given above. The structure of the image is best understood by use of the rotational symmetric perspective transformation T3 in fig. 6(d). One sees that the interaction point is not exactly in the centre of the coordinate system, which leads for transformation T1 to the high curvature for small R and to a slight inclination of all tracks if one uses T2. The transformations T2 and T3 conserve the straightness of most tracks quite well, which is not the case for T1. By comparing figs 6(c) and (d) one will prefer one or the other depending on what one is looking for. With transformation T3 one may display the image down to $R = 0$, which is not possible for T2.

If one is interested in a small sector only the perspective transformation (formula (6)) may again be used. However, the non linear

(*) The transformation T3 may also be obtained by transforming an image forward (T2) and backward ($T2^{-1}$) using T2 but with different values of R_1 ($R_1 \rightarrow R'_1$) for the inverse transformation.

perspective transformation of all image points inside the sector must be recalculated, whenever the position of the sector is changed. This means in practice a rotation/translation/scaling of the relevant image points followed by the non-linear perspective transformation. By use of T2 or T3, which imposes some constraints on the transformation parameters H, B and T, the two steps may be interchanged. All image points may be transformed only once followed by a rotation/translation/scaling in the transformed image space.

5. IMPLEMENTATION BY ELECTRONIC AND MECHANICAL MEANS

If the images are not yet available in digital form but are stored on film the question arises how to generate the perspectively transformed images. The following methods of implementation may be used:

- (a) There exist many machines to digitize bubble chamber film. On these machines the coordinates and diameter of bubble images with an intensity exceeding a certain threshold are determined. These coordinates may be transformed into new ones and displayed. Fig. 7 shows the image of fig. 1(a) treated in such a way by the flying spot device ERASME [4].
- (b) The image in fig. 1(d) was gained by digitizing several TV images taken from the photo at different positions [5]. Then the intensity of each pixel on the monitor screen is calculated by interpolating (at the bottom of the image) or by averaging (at the top of the image) the intensity of the pixels of the TV image at the positions calculated by the inverse of formula (6) (see Appendix A formula (A.1)). If the TV camera is not distortion free, it may be necessary to use only the centre part of the digitized images, which means however that more TV images have to be recorded. For the images shown here 130 TV lines in the centre were used.
- (c) In the extreme case only one line of the TV image is used so that for the construction of each monitor line a new picture must be taken at a different position. This means that the TV camera is moved in the x direction over the image. If one wants to generate the new image in a short time the image of the TV camera should be read out in

flight. In this case one may also replace the TV camera by a photodiode array, which may have better resolution than the TV camera. The intervals $(X_{n+1} - X_n)$ at which the photodiodes (or TV camera) are read out increase when moving from bottom to top (see fig. 8(a), where it is assumed for simplicity that the monitor picture is formed by five lines only). Each time a different scaling of the data from the photodiodes is necessary according to formula (A.5). If the length of the diode array or TV image is not sufficient empty triangles to the right and left at the top of the image are produced as is seen in figs 1(d), 3(a) and 4(b).

- (d) The generation of the image may be easier if one uses the approximate perspective transformation according to formula (10). In this case the photodiodes are turned around the origin of the polar coordinate system (see figs 2(a) and 8(b)) and read out in equidistant intervals, which avoids the scaling discussed in the preceding paragraph. As may be calculated from formula (10) a varying number of cells of the photodiode arrays, increasing from top to bottom but independent of ϕ , must be averaged to form the image. If the resolution of the array is not sufficient one may combine several arrays together, of which each forms a section of the new image (see fig. 8(b)).

If the image is generated by use of a TV camera or a photodiode array the new image must be stored on a storage tube or on a video memory and displayed from it.

6. IMPLEMENTATION BY OPTICAL MEANS

The most direct way to transform an image by the perspective transformation is to transfer it by use of a lens from one plane to another one. Except for the trivial case $B = T = H$ these planes are not parallel to each other. If the lens is assumed to be thin and lying in a third plane, then - due to the Scheimpflug principle - all three planes must meet in the same straight line for exact focalisation of the first plane onto the second one. See fig. 9(a) where the image between the points 1 and 2 is transferred to the image between the points 1' and 2'.

It is shown in Appendix B that a direct projection of the image from normal size film onto a vidicon cannot be realized because the focal length of the lens gets very small and the acceptance angle γ_2 (see fig. 9(a)) is much too big.

A realization is possible if the original image is projected in a first step onto an intermediate screen or table to increase its size by the magnification factor S (see fig. 9(b)). In a second step the new picture (lying between the points 1 and 2 in fig. 9(a)) is transferred via the inclined lens to the vidicon (between the points 1' and 2'). In this case one may use a lens with a longer focal length. In order to avoid astigmatism in the glass plate covering the vidicon the angle ϵ' (see fig. 9(a)) should be sufficiently big. Therefore a possible realization may impose some constraints on the original parameters H , B , T . This is discussed in detail in Appendix B.

The table shows several examples for such calculations. The first one gives the values with which fig. 9 was drawn for $S = 1$. In this case however B and T are chosen too large as compared to H , to really improve the picture quality as in fig. 1(d). The second example shows the unrealistic small focal length and the large acceptance angle if one chooses for H , B and T similar values to those used for the generation of the image in fig. 1(d) but keeping S equal to 1. The next examples give values for possible realisations with $S = 40$, where only the last one has a sufficiently small acceptance angle (γ_2) and a sufficiently big entry angle (ϵ') into the vidicon.

TABLE

S	H	B	T	F	L	L'	L ₀	θ	θ'	γ_2	ϵ'_2
1.	40.	15.0	30.0	20.	72.	42.0	47.3	39.	42.	30.	78.
1.	30.	1.6	5.8	2.	97.	6.8	86.6	1.	65.	86.	111.
40.	40.	1.5	5.8	80.	670.	85.1	90.1	46.	81.	43.	51.
40.	30.	1.5	5.8	80.	502.	85.1	36.9	73.	81.	16.	25.
40.	30.	1.6	5.8	80.	543.	93.0	68.3	69.	65.	19.	44.
[mm]								[degrees]			

Fig. 10(a) shows an image generated in this way. It shows the same picture as the one in fig. 1(d). A photo from the experimental set-up on a scanning table is shown in fig. 10(b). The TV camera at the side of the table looks via the inclined lens and a small mirror in the centre onto the table.

7. CONCLUSIONS

Bubble chamber pictures are in most laboratories projected on large tables. Due to the complexity and jet structure of the interactions the best way to examine them is to look at them downstream with a grazing (narrow) incident view. It is known for a long time that this way of looking at the events is much superior to a projection of the image onto a screen.

If the possibility of a projection of the image onto a table is lost the perspective transformation of the image shows the interaction in a similar way as if the operator looks at it on the table. The great freedom in the choice of the parameters of the transformation may in some cases even lead to a superior presentation. The method is applicable for most high energy physics images from whatever detector they originate. In this way one may reach the point where the ease of examination is not limited by the displaying system but by the detector.

The methods discussed above may also be successfully applied to experiments where the human pattern recognition plays no role but where the resolution is limited by a recording TV camera or/and an image intensifier or by the way of data storage. If the image seen by the TV camera etc. is optically transformed beforehand to cover better the photocathode the resolution may be enhanced.

The method of geometrical image transformation is complementary to local image treatment methods on the one side and to pattern recognition methods on the other side.

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- [5] The TV-image was digitized using the IP6400 image processing system from GOULD Inc., De Anza Imaging and Graphics Division.

APPENDIX A

The inverse of the perspective transformation is given by:

$$\begin{aligned} X &= X' \cdot \frac{H \cdot B}{H' \cdot T - X' \cdot (T - B)} \\ Y &= Y' \cdot \frac{T \cdot B}{H' \cdot T - X' \cdot (T - B)} \end{aligned} \quad (A.1)$$

The sequential application of two of these transformations yields a transformation of the same kind.

Using eq. (3) the straight line $Y = P \cdot (X - X_0)$ is transformed into the straight line

$$Y' = - \frac{X_0 \cdot P \cdot H'}{B} + X' \cdot P \cdot \left(\frac{H}{T} + \frac{X_0}{B} - \frac{X_0}{T} \right) \quad (A.2)$$

The angular magnification M_α for a straight line passing through $X = X_0$ and $Y = 0$ is therefore given by:

$$M_\alpha = \frac{H}{T} + \frac{X_0}{B} - \frac{X_0}{T} \quad (A.3)$$

The magnification in X and Y-direction are given by:

$$M_X(X) = \frac{H' \cdot H \cdot T \cdot B}{(H \cdot B + X \cdot (T - B))^2} \quad (A.4)$$

$$M_Y(x) = \frac{H' \cdot H}{H \cdot B + X \cdot (T - B)} \quad (A.5)$$

The approximation (formula (10)) is derived from formula (6) by expressing X and Y in polar coordinates (see fig. 2(a)):

$$X = R \cdot \cos(\phi) - R_1 \quad Y = R \cdot \sin(\phi) \quad \text{with } R_1 = \frac{H \cdot B}{T - B} \quad (A.6)$$

Then formula (10) changes into:

$$\begin{aligned} X' &= H' \cdot \frac{1 - \frac{R_1}{R \cdot \cos \phi}}{1 - \frac{1}{R_2}} \\ Y' &= H' \cdot \frac{\text{tg} \phi}{c} \end{aligned}$$

with

$$C = \frac{T - B}{H} = 2 \cdot \text{tg}\left(\frac{\beta}{2}\right) \sim \beta \quad (A.7)$$

From this one gets with $\operatorname{tg} \phi = \phi$ and $\cos \phi = 1$ the approximation:

$$\begin{aligned} X' &\sim H' \cdot \frac{1 - \frac{R_1}{R}}{1 - \frac{1}{R_2}} & Y' &= H' \cdot \frac{\phi}{\beta} \end{aligned} \quad (10)$$

APPENDIX B

A thin lens transforms a point in the system U, V, W to a point in the system U', V', W' (see fig. 9(a)) with:

$$\frac{U'}{F} = \frac{F}{U} = \frac{V'}{V} = \frac{W'}{W} \quad (B.1)$$

To execute the transformation given by formula (6) for the two points 1 and 2 one sets:

$$W_1 = \frac{1}{2} \cdot B \cdot S \quad W_2 = \frac{1}{2} \cdot T \cdot S \quad W'_1 = \frac{1}{2} \cdot H' \quad W'_2 = \frac{1}{2} \cdot H' \quad (B.2)$$

$$H \cdot S = \sqrt{(V_2 - V_1)^2 + (U_2 - U_1)^2} \quad (B.3)$$

$$H' = \sqrt{(V'_2 - V'_1)^2 + (U'_1 - U'_2)^2} \quad (B.4)$$

where S is a scale factor, which is equal to 1 if the original image is positioned between the points 1 and 2 (see fig. 9(a)). The image is then transferred onto the face of the TV tube between the points 1' and 2'.

Given the transformation parameters H, T, B, H', the focal length F and the scale factor S, one can derive from formula (B.1):

$$V_1 = F \cdot \left(\frac{P \cdot S \cdot B}{H'} - Q \right) \quad (B.5)$$

$$V_2 = F \cdot \left(\frac{P \cdot S \cdot T}{H'} - Q \right) \quad (B.6)$$

with

$$P = \sqrt{\frac{F^2_{\max}}{F^2} - 1} = \text{ctg} \theta \quad (B.7)$$

and

$$Q = \sqrt{\frac{S^2}{S^2_{\min}} - 1} = \text{ctg} \theta' \quad (B.8)$$

$$F_{\max} = \frac{H \cdot H'}{T - B} \quad (B.9)$$

$$S_{\min} = F \cdot \left(\frac{1}{B} - \frac{1}{T} \right) \quad (B.10)$$

By use of the equations above all other distances may then be calculated. However a solution exists only if $F < F_{\max}$ and $S > S_{\min}$.

If an image recorded on film should be directly transferred onto a vidicon the scale factor S must be set to 1. For $1 = S > S_{\min}$ it follows from formula (B.10) that

$$F < \frac{T \cdot B}{T - B} \quad (\text{B.11})$$

which may lead to very small values of F , if one uses similar parameters as were used for the transformation in fig. 1(d).

A critical value for the realisation is the acceptance angle γ_2 which is given by:

$$\text{tg} \gamma_2 = \frac{S \cdot T \cdot P - H \cdot Q}{S \cdot T + H'} \quad (\text{B.12})$$

For jets similar to the one in fig. 1(a) recorded on normal size film one may assume $T < H'$. In this case a lower limit for γ_2 may be derived for $S = 1$:

$$\text{tg} \gamma_2 > \frac{H - B}{2 \cdot B} \quad (\text{B.13})$$

The acceptance angle must be sufficiently small in order not to lose too much light by vignetting the light from the image near to point 2. However as B is much smaller than H the acceptance angle gets very large.

Therefore a simple realization is possible only if S is much bigger than 1 which necessitates the projection of the image onto an intermediate screen (or table) (see fig. 9(b)). In this case one may use a lens with a longer focal length, which leads to a higher value of $S_{\min}$ according to formula (B.10). In order that S is not too large it should be chosen near to $S_{\min}$. In this case the acceptance angle is given approximately by:

$$\text{tg} \gamma_2 \sim \sqrt{\frac{F_{\max}^2}{F^2}} - 1 = \text{ctg} \theta \quad (\text{B.14})$$

which means that the angle λ by which the "camera" looks onto the table is very small (fig. 9(b)).

In order to keep γ_2 small, F should be near to $F_{\max}$. The acceptance angle is minimum for $F = F_{\max}$ and $S = S_{\min}$ from which follows for a given value of S and H' a constraint for the parameters H , B and T .

$$\frac{H}{B \cdot T} = \frac{S}{H'} \quad (\text{B.15})$$

Another critical value is the angle at which the light falls onto the vidicon surface (ϵ'):

$$\epsilon_{1,2} = 90^\circ - \theta' + \gamma_{1,2} \quad (\text{B.16})$$

If ϵ' is too small the glass plate covering the photo cathode of the vidicon produces astigmatism.

FIGURE CAPTIONS

- Fig. 1 (a) Untreated bubble chamber picture 25 mm long.
 (b) Drawing of the relevant tracks.
 (c) Anamorphically transformed picture 44 mm long, 2.2 mm wide.
 (d) Perspective transformed picture 44 mm long, bottom width = 1.4 mm, top width = 5.6 mm. The coordinate system is defined in fig. 2(b).
- Fig. 2 (a) Definition of coordinates and parameters of the perspective transformation.
 (b) Non-orthogonal coordinate system as a result of the perspective transformation.
- Fig. 3 (a) Image of an interaction originally recorded by holography modified by the perspective transformation. The coordinate system is defined in fig. 2(b).
 (b) Untransformed drawing of the relevant tracks of the interaction.
- Fig. 4 (a) Image of an interaction in the bubble chamber BEBC.
 (b) The same interaction after perspective transformation. The coordinate system is defined in fig. 2(b).
- Fig. 5 (a) Untransformed drawing of a fixed target experimental apparatus with 16 units (A to P) similar to the European Hybrid Spectrometer.
 (b) Drawing of the apparatus after the perspective transformation.
- Fig. 6 (a) Simulated collider interaction with straight tracks.
 (b) Interaction in polar coordinates.
 (c) Interaction after approximated perspective transformation.
 (d) Interaction after rotational perspective transformation.
- Fig. 7 Perspective transformation of the image in fig. 1(a) generated by the flying spot device ERASME.

FIGURE CAPTIONS

Fig. 8 Principle of the generation of the perspective transformation by a linear movement of a TV camera or photodiodes (8(a)) or by a rotation of photodiodes (8(b)).

Fig. 9 (a) Optical principle to generate the perspective transformation.

(b) Principle of realisation.

Fig. 10 (a) The image of fig. 1(a) as seen by a TV camera via an inclined lens.

(b) The image projected on a scanning table is seen via a mirror by a TV camera and shown on a monitor.

Fig. 1(a)

Fig. 1(b)

Fig. 2(a)

Fig. 2(b)

Fig. 3(a)

Fig. 3(b)

Fig. 4(a)

P1L032A1.592
44000
1400
5600

HOLMES

Fig. 4(b)

Fig. 1(c)

P1L457B3.554
44000
1400
5600

HOLMES

Fig. 1(d)

Fig. 5

Fig. 6(b)

Fig. 6(c)

Fig. 6(d)

EXP. 27 F

Roll 457
Frame 3554
View 3

Fig. 7

Fig. 8

(a)

(b)

Fig. 10