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A MODEST PROPOSAL FOR GENERATING YUKAWA COUPLINGS

FROM GAUGE INTERACTIONS

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A B S T R A C T

We propose that the quark and lepton Yukawa superpotential couplings to Higgs supermultiplets arise from non-perturbative gauge interactions. This is possible in models with an $SU(N) \times G$ gauge group. We present a three-generation model based on $SU(8) \times G$, and indicate how such a scenario could lead to a realistic hierarchy of quark and lepton masses.

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The success of the standard model poses three fundamental problems. These are the hierarchy problem - which we believe can only be solved with the protection of supersymmetry¹⁾, gauge unification - which we believe occurs in a grand unified theory²⁾ whose details cannot yet be specified, and the flavour problem. Included in this latter are the bewildering proliferation of fundamental fermions with masses and mixing angles which look quite random. In the standard model with six flavours each of quarks and leptons, there are 13 such seemingly arbitrary parameters in the fermion sector. They are supposed to be linked to correspondingly random-seeming Yukawa couplings of Higgs bosons, whose self-couplings introduce at least two more parameters. Neither supersymmetry nor grand unification is of much use in reducing this apparent arbitrariness. Supersymmetry retains the same set of unspecified Yukawa coupling parameters, while grand unification provides some modest relations between quark and lepton couplings³⁾ which are not uniformly successful. The market is therefore open to suggestions for simplifying Yukawa couplings or for deriving them from some higher principle.

It is attractive to surmise that the only truly fundamental interactions are gauged. Then the required Yukawa couplings should be derived indirectly from gauge interactions in some way. We are aware of two classes of proposals how this might happen. One is that condensates of fermions with new strong interactions break chiral symmetry and feed masses down to our known fermions through massive gauge boson exchange. Proposed⁴⁾ in the context of models of dynamical breaking of weak gauge symmetry, this extended technicolour scenario had problems in its original form because the new strong interactions were forced to have a mass scale of order 1 TeV, the massive gauge bosons could not be too much heavier and thus gave rise to unacceptable flavour-changing neutral interactions⁵⁾. This phenomenological difficulty seems to be the bane of attempts to solve the flavour problem at a mass-scale of order 1 TeV. It provides an argument for thinking that the flavour problem may only be solved at a much higher mass scale, perhaps of order the Planck mass. In direct contrast, the naturalness aspect of the hierarchy problem required a remedy around 1 TeV. A second way of generating new interactions is through non-perturbative gauge interactions. 't Hooft⁶⁾ showed how effective multifermion interactions were generated at low energies ($\ll m_W$) by weak instantons with a characteristic scale size of $O(m_W)$. A similar phenomenon is now known to occur in supersymmetric theories⁷⁾, with non-perturbative gauge interactions generating a non-trivial superpotential for the chiral superfields. It has been discussed⁷⁾ whether this phenomenon could be useful for inducing dynamical supersymmetry breaking in realistic models, though such a possibility is not our interest here.

In this paper we propose that the mysterious Yukawa interactions of the known quarks and leptons may arise from such non-perturbative gauge interactions. We make this modest proposal⁸⁾ in the context of supersymmetric grand unified theories whose couplings become strong around the Planck scale. Because of the specific chiral selection rules derived⁷⁾ for such non-perturbative superpotential terms, we must actually postulate another set of strong gauge interactions around the Planck scale, and study gauge groups of the form $G_{\text{GUT}} \times G_{\text{Hidden}}$. The simplest three-generation model with no obvious phenomenological defects is based on an $SU(8)$ GUT group and yields a superpotential which is the product of several hundred chiral fields. The vast majority of these must acquire vacuum expectation values of order m_p in order to give the low energy cubic superpotential terms. The different powers with which fields in representations of different dimensions appear in the superpotential yield a natural mechanism for providing hierarchically different fermion masses, and we show how a realistic hierarchy of quark masses can be obtained from our $SU(8)$ toy model.

As a start let us assume the existence of a simple GUT group G and chiral multiplets $\sum_i N_i \phi_i(R_i)$, where R_i denotes the dimensionality of the GUT representation in which ϕ_i sits, and N_i is its multiplicity. The global symmetry of the model at the classical level is

$$H \equiv \prod_i [SU(N_i) \times U(1)_i] \times U(1)_\lambda \quad (1)$$

with the $U(1)$ charge assignments shown in Table 1. However, the $U(1)$ currents J_1^μ and J_λ^μ are anomalous at the quantum level:

$$\partial_\mu J_i^\mu = 2T_i N_i q \quad , \quad \partial_\mu J_\lambda^\mu = 2C_2(G)q \quad (2)$$

where

$$T_i \delta^{ab} \equiv T_r (T^a(R_i) T^b(R_i)) \quad , \quad q \equiv \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} \quad (3)$$

Hence the following linear combinations of $U(1)$ currents are anomaly free:

$$\left. \begin{aligned} J_{ij}^\mu &\equiv \frac{1}{N_i T_i} J_i^\mu - \frac{1}{N_j T_j} J_j^\mu \\ J_R^\mu &\equiv J_\lambda^\mu - \frac{C_2(G)}{\sum_i T_i N_i} \sum_i J_i^\mu \end{aligned} \right\} \quad (4)$$

and any non-perturbative effects must have

$$H' \equiv \prod_i SU(N_i) \times \prod_{i,j} U(1)_{ij} \times U(1)_R \quad (5)$$

as an unbroken global symmetry.

We look for a superpotential of the form

$$W = k \Lambda^{3 - \sum_i \gamma_i} \prod_i [\phi_i]^{\gamma_i} \quad (6)$$

where the γ_i are powers to be determined, Λ is the scale of order m_p at which the G gauge interactions become strong, and k is a dimensionless numerical constant supposed to be $O(1)$. We can imagine W (6) as being generated by "small instantons" of size $\rho \sim 1/\Lambda$ with action $\exp(-8\pi^2/g^2) = O(1)$. The requirement (5) of $U(1)_{ij}$ symmetry enforces the universality

$$\frac{\gamma_i}{T_i N_i} = \frac{\gamma_j}{T_j N_j} \equiv a \quad (7)$$

The discrete charges of instantons

$$\nu = \int d^4x \, q(x) = 0, \pm 1, \pm 2, \dots \quad (8)$$

substituted into the anomalous conservation equations (2) give us discrete changes in the $U(1)_i$ charges

$$\Delta Q_i = 2 T_i N_i \nu \quad (9)$$

Comparing with the generic form (6) of the superpotential which has

$$\Delta Q_i = \gamma_i = a T_i N_i \quad (10)$$

we deduce that

$$\gamma_i = 2T_i N_i \nu \quad (11)$$

Finally, $U(1)_R$ invariance of the F-term derived from the superpotential W (6) imposes

$$\left(1 - \frac{C_2(G)}{\sum_i T_i N_i}\right) \sum_i \gamma_i - 2 = 0 \quad (12)$$

so that we have the key constraint

$$1 + \nu \left(C_2(G) - \sum_i T_i N_i\right) = 0 \quad (13)$$

Turning now to the $SU(N_i)$ invariances of the superpotential (6) we see that they require

$$W \sim \prod_i [\phi_i]^{2T_i N_i \nu} = \prod_i \left\{ [\phi_i]_A^{N_i} \right\}^{2T_i \nu} \quad (14)$$

where the antisymmetric product

$$[\phi_i]_A^{N_i} \equiv \epsilon_{\alpha_1 \alpha_2 \dots \alpha_{N_i}} \phi_i^{\alpha_1} \phi_i^{\alpha_2} \dots \phi_i^{\alpha_{N_i}} \quad (15)$$

is an $SU(N_i)$ singlet. Thus the form of the superpotential is almost completely determined:

$$W = k \Lambda^{3-2\nu \sum_i T_i N_i} \prod_i \left\{ [\phi_i]_A^{N_i} \right\}^{2T_i \nu} \quad (16)$$

subject to the constraint (13) for $\nu = \pm 1, \pm 2, \dots$.

Before proceeding to model-building we must note a severe difficulty with the assumption of a simple gauge group G . The $U(1)_R$ constraint (13) requires

$$C_2(G) - \sum_i T_i N_i = \pm 1 \left(\pm \frac{1}{2} \right) \text{ for } \nu = \mp 1 (\mp 2) \quad (17)$$

Therefore higher dimensional representations are forbidden, and even one adjoint supermultiplet with $T_1 = C_2(G)$ almost saturates the condition (17). Therefore it is impossible to make a realistic model with a simple gauge group, and we adopt the $G = G_{\text{GUT}} \times G_{\text{Hidden}}$ philosophy mentioned in the introduction. Here G_{Hidden} may be decoupled from conventional observable physics at energies $\ll m_p$, and we imagine it as being confined at some energy scale $\mu = O(m_p)$. The superpotential (6) now becomes

$$W \sim \prod_i \left\{ [\phi_i]_A^{N_i} \right\}^{2T_i \nu} \times W_{\text{Hidden}} \quad (18)$$

and we presume that non-perturbative G_{Hidden} dynamics yield $\langle W_{\text{Hidden}} \rangle \neq 0$. The $U(1)_R$ constraint (13) is now modified to become

$$1 + \nu (C_2(G) - \sum_i T_i N_i) + (\text{analogous } G_{\text{Hidden}} \text{ terms}) = 0 \quad (19)$$

and it is possible to arrange the hidden sector so that $1 + \nu(C_2(G) - \sum_i T_i N_i)$ for the observable GUT sector can take any value. While most supermultiplets could transform as $(R_i, 1)$ or $(1, R'_i)$ representations of $G_{\text{GUT}} \times G_{\text{Hidden}}$, to avoid an unwanted additional $U(1)$ global symmetry, at least one supermultiplet must transform non-trivially under both G_{GUT} and G_{Hidden} . The need for a G_{Hidden} sector with this structure does not shock us, since it may emerge naturally from a larger theoretical context such as $N = 8$ supergravity⁹⁾: $G = SU(8)_{\text{GUT}} \times SO(8)_{\text{Hidden}}$, or from the superstring¹⁰⁾: $G = E_{6\text{GUT}} \times (SU(3) \times E_8)_{\text{Hidden}}$.

To construct a phenomenologically realistic model we need at least three $(\bar{5}+10)$ representations of an $SU(5)$ GUT, G must be free of anomalies, and we want no light exotics. Therefore our G_{GUT} multiplets must have the $SU(5)$ decomposition

$$\sum_i N_i \phi_i(R_i) \rightarrow N_g (\bar{5}+10) + \sum_m (r_m + \bar{r}_m) + \sum \text{real} \quad (20)$$

with $N_g \geq 3$. The superpotential W (16) is clearly non-renormalizable, and in order to get a renormalizable cubic effective low energy potential, we must replace most of the ϕ_i factors by vacuum expectation values $\langle \phi_i \rangle$ which are $SU(3) \times SU(2) \times U(1)$ singlets. Then we have

$$W_{\text{eff}} = \frac{1}{2} \phi_a \phi_b \left\langle \frac{\partial^2 W}{\partial \phi_a \partial \phi_b} \right\rangle + \frac{1}{3!} \phi_a \phi_b \phi_c \left\langle \frac{\partial^3 W}{\partial \phi_a \partial \phi_b \partial \phi_c} \right\rangle \quad (21)$$

and we see from the general form of W (16) that we will need

$$\left\langle [\phi_i]_A^{N_i} \right\rangle \neq 0 \quad (22)$$

Therefore any G representation R_i appearing in the model with multiplicity N_i must contain at least N_i $SU(3) \times SU(2) \times U(1)$ singlet components which can acquire non-vanishing vacuum expectation values of order m_p .

Now we are ready for model-building, and try as a first step $G_{\text{GUT}} = \text{SU}(N)$. In general, $N > 5$ and high-dimensional representations of G_{GUT} are required. For example, if we restrict ourselves to totally antisymmetric representations (one column Young tableaux), then $G_{\text{GUT}} = \text{SU}(6)$, $\text{SU}(7)$ and $\text{SU}(8)$ do not yield models with $N_g > 2$. We must go to at least $G_{\text{GUT}} = \text{SU}(9)$ to get $N_g > 3$, and even here we must introduce additional real representations to break $\text{SU}(9) \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$, as well as further high-dimensional representations to give superheavy masses $O(m_p)$ to all the unwanted stuff in Eq. (20).

The obvious alternative is to try $\text{SU}(N)$ representations with two-column Young tableaux. In this case there is no $\text{SU}(6)$ model with $N_g \neq 0$, but one can construct models with $N_g < 2$ using $\text{SU}(7)$. Many models with $N_g > 3$ are possible using $\text{SU}(8)$. If we look for models with $N_1 < 1$ for all G_{GUT} representations R_1 , we find no $\text{SU}(8)$ model with $N_g > 3$, and an $N_g = 3$ model which is unique apart from the possible addition of real representations:

$$\sum_i \phi_i = \underline{8} + \underline{56} + \underline{36} + \underline{378} + \underline{504} + \underline{216} + \underline{336} + \underline{1008} + \underline{1512} + \underline{1344} \quad (23)$$

The $\text{SU}(5) \times \text{SU}(3) \times \text{U}(1)$ decompositions of the irreducible representations in this model are shown in Table 2. It yields the following $\text{SU}(5)$ representations:

$$3(\underline{5} + \underline{10}) + 33(\underline{5} + \underline{5}) + 40(\underline{10} + \underline{10}) + 11(\underline{15} + \underline{15}) + 15(\underline{40} + \underline{40}) + 15(\underline{45} + \underline{45}) + 4(\underline{50} + \underline{50}) + 34\underline{1} + 16\underline{24} + 7\underline{75} \quad (24)$$

among which are all those desired to get realistic values of m_d/m_e and $m_s/m_\mu^{11)}$, the missing partner mechanism¹²⁾, neutrino masses, inflation, etc. The superpotential is

$$W \sim \left\{ [\underline{8}]^{15} \cdot [\underline{56}]^{10} \cdot [\underline{36}]^{156} \cdot [\underline{378}]^{215} \cdot [\underline{504}]^{75} \cdot [\underline{216}]^{160} \cdot [\underline{336}]^{526} \cdot [\underline{1008}]^{804} \cdot [\underline{1512}]^{680} \cdot [\underline{1344}] \right\}^{25}$$

The large powers appearing in this superpotential are due to the large quadratic Casimirs T_1 of the higher-dimensional representations of $\text{SU}(8)$ appearing in the model (23).

Note that the vacuum expectation values of the $\underline{1}$, $\underline{24}$ and $\underline{75}$ representations of $\text{SU}(5)$ listed in Table 2 are sufficient to break $\text{SU}(8) \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$. In

order to give superheavy masses to all the unwanted multiplets in (24), we must check that their mass terms $W_m \sim \phi_a \phi_b \langle \partial^2 W / \partial \phi_a \partial \phi_b \rangle$ are not forbidden by SU(8). This can be checked by examining the subgroup SU(5)×SU(3)×U(1), and it is easy to check that no problems arise from the SU(5)×SU(3) subgroup. However, U(1) charge conservation is not automatic, and this was the downfall of the SU(9) single column model. The SU(8) model (23) survives this test because the 1008 contains $(\underline{1}, \underline{\bar{3}}, -25)$ and $(\underline{75}, \underline{1}, 15)$ representations of SU(5)×SU(3)×U(1). Therefore

$$\langle \underline{1008} \rangle^{526} \supset \sum_{p=0}^{526} \langle \underline{1}, \underline{\bar{3}}, -25 \rangle^p \langle \underline{75}, \underline{1}, 15 \rangle^{526-p} \quad (26)$$

and we can adjust the U(1) charge so as to get all required mass terms with some appropriate choice of the integer p. In the same way we can get all the trilinear Yukawa couplings required to give quark and lepton masses

$$W_Y \sim \bar{F}_i T_j \bar{H} \left\langle \frac{\partial^3 W}{\partial \bar{F}_i \partial T_j \partial \bar{H}} \right\rangle + T_i T_j H \left\langle \frac{\partial^3 W}{\partial T_i \partial T_j \partial H} \right\rangle \quad (27)$$

where the $\bar{F}_i$ and T_j are $N_g = 3$ generations of $\underline{\bar{5}}$ and $\underline{10}$ chiral supermultiplets, and H and $\bar{H}$ are $\underline{5}$ and $\underline{\bar{5}}$ representations of Higgses.

It is evident that the Yukawa coupling of any three supermultiplets $\phi_{a,b,c}$ is

$$\Gamma_{abc} \equiv \left\langle \frac{\partial^3 W}{\partial \phi_a \partial \phi_b \partial \phi_c} \right\rangle = \gamma_a \gamma_b \gamma_c k \left\langle \pi_i \left[\frac{\phi_i}{\Lambda} \right]^{\tilde{\gamma}_i} \right\rangle \quad (28)$$

where $\tilde{\gamma}_i \equiv \gamma_i - 1$ for $i = a, b, c$ and $\tilde{\gamma}_i \equiv \gamma_i$ otherwise. Thus the quark and lepton mass matrices

$$M_{ab} = \langle \phi_c \rangle \Gamma_{abc} = \gamma_a \gamma_b N_{ab} : N_{ab} \equiv \gamma_c \langle \phi_c \rangle k \left\langle \pi_i \left[\frac{\phi_i}{\Lambda} \right]^{\tilde{\gamma}_i} \right\rangle \quad (29)$$

where ϕ_c is a Higgs superfield. We cannot specify the detailed structure of N_{ab} because the superpotential (25) is horrendously complicated, with a large number of unknown vacuum expectation values all $O(\Lambda) = O(m_p)$, and there are many ways to contract SU(8) indices. Nevertheless, it seems reasonable to consider N_{ab} to be a "pseudo-random" matrix with no strong hierarchical structure. However, the

prefactors $\gamma_{a,b}$ which were exponents in the superpotential (25) are very different for different SU(8) representations. Therefore we can get hierarchical ratios between the masses of fermions in different generations if the $\bar{F}_i$ and T_j are assigned to representations among (23) (see Table 2) which have very different quadratic Casimirs and hence exponents γ_i (11). If we define

$$\epsilon_1 \equiv \frac{\gamma(F_1)}{\gamma(\bar{F}_2)}, \epsilon_2 \equiv \frac{\gamma(\bar{F}_2)}{\gamma(\bar{F}_3)}; \theta_1 \equiv \frac{\gamma(T_1)}{\gamma(T_2)}, \theta_2 \equiv \frac{\gamma(T_2)}{\gamma(T_3)} \quad (30)$$

then the charge +2/3 quark mass matrix has the generic form

$$m_{+2/3} \sim \begin{pmatrix} \theta_1^2 \theta_2^2 & \theta_1 \theta_2^2 & \theta_1 \theta_2 \\ \theta_1 \theta_2^2 & \theta_2^2 & \theta_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix} \\ \sim \begin{pmatrix} 1 & \theta_1 & \theta_1 \theta_2 \\ \theta_1 & 1 & \theta_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix} \begin{pmatrix} \theta_1^2 \theta_2^2 & 0 & 0 \\ 0 & \theta_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_1 & \theta_1 \theta_2 \\ \theta_1 & 1 & \theta_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix} \quad (31)$$

and the charge -1/3 quark mass matrix has the generic form

$$m_{-1/3} \sim \begin{pmatrix} \epsilon_1 \epsilon_2 \theta_1 \theta_2 & \epsilon_1 \epsilon_2 \theta_2 & \epsilon_1 \epsilon_2 \\ \epsilon_2 \theta_1 \theta_2 & \epsilon_2 \theta_2 & \epsilon_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix} \\ \sim \begin{pmatrix} 1 & \epsilon_1 & \epsilon_1 \epsilon_2 \\ \epsilon_1 & 1 & \epsilon_2 \\ \epsilon_1 \epsilon_2 & \epsilon_2 & 1 \end{pmatrix} \begin{pmatrix} \epsilon_1 \epsilon_2 \theta_1 \theta_2 & 0 & 0 \\ 0 & \epsilon_2 \theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_1 & \theta_1 \theta_2 \\ \theta_1 & 1 & \theta_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix} \quad (32)$$

Thus we find the following qualitative form for the mass eigenvalues and for the Kobayashi-Maskawa matrix

$$\left. \begin{aligned}
 m_u : m_c : m_t &\sim \theta_1^2 \theta_2^2 : \theta_2^2 : 1 \\
 m_d : m_s : m_b &\sim \epsilon_1 \epsilon_2 \theta_1 \theta_2 : \epsilon_2 \theta_2 : 1 \\
 U &\sim \begin{pmatrix} 1 & \theta_1 & \theta_1 \theta_2 \\ \theta_1 & 1 & \theta_2 \\ \theta_1 \theta_2 & \theta_2 & 1 \end{pmatrix}
 \end{aligned} \right\} \quad (33)$$

As an exercise to illustrate this idea, consider the example (denoting $\bar{\gamma} = \gamma/v$)

$$\left. \begin{aligned}
 \bar{F}_1 &\in \underline{216} : \bar{\gamma} = 75 ; T_1 \in \underline{56} : \bar{\gamma} = 15 \\
 \bar{F}_2 &\in \underline{336} : \bar{\gamma} = 160 ; T_2 \in \underline{378} : \bar{\gamma} = 156 \\
 \bar{F}_3 &\in \underline{1512} : \bar{\gamma} = 804 ; T_3 \in \underline{1512} : \bar{\gamma} = 804
 \end{aligned} \right\} \quad (34)$$

With this choice we find

$$\left. \begin{aligned}
 \frac{m_c}{m_u} &\sim \theta_1^2 \sim \left(\frac{156}{15}\right)^2 \sim 108 ; \frac{m_t}{m_c} \sim \theta_2^2 \sim \left(\frac{804}{156}\right)^2 \sim 27 \\
 \frac{m_s}{m_d} &\sim \epsilon_1^{-1} \theta_1^{-1} \sim \frac{156 \cdot 160}{15 \cdot 75} \sim 22 ; \frac{m_b}{m_s} \sim \epsilon_2^{-1} \theta_2^{-1} \sim \frac{804 \cdot 804}{156 \cdot 160} \sim 26 \\
 \theta_c &\sim \theta_1 \sim \left(\frac{15}{156}\right) \sim 0.1 ; \theta_{bc} \sim \theta_2 \sim \left(\frac{156}{804}\right) \sim 0.2
 \end{aligned} \right\} \quad (35)$$

These results are certainly qualitatively reasonable, particularly when we recall the numerical factors of $O(1)$ which could easily appear in ratios of the elements of the matrix N_{ab} (29).

In this paper we have presented a modest proposal⁸⁾ for the origin of the Yukawa interactions of the standard model, and speculated how it would lead to a hierarchy of quark and lepton masses. We are aware that our simplest model (23) is not very economical, and that the superpotential (25) is frankly horrifying. We hope that subsequent authors may be able to improve on our modest efforts. As already mentioned, it is natural to suggest that some such non-perturbative mechanism could play a role in any theory of everything^{9),10)}, and in the low energy sector of the superstring¹⁰⁾ in particular.

Table 1 - U(1) charges

	Q_1	Q_λ
ϕ_j	δ_{1j}	1
Gauginos	0	1
Superspace θ	0	-1

Table 2 - SU(5)×SU(3)×U(1) decompositions of SU(8)
representations in the model (23)

$$\begin{aligned}
 \underline{\bar{8}} &= (\underline{\bar{5}}, \underline{1}, -3) + (\underline{1}, \underline{\bar{3}}, 5) \\
 \underline{56} &= (\underline{1}, \underline{1}, -15) + (\underline{5}, \underline{\bar{3}}, -7) + (\underline{10}, \underline{3}, 1) + (\underline{\bar{10}}, \underline{1}, 9) \\
 \underline{\bar{36}} &= (\underline{\bar{15}}, \underline{1}, -6) + (\underline{\bar{5}}, \underline{\bar{3}}, 2) + (\underline{1}, \underline{\bar{6}}, 10) \\
 \underline{378} &= (\underline{1}, \underline{3}, -20) + (\underline{5}, \underline{1+8}, -12) + (\underline{10}, \underline{\bar{3}+6}, -4) + (\underline{15}, \underline{\bar{3}}, -4) \\
 &\quad + (\underline{\bar{10}}, \underline{3}, 4) + (\underline{40}, \underline{3}, 4) + (\underline{45}, \underline{1}, 12) \\
 \underline{\bar{504}} &= (\underline{24}, \underline{1}, -15) + (\underline{45}, \underline{\bar{3}}, -7) + (\underline{5}, \underline{\bar{3}}, -7) + (\underline{40}, \underline{3}, 1) \\
 &\quad + (\underline{10}, \underline{\bar{3}+6}, 1) + (\underline{15}, \underline{1}, 9) + (\underline{\bar{10}}, \underline{1+8}, 9) + (\underline{\bar{5}}, \underline{\bar{3}}, 17) \\
 \underline{216} &= (\underline{\bar{10}}, \underline{3}, -11) + (\underline{\bar{5}}, \underline{1+8}, -3) + (\underline{45}, \underline{1}, -3) + (\underline{1}, \underline{\bar{3}+6}, 5) \\
 &\quad + (\underline{24}, \underline{\bar{3}}, 5) + (\underline{5}, \underline{3}, 13) \\
 \underline{\bar{336}} &= (\underline{50}, \underline{1}, -12) + (\underline{40}, \underline{\bar{3}}, -4) + (\underline{15}, \underline{\bar{6}}, 4) + (\underline{\bar{10}}, \underline{3}, 4) \\
 &\quad + (\underline{\bar{5}}, \underline{8}, 12) + (\underline{1}, \underline{6}, 20) \\
 \underline{1008} &= (\underline{1}, \underline{\bar{3}}, -25) + (\underline{5}, \underline{\bar{3}+6}, -17) + (\underline{10}, \underline{1+8}, -9) + (\underline{15}, \underline{8}, -9) + (\underline{\bar{10}}, \underline{\bar{3}}, -1) \\
 &\quad + (\underline{40}, \underline{\bar{3}+6}, -1) + (\underline{45}, \underline{3}, 7) + (\underline{50}, \underline{3}, 7) + (\underline{75}, \underline{1}, 15) \\
 \underline{1512} &= (\underline{45}, \underline{1}, -18) + (\underline{75}, \underline{\bar{3}}, -10) + (\underline{24}, \underline{\bar{3}}, -10) + (\underline{50}, \underline{3}, -2) + (\underline{45}, \underline{\bar{3}+6}, -2) \\
 &\quad + (\underline{5}, \underline{3}, -2) + (\underline{40}, \underline{1+8}, 6) + (\underline{10}, \underline{1+8}, 6) + (\underline{15}, \underline{\bar{3}}, 14) + (\underline{\bar{10}}, \underline{\bar{3}+6}, 14) \\
 &\quad + (\underline{\bar{5}}, \underline{3}, 22) \\
 \underline{1344} &= (\underline{10}, \underline{\bar{3}}, -19) + (\underline{\bar{10}}, \underline{\bar{3}+6}, -11) + (\underline{40}, \underline{3}, -11) + (\underline{\bar{5}}, \underline{1+8}, -3) + (\underline{45}, \underline{1+8}, -3) \\
 &\quad + (\underline{50}, \underline{1}, -3) + (\underline{1}, \underline{\bar{3}}, 5) + (\underline{24}, \underline{\bar{3}+6}, 5) + (\underline{75}, \underline{\bar{3}}, 5) + (\underline{5}, \underline{3}, 13) \\
 &\quad + (\underline{45}, \underline{3}, 13) + (\underline{10}, \underline{1}, 21)
 \end{aligned}$$

REFERENCES

- 1) M. Gel'fand and E.P. Likhtman - Pis'ma Zh.Eksp.Teor.Fiz.13 (1971) 323;
D. Volkov and V.P. Akulov - Phys.Lett. 46B (1973) 109;
J. Wess and B. Zumino - Nucl.Phys. B70 (1974) 39.
- 2) J.C. Pati and A. Salam - Phys.Rev. D8 (1973) 1240;
H. Georgi and S.L. Glashow - Phys.Rev.Lett. 32 (1974) 438.
- 3) M.S. Chanowitz, J. Ellis and M.K. Gaillard - Nucl.Phys. B128 (1977) 506;
A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos - Nucl.Phys. B135 (1978) 66.
- 4) S. Dimopoulos and L. Susskind - Nucl.Phys. B155 (1979) 237;
E. Eichten and K.D. Lane - Phys.Lett. 90B (1980) 125.
- 5) S. Dimopoulos and J. Ellis - Nucl.Phys. B182 (1981) 505.
- 6) G. 't Hooft - Phys.Rev.Lett. 37 (1976) 8; Phys.Rev. D14 (1976) 3432.
- 7) I. Affleck, M. Dine and N. Seiberg - Phys.Rev.Lett. 51 (1983) 1326;
Phys.Rev.Lett. 52 (1984) 1677;
T. Taylor, G. Veneziano and S. Yankielowicz - Nucl.Phys. B218 (1983) 493;
D. Amati, P. Rossi and G. Veneziano - Nucl.Phys. B249 (1985) 1.
- 8) J. Swift - "A modest Proposal for Preventing the Children of Poor People from being a Burthen to their Parents, or the Country, and for Making them Beneficial to the Publick" (1729).
- 9) E. Cremmer and B. Julia - Phys.Lett. 80B (1978) 48; Nucl.Phys. B159 (1979) 141;
B. de Wit and H. Nicolai - Phys.Lett. 108B (1981) 285; Nucl.Phys. B208 (1982) 323.
- 10) M.B. Green and J.H. Schwarz - Caltech Preprints CALT-68-1182 (1984); CALT-68-1194 (1984);
P. Candelas, G.T. Horowitz, A. Strominger and E. Witten - Institute for Theoretical Physics, Santa Barbara Preprint (1984).
- 11) H. Georgi and C. Jarlskog - Phys.Lett. 86B (1979) 297;
H. Georgi and D.V. Nanopoulos - Phys.Lett. 82B (1979) 392.
- 12) A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida - Phys.Lett. 115B (1982) 380;
B. Grinstein - Nucl.Phys. B206 (1982) 387.