CERN-TH.4138/85

S MATRIX VIEW OF THE TOPONIUM- Z^0 INTERFERENCE PATTERN

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A B S T R A C T

We show that the Kühn-Zerwas-Franzini-Gilman-Renard interference pattern of Toponium and Z^0 which occurs when their mass difference is of the order of the Z^0 width can be understood very naturally in the framework of S matrix theory by imposing analyticity, and unitarity requirements together with smoothness assumptions.

CERN-TH.4138/85

March 1985

Recently Kühn and Zerwas and, also Gilman and Franzini¹⁾ have shown, by an explicit calculation, the existence of a remarkable Toponium- Z^0 interference pattern in e^+e^- collisions if the Toponium- Z^0 mass difference is less than or of the order of the Z^0 width, i.e., ~ 3 GeV. In particular they showed that in the case of exact degeneracy of one of the Toponium levels with the Z^0 one observes a dip at this particular energy. For most people this came as a surprise, though a particular case of this phenomenon was already exhibited in the article of Renard²⁾.

The calculation necessitates working with Feynman graphs for unstable particles, a thing for which some people feel somewhat uneasy especially if imaginary contributions are dominant in propagator denominators. In this paper I want to show, after making a few minor simplifying assumptions, that this kind of phenomena can be understood from a pure S matrix point of view.

We shall look at an n channel T matrix, simulating the reactions

$$f^+ f^- \rightarrow f^+ f^-,$$

where f stands for any kind of fermion, lepton or quark. This means that we forget about three gluons and single-quark decay final states (which contribute little to the total width if $M_{t\bar{t}}$ is very close to M_{Z^0}) and treat quarks as real particles like every QCD-ist does.

We also assume that the T matrix is factorizable:

$$T_{ij} = \alpha_i \alpha_j t(E). \quad (1)$$

This assumption holds in the physical problem considered only if the photon intermediate state is negligible and if all couplings of fermions go through the Z^0 . This is the case only if Toponium is very near the Z^0 (less than 4 GeV away). Otherwise it is possible to make a more complicated treatment.

If the T matrix is factorizable it is possible to reduce the problem to a one-channel problem. Without loss of generality we can assume

$$\sum_{i=1}^n |\alpha_i|^2 = 1 \quad (2)$$

The unitarity condition reads

$$\text{Im } T_{ij} = \sum_k T_{ik}^* T_{kj} \quad (3)$$

i.e., substituting (1), and dividing by $\alpha_i \alpha_j$:

$$\text{Im } t = |t|^2, \quad (4)$$

which is the one-channel unitarity condition. Therefore

$$t(E) = \frac{S(E) - 1}{2i}$$

with $|S(E)| = 1$. (5)

The requirement of analyticity is that $S(E)$ has poles in the lower half complex plane corresponding to the N Toponium bound states and the Z^0 . Notice that these poles are the true poles, i.e., after the mixing effects of Toponium with Z^0 have been taken into account.

So we have

$$S(E) = \prod_{i=1}^N \left(\frac{E - E_i - i\gamma_i}{E - E_i + i\gamma_i} \right) \left(\frac{E - M_Z - i\Gamma}{E - M_Z + i\Gamma} \right) \cdot f(E)$$

with $|f(E)| = 1$. (6)

The cross-section is proportional to

$$1 - \text{Re}(S(E)) \quad (7)$$

However, we want the cross-section to be small for $E - \inf(E_1, M_Z) \ll \Gamma$ and for $E - \sup(E_N, M_Z) \gg \Gamma$. This forces $f(E)$ to be very close to unity in these regions, and also, because it is smooth, in the resonance region. So we end up with

$$S(E) \simeq \prod_{i=1}^N \left(\frac{E - E_i - i\gamma_i}{E - E_i + i\gamma_i} \right) \left(\frac{E - M_Z - i\Gamma}{E - M_Z + i\Gamma} \right) \quad (8)$$

Now we consider the following case which corresponds to the one treated by Kühn and Zerwas¹⁾ and Renard²⁾:

1) All γ_i 's are very small with respect to energy differences. More specifically

$$\forall j, \sum_{i \neq j} \frac{\gamma_i}{|E_i - E_j|} \ll 1. \quad (9)$$

This assumption is always satisfied, the worst case being³⁾

$$\frac{\gamma_1}{E_2 - E_1} \lesssim \frac{1}{50},$$

if the Toponium mass coincides with the Z^0 mass.

2) $E_1 - M_Z$ and $M_Z - E_N$ are of the same order of magnitude or less than Γ , the total width of the Z^0 . If one of the two properties hold, the second holds since $E_{11} - E_1$ is of the order of 2 GeV³⁾.

Let us assume that

$$E_1, E_2, \dots, E_p < M_Z < E_{p+1}, \dots, E_N. \quad (10)$$

It is then very easy to locate the zeros of T matrix by following the phase variation of $S(E)$ when E goes from $-\infty$ to $+\infty$. The zeros of T correspond to $\text{Arg}(S(E)) = 2n\pi$ (see Fig. 1). When E goes through E_i the phase of $S(E)$ increases of π over an interval of the order of magnitude of γ_i . Closer examination shows that there is a zero of T close to the right of $E_1, E_2, \dots, E_p$ and close to the left of $E_{p+1}, E_{p+2}, \dots$.

To demonstrate this it is in fact sufficient to consider a two-pole S matrix. Indeed, because of assumption (9) the S matrix in the neighbourhood of E_i , or more specifically for

$$\frac{E_{i-1} + E_i}{2} < E < \frac{E_i + E_{i+1}}{2}$$

is well approximated by

$$S = \left(\frac{E - E_i - i\gamma_i}{E - E_i + i\gamma_i} \right) \left(\frac{E - M_Z - i\Gamma}{E - M_Z + i\Gamma} \right)$$

Then looking at Fig. 2, we can construct E_1^0 such that $t(E) = 0$ and E_1^M such that $t(E) = 1$ (i.e., $\text{Arg}S = (2n+1)\pi$). In the approximation defined by (9) and if $E_1 \neq M_Z$ we have:

$$E_i^0 \cong E_i + \frac{\gamma_i}{\Gamma} (M_Z - E_i) \quad (11)$$

$$E_i^M \cong E_i - \frac{\gamma_i \Gamma}{M_Z - E_i} \quad (12)$$

Therefore the magnitude of t drops very rapidly in an interval

$$\gamma_i \left[\frac{M_Z - E_i}{\Gamma} + \frac{\Gamma}{M_Z - E_i} \right]. \quad (13)$$

In passing notice that Eqs. (11) and (12) show that the drawing in Ref. 2) is not completely correct.

We must notice that the peak at E_1^0 and the dip at E_1^M are extremely asymmetric. For instance, if we take $(M_Z - E_1)/\Gamma = 1$, we have, in the approximation

$\gamma_i \ll \Gamma$:

$$\begin{cases} |t(E_i - 10\gamma_i)| = 0.77 \\ |t(E_i - 2\gamma_i)| = 0.95 \\ |t(E_i - \gamma_i)| = 1 \\ |t(E_i)| = 0.71 \end{cases} \quad \begin{cases} |t(E_i)| = 0.71 \\ |t(E_i + \gamma_i)| = 0 \\ |t(E_i + 2\gamma_i)| = 0.32 \\ |t(E_i + 10\gamma_i)| = 0.63 \end{cases}$$

This asymmetry is essential because it means that after smearing by the energy resolution of the machine as was done by Kühn and Zerwas¹⁾ and by Kühn⁴⁾, the maximum and the dip will move apart, in addition to being less marked.

In the special case of $E_1 = M_Z$, Eq. (11) still applies but (12) makes no sense because one cannot neglect the phase difference of $M_Z + i\Gamma - E_i$ and $M_Z + i\Gamma - E_i^M$.

We get (see Fig. 2b)

$$E_i^0 = E_i = M_Z$$

$$E_i^M = M_Z \pm \sqrt{\gamma_i \Gamma}$$

Therefore we have a dip between two maxima. Again these peaks have a very asymmetric structure. We have

$$|t(M_z)| = 0$$

$$|t(M_z \pm \sqrt{\gamma_i \Gamma})| = 1$$

$$|t(M_z \pm 2\sqrt{\gamma_i \Gamma})| = 1 - \frac{9}{8} \cdot \gamma_i / \Gamma.$$

However, here at least in the case of the ground state of Toponium, where $\gamma_1 \approx 10$ MeV, the energy smearing of the order of 30 to 60 MeV will only make the dip less marked since $\sqrt{\gamma_1 \Gamma} \sim 170$ MeV.

In Figs. 3 and 4 we illustrate two situations with only three Toponium poles (for simplicity).

ACKNOWLEDGEMENTS

The author acknowledges discussions with W. Bartel, W. Buchmüller, M. Jacob, H. Kühn and P. Zerwas.

REFERENCES

- 1) J.H. Kühn and P.M. Zerwas - CERN Preprint TH. 4089 (1985);
Frederik Gilman and Paula Franzini - SLAC Pub 3541 (1985).
- 2) F.M. Renard - Z. für Physik C, Particles and Fields, 1 (1979) 225.
- 3) See, for instance: A. Martin - Toponium Physics, CERN Preprint TH. 4060 (1984) to appear in the Proceedings of the 22nd Course of the International School of Subnuclear Physics, Erice (1984).
- 4) J.H. Kühn - Contribution to the LEP Working group on Toponium, to appear in W. Buchmüller's summary.

FIGURE CAPTIONS

Fig. 1 The poles and the zeros of the S matrix.

Fig. 2 The two-pole case:

- a) $E_1 = M_Z$ (the scale is not respected; the Z^0 would be much further away);
- b) $E_1 = M_Z$.

Fig. 3 $|t|^2$ for the case $E_2 = M_Z$ with three poles.

Fig. 4 $|t|^2$ for a case $E_1 > M_Z$, with three poles.

Fig. 1

Fig. 2

Fig. 3

Fig. 4