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K^+N SPIN-ORBIT INTERACTION IN THE
CONSTITUENT QUARK MODEL

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ABSTRACT

We calculate P-wave phase shifts of the KN -system in the framework of the non-relativistic constituent quark model. We show that the symmetric spin-orbit potential from gluon exchange between quarks leads to KN spin-orbit potentials for the $I = 0$ and $I = 1$ states with the correct sign and magnitude. We also discuss the spin-orbit contributions from the naive confinement potential which give the wrong behaviour for the hadron-hadron spin-orbit potential.

1. - INTRODUCTION

The advent of the renormalizable non-Abelian colour gauge field theory named quantum chromodynamics (QCD) has ushered in a new approach to the understanding of physics of strong interactions in the light of structures more elementary than hadrons. All hadrons are thus bound states of a quark and an antiquark (meson) or of three quarks (baryon) appropriately picked from among the eighteen quark species with the two quark interaction being governed dynamically by the colour rather than the flavour degrees of freedom. The quantum numbers of hadrons uniquely fix colour, spin and isospin parts of their wave functions. Baryon and meson spectroscopy substantiates this so-called constituent quark model^{1),2)} where all subtleties and complications of non-perturbative QCD are tacitly tucked away in the constituent quark mass and a confining potential. The study of hadron scattering in terms of QCD residual forces is complementary to hadron spectroscopy.

The present paper investigates K^+N -scattering. The situation is very much favoured in K^+N -scattering to study QCD forces, since for kaonic interactions, long range interactions are absent or weak, namely:

- a) one-pion exchange is forbidden because two pseudoscalars cannot couple to a third one;
- b) two-pion exchange is suppressed because the $KK^*\pi$ coupling is smaller than the $N\Delta\pi$ coupling with K^* having half the width of Δ in spite of the larger phase space in $K^* \rightarrow K\pi$ compared to $\Delta \rightarrow N\pi$. Also the t-channel dispersion integral of $\bar{K}K \rightarrow \pi\pi$ times $\pi\pi \rightarrow \bar{N}N$ amplitudes may vary in sign as a function of t in contrast to NN scattering.

The K^+ induced scattering has some additional advantages over the K^- case, namely, the K^+ interacting with a nucleon resembles the exotic NN system inasmuch as no annihilation of valence quarks can occur in the direct channel, the only antiquark being a strange one. This avoids difficulties involved in handling annihilation within the QCD framework. In Ref. 3) it was shown that there is a pronounced channel-isospin selectivity for the K^+N system. The hard core repulsion for $I = 1$ is due mostly to antisymmetry of the many-quark wave function (quark rearrangement) whereas the $I = 0$ channel is weakly repulsive originating solely from chromomagnetic interactions (one quark - one gluon exchange only). Thus, different isospins exhibit complementary features in the KN cluster model enabling an investigation of the relative importance of QCD hyperfine interaction on the one hand and quark antisymmetrization on the other.

It was found³⁾ that if one started with an interaction potential composed of confinement, Coulomb, contact spin-spin and Darwin terms, s-wave phase shifts for both channels came out repulsive, in accord with experiment, with the $I = 1$ phase shifts more repulsive than the $I = 0$ ones. For a reasonable choice of various parameters, the agreement between theory and experiment was excellent for $I = 0$. The s-wave calculation thus indicated that QCD quark-gluon exchange describes the major features of this channel without having to accommodate 2π -exchange. In contrast, the fact that the $I = 1$ s-wave phase shifts came out too negative was ascribed by the authors to having overestimated the repulsive shell effect associated with quark exchange. Extending their calculations to higher partial waves the authors found that the d-wave phase shifts came out with the right sign, albeit with very small magnitude and especially for $I = 0$ too small. Worse still, neglect of spin-orbit (SO) coupling in QCD residual interaction completely failed to account for the p-wave phase shifts with even the sign coming out wrong.

In the present work, a one-channel p-wave formulation is set up in Section 2 taking the SO part of one gluon exchange (OGE) along with terms mentioned earlier to see whether $J = 1/2$ and $3/2$ phase shifts for K^+N scattering can be explained by this additional term as conjectured out of phenomenological considerations in Ref. 4). In that work an SO interaction, additive in spin and isospin of the exchanged light quark explained the systematics of baryon-nucleus SO potential and of p-wave KN scattering on the same footing. For $I = 1$ the nuclear type SO potential with attraction for $J = \ell + 1/2$ was obtained whereas $I = 0$ gave the opposite sign. This additive quark model was first introduced in Ref. 5) where it was argued that the nuclear SO potential may arise largely from the short-range part of the NN interaction and may thus be calculable in terms of quark-quark SO exchange kernels. The baryon-nucleus SO potential was then obtained by adding the individual SO terms of each light exchanged quark for each nucleon in the nucleus. This additive quark model has explained the vanishing Λ -nucleus spin-orbit potential and made predictions for the Σ -nucleus spin-orbit potential. In these phenomenological models certain exchange diagrams are missing. Part of the motivation of this paper is to investigate the importance of these contributions.

The generator co-ordinate method (GCM)^{6),7)} is a useful approximation for treating K^+N scattering starting from quark-quark forces. In Section 3, GCM kernels for the many quark system are calculated from the SO part of the OGE force. We make a local approximation to these kernels obtaining spin-orbit potentials which can be used in a Schrödinger equation. A criticism applies to

all constituent quark models. It may not be possible to calculate the hadronic potential from a sum of two-body quark-quark forces. For instance, in the colour dielectric model⁸⁾ or the soliton bag model⁹⁾, the confining soliton field depends on the total quark density. The dielectric field also modifies the one-gluon exchange potential. In the constituent quark model a more realistic confinement interaction has been proposed in Ref. 10). The present paper does not take into account these complexities. Following the philosophy of earlier work on the NN and KN interaction, it uses a local two-body confinement interaction. Previously, it has been considered as an advantage that this confinement mechanism does not influence the scattering phase shifts. For the p-wave interaction this is no longer true if one includes relativistic corrections from confinement.

Already in spectroscopy the SO-splittings of p-wave baryon states are rather obscure. A moderately good description is possible with symmetric $(\vec{\sigma}_1 + \vec{\sigma}_2)$ SO terms from one gluon exchange and from the confinement potential. In this approach the two contributions cancel almost totally, and therefore may be left out altogether¹¹⁾. Experimentally, the splitting of the $\Lambda_{3/2}$ (1520 MeV) and $\Lambda_{1/2}$ (1405 MeV) then remains unexplained. Theoretically, there seems to be no reason to exclude the antisymmetric $(\vec{\sigma}_1 - \vec{\sigma}_2)$ SO terms¹²⁾. These terms add for the baryon spectra and give phenomenologically unacceptable large spin-orbit splittings. We will therefore only include the symmetric terms whereby we give the relativistic SO correction from confinement only for completeness. As mentioned above a two-body confinement interaction with strong effects on the scattering dynamics has to be taken with caution.

For the p-wave baryon-baryon interaction, Warke and Shanker¹³⁾ did pioneering work with a detailed treatment of tensor and SO terms. More recently, Suzuki and Hecht¹⁴⁾ and Morimatsu, Ohta, Shimizu and Yazaki¹⁵⁾ have calculated spin-orbit baryon-baryon interactions in the quark model. Our paper follows more the lines of Ref. 15) but we go beyond both papers in using wave functions which include SU(3) symmetry breaking due to the heavier strange quark mass. This effect has been shown to be important in baryon spectroscopy¹⁶⁾ in order to understand the low position of the Λ (1405 MeV).

Our paper is outlined as follows. Section 2 describes the Hamiltonian of constituent quarks focusing on the spin-orbit parts of the quark-quark interaction. In this section also the cluster wave functions are defined. In Section 3, we calculate the orbital part of the generator co-ordinate kernels from the

quark-quark spin-orbit interaction. Section 4 separately discusses the spin-isospin part of the interaction kernels. In Section 5 we make a local approximation to the cluster-cluster kernels that can be used in a Schrödinger equation. We present and discuss the final results for the KN P-wave phase shifts in Section 6.

2. - HAMILTONIAN AND CLUSTER WAVE FUNCTIONS

The effective Hamiltonian in the constituent quark model for K^+N scattering is

$$\mathcal{H} = \sum_{i=1}^4 \frac{\vec{p}_i^2}{2m_q} + \frac{\vec{p}_5^2}{2m_s} + \sum_{i < j} (V_{ij}^{QCD} + V_{ij}^{conf}) \quad (2.1)$$

The momenta $\vec{p}_i$ and positions $\vec{x}_i$ of the light quarks are numbered from 1 to 4; the strange antiquark carries the number 5. The masses of the light quarks are $m_q \approx 0.3$ GeV and the heavier strange quark mass is

$$m_s = \kappa m_q \approx 0.5 \text{ GeV} \quad (2.2)$$

The Hamiltonian contains the confining potential V_{ij}^{conf} with quadratic confinement and the one-gluon exchange terms like Coulomb, spin-spin and Darwin [cf., Ref. 3)]. For this paper we concentrate on the spin-orbit part of the quark-quark interaction with the one-gluon exchange Fermi-Breit reduction giving an SO-term of the following form:

$$\begin{aligned} V_{ij}^{QCD-SO} &= -\frac{1}{4} \alpha_s \vec{\lambda}_i \vec{\lambda}_j \frac{1}{|\vec{r}_{ij}|^3} \cdot \\ &\left\{ \frac{1}{2m_i m_j} \cdot \vec{r}_{ij} \times \vec{p}_{ij} \cdot \left[\vec{\sigma}_i \left(1 + \frac{m_j}{2m_i} \right) + \vec{\sigma}_j \left(1 + \frac{m_i}{2m_j} \right) \right] \right. \\ &\quad \left. - \frac{1}{m_i + m_j} \vec{r}_{ij} \times (\vec{p}_i + \vec{p}_j) \cdot \left(\frac{\vec{\sigma}_i}{m_i} - \frac{\vec{\sigma}_j}{m_j} \right) \right\} \end{aligned} \quad (2.3)$$

with

$$\vec{p}_{ij} = \frac{m_j \vec{p}_i - m_i \vec{p}_j}{m_i + m_j},$$

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

The spin-dependence of the first term is given approximately as

$$\vec{\sigma}_i \left(1 + \frac{m_j}{2m_i}\right) + \vec{\sigma}_j \left(1 + \frac{m_i}{2m_j}\right) \approx \frac{3}{2} (\vec{\sigma}_i + \vec{\sigma}_j) \quad (2.4)$$

The confinement interaction only leads to a spin-orbit term, but to neither spin-spin nor tensor forces, if one assumes that it originates from a scalar exchange in a Bethe-Salpeter equation. The leading term of the confinement potential has the form [cf. Ref. 3)]

$$\begin{aligned} V_{ij}^{conf} &= -\frac{1}{2} D \vec{\lambda}_i \vec{\lambda}_j r_{ij}^2 && \text{for } q\bar{q} \\ &= -\frac{1}{2} D (-\vec{\lambda}_i \vec{\lambda}_j^T) r_{ij}^2 && \text{for } q\bar{q} \end{aligned} \quad (2.5)$$

with $D \approx 0.0135 \text{ GeV}^3 \approx 0.33 \text{ GeV fm}^{-2}$. The relativistic spin-orbit interaction resulting from scalar confinement would have the form

$$\begin{aligned} V_{ij}^{conf-so} &= -\frac{1}{4r_{ij}} \frac{\partial V_{ij}^{conf}}{\partial r_{ij}} \left\{ (\vec{r}_{ij} \times \vec{p}_{ij}) \cdot \left(\frac{\vec{\sigma}_i}{m_i^2} + \frac{\vec{\sigma}_j}{m_j^2} \right) \right. \\ &\quad \left. + (\vec{r}_{ij} \times (\vec{p}_i + \vec{p}_j)) \frac{1}{m_i + m_j} \left(\frac{\vec{\sigma}_i}{m_i} - \frac{\vec{\sigma}_j}{m_j} \right) \right\}^{(2,6)} \end{aligned}$$

It can be easily seen that the spin symmetric and antisymmetric terms have different behaviour in the one-gluon exchange potential and in the confinement potential. The symmetric terms have opposite signs which may be connected to the observed small spin-orbit splittings in baryon spectroscopy. The common signature of the antisymmetric terms (both positive) leads to a catastrophe because these terms produce large splittings in contradiction with experiment. Therefore their physical meaning can be questioned. Their theoretical derivation is generally acknowledged and well understood in the weak binding limit due to the

work of Close¹²⁾, Osborn¹⁷⁾ and others. It is just an outcome of the Wigner rotation arising when boosting the n-quark c.m. system to the c.m. subsystem of quark i and j.

The effective Hamiltonian of Eq. (2.1) acts on the KN-system represented by their quark cluster wave functions. For the trial GCM wave function we make the variational³⁾ ansatz

$$\psi = A \int c(\vec{R}) \psi_{\vec{R}}^{m_s} d^3\vec{R} \quad (2.7)$$

$$\psi_{\vec{R}}^{m_s} = \phi_{\vec{R}}(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5) \chi_{m_s}^{SIG} \quad (2.8)$$

The total $\psi_{\vec{R}}^{m_s}$ wave function separates in a spatial wave function $\phi_{\vec{R}}$ depending on the five quark co-ordinates and a spin-isospin colour wave function $\chi_{m_s}^{SIG}$ with $m_s (= \pm 1/2)$, being the magnetic projection of nucleon spin. The spatial co-ordinate $\vec{R}$ is to be identified with the generator co-ordinate of the many quark scattering problem. The antisymmetrization operator A is defined as

$$A = 1 - P_{14} - P_{24} - P_{34} \quad (2.9)$$

It exchanges all light quarks (1,2,3) in the nucleon with the light quark (4) in the kaon. Noting that the eigenfunctions of a Hamiltonian with a quadratic coupling potential are harmonic oscillator wave functions, the spatial wave function ϕ is then

$$\phi_{\vec{R}} = \psi_{os}(\vec{x}_1 - \vec{x}) \psi_{os}(\vec{x}_2 - \vec{x}) \psi_{os}(\vec{x}_3 - \vec{x}) \psi_{os}(\vec{x}_4 - \vec{y}) \eta_{os}(\vec{x}_5 - \vec{y}) \quad (2.10)$$

where $\vec{x}_1, \vec{x}_2, \vec{x}_3$ are the positions of the three nucleonic quarks with the Gaussian cluster centered around $\vec{x}$, while $\vec{x}_4$ and $\vec{x}_5$ are the positions of the light quark and the heavy antistrange quark respectively with the Gaussian cluster for the kaon peaked around $\vec{y}$. For the lighter quarks, ψ_{os} is the normalized s-wave shell model type wave function with

$$\psi_{os}(\vec{x}_i - \vec{x}) = \left(\frac{1}{\pi b^2} \right)^{3/4} \exp \left(- \frac{1}{2b^2} (\vec{x}_i - \vec{x})^2 \right) \quad (2.11)$$

and similarly η for the heavy quarks

$$\eta_{os}(\vec{x}_5 - \vec{y}) = \left(\frac{\kappa}{\pi b^2} \right)^{3/4} \exp \left(- \frac{\kappa}{2b^2} (\vec{x}_5 - \vec{y})^2 \right) \quad (2.12)$$

where $\kappa = m_s/m_q$ and b is the oscillator parameter obtained from the first radial excitation of the baryon^{*}). The centres of the two hadrons

$$\vec{x} = - \frac{1+\kappa}{4+\kappa} \vec{R}, \quad \vec{y} = \frac{3}{4+\kappa} \vec{R}$$

show the inherent asymmetry of the K-N problem in contrast to the NN case owing to the massive strange quark, which is more localized at the K cluster because $\kappa = 1.67$, and hence does not participate as much in peripheral partial waves. Besides, due to the mass ratio $1/\kappa^2 = (m_q/m_s)^2$, the relativistic coupling of the antistrange quark is reduced. These two arguments notwithstanding, the role of the spectator strange quark is found to be very important in KN scattering even though it is inhibited from partaking in exchange, because within the GCM framework diagrams involving OGE between the strange quark and a nucleonic quark also contribute.

3. - GCM KERNELS FROM THE QUARK-QUARK SPIN-ORBIT INTERACTION

The spin-orbit potential kernel is obtained from the matrix elements of the two spin-orbit interactions [Eqs. (2.3) and (2.6)] with the cluster wave function

$$V^{SO}(\vec{R}', \vec{R}) = \int \prod_{i=1}^5 d\vec{x}_i \Psi_{\vec{R}'} \left(\sum_{ij} V_{ij}^{QCD-SO} + \sum_{ij} V_{ij}^{conf-SO} \right) A \Psi_{\vec{R}} \quad (3.1)$$

Because of the symmetries of the Hamiltonian and the cluster wave function [Eqs. (2.1) and (2.10)], the antisymmetrization operator reduces to

$$A \Psi_{\vec{R}} = (1 - 3P_{34}) \Psi_{\vec{R}} \quad (3.2)$$

In general, there are direct and exchange parts for each kernel. For the spin-orbit interaction, the direct parts of the kernel vanish either because of colour neutrality of the clusters or because of the s-state cluster wave functions. Therefore

^{*}) In Ref. 3) $\Omega = 1/2b^2$.

$$V^{so}(\vec{R}', \vec{R}) = - V_{ex}^{QCD-so}(\vec{R}', \vec{R}) - V_{ex}^{conf-so}(\vec{R}', \vec{R}) \quad (3.3)$$

with

$$V_{ex}^{QCD-so}(\vec{R}', \vec{R}) = \int \prod_{i=1}^5 d\vec{x}_i \bar{\Psi}_{\vec{R}'} \sum_{i < j} V_{ij}^{QCD-so}(3P_{34}) \Psi_{\vec{R}} \quad (3.4)$$

and

$$V_{ex}^{conf-so}(\vec{R}', \vec{R}) = \int \prod_{i=1}^5 d\vec{x}_i \bar{\Psi}_{\vec{R}'} \sum_{i < j} V_{ij}^{conf-so}(3P_{34}) \Psi_{\vec{R}} \quad (3.5)$$

Since there are five quarks in the KN-system, there are $\binom{5}{2} = 10$ possible two-body OGE diagrams. Since quark number two plays the same role as quark number one in the baryon, these ten diagrams actually reduce to seven distinct ones. In Fig. 1 we show the quark-quark interactions contributing to the total interaction kernel $V^{QCD}(\vec{R}, \vec{R}')$ [cf. Ref. 3)]. In line with the philosophy we only keep the symmetric spin-orbit quark-quark forces of Eqs. (2.3) and (2.6). Then we obtain:

$$\begin{aligned} V_{ex}^{QCD-so}(\vec{R}', \vec{R}) = & 3\alpha_s (\Delta(\vec{R}', \vec{R}))^2 \varepsilon(\vec{R}', \vec{R}) \varepsilon^{ex}(\vec{R}', \vec{R}) \varepsilon^{ex}(\vec{R}, \vec{R}') \\ & \cdot \sum_{i=1}^7 C_i^{ex} \cdot n_i \cdot S_i^s \cdot \vec{I}_{i2}(\pm, \vec{p}_i) \cdot \vec{\sigma}_N \end{aligned} \quad (3.6)$$

and

$$\begin{aligned} V_{ex}^{conf-so}(\vec{R}', \vec{R}) = & 3 \cdot D (\Delta(\vec{R}', \vec{R}))^2 \varepsilon(\vec{R}', \vec{R}) \varepsilon^{ex}(\vec{R}', \vec{R}) \varepsilon^{ex}(\vec{R}, \vec{R}') \\ & \cdot \sum_{i=1}^7 C_i^{ex} \cdot n_i \cdot T_i^s \cdot \vec{J}_i(\pm, \vec{p}_i) \cdot \vec{\sigma}_N \end{aligned} \quad (3.7)$$

The exchange overlap functions Δ , ε , ε^{ex} are defined as in Ref. 3):

$$\Delta(\vec{R}', \vec{R}) = \exp \left[-\frac{1}{4b^2} \left(\frac{1+\kappa}{4+\kappa} \right)^2 (\vec{R}' - \vec{R})^2 \right] \quad (3.8)$$

$$\varepsilon(\vec{R}', \vec{R}) = \exp \left[-\frac{\kappa}{4b^2} \left(\frac{3}{4+\kappa} \right)^2 (\vec{R}' - \vec{R})^2 \right] \quad (3.9)$$

and

$$\varepsilon^{\text{ex}}(\vec{R}', \vec{R}) = \exp \left[-\frac{1}{4b^2} \left(\frac{1+\kappa}{4+\kappa} \vec{R}' + \frac{3}{4+\kappa} \vec{R} \right)^2 \right] \quad (3.10)$$

The factors C_i^{ex} give the colour overlap and n_i the multiplicity for each diagram [cf. Ref. 3)]. The calculation of the spin matrix elements S_i^s and T_i^s is summarized in Section 4. The values S_2 , S_4 , T_2 , T_4 are given in Table 1. Only the diagrams 1.2 ($i = 2$) and 1.4 ($i = 4$) shown in Fig. 1 do not vanish because of the orbital integrations. The functions $\vec{I}_i$ and $\vec{J}_i$ are derived by transforming to c.m. and relative co-ordinates of the interacting quarks in the standard way. They have the form

$$\begin{aligned} \vec{I}_{i\lambda}(\pm, \vec{\rho}_i) &= \left(\frac{\lambda}{1+\lambda} \right)^{3/2} \left(\frac{1}{\pi b^2} \right)^{3/2} \int d\vec{\xi} \exp \left[-\frac{\lambda}{1+\lambda} \frac{1}{b^2} \left(\vec{\xi} + \frac{1}{2} \vec{\rho}_i \right)^2 \right] \quad (3.11) \\ &\cdot (\pm) \frac{3}{16} \frac{1}{m^2 b^2} \frac{1}{|\vec{\xi}|^3} i (\vec{\xi} \times \vec{R}) \frac{1}{1+\lambda} \end{aligned}$$

$$J_{ia}(\pm, \vec{p}_i) = \left(\frac{\lambda}{1+\lambda}\right)^{3/2} \left(\frac{1}{\pi b^2}\right)^{3/2} \int d\vec{\xi} \exp \left[-\frac{\lambda}{1+\lambda} \frac{1}{b^2} \left(\vec{\xi} + \frac{1}{2} \vec{p}_i \right)^2 \right] \cdot \begin{pmatrix} + \\ - \end{pmatrix} \left(-\frac{1}{4m^2 b^2} \right) i (\vec{\xi} \times \vec{R}) \frac{\lambda}{1+\lambda} \quad (3.12)$$

There are only the two contributions ($i = 2, 4$) because all the other ones reduce to applying the angular momentum operator $\vec{r}_{ij} \times \vec{p}_{ij}$ on a pair of quarks which are in an s-state cluster. Naturally the same simplification would not follow for the asymmetric spin-orbit terms which depend on $\vec{r}_{ij} \times (\vec{p}_i + \vec{p}_j)$. The evaluation of the integrals in Eqs. (3.11) and (3.12) yields

$$I_{ia}(\pm, \vec{p}_i) = \begin{pmatrix} + \\ - \end{pmatrix} i \vec{R}' \times \vec{R} \frac{3}{4m^2 \lambda} \alpha_{ia} \left[\frac{\exp(\sqrt{\alpha_{ia}} \frac{p_i}{2})}{p_i^3} - \sqrt{\frac{\alpha_{ia}}{\pi}} \frac{\exp(-\frac{\alpha_{ia} \vec{p}_i^2}{4})}{p_i^2} \right] \quad (3.13)$$

$$\alpha_2 = \frac{1}{2b^2} \quad , \quad \alpha_4 = \frac{\lambda}{1+\lambda} \frac{1}{b^2}$$

$$J_{ia}(\pm, \vec{p}_i) = (\pm i) (\vec{R}' \times \vec{R}) \frac{1}{8m^2 b^2} \left(-\frac{\lambda}{1+\lambda} \right) \quad (3.14)$$

Since the expression in brackets of Eq. (3.13) is positive definite, the difference in sign between QCD and confinement parts is conserved on the level of hadron-hadron interactions.

4. - SPIN-ISOSPIN MATRIX ELEMENTS FOR KN

In the previous section the KN quark exchange potential was evaluated in coordinate space. The colour matrix elements were already presented in Ref. 3). This section is devoted to evaluating the spin-isospin part of the SO matrix elements. The techniques used are very similar to the ones developed by Warke and Shanker in their NN work¹³⁾ adapted to the KN-system in the present calculation. The fact that unlike the NN case there is an antistrange quark present here does not affect the evaluation.

A notable simplification is obtained since the kaon has spin zero. Further, as mentioned in the Introduction, we do not consider the coupled channels K^*N , $K^*\Delta$ and $K\Delta$ in this work. In what follows the Wigner-Eckart theorem together with the symmetry of the matrix elements play the major role for the calculation. Thus, the isospin matrix elements are diagonal in the intermediate coupling quantum number of the first two nucleonic quarks 1 and 2, since the particle exchange operator P_{34}^{SI} does not act on either of them. This in turn makes the spin matrix elements diagonal in $\vec{l} \cdot \vec{s}$ even though the spin operator acts on quarks 1 and 2. The rest consists of changing the basis to one where P_{34}^I is diagonal through recoupling with 9j symbols. The Wigner-Eckart theorem then gives

$$\begin{aligned} & \langle \chi^{SI}(m'_s; I, M_I) | \vec{\sigma}_j P_{34}^{SI} | \chi^{SI}(m_s; I, M_I) \rangle \\ &= \langle m'_s | \vec{\sigma}_N | m_s \rangle s_j(I). \end{aligned} \quad (4.1)$$

where $\langle m'_s | \vec{\sigma}_N | m_s \rangle$ are the matrix elements of the nucleon Pauli matrix between states with $m_{s_1} m'_s = \pm 1/2$. By symmetry considerations one can write down four relations between the different s_i 's which are all real

$$s_4(I) + s_5(I) = 0$$

$$s_1(I) = s_2(I)$$

$$s_1(I) + s_2(I) + s_3(I) = \langle \chi_{m_s}^{SI} | P_{34}^{SI} | \chi_{m_s}^{SI} \rangle \quad (4.2)$$

$$s_3(I) = s_4(I)$$

The coefficient $s_3(I)$ is then calculated explicitly

$$s_3(I) = \frac{1}{2} \sum_{\ell=0}^1 \left\{ \begin{matrix} \ell & 1/2 & 1/2 \\ I & 1/2 & 1/2 \end{matrix} \right\} - \frac{2}{3} \left\{ \begin{matrix} 1 & 1/2 & 1/2 \\ I & 1/2 & 1/2 \end{matrix} \right\} \quad (4.3)$$

Using the relations (4.2), one can evaluate the s_j 's for the two isospin channels by using standard tables of 6j symbols. The results are shown in Table 2. As can be seen from the table, the matrix elements $J_4 + J_5 = \langle (\vec{\sigma}_4 + \vec{\sigma}_5) P_{34}^{SI} \rangle$ and $J_3 + J_5 = \langle (\vec{\sigma}_3 + \vec{\sigma}_5) P_{34}^{SI} \rangle$ vanish as a consequence of the zero spin of the K^+ ground state.

5. - LOCAL APPROXIMATION

In cluster model calculations of hadronic forces, the quark-quark interaction manifests itself as an intrinsically non-local interaction which does not render itself to direct physical interpretation. Clearly, to understand the results of Section 3 the non-local quark exchange kernels ought to be converted to equivalent local forms. Several different methods have been used^{14),15)} and here we apply the moment expansion technique already described in Ref. 3) to the spin-orbit parts of the interaction. In the non-local Hill-Wheeler equation

$$\int [\mathcal{H}(\vec{R}', \vec{R}) - EN(\vec{R}', \vec{R})] \Psi(\vec{R}) d^3R = 0 \quad (5.1)$$

we assume the wave function $\psi(\vec{R})$ to vary slowly compared to the intrinsic cluster size. Then we can interpret $\psi(\vec{R})$ as the wave function of relative motion and expand the kernels around $\vec{R}' = \vec{R}$. With the co-ordinates

$$\vec{x} = \frac{1}{2} (\vec{R}' + \vec{R}) \quad , \quad \vec{y} = \vec{R}' - \vec{R} \quad (5.2)$$

the SO kernels have the general form

$$\begin{aligned} V^{LS}(\vec{x}, \vec{y}) &= i \vec{\sigma}_N \cdot (\vec{R}'(\vec{x}, \vec{y}) \times \vec{R}(\vec{x}, \vec{y})) w_1(\vec{x}^2) w_2(\vec{y}^2) \\ &= -i \vec{\sigma}_N \cdot (\vec{x} \times \vec{y}) \cdot w_1(\vec{x}^2) w_2(\vec{y}^2) . \end{aligned} \quad (5.3)$$

Expanding the spin-orbit kernel together with the norm kernel, we obtain

$$V^{LS}(\vec{x}, \vec{y}) = V_0(\vec{x}) \delta(\vec{y}) + V_i^{(1)}(\vec{x}) \frac{d}{dy_i} \delta(\vec{y}) + \dots \quad (5.4)$$

$$N(\vec{x}, \vec{y}) = N_0(\vec{x}) \delta(\vec{y}) + \frac{1}{2} N_{em}^{(2)}(\vec{x}) \frac{d}{dy_e} \frac{d}{dy_m} \delta(\vec{y}) + \dots$$

with the moments

$$V_0(\vec{x}) = \int V(\vec{x}, \vec{y}) d^3y \quad , \quad V_i^{(1)}(\vec{x}) = - \int y_i V(\vec{x}, \vec{y}) d^3y \quad (5.5)$$

$$N_0(\vec{x}) = \int N(\vec{x}, \vec{y}) d^3y \quad , \quad N_{em}^{(2)}(\vec{x}) = \int y_e y_m N(\vec{x}, \vec{y}) d^3y$$

Since the function $w_2(\vec{y})$ [cf. Eq. (5.3)] is an even function of $\vec{y}$, only the odd moments contribute

$$V_i^{(1)}(\vec{x}) = w_1(\vec{x}) i (\vec{\sigma} \times \vec{x})_i \cdot \frac{1}{3} \int \vec{y}^2 w_2(\vec{y}^2) d^3y = i (\vec{\sigma} \times \vec{x})_i w_1(\vec{x}) \cdot \frac{1}{3} \langle \vec{y}^2 \rangle_{w_2} \quad (5.6)$$

To bring the SO potential [Eq. (5.3)] into canonical form we substitute Eq. (5.4) into Eq. (5.1) and obtain the following SO potential to be used in a Schrödinger type equation [cf. Ref. 3)]

$$V^{SO}(\vec{R}) = - \frac{w_1(\vec{R})}{\tilde{N}_0(R)} \cdot \vec{\sigma} \cdot \vec{L} \frac{1}{3} \langle \vec{y}^2 \rangle w_2 \quad (5.7)$$

with $\vec{L} = \vec{R} \times \vec{P}$ and the norm kernel

$$\tilde{N}_0 = N_0 + \frac{1}{8} \frac{d}{dx_e} \frac{d}{dx_m} N_{em}^{(2)}$$

Explicitly the modified overlap function $\tilde{N}_0$ is given as

$$\tilde{N}_0(R) = (4\pi b^2)^{3/2} \left\{ \frac{1}{\alpha^{3/2}} - \frac{1}{3} \delta_{I1} e^{-R^2/2b^2} \left[\frac{1}{\beta^{3/2}} + \frac{1}{\beta^{1/2}} \left(\frac{R^2}{4b^2} - \frac{3}{4} \right) \right] \right\} \quad (5.8)$$

where

$$\alpha = \frac{3(1+\kappa)}{4+\kappa}, \quad \beta = \frac{2+5\kappa}{2(4+\kappa)}$$

Following Eq. (5.7), we evaluate the moments of the SO kernels given in Eqs. (3.3), (3.6), (3.7), (3.12) and (3.13) and obtain the local spin-orbit potential from quark-gluon exchange and confinement

$$V^{SO}(\vec{R}) = \vec{s} \cdot \vec{\ell} \left[V_{QCD}(\vec{R}) + V_{conf}(\vec{R}) \right] \quad (5.9)$$

with

$$\begin{aligned}
 V_{QCD}(\vec{R}) = & e^{-R^2/2b^2} \cdot \frac{\alpha_s}{2m^2 b^2 \tilde{N}_0(R)} \cdot \\
 & \left\{ -C_2 S_2 n_2 \cdot \frac{9}{2} t_1 + \right. \\
 & + C_4 S_4 n_4 \frac{9}{1+\kappa} \left[\frac{\text{erf}\left(\sqrt{\frac{\kappa}{1+\kappa}} \frac{R}{b}\right)}{8R^3} - \frac{\sqrt{\frac{\kappa}{(1+\kappa)\pi b^2}} \cdot e^{-\left(\frac{\kappa}{1+\kappa}\right) R^2/b^2}}{4R^2} \right] t_2 \left. \right\}
 \end{aligned} \quad (5.10)$$

$$\begin{aligned}
 V_{conf}(\vec{R}) = & e^{-R^2/2b^2} \cdot \frac{D}{2m^2 b^2 \tilde{N}_0(R)} \cdot \\
 & \left\{ C_2 T_2 n_2 \cdot \frac{3}{4} - C_4 T_4 n_4 \frac{3\kappa}{2(1+\kappa)} \right\} t_2.
 \end{aligned} \quad (5.11)$$

The coefficients t_1 and t_2 are the second moments of the y -dependent functions which are contained in the kernels of Eqs. (3.6) and (3.7)

$$\begin{aligned}
 t_1 = & \frac{1}{3} \int \vec{y}^2 \frac{d^3 \vec{y}}{|\vec{y}|^3} \left[\text{erf}\left(\frac{|\vec{y}|}{2b\sqrt{2}}\right) - |\vec{y}| \sqrt{\frac{1}{2\pi b^2}} e^{-\vec{y}^2/8b^2} \right] e^{-\beta \vec{y}^2/4b^2} \\
 = & \frac{2\sqrt{2} b^2 \pi}{3 \beta \alpha^{3/2}}.
 \end{aligned} \quad (5.12)$$

$$t_2 = \frac{1}{3} \int \vec{y}^2 e^{-\beta \vec{y}^2/4b^2} d^3 \vec{y} = \frac{\pi^{3/2} (4b^2)^{5/2}}{2 \beta^{5/2}}. \quad (5.13)$$

The SO potentials are plotted in Figs. 2 and 3. Let us first discuss the QCD part of the potential. It consists of two parts from diagram (34) and diagram (25) (cf. Fig. 2). The long range part is clearly dominated by the (34) diagram considered in earlier work^{4),5)}. At short distances $r \leq 0.5$ fm, the contribution of the strange quark interaction is at least as large as the (34) diagram. Its contribution has the same sign as that of the light quark. Using the expectation

values of $\vec{\ell} \cdot \vec{s}$ in $j = 3/2$ and $j = 1/2$ states,

$$\begin{aligned} \langle \vec{\ell} \cdot \vec{s} \rangle_{j=1/2} &= -1 \\ \langle \vec{\ell} \cdot \vec{s} \rangle_{j=3/2} &= +1/2 \end{aligned} \tag{5.14}$$

one sees that for $j = 3/2$ the $I = 0$ potential is repulsive and the $I = 1$ potential attractive in agreement with the sign of the phase shifts. The comparison for $j = 1/2$ shows the same agreement of signs. If one considers the effect of confinement in the naive model presented in Section 2, the resulting spin-orbit contributions would be large and generally would give potentials with opposite sign to the QCD part (Fig. 3). If they are added they would totally reverse the behaviour of the KN-SO potential. A similar behaviour has already been observed in calculations of the nucleon-nucleon spin-orbit potential. The naive phenomenological SO potential from confinement weakens or destroys the QCD contribution. Since the complex confinement interaction is only crudely described in the constituent quark model, we doubt very much the SO contribution resulting from confinement. It would be very interesting to have a better description for confinement which fits light hadron spectroscopy and can be applied to hadron-hadron collisions. Subsequently we will only discuss the QCD part of the spin-orbit potential.

6. - RESULTS AND DISCUSSION

In order to compare the derived SO potentials with the experimental data, we calculate the K^+N phase shifts. As indicated in the Introduction, the K^+N system does not allow long range one-pion exchange. For s-wave scattering good agreement between the QCD calculation and experiment was achieved in Ref. 3). Here we use the same parameters, namely $m_q = 0.3$ GeV and $\kappa = m_s/m_q = 1.67$, $\alpha_s = 0.66$ and $1/2b^2 = 2.3 \text{ fm}^{-2}$. The resulting p-wave phase shifts $\delta_{I,2J+1}$ for $I = 0$ and $I = 1$ KN scattering from the spin-orbit QCD potential (5.1) are shown in Fig. 4. They are compared with the data of Refs. 18) and 19). Note the especially large scale for the $I = 0$, $J = 1/2$ channel. The sign and magnitude of the phase shifts are well reproduced for all channels besides the very attractive $I = 0$, $J = 1/2$ channel. If we include the other quark-gluon exchange contributions determined in Ref. 3), namely the Coulomb, spin-spin and Darwin term in the local approximation [cf. Fig. 4, Ref. 3)], the theoretical phases for $I = 0$ are not visibly

changed, whereas the $I = 1$ phases both become repulsive. The theoretical δ_{13} phase would be overlapping with the δ_{11} phase of Fig. 4, whereas δ_{11} would be approximately twice as big as in Fig. 4. We already have seen such a tendency when we calculated³⁾ linear combination of amplitudes in such a way that the S0 potentials drop out. For the peripheral waves this disagreement of the central potential points to additional long-range forces not included in the present calculation.

The major discrepancy is in the $I = 0, J = \frac{1}{2}$ channel (δ_{01}) (cf. Fig. 4). For these quantum numbers Refs. 20), 21), predict a large low energy attraction from the inelastic K^*N -channel. We therefore expect that its inclusion will also improve our calculation here. Quantitatively this depends on how much the K^* is coupled to the K - via short range quark gluon exchange or via long range π -exchange. The latter contribution would still be missing in a calculation with the Hamiltonian of the constituent quark model. Note also that the single channel analysis in Ref. 22) obtains a strong attraction in the P_{01} wave principally coming from ρ -exchange.

In conclusion, we would like to mention that, like the K^*N -system, spin-orbit effects in the baryon-baryon interaction are well derived^{14), 15), 23)} from the QCD spin symmetric residual quark-quark interaction. A serious problem, however, comes from the naive confinement S0 potential which reverses the signs of the theoretical spin-orbit differences. This same Thomas spin-orbit term of the naive confinement potential is successful in suppressing spin-orbit splittings in baryon spectra. It remains to see whether more realistic models of confinement can give good baryon p-wave spectra and at the same time eliminate the unwanted hadron-hadron S0 contributions. Until then spin-orbit phenomena in nuclear and hypernuclear physics⁵⁾ only give an indication but not a clear proof of the importance of quark degrees of freedom in low energy hadron physics.

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Table 1

The spin-isospin operators S_i and T_i , colour operators C_i , multiplicities n_i of the diagrams 1.2 and 1.4 with quark exchange for $I = 0$ and $I = 1$. For the definitions of C_i , see Ref. 3). The integrals have the form $\tilde{I}_{i\lambda}(\pm, \vec{\rho}_i)$ and $\tilde{J}_{i\lambda}(\pm, \vec{\rho}_i)$ for the QCD and confinement two-body SO interaction. The co-ordinates $\vec{\rho}_i$ and signatures $\pm$ are given in the last column. All the other integrals for the spin-orbit interactions $i = 1, 3, 5, 6, 7$ (Fig. 1) corresponding to quark-quark interactions (45), (24), (35), (12) and (23) are zero.

Spin-Isospin Operator		$I = 0$ $\langle m_s \vec{\sigma}_N m_s' \rangle$	$I = 1$ $\langle m_s \vec{\sigma}_N m_s' \rangle$	n_i	C_i	Integrals
S_2^S	$(\vec{\sigma}_3 + \vec{\sigma}_4) P_{34}^{SI}$	-2/3	+4/9	1	16/9	$\tilde{I}_1(+, \vec{R}' - \vec{R})$
T_2^S	$(\vec{\sigma}_3 + \vec{\sigma}_4) P_{34}^{SI}$	-2/3	+4/9			$\tilde{J}_1(+, \vec{R}' - \vec{R})$
S_4^S	$(\vec{\sigma}_2 + \vec{\sigma}_5) P_{34}^{SI}$	1/2	-1/6	2	8/9	$\tilde{I}_K(-, \vec{R}' + \vec{R})$
T_4^S	$(\vec{\sigma}_2 + \vec{\sigma}_5 / \kappa^2) P_{34}^{SI}$	0.28	-0.02			$\tilde{J}_K(-, \vec{R}' + \vec{R})$

Table 2

Spin-matrix elements $s_i(I) = \langle \chi^{SI} | \vec{\sigma}_j P_{34}^{SI} | \chi^{SI} \rangle / \langle m_s' | \vec{\sigma}_N | m_s \rangle$ for the two isospin channels $I = 0$ and $I = 1$.

	$J_1(I)$	$J_2(I)$	$J_3(I)$	$J_4(I)$	$J_5(I)$
$I = 0$	1/6	1/6	-1/3	-1/3	+1/3
$I = 1$	1/18	1/18	2/9	2/9	-2/9

FIGURE CAPTIONS

- Fig. 1: Graphical representation of the contribution to the interaction kernel $V_{\text{ex}}^{\text{QCD}}(\vec{R}', \vec{R})$ [Eq. (3.6)]. Solid lines denote fermions, dotted lines two-body quark-quark interactions.
- Fig. 2: Local SO potentials $V_{\text{QCD}}(r)$ from quark-gluon exchange [Eq. (5.10)] for the $I = 0$ and $I = 1$ KN-system. The potentials V_{34} give the part of the potential due to the interaction of the light quarks (3) and (4). V_{QCD} is the sum of V_{34} and V_{25} .
- Fig. 3: Local SO potentials $V_{\text{conf}}(r)$ from the confinement [Eq. (5.1)] potential for the $I = 0$ and $I = 1$ KN-system.
- Fig. 4: K^+N p-wave phase shifts $\delta_{I,2J}$ from the QCD spin-orbit potential [Eq. (5.10)].

Fig. 1

Fig. 2

Fig. 3

Fig. 4