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NEW ISR AND SPS COLLIDER MULTIPLICITY DATA
AND THE GOLOKHAVSTOV GENERALIZATION OF THE KNO SCALING

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ABSTRACT

The generalization of KNO scaling proposed by Golokhvastov (KNO-G scaling) is tested using pp multiplicity data, in particular results of the new high precision ISR measurements.

Since the data obey KNO-G scaling over the full energy range $\sqrt{s} = 2.51\text{--}62.2$ GeV with the scaling function $\psi(z)$, having only one free parameter, the superiority of the KNO-G over the standard approach is clearly demonstrated.

The extrapolation within KNO-G scaling to the SPS Collider energy range and a comparison with the recent UA5 multiplicity results is presented.

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1 - INTRODUCTION

Since its introduction in 1972 [1] the KNO scaling is used as a useful phenomenological framework to describe and predict the multiplicity distributions at a given energy.

The original formulation of KNO scaling (we call it hereafter 'standard KNO' or simply 'KNO') states that at asymptotic energies the following relations hold:

$$P_n = \frac{1}{\langle n \rangle} \psi\left(\frac{n}{\langle n \rangle}\right), \quad (\langle n \rangle = \sum_n n P_n) \quad (1.1)$$

where n is the number of secondaries produced in an inelastic interactions, P_n is the probability of producing n secondaries in a final state, and $\psi(z)$ is an energy-independent function normalized by the following equations:

$$\int_0^\infty \psi(z) dz = \int_0^\infty z \psi(z) dz = 1 \quad (z = \frac{n}{\langle n \rangle}). \quad (1.2)$$

The application of the KNO scaling to the existing pp data [2,3] showed that the data obey approximately the KNO scaling even at small energies, where most of the approximations used for its derivation [1] simply cannot work. Nevertheless, the agreement between the data and KNO predictions was never perfect. In fact, the shape of the scaling function $\psi(z)$ has to be changed considerably with the energy in order to obtain the best fit to the data [3]. A great deal of effort was undertaken to generalize KNO scaling or to find a way of enabling description of the multiplicity data in a full energy range by a single scaling function. For a recent review on the standard approach to the KNO scaling the reader is referred to ref. [4].

A rather successful generalization of the KNO scaling in a non-standard way was proposed in 1977 by Golokhvastov [5] (we call it hereafter KNO-G). KNO-G scaling needs only one assumption, namely that there exists a probability distribution $P(\tilde{n})$ and that it is a continuous function of a continuous parameter $\tilde{n}$. The discrete probability P_n measured in an experiment is then defined in the following way:

$$P_n = \int_n^{n+1} P(\tilde{n}) d\tilde{n} = \frac{(n+1)/\langle \tilde{n} \rangle}{n/\langle \tilde{n} \rangle} \int_0^\infty \psi(\tilde{z}) d\tilde{z}, \quad P(\tilde{n}) = \frac{1}{\langle \tilde{n} \rangle} \psi\left(\frac{\tilde{n}}{\langle \tilde{n} \rangle}\right),$$

$$(\tilde{z} = \frac{\tilde{n}}{\langle \tilde{n} \rangle}, \langle \tilde{n} \rangle = \int_0^\infty \tilde{n} \psi(\tilde{z}) d\tilde{z}), \quad (1.3)$$

where $\psi(\tilde{z})$ is a scaling function normalized by the equations:

$$\int_0^\infty \psi(\tilde{z}) d\tilde{z} = \int_0^\infty \tilde{z} \psi(\tilde{z}) d\tilde{z} = 1 . \quad (1.4)$$

It is worth noting that the normalization relations for the KNO-G [eq. (1.4)] can always be fulfilled (even for small $\langle n \rangle$), which is not the case for the standard KNO [5], since at small $\langle n \rangle$ the normalization condition $\int z \psi(z) dz = 1$ is a poor approximation of a $\sum n / \langle n \rangle \psi(n / \langle n \rangle) = 1$. Obviously, the above definition of P_n is for the case where $n = 0, 1, 2, 3, \dots$. In general we can measure the multiplicity $m = k, k+l, k+2l, k+n_l, \dots$. But we always can reduce m to n . The following relations hold:

$$P(\tilde{m}) = \frac{1}{\sigma_{in}} \frac{d\sigma}{d\tilde{m}} = \frac{1}{\sigma_{in}} \frac{d\sigma}{d\tilde{n}} \frac{d\tilde{n}}{d\tilde{m}} = \frac{1}{l} P(\tilde{n})$$

and

$$P_m \equiv \int_m^{m+l} P(\tilde{m}) d\tilde{m} = \int_n^{n+1} P(\tilde{n}) d\tilde{n} \equiv P_n.$$

Since n is primary to m , we will often use the number of negative secondaries (n) as a measure of the multiplicity instead of the most often used variable n_{ch} ; n_{ch} is the number of charged particles and $n_{ch} = 2 + 2n$, so $k = 2$ and $l = 2$.

The plan of the paper is the following. In Section 2 we re-examine the applicability of the standard KNO and KNO-G approach to the pp multiplicity data and we show that all the available multiplicity data obey perfectly KNO-G with the same scaling function. In Section 3 we demonstrate that a long-standing contradiction between the 'Wroblewski relation' [3] and the KNO scaling disappears if the KNO-G framework is taken into account. Moreover, the 'Wroblewski relation' is a simple derivative of the KNO-G. We point out in Section 4 that a present way of testing KNO scaling at SPS Collider energies by using the scaling function $\psi(z)$ fitted at the ISR energies may be meaningless because of the hypothetical 'locality' of the standard KNO. Since KNO-G with the same scaling function works over the full energy range, it can be considered as a useful framework to extrapolate from existing non-diffractive pp data, in particular from the new measured ISR data [6], to the SPS Collider energy. We compare such extrapolation with UA5 [7] data.

In the present paper we use pp data ranging from $p_{lab} = 2.23$ GeV/c up to $p_{lab} = 2062$ GeV/c ($\sqrt{s} = 2.5-62.2$ GeV) [6,8].

The conclusions are given in the last section.

2 - COMPARISON OF THE STANDARD KNO AND KNO-G

2.1 - GRAPHICAL TESTS OF THE KNO AND KNO-G

There are two usual ways of testing KNO. In the first, the overlap of the data at different energies in the variables $\langle n \rangle P_n$ and $n/\langle n \rangle$ is checked. In the second, the dependence of the normalized statistical moments on energy is verified. Neither is possible in the KNO-G case. Instead, it is easy to see [5] that the data at different energies should overlap, if sums

$$S_n (S_n = \sum_{i=n}^{\infty} P_i) \text{ are plotted versus } \tilde{z} (\tilde{z} = \frac{n}{\langle \tilde{n} \rangle}).$$

Figures 1a,b show in a linear and logarithmic scale the usual graphical test of the standard KNO. Since the whole data set ($\sqrt{s} = 2.5-62.2$ GeV) [6,8] is plotted, the well-known deviations from KNO are visible. The same data are presented in Figs. 1c,d in the improved variables, proposed by Wroblewski [3]. This time the data are slightly less scattered.

A graphical test of the KNO-G is shown in Figs. 1e,f. It can be seen that all data points lie on a single curve and the only scattering seen is at the tails of multiplicity distributions where the largest experimental (statistical) errors are expected.

2.2 - QUANTITATIVE COMPARISON OF THE KNO AND KNO-G

Before presenting a quantitative comparison of the KNO, improved KNO [with $z = (n-\alpha)/(\langle n \rangle - \alpha)$] [3], and the KNO-G, we would like to point out two features of the KNO-G which facilitate the study. The first one is that the following equation is fulfilled with very good precision, already for $\langle n \rangle \geq 1$ [5]:

$$\langle \tilde{n} \rangle \approx \langle n \rangle + 0.5. \quad (2.1)$$

The relation (2.1) can be derived from the definition of $\langle n \rangle$ in the following way:

$$\langle \tilde{n} \rangle = \int_0^{\infty} \tilde{n} \psi(\tilde{z}) d\tilde{z} = \sum_{n=0}^{\infty} \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} ([\tilde{n}] + \Delta \tilde{n}) \psi(\tilde{z}) d\tilde{z},$$

where $[\tilde{n}]$ is an integer part of a continuous $\tilde{n}$.

Now

$$\sum_{n=0}^{\infty} \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} [\tilde{n}] \psi(\tilde{z}) d\tilde{z} = \sum_{n=0}^{\infty} n \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} \psi(\tilde{z}) d\tilde{z} \equiv \langle n \rangle$$

and for large $\langle n \rangle$

$$\begin{aligned} \sum_{n=0}^{\infty} \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} \Delta \tilde{n} \psi(\tilde{z}) d\tilde{z} &\approx \sum_{n=0}^{\infty} \psi(\tilde{z}) \int_0^1 \Delta \tilde{n} \frac{d(\Delta \tilde{n})}{\langle \tilde{n} \rangle} \\ &\approx 0.5 \int_0^{\infty} \psi(\tilde{z}) d\tilde{z} = 0.5 \end{aligned}$$

so

$$\langle \tilde{n} \rangle \approx \langle n \rangle + 0.5.$$

The second feature is an observation that instead of using the function $\psi(z)$ in numerical calculations, function $\phi(z)$ defined below can be used:

$$\phi(\tilde{z}) = \int_0^{\tilde{z}} \psi(\tilde{z}) d\tilde{z} \quad (2.2)$$

It is seen that

$$P_n = \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} \psi(\tilde{z}) d\tilde{z} = \phi\left(\frac{n+1}{\langle \tilde{n} \rangle}\right) - \phi\left(\frac{n}{\langle \tilde{n} \rangle}\right) \quad (2.2)$$

and

$$S_n = \sum_{i=n}^{\infty} P_i = \int_{n/\langle \tilde{n} \rangle}^{\infty} \psi(\tilde{z}) d\tilde{z} = -\phi(\tilde{z}), \text{ if } \phi(\infty) = 0. \quad (2.3)$$

So, the knowledge of a $\phi(\tilde{z})$ and eq. (2.1) is sufficient to obtain all of the multiplicity distributions. The normalization conditions [eq. (1.4)] are redefined for a function $\phi(z)$ in the following way:

$$\int_0^{\infty} \psi(\tilde{z}) d\tilde{z} = 1 = \phi(\infty) - \phi(0), \quad \phi(0) = -1. \quad (2.4)$$

and

$$\int_0^{\infty} \tilde{z} \psi(\tilde{z}) d\tilde{z} = 1 = -\int_0^{\infty} \phi(\tilde{z}) d\tilde{z}. \quad (2.5)$$

It is also easy to see that, owing to eq. (2.3), the observation that the data in Fig. 1f lie on a straight line in the variables $\log S_n [\equiv \log\{-\phi(\tilde{z})\}]$ versus $\tilde{z}^2$, suggests $\phi(\tilde{z})$ in the form .

$$-\phi(\tilde{z}) \sim e^{-b\tilde{z}^2}.$$

Available pp multiplicity data [6,8] were fitted with several functions of the form

$$\phi(\tilde{z}) = -(1 + \sum a_l \tilde{z}^l) e^{-\sum b_k \tilde{z}^k}.$$

It was found that in the case of more complicated functions than a simple one,

$$\phi_o(\tilde{z}) = -(1 - a\tilde{z}) e^{-b\tilde{z}^2}, \quad (2.6)$$

the χ^2 has not been improved. It should be stressed that ϕ_o has only one free parameter since from the normalization condition [eq. (2.5)] one obtains : $a = \sqrt{\pi} \sqrt{b} - 2b$.

Results of the fits to the pp data within KNO-G with the function given by eq. (2.6) (negative multiplicities) and with a 'Slattery type' [2] parametrization of $\psi_1(z)$, i. e. $\psi_1(z) = (a_0 + a_1 z + a_2 z^3 + a_3 z^5 + a_4 z^7) e^{-bz}$ ($z = n_{ch} / \langle n_{ch} \rangle$) are presented in Table 1 together with 'improved' KNO scaling with a function $\psi_2(z)$ having shifted argument $z = (n_{ch} - \alpha) / (\langle n_{ch} \rangle - \alpha)$.

Although the scaling function in the KNO-G framework has only one free parameter, it fits very well the data ranging from the threshold energy for pion production in pp collision to the highest ISR energy. As can be seen in Table 1, neither the standard KNO scaling function (4 free parameters) nor the one with shifted argument (5 free parameters) give equally small χ^2 . It should be mentioned that fits of ψ_1 and ψ_2 were done taking into account the normalization constraints [eq. (1.4)]. The fits performed without the normalization constraints yielded somewhat smaller overall χ^2 .

2.3 - STATISTICAL MOMENTS OF THE MULTIPLICITY DISTRIBUTIONS

Figures 2 and 3 present the energy dependence of the various statistical moments of the multiplicity distributions. The following statistical moments are used:

$D = \sqrt{\mu_2}$	dispersion
$\phi = \langle n \rangle / D$	
$\gamma_1 = \mu_3 / D^3$	skewness
$\gamma_2 = \mu_4 / D^4$	kurtosis
$C_K = \langle n^K \rangle / \langle n \rangle^K$	normalized algebraic moments
$\mu_K = \sum (n - \langle n \rangle)^K P_n$	central moments.

The solid curve corresponds to the moments of the multiplicity distributions obtained using the fitted scaling function ϕ_0 ; the dashed curve corresponds to the ψ_1 and the dotted one to the ψ_2 function. The generation of P_n^{th} from ϕ_0 was straightforward

$$P_n^{th}(\langle n \rangle) = \phi_0\left(\frac{n+1}{\langle \tilde{n} \rangle}\right) - \phi_0\left(\frac{n}{\langle \tilde{n} \rangle}\right),$$

where $\langle \tilde{n} \rangle = \langle n \rangle + 0.5$ for $\langle n \rangle > 1$ and for $\langle n \rangle < 1$ a simple iteration procedure was used to obtain $\langle \tilde{n} \rangle$. No extra normalization or arbitrary cuts were used. For the moments obtained from ψ_1 and ψ_2 it was necessary to apply a 'correcting' procedure.

$$P_{n_{ch}}^{th}(\langle n_{ch} \rangle) = \frac{1}{\langle n_{ch} \rangle} \psi\left(\frac{n_{ch}}{\langle n_{ch} \rangle}\right).$$

The multiplicity distributions were renormalized ($P_n^{th} = P_n^{th} / \sum P_n^{th}$) and $\langle n_{ch} \rangle$ was recalculated. The necessity of applying this procedure is due to the internal inconsistency of the standard KNO formulation for small $\langle n \rangle$ (see Introduction); a properly normalized function ψ does not lead to the properly normalized P_n^{th} multiplicity distributions. It is seen that only multiplicity distributions generated within the KNO-G framework reproduce the data very well.

It may be quite surprising that the 'improved KNO' (ψ_2) framework fails to reproduce the higher order moments while the χ^2 obtained during the fitting procedure (see Table 1) was relatively good.

The reason is the following: during a fitting procedure a possible long tail of the $P_n^{th}(\langle n \rangle)$, $n > n_{max}$ (n_{max} - maximal measured multiplicity) is not counted, while it contributes when $P_n^{th}(\langle n \rangle)$ distribution is generated. Figure 3a shows a similar comparison to Fig. 3, but generated multiplicity distributions are cut at a given n , $n = n_{max}(\langle n \rangle)$. As is seen, the moments are reproduced much better in this case and the line representing 'improved KNO' totally overlaps with the curve generated within KNO-G.

Concluding this section one can say that the KNO-G scaling describes the data excellently since it gives absolutely normalized distributions and multiplicity distributions approach zero for large n exactly in the same way as real data do. Furthermore, the ϕ_0 function has only one free parameter. It is seen in Figs. 3b that 'improved KNO' scaling ($z = (n_{ch} - \alpha) / (\langle n_{ch} \rangle - \alpha)$, $\alpha \approx 1$) can be treated also as a good approximation of the real data, but only if the multiplicity cut ($n \leq n_{max}(\langle n \rangle)$) and "correcting" procedure is provided. However, this happens due to the fact that "improved KNO" scaling is a simple approximation of the KNO-G what can be seen in the following :

$$P_n = \frac{1}{\langle \tilde{n} \rangle} \frac{\langle n \rangle}{\langle \tilde{n} \rangle} \int \frac{(n+1)/\langle \tilde{n} \rangle}{\psi(\frac{n}{\langle \tilde{n} \rangle})} d\tilde{n} \approx \frac{1}{\langle \tilde{n} \rangle} \psi(\frac{n+0.5}{\langle \tilde{n} \rangle}),$$

since $\langle \tilde{n} \rangle \approx \langle n \rangle + 0.5$

$$P_n \approx \frac{1}{\langle n \rangle + 0.5} \psi(\frac{n+0.5}{\langle n \rangle + 0.5}) \quad \text{or} \quad P_{n_{ch}} \approx \frac{2}{\langle n_{ch} \rangle - 1} \psi(\frac{\langle n_{ch} \rangle - 1}{\langle n_{ch} \rangle - 1}).$$

3 - WROBLEWSKI RELATION

As was pointed out first by Wroblewski [3], the exact KNO scaling implies that $D/\langle n_{ch} \rangle = \text{const}$, but in fact the data show that $D/(\langle n_{ch} \rangle - 1) = \text{const}$. Thus the excellent support of the 'Wroblewski relation' $D = A(\langle n_{ch} \rangle - 1)$ or more general $D = A(\langle n_{ch} \rangle - \alpha)$ by the data [6,9,10], was always understood as a direct contradiction with a standard KNO. This was a reason why 'improved KNO' with a shifted argument $z = (n_{ch} - \alpha) / (\langle n_{ch} \rangle - \alpha)$ was proposed [3]. In Section 2.3 we have shown that even highest order moments, are reproduced very well when applying the KNO-G. This was indirect proof that the 'Wroblewski relation' is also a simple consequence of KNO-G.

In the following we show for completeness a derivation of the 'Wroblewski relation' using only the definition of P_n within KNO-G. Firstly, we evaluate $\langle n^2 \rangle$:

$$\begin{aligned} \langle \tilde{n}^2 \rangle &\equiv \int_0^\infty \tilde{n}^2 \psi(\tilde{z}) d\tilde{z} = \sum_{n=0}^\infty \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} ([\tilde{n}] + \Delta \tilde{n})^2 \psi(\tilde{z}) d\tilde{z} \\ &= \sum_{n=0}^\infty \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} [\tilde{n}]^2 \psi(\tilde{z}) d\tilde{z} + \sum_{n=0}^\infty \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} 2[\tilde{n}] \Delta \tilde{n} \psi(\tilde{z}) d\tilde{z} \\ &\quad + \sum_{n=0}^\infty \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} (\Delta \tilde{n})^2 \psi(\tilde{z}) d\tilde{z} . \end{aligned}$$

Using similar approximations as for the derivation of eq. (2.1), we obtain the relation

$$\langle \tilde{n}^2 \rangle \approx \langle n^2 \rangle + \langle n \rangle + \frac{1}{3} .$$

Because $\langle n \rangle \approx \langle n \rangle - 0.5$, we finally get

$$D^2 = \langle n^2 \rangle - \langle n \rangle^2 \approx \langle \tilde{n}^2 \rangle - \langle \tilde{n} \rangle^2 .$$

and with $\tilde{C}^2 = \langle \tilde{n}^2 \rangle / \langle \tilde{n} \rangle^2 = \int_0^\infty \langle \tilde{n}^2 \rangle / \langle \tilde{n} \rangle^2 \psi(\tilde{z}) d\tilde{z}$ being a constant

$$D^2 \approx (\tilde{C}_2^2 - 1) \langle \tilde{n}^2 \rangle$$

For large $\langle n \rangle$ we get finally

$$D \approx A (\langle n \rangle + 0.5) . \quad (3.1)$$

Since in the experiment the multiplicity m ($m = k + n\ell$, $n = 0, 1, 2, 3, \dots$) is often measured, we extend the result obtained for this more general case. It can easily be shown that

$$D_m^2 = \ell^2 D_n^2$$

so

$$D_m = \ell D_n \approx \ell A \left(\langle n \rangle + \frac{1}{2} \right) = A \left(\langle m \rangle - k + \frac{\ell}{2} \right) . \quad (3.2)$$

Although, in the present paper we have not shown that KNO-G is a valid concept for other than pp multiplicity data, still we show a few examples of how the formula (3.2) works for collisions of various interacting particles. Figures 4 and 5 show the dispersion versus average multiplicity dependence for π^+p , K^+p , and π^-p , K^-p , $\bar{p}p$ multiplicity data [9]. The π^-p , K^-p and $\bar{p}p$ data (Fig. 5) are also shown without 0-prong inelastic events, which usually have the largest experimental uncertainties. The intercept with the $\langle n_{ch} \rangle$ axis is always fixed by eq. (3.2). All the used data seem to support formula (3.2).

4 - EXTRAPOLATION OF KNO-G TO THE SPS COLLIDER ENERGY

4.1 - THE ASYMPTOTIC ENERGY DEPENDENCE OF $\langle n \rangle$

As is well known [6,10] the mean charged multiplicity rises faster than logarithmically with energy. This is illustrated in Fig. 6, where two different lines are plotted for inelastic (black circles) and non-diffractive (open circles) data samples. The solid line corresponds to the dependence of the form $\langle n \rangle = A + B \ln s + C (\ln s)^2$; the dotted one is just a linear fit $\langle n \rangle = A + B \ln s$ to the highest energy data ($\sqrt{s} > 15$ GeV). As is seen in Fig. 6, a simple linear dependence of $\langle n \rangle$ on $\ln s$ for high enough energies ($\sqrt{s} > 15$ GeV) cannot be excluded.

4.2 - COMPARISON OF UA5 MULTIPLICITY DATA WITH KNO-G EXTRAPOLATION

We would like to stress that an early KNO scaling was never proved, since new data at higher energies always forced a change of a parametrization of the scaling function $\psi(z)$. Having no idea how far we are at the ISR energies from the asymptotic energy, where $\psi(z)$ should be finally fixed (even having no proof that such an energy exists), any extrapolation from the ISR data to the SPS Collider energies may produce a fortuitous result. The situation is completely different if one takes into account the KNO-G scaling. Since the KNO-G scaling is really perfect in the whole energy range, it should be relatively safe to use it for extrapolation.

Furthermore, it should be noted that it was not necessary to subtract a diffractive component from the data to obtain perfect KNO-G scaling. So, an experiment measuring multiplicity distributions for all inelastic events at the SPS Collider is very much required. Again, it should be stressed that so called 'non-diffractive' or 'non- single-diffractive' data samples usually have very large uncertainties because of differences in the subtraction of the diffractive component. Unfortunately, the only existing SPS Collider multiplicity data measuring secondaries approximately in a full solid angle are of the non-diffractive type [7].

In order to compare the extrapolation within KNO-G with the recent UA5 data [7], the validity of KNO-G should be checked for lower energy non-diffractive data, which should lead to the scaling function ϕ_o^{nd} and then extrapolation could be performed. Before doing all this, we have to make another comment.

It was observed (see, for example [6]) that for non-diffractive data below $\sqrt{s} = 62.2$ GeV [11,12], a linear dependence of the dispersion on the mean charged multiplicity takes the form $D \approx 0.44 \langle n_{ch} \rangle$, whereas the KNO-G implies that $D \approx A(\langle n_{ch} \rangle - 1)$. A careful investigation of the problem shows that as is illustrated by Fig. 7, the D dependence on $\langle n_{ch} \rangle$ being of the form $D = A(\langle n_{ch} \rangle - 1)$ cannot be excluded, since only lowest energy data do not obey this dependence. In our opinion all uncertainties in the subtraction of the single diffractive component, especially in the low-energy data (see, for example, [11]), make KNO tests based on non-diffractive data of a much lower quality than similar tests for all inelastic events.

Furthermore for KNO-G tests we use only the Fermilab [12] and ISR [6] non-diffractive data. Figure 8 presents Fermilab and ISR non-diffractive data together with recent UA5 results [7] in a plot similar to that presented in Fig. 1 f. All data for $\sqrt{s} < 62.2$ GeV form a single curve, but UA5 data deviate strongly from them. It is interesting to check in which multiplicity region the deviation of the UA5 data from KNO-G extrapolation is largest. We get the following best fit of the ϕ_o^{nd} :

$$\phi_o^{nd}(z) = -(1 + 0.1104 z) e^{-0.8924 z^2}, \quad \chi^2 / ND = 1.42.$$

Now we only need to know the average multiplicity to be expected at $\sqrt{s} = 540$ GeV. We take three extreme assumptions:

- a) $\langle n \rangle = 10.51$ extrapolation with $A + B \ln s$ dependence
- b) $\langle n \rangle = 12.11$ extrapolation with $A + B \ln s + C(\ln s)^2$ dependence
- c) $\langle n \rangle = 14.55$ value measured by UA5 experiment

A comparison of the KNO-G predictions with the UA5 data is shown for the above three mean multiplicities in Fig. 9. It is clearly seen that the experimental data differ from KNO-G predictions only at the high multiplicity tail when the theoretical P_n^{th} distribution is generated within KNO-G with $\langle n \rangle$ obtained using the ' $(\ln s)^2$ ' formula. Also seen are rather worrying fluctuations in the data. Assuming that the UA5 data are correct, one can claim that on top of the usual particle production mechanism responsible for the KNO-G scaling, there is evidence for an extra process contributing to the high multiplicities.

5 - CONCLUSIONS

The KNO scaling derived by Koba, Nielsen and Olesen for asymptotic energies is very often applied to the low-energy data, where its formulation is not self-consistent. Golokhvastov proposed a minimal possible extension of KNO making it consistent for all energies. In fact a cancellation of the last approximation in the original KNO derivation (see eq. (2.16) in Ref.[1]) leads to the KNO-G.

Using new ISR data for all inelastic and non-diffractive event samples we have re-examined the standard approach to the KNO and KNO-G in the whole energy range. The data for all inelastic events obey perfectly KNO-G.

Moreover, the features of the multiplicity distributions like "Wroblewski relation" or a rise with energy of the normalized multiplicity moments do not contradict scaling in multiplicity, provided that the scaling is properly formulated for all energies. In fact properly formulated multiplicity scaling (KNO-G $\rightarrow$ KNO, for $\sqrt{s} \rightarrow \infty$) implies the above mentioned features of the data exactly in the way there are observed in the experiments.

It should be stressed that it was not necessary to exclude the single diffractive component from the data in order to obtain perfect validity of the KNO-G.

A firm test of the KNO-G validity at SPS Collider energy needs new multiplicity data measured for all inelastic events in a full phase space.

An extrapolation from lower energy non-diffractive data to $\sqrt{s} = 540$ GeV was undertaken. The comparison with the UA5 multiplicity results shows that only the tail of the multiplicity distribution ($n_{ch} > 40$) is not consistent with extrapolated values.

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TABLE I

		Number of parameters	χ^2	χ^2/ND
ψ_1	$b = 3.303 \pm 0.003$ $a_0 = -0.487$ $a_1 = 3.426$ $a_3 = 52.10 \pm 0.08$ $a_5 = -10.62 \pm 0.01$ $a_7 = 0.5304 \pm 0.0012$	4	3845	13.1
ψ_2	$b = 2.967 \pm 0.007$ $a_0 = 0.447$ $a_1 = 3.476$ $a_3 = 25.4 \pm 0.4$ $a_5 = -3.48 \pm 0.08$ $a_7 = 0.119 \pm 0.004$ $\alpha = -0.864 \pm 0.010$	5	505	1.73
ϕ_0	$b = 0.6276 \pm 0.0041$ $a = 0.1489 \pm 0.0038$	1	377	1.26

$$\psi_1 = (a_0 + a_1 z + a_3 z^3 + a_5 z^5 + a_7 z^7) e^{-bz}, \quad z = n_{\text{ch}} / \langle n_{\text{ch}} \rangle$$

$$\psi_2 = (a_0 + a_1 z + a_3 z^3 + a_5 z^5 + a_7 z^7) e^{-bz}, \quad z = (n_{\text{ch}} - \alpha) / (\langle n_{\text{ch}} \rangle - \alpha)$$

a_0 and a_1 are calculated from the normalization constraints.

$$\phi_0 = - (1 - az) e^{-bz^2}$$

$a = \sqrt{\pi} \sqrt{b} - 2b$ calculated from normalization constraint.

FIGURE CAPTIONS

- Fig. 1: Graphical test of KNO (a,b), 'improved KNO' (c,d), and KNO-G (e,f) for inelastic event samples; n_{ch} denotes all charged secondaries, $z = n/\langle n \rangle$, where $\langle n \rangle$ is a mean continuous multiplicity of negative secondaries. In all cases the whole data sample [6,8] was used.
- Fig. 2: Statistical moments: $D^2 = \langle n_{ch}^2 \rangle - \langle n_{ch} \rangle^2$, $\phi = \langle n_{ch} \rangle / D$, $\gamma_1 = \mu_3 / D^3$, $\gamma_2 = \mu_4 / D^4$ where $\mu_K = \sum (n_{ch} - \langle n_{ch} \rangle)^K P_n$ dependence on average $\langle n_{ch} \rangle$ for pp data in a whole energy range [6,8]. The solid, dashed, and dotted lines correspond to the statistical moments generated within KNO-G, 'improved KNO', and KNO, respectively, as described in the text.
- Fig. 3: The same dependence as in Fig. 2 but for normalized algebraic moments ($C_K = \langle n^K \rangle / \langle n \rangle^K$); (a) without multiplicity cut and (b) with multiplicity cut, as described in the text.
- Fig. 4: Dispersion versus average charged multiplicity is plotted for $\pi^+ p$, and $K^+ p$ data [9]. The lines represent fits to the data using formula (3.2).
- Fig. 5: Dispersion versus average charged multiplicity is plotted for $\pi^- p$, $K^- p$, and $p\bar{p}$ data [9] for full multiplicity distributions (a,c,e) and for distributions without 0-prong inelastic events (b,d,f). The lines represent fits to the data using formula (3.2).
- Fig. 6: The energy dependence of average multiplicity for all inelastic (full circles) and non-diffractive (open circles) events. The plotted lines represent fits to the data done with the following formulas: solid line $\langle n \rangle = A + B \ln s + C(\ln s)^2$ (inelastic data : $A = -0.495$, $B = 0.179$, $C = 0.0636$, non-diffractive data : $A = -1.17$, $B = 0.456$, $C = 0.0476$); dotted line $\langle n \rangle = A + B \ln s$, only for data with $\sqrt{s} > 15$ GeV (inelastic data : $A = -2.97$, $B = 0.979$, non-diffractive data : $A = -3.29$, $B = 1.097$).
- Fig. 7: Dispersion versus average charged multiplicity dependence plotted for the non-diffractive data [6,11,12]. The dashed line represents a fit to the data of the form $D = A(\langle n_{ch} \rangle - 1)$ (4 lowest energy data [11] are omitted in the fit). The solid one shows the relation $D \approx 0.44 \langle n_{ch} \rangle$.
- Fig. 8: A graphical test of the KNO-G scaling. The non-diffractive Fermilab [12], ISR [6] (black circles), and SPS Collider [7] (open circles) data are plotted.
- Fig. 9: Comparison of UA5 multiplicity data [7] with extrapolation done within KNO-G for three different $\langle n \rangle$ values expected at $\sqrt{s} = 540$ GeV: (a,b) extrapolation with $A + B \ln s$ dependence, $\langle n \rangle = 10.51$; (c,d) extrapolation with $A + B \ln s + C(\ln s)^2$ dependence, $\langle n \rangle = 12.11$; (e,f) extrapolation with value measured by UA5 experiment, $\langle n \rangle = 14.55$.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9