HADRON MASSES IN QUENCHED QUANTUM CHROMODYNAMICS

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A B S T R A C T

Hadron masses are calculated by numerical methods in the quenched approximation to lattice QCD. The analysis is performed at $\beta = 6.0$ on a large $(16^3 \times 32)$ lattice, with Kogut-Susskind fermions and bare quark masses down to $m_q a = 0.01$. The results are in satisfactory agreement with experimental data and the value of the scale parameter, determined by fixing the mass of the ρ -meson, is consistent with independent estimates based on the calculation of the string tension.

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Numerical calculations, performed in the context of the lattice formulation of a quantized field theory, have provided very valuable results for many observables of quantum chromodynamics (QCD). Certainly, the masses in the hadronic spectrum are among the most fundamental quantities that one may wish to compute and several research efforts have already gone in this direction (for the earliest results, see Ref. 1). However, calculations of masses, which rely on a finite, periodic box to contain the hadron and on a propagation for a limited extent of time to extract the lowest mass eigenvalues, can be affected by substantial errors if the lattice is not sufficiently big. One can get an idea on how large the lattice ought to be by considering that various calculations allow one to estimate a lattice spacing a of the order of 0.1 fm at the point where scaling appears to set in and that 1 fm is a typical scale for hadronic phenomena. Thus lattices with a linear extent of at least 10 sites, and possibly larger, should be used for reliable calculations. While numerical simulations embodying all dynamical features of QCD are only marginally feasible on lattices of such size, calculations in the quenched approximation are well within the grasp of today's supercomputers. In the quenched approximation the integration over gauge field configurations is performed according to the pure gauge field measure, $\exp\{-S_G\}$, neglecting the factor $\text{Det}(D+m)$, $D+m$ being the lattice Dirac's operator, which would be induced by the integration over the quark fields. This tantamounts to neglecting vacuum polarization effects from the creation of virtual quark-antiquark pairs, an approximation which has found several theoretical and phenomenological justifications. In this letter we present results for the hadronic mass spectrum obtained

in the quenched approximation on a lattice extending for 16 sites in each of the spatial directions and 32 sites in the temporal one.

In a general program of research aimed at taking full advantage of the capabilities of supercomputers with vector architectures, we have developed a series of very efficient codes for performing Monte Carlo calculations. We already reported about the codes for the basic Monte Carlo simulation⁽²⁾ on the code for finding the quark propagators⁽³⁾ and on the results for the force between static quark and antiquark⁽⁴⁻⁵⁾. The procedure to evaluate the lowest masses in the hadronic spectrum is rather well known. By Monte Carlo simulation one generates N configurations C_i of the gauge field variables U_x^μ distributed according to the measure $\exp\{-S_G\}$ (quenched approximation). S_G is the lattice action, which we have taken to be of Wilson's form:

$$S_G = \beta \sum_x \sum_{\mu \leq \nu} \left(1 - \frac{1}{3} \text{Re Tr}(U_x^{\nu\dagger} U_{x+\hat{\nu}}^\mu U_{x+\hat{\mu}}^\nu U_x^\mu)\right) \quad (1)$$

(β , the coupling parameter, is related to the bare coupling constant g by $\beta = 6/g^2$). On the individual configurations one calculates the propagators for the quarks

$$P_{xc, x_0 c_0} = (D+m)_{xc, x_0 c_0}^{-1} \quad (2)$$

(c, c_0 stand for color indices). Out of these, one forms propagators for mesons and baryons:

$$P_{xx_0}^{m(k)} = \sum_{cc_0} S_x^{(k)} P_{xc, x_0 c_0} P_{xc, x_0 c_0}^* \quad (3)$$

and

$$p_{xx_0}^{b(k)} = \sum_{c_1 c_2 c_3} S_x^{(k)} \epsilon_{c_1 c_2 c_3} p_{xc_1, x_0 1} p_{xc_2, x_0 2} p_{xc_3, x_0 3} , \quad (4)$$

where $S_x^{(k)}$ are suitable spin factors (see later). Finally one sums over the spatial coordinates $\vec{x}$ to extract zero momentum components

$$G_{tt_0}^{m,b(k)} = \sum_{\vec{x}} p_{\vec{x}t, \vec{x}_0 t_0}^{m,b(k)} . \quad (5)$$

The quantum averages

$$\langle G_{tt_0}^{m,b(k)} \rangle \approx \frac{1}{N} \sum_{\{C_i\}} G_{tt_0}^{m,b(k)}(C_i) \quad (6)$$

will exhibit a rate of decay in the separation $(t-t_0)$ determined by the contributions from physical intermediate states

$$\langle G_{tt_0}^{m,b(k)} \rangle = \sum_n A_n^{m,b(k)} \exp(-m_n^{m,b(k)}(t-t_0)) . \quad (7)$$

From an analysis of the rate of decay of $\langle G \rangle$ one can obtain information on the lowest masses in the spectrum.

We adopted the Kogut-Susskind formulation of lattice fermions. It describes four quark flavors, preserves a one-parameter non-singlet continuous chiral symmetry for vanishing quark mass and effectively splits the 4×4 spin and flavor components of the quark fields among the 16 points of a cell of side 2. The Dirac operator $D+m$ is defined through the action of D on ψ , namely (leaving color indices implicit):

$$(D\psi)_x = \frac{1}{2a} \sum_{\mu} s_x^{\mu} (U_x^{\mu\dagger} \psi_{x+\hat{\mu}a} - U_{x-\hat{\mu}a}^{\mu} \psi_{x-\hat{\mu}a}) , \quad (8)$$

where $s_x^1 = (-1)^{(x_2+x_4)/a}$, $s_x^2 = (-1)^{(x_3+x_4)/a}$, $s_x^3 = (-1)^{(x_1+x_4)/a}$,
 $s_x^4 = 1$.

The spin factors $s_x^{(k)}$ in Eqns. 3 and 4 act to give different quantum numbers to the local sources (at least insofar as the mesons are concerned, for baryons see later discussion), and are given by

$$\begin{aligned} s_x^{(1)} &= 1 , \\ s_x^{(2)} &= (-1)^{x_1/a} + (-1)^{x_2/a} + (-1)^{x_3/a} , \\ s_x^{(3)} &= (-1)^{(x_2+x_3)/a} + (-1)^{(x_3+x_1)/a} + (-1)^{(x_1+x_2)/a} , \\ s_x^{(4)} &= (-1)^{(x_1+x_2+x_3)/a} . \end{aligned} \quad (9)$$

The results we present here have been obtained at $\beta = 6$ and for bare quark masses $m_q = 0.5, 0.16, 0.09, 0.04$ and 0.01 , in units of a^{-1} . They are based on averages of 15 propagators (at each value of m_q) calculated with 3 different source points x_0 in 5 configurations $C_1 - C_5$. C_1 was the final configuration of the long Monte Carlo simulation used to calculate the force between static quarks⁽⁴⁾ . It had been thermalized by 1100 iterations at $\beta = 6$. Prior to this, its history consisted of 500 more thermalizing iterations of the half-lattice (16^4) , always at $\beta = 6$, and 1500 earlier iterations (always of the half-lattice) at $\beta = 5.6$ and $\beta = 5.8$. Configurations

$C_2 - C_5$ have been obtained performing 800 Monte Carlo iterations starting from C_1 and selecting configurations separated by 200 iterations. We used periodic boundary conditions in all directions. The source points have been chosen all with the same spatial location and time coordinates $x_4 = a, 12a$ and $22a$. This particular selection has been motivated by our intention to analyze the quark charge distribution inside the mesons, an observable about which we plan to report in a future publication. The separation of the sources inside the same configuration is sufficiently larger than the correlation length (inverse glueball mass) at $\beta = 6$ that the propagators with different sources can be considered independent. We calculated the quark propagators with the algorithm of conjugate gradients⁽³⁾. We did most of the calculation in 32-bit arithmetic, but we made several tests using full 64-bit arithmetic to make sure that the half-word size did not produce any significant errors. For each source point and color of the source we performed 40 iterations at $m_q a = 0.5$, followed by 80, 120, 250 and 500 iterations at the progressively lower values of m_q . We found convergence with errors, defined $((D+m)\psi - \delta_{xc, x_0} c_0) / |\psi|$, of the order of 10^{-3} or smaller.

The propagators $\langle G_{tt_0}^{m,b} \rangle$ are expected to obey identities, which follow from various symmetries of the lattice theory. In particular one should have⁽⁶⁻⁷⁾

$$\text{Im} \langle G_{tt_0}^{b(k)} \rangle = 0 \quad (11)$$

($\text{Im} \langle G_{tt_0}^{m(k)} \rangle = 0$ is trivially true),

$$\langle G_{tt_0}^{b(2)} \rangle = \langle G_{tt_0}^{b(3)} \rangle = 3 \langle G_{tt_0}^{b(1)} \rangle = 3 \langle G_{tt_0}^{b(4)} \rangle, \quad (12)$$

$$\langle G_{32a+2t_0-t, t_0}^{m(k)} \rangle = \langle G_{t, t_0}^{m(k)} \rangle \quad (13)$$

and

$$\langle G_{32a+2t_0-t, t_0}^{b(k)} \rangle = (-1)^{(t-t_0)/a} \langle G_{t, t_0}^{b(k)} \rangle. \quad (14)$$

We have verified that all of the above equations are well satisfied within the statistical accuracy. We have then used Eqn. 12 to define a unique propagator for the baryons*

$$\langle G_{tt_0}^b \rangle = \sum_k \langle G_{tt_0}^{b(k)} \rangle \quad (15)$$

and Eqns. 13, 14 to fold over the propagations forward and backward in time into a single propagator.

In the following we shall denote the resulting propagators simply by $G_t^{m(k)}$ and G_t^b , dropping the brackets and identifying the subscript t ($0 \leq t \leq 16a$) with the time difference $t-t_0$.

Because of the periodicity in time one expects the following behavior for G_t :

$$G_t = \sum_n A_n (e^{-m_n t} + e^{-m_n(32a-t)}) + (-1)^{t/a} \sum_n \tilde{A}_n (e^{-\tilde{m}_n t} + e^{-\tilde{m}_n(32a-t)}). \quad (16)$$

*Eqn. 12 follows from the fact that all contributions to $\langle G_{tt_0}^{b(k)} \rangle$ from the points having any of the differences $x_\mu - x_{0\mu}$ equal to an odd number of lattice spacings cancel in pairs. In the combination of Eqn. 15 only the contributions where all differences $x_\mu - x_{0\mu}$ are an even multiple of a survive.

The terms alternating in sign are a peculiarity of the Kogut-Susskind formulation of lattice fermions and correspond to contributions from intermediate state of opposite parity than those contributing to the first sum. It will be convenient to adopt the terminology of states in the direct channel and states in the opposite parity channel to characterize the states contributing to the two sums in the r.h.s. of Eqn. 16. We made fits to the propagators G_t in terms of one, two or three intermediate states. We used the CERN Minuit program to minimize the χ^2 and evaluate the associated errors in the parameters. Fitting the numerical results is not straightforward. If one includes too many states one of course loses any amount of significant information. If, on the other hand, one keeps too few contributions to the sums and pretends to fit the whole range of values of t , one is bound to overestimate the mass parameter (and, if the statistical errors are small, will also obtain an unacceptable χ^2). The correct thing to do is, in this case, to drop from the fit the points closest to the source, until the results for the mass become stable. Here also, however, if too many points are left out, the error induced by statistical fluctuations may become too high. The procedure we have followed has been to fit the numerical data dropping subsequently more and more points near the source, i.e., restricting the fit to the points $t_{\min} \leq t \leq 16a$, until we found stability in the mass (or masses) and a reasonable χ^2 . We have systematically checked the consistency of the fits with smaller number of masses (and generally large $t_{\min}$) with the fits making use of a larger number of states (where $t_{\min}$ could be taken closer, if not coincident with the source) and the agreement turned out to be always

satisfactory. Another matter to be considered in the fit is the proper definition of the χ^2 . A straightforward definition gives

$$\chi^2 = \sum_{t_{\min} \leq t \leq 16a} \frac{(G_t^{\text{fit}} - G_t^{\text{data}})^2}{(\Delta G_t^{\text{data}})^2}, \quad (17)$$

(ΔG being the standard deviation) but this assumes that the data points are uncorrelated. One can correct for correlations between neighboring points (in t) up to some maximum separation la . Taking into account such correlations tends to reduce, slightly, the estimated error in the parameters, but, as we have again systematically verified, the values obtained for masses and coefficients are consistent. In general, then, we used the definition of χ^2 where the data are considered uncorrelated so as to be on the safe side in the estimate of the statistical errors, apart from occasional propagators where the error analysis performed by the Minuit program tended to give unstable results unless the χ^2 were corrected for some amount of correlations. We plan to present a more detailed account of our error analysis in a future, longer publication.

Fig. 1 exemplifies our data points and the quality of the fits. Exhibited are the propagators for the mesons with the quantum numbers of the π in the direct channel (spin factor $S^{(1)}$), those with the quantum numbers of the ρ (spin factor $S^{(2)}$) and the baryon propagator, with the quantum numbers of the nucleon always in the direct channel, all at the quark mass value $m_q = 0.09 a^{-1}$.

We turn now to a discussion of the results in the various channels. In Table 1 we reproduce the values and errors we estimate from the fits for the masses and coupling parameters in correspondence to the various quark masses and mesonic spin factors (or of the baryonic propagators,

indicated by the entry b). The number of states used in the fits is clear from the table and the minimum time coordinate is also indicated. All dimensional quantities are in units of the appropriate power of the lattice spacing.

If we consider the propagators with spin factor $S^{(1)}$, the intermediate states have the quantum numbers of the π in the direct channel, ought to have $J^{PC} = 0^{+-}$ in the opposite parity channel instead. The latter are conspicuously absent (all $\tilde{A}_1$ are consistent with 0), in agreement with quark model expectations. In the direct channel one notices a clear signal from the lowest lying meson (π -like) and a distinct contribution from states of higher mass. Fits performed with a single state, but larger $t_{\min}$, give, as we mentioned already, consistent results. For instance, with $m_q a = 0.01$ and $t_{\min} = 5a$ find $A_1 = 0.629(36)$, $m_1 a = 0.252(5)$ versus $A_1 = 0.595(29)$, $m_1 a = 0.249(4)$ as in the Table. A plot of the lowest mass versus $\sqrt{m_q} a$ exhibits a remarkably linear behavior for $0 < m_q a < 0.16$, as one would expect for a Goldstone boson associated with spontaneous breaking of chiral symmetry. We recall that the Kogut-Susskind formulation preserves a non-singlet continuous chiral symmetry on the lattice for $m_q = 0$. A fit to m_1 of the type

$$m_1 a = c_1 (m_q a)^{1/2} + c_2 (m_q a)^{3/2}, \quad (18)$$

in the range $0 \leq m_q a \leq 0.16$, produces

$$c_1 = 2.393 \pm 0.012 \quad (19)$$

with an irrelevant $(m_q a)^{3/2}$ term. Only if the highest value of the

quark mass is included in the range one notices a slight departure from the linear behavior. All of this is exhibited in Fig. 2a. The continuous line in the graph should be explained. We performed separate fits of the form as in Eqn. 18, in the ranges $0 \leq m_q a \leq 0.16$ and in the full range. The continuous line has been obtained merging the two fits with a weight factor, such that for $m_q a \leq 0.1$ the line corresponds essentially to the first fit and then gradually interpolates toward the second. This procedure, which has been used also in Fig. 2b, has been followed to obtain a unique line interpolating throughout the rather wide range of quark masses, without introducing too many parameters and without spoiling the accuracy obtained at the lowest quark masses. It is, in any event, of indicative value only, since all the numbers which we shall quote later, and which pertain to rather small quark masses, have been obtained using the more accurate fits through the range $0 \leq m_q a \leq 0.16$.

The propagators with spin factor $S^{(2)}$ receive contributions from states with the quantum numbers of the ρ in the direct channel, from states with $J^{PC} = 1^{+-}$, as for the B meson, in the opposite parity channel. The 3-mass fit gives a clear signal for a ρ -like meson state, a sizable contribution from excited states in the same channel and also an appreciable contribution from states in the opposite parity channel. The masses of these last states appear however rather high and are not stable if the fit is replaced with a 2-mass fit and $t_{\min}$ is increased. A 2-mass fit, with higher $t_{\min}$, gives rather consistent results for the ρ -like meson, but indicates substantially smaller masses for the 1^{+-} states. The couplings of the latter, however, become almost undistinguishable from zero within the statistical errors. For this reason,

although the values for the masses of the 1^{+-} particle appear quite reasonable, we have not included them in Fig. 2b nor considered in the further analysis.

All of the other results correspond to fits making use of two masses. (We found the 3-mass fits, although generally consistent, too unconstrained for the values of $t_{\min}$ leading to stable results). With spin factor $S^{(3)}$ one expects contributions from 1^{--} states in the direct channel, where one finds masses compatible with those previously determined for the ρ -like particle, and contributions from states with $J^{PC} = 1^{++}$ (like the A_1 meson) in the other channel. The signal for the latter is very clear. Similarly, the propagators with spin factor $S^{(4)}$ give distinct signals for the π -like state in the direct channel, with consistent masses, and for a flavor-singlet state with $J^{PC} = 0^{++}$ (like the S resonance) in the channel with opposite parity. Finally in the baryonic channel one sees well defined contributions from states with the quantum numbers of the nucleon (direct channel) and of $J^P = \frac{1}{2}^-$ baryonic resonances.

In Fig. 2b we display all the values found for the masses as well as the results of quadratic fits for m_a as a function of $m_q a$. The fits interpolating through the points with $m_q a \leq 0.16$ give intercepts

$$\begin{aligned} m_a = & 0.457(37), 0.896(97), 0.636(47), \\ & 0.635(54) \text{ and } 0.772(77) \end{aligned} \quad (20)$$

for the states with the quantum numbers of the ρ , $A_1(1^{++})$, $S(0^{++})$, N and $N^*(\frac{1}{2}^-)$ respectively. If we use the physical values of the π and ρ mesons to fix lattice spacing and bare quark mass, we find

$$a^{-1} = 1659 \pm 134 \text{ MeV} \quad (21)$$

and
$$m_{u,d} = 0.001208(6)a^{-1} \quad (22)$$

(i.e. $m_{u,d} \approx 2.0$ MeV). This allows us to determine in turn the masses of the other particles, for which we find

$$\begin{aligned} m_{A_1} &= 1497 \pm 162 \text{ (exp 1275) MeV ,} \\ m_{S(0^{++})} &= 1063 \pm 79 \text{ (exp 975) MeV ,} \\ m_N &= 1073 \pm 91 \text{ (exp 939) MeV ,} \\ m_{N^*(\frac{1}{2}^-)} &= 1300 \pm 130 \text{ (exp 1405) MeV }^* . \end{aligned} \quad (23)$$

(The errors do not include the contribution from the error in the determination of a^{-1} , which would add a further relative error of 8.1%).

If one assumes the asymptotic formula

$$\Lambda = \left(\frac{8\pi^2\beta}{33}\right)^{\frac{51}{121}} \exp\left(-\frac{4\pi^2\beta}{33}\right)a^{-1} \quad (24)$$

for the relation between Λ and a , Eqn. 21 then gives

$$\Lambda = 3.89 \pm 0.32 \text{ MeV} \quad (25)$$

for the lattice scale parameter. In terms of the string tension σ (assuming $\sqrt{\sigma} = 420$ MeV from phenomenology) this implies

$$\Lambda = (9.27 \pm 0.75)10^{-3}\sqrt{\sigma} , \quad (26)$$

*The states in this channel would have quantum numbers appropriate for a $\Lambda(\frac{1}{2}^-)$ resonance, but with $m_s = m_u = m_d$. The "experimental" mass 1405 MeV has been inferred correcting the true experimental value for the $m_s - m_{u,d}$ splitting. We thank J. Smit for bringing this point to our attention.

in good agreement with the direct determination of $\Lambda/\sqrt{\sigma}$ from the analysis of Wilson loop factors. (The most recent numerical evaluations of $\sigma^{(4,5,8,9,10)}$, done on large lattices, give for $\Lambda/\sqrt{\sigma}$ values in the range $9.5 - 10.5 \times 10^{-3}$.)

If we adjust m_q so as to obtain a state with the mass of the K-meson in the appropriate channel (spin factor $S^{(1)}$), we find

$$m_q = \frac{m_{u,d} + m_s}{2} \approx 25.8 \text{ MeV} \quad (27)$$

and

$$m_s \approx 49.7 \text{ MeV} . \quad (28)$$

Inserting these numbers in the formula for the masses of the $J^{PC} = 1^{--}$ states, we obtain

$$m_{K^*} \approx 890 \text{ MeV} , m_{\phi} \approx 1005 \text{ MeV} \quad (29)$$

in remarkable agreement with the experimental values (892 and 1020 MeV, respectively).

The final observable we shall comment about is the value of the quark condensate. For 2 flavors it is given by

$$\langle \bar{\psi} \psi \rangle_{2f} = \frac{1}{2} \left\langle \sum_c P_{x_0 c, x_0 c} \right\rangle , \quad (30)$$

the bracket in the r.h.s. denoting an average over all source points and configurations. The values we find for $\langle \bar{\psi} \psi \rangle_{2f}$, for the 4 lowest values of m_q , are displayed in Fig. 3. The lines represent quadratic fits through all points (full line), and through the points with the

exclusion of $m_q = 0.01$. Since the values of the condensate at very low quark masses are quite sensitive to infrared effects, one can use the difference between the two fits as an indication of the possible systematic errors. The intercepts are at $a^3 \langle \bar{\psi} \psi \rangle_{2f} = 0.0364$ and 0.0483 respectively. Correspondingly, the current algebra relation $f_{\pi}^2 m_{\pi}^2 = m_q \langle \bar{\psi} \psi \rangle$ gives $f_{\pi} = 93.6$ and 107.8 MeV, respectively, in good agreement with the experimental value (≈ 95 MeV).

With numerical results like the ones we have presented it is very important to appraise the possible sources of error. We believe that the errors of purely statistical nature are reasonably under control. It is more difficult to evaluate the magnitude of the systematic errors. These can result from being still away from the continuum limit (ultraviolet effects), from the finite spatial volume (infrared effects) and from the limited propagation in time (which may produce overestimates of the masses). Insofar as ultraviolet and infrared effects are concerned, in absence of calculations done at larger β or with larger lattices, we can only notice that several independent investigations of different observables place $\beta = 6$ within, albeit at the very beginning, the asymptotic scaling region and that the total size of the lattice, $\approx (1.9 \text{ fm})^3$, should be large enough to contain the hadrons. About systematic effects due to the limited propagation in time, these may still be there; we found essential constancy (relative to the statistical errors) when we raised $t_{\min}$ to the maximum value considered, but the general trend of the masses is of a rather regular, albeit very slow, decrease for increasing $t_{\min}$. Thus, systematic errors of a few percent could easily be present because of finite time effects. Ultimately, it is again because of the statistical errors that such systematic errors

cannot be reduced beyond a certain point; indeed, if one makes $t_{\min}$ too large, the statistical fluctuations increase the degree of uncertainty in the evaluation of the parameters.

The comparison of our results with previous results obtained at similar values of β and on reasonably large lattices with Kogut-Susskind fermions^(11,12) reveal some discrepancies. We attribute these to systematic effects in the fits and extrapolations, more than to disagreements in the "raw" numerical data. For instance, the results for the masses of the 1^{--} mesons (which in both studies eventually determine the scale) in Ref. 11 are reasonably consistent with ours, where the ranges of quark masses overlap. But the calculations are done at rather large quark masses in Ref. 11 and our analysis shows how crucial it is to have data for very low quark masses. Indeed, being able to perform the calculations at very small m_q and to follow the propagation for very long extents of time appear to be the most relevant factors for the accuracy of the results.

Altogether, we find our results rather satisfactory. The dynamical breaking of chiral symmetry and the Goldstone nature of the π are well verified; the overall scale is in agreement with what can be inferred from the calculation of totally different observables; the dependence of the masses on the bare quark-mass values exhibit an interesting crossover from the heavy quark regime, where meson and baryon masses are in a ratio of approximately 2 to 3, to the light quark regime, where the mass of the nucleon becomes comparable to the masses of mesonic resonances. Also the absolute values of the hadron masses turn out compatible with the experimental values. But here one should not push the quest for agreement beyond a certain point. We must not forget that the calculation still relies on an approximation, the neglect of virtual $q-\bar{q}$ creation and

annihilation, which, although it may be justifiable, eventually must be felt. In our opinion, what is important at the present stage is being able to push the accuracy of the calculations done in the quenched approximation to the point that the numerical results may be considered with confidence as representative of the exact values of the observables in this interesting, simplified version of QCD. Our calculation, we believe, goes some way in this direction. It also shows that quenched calculations with an accuracy of few percent are well within the grasp of the supercomputers which will become available in the very near future.

Acknowledgements

This calculation required a very substantial amount of computer time on a CDC CYBER 205, which was granted by Control Data Corporation and ETA Systems, Inc. This is gratefully acknowledged with particular thanks to L.M. Thorndyke, B. Robertson and L.K. Steiner.

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Table Caption

Table 1. Values for masses and couplings as determined from the fits to the propagators. All dimensional quantities are in units of the appropriate power of the lattice spacing a . The parameters are defined as in Eqn. 16.

Figure Captions

- Fig. 1. Numerical results and fits to three typical propagators.
- Fig. 2a. Numerical result for the mass of the lowest π -like particle as function of the square root of the quark mass and interpolation.
- Fig. 2b. Numerical results for the particles with the quantum numbers of the $\pi(\Delta)$, $\rho(x)$, $S(0^{++})(\square)$, $A_1(1^{++})(+)$, nucleon (0) and $N_{1-}(\Delta)$ and interpolations.
- Fig. 3. Results for the quark condensate and interpolations, using all points $(-)$ and leaving out of the fit the point of $m_q a = 0.01$ $(---)$.

Table 1

type	m_q	t_m	A_1	m_1	A_2	m_2	$\tilde{A}_1$	$\tilde{m}_1$
$s^{(1)}$	0.5	2	1.341(25)	1.652(2)	0.218(76)	2.306(332)	-0.005(51)	2.832(121)
	0.16	2	0.983(29)	0.966(2)	0.906(53)	1.367(42)	-0.003(5)	7.077(2.174)
	0.09	2	0.771(21)	0.723(2)	1.382(50)	1.251(26)	-0.017(20)	1.539(65)
	0.04	2	0.551(20)	0.474(3)	1.500(88)	1.049(28)	-0.002(66)	6.240(203)
	0.01	2	0.595(29)	0.249(4)	1.964(68)	1.046(42)	0.003(4)	0.462(207)
$s^{(2)}$	0.5	1	2.160(119)	1.823(6)	0.705(95)	2.499(274)	-0.591(84)	2.293(101)
	0.16	1	0.590(62)	1.116(9)	2.223(78)	1.720(48)	-0.838(79)	1.694(62)
	0.09	1	0.332(34)	0.869(10)	2.354(75)	1.594(38)	-0.857(77)	1.520(55)
	0.04	1	0.192(28)	0.656(17)	2.371(70)	1.505(38)	-0.843(74)	1.378(51)
	0.01	1	0.120(41)	0.498(51)	2.370(76)	1.453(56)	-0.817(85)	1.303(67)
$s^{(2)}$	0.5	5	2.185(83)	1.824(5)	-	-	-0.110(114)	2.020(173)
	0.16	5	0.782(46)	1.138(6)	-	-	-0.040(38)	1.260(149)
	0.09	5	0.456(32)	0.897(8)	-	-	-0.062(48)	1.116(138)
	0.04	5	0.296(31)	0.704(14)	-	-	-0.123(104)	1.048(162)
	0.01	5	0.249(77)	0.606(57)	-	-	-0.248(752)	1.166(647)
$s^{(3)}$	0.5	5	1.363(93)	1.878(9)	-	-	-0.523(247)	2.012(80)
	0.16	5	0.322(35)	1.139(13)	-	-	-0.831(229)	1.441(55)
	0.09	5	0.170(25)	0.891(17)	-	-	-0.937(238)	1.275(53)
	0.04	5	0.060(13)	0.614(25)	-	-	-0.808(203)	1.093(51)
	0.01	5	0.024(11)	0.388(60)	-	-	-0.657(288)	0.942(95)
$s^{(4)}$	0.5	5	0.277(38)	1.826(17)	-	-	-0.480(90)	1.943(29)
	0.16	5	0.082(19)	1.060(22)	-	-	-0.898(63)	1.229(10)
	0.09	5	0.041(14)	0.776(29)	-	-	-0.962(65)	0.988(10)
	0.04	6	0.016(11)	0.483(60)	-	-	-0.987(144)	0.785(21)
	0.01	6	0.003(2)	0.178(53)	-	-	-1.588(571)	0.769(83)
b	0.5	5	0.095(7)	2.866(9)	-	-	-0.033(19)	3.012(95)
	0.16	7	$58(9) \cdot 10^{-4}$	1.781(15)	-	-	$-42(32) \cdot 10^{-4}$	1.936(88)
	0.09	7	$16(3) \cdot 10^{-4}$	1.381(14)	-	-	$-11(8) \cdot 10^{-4}$	1.502(76)
	0.04	7	$39(9) \cdot 10^{-5}$	0.999(23)	-	-	$-46(31) \cdot 10^{-5}$	1.158(80)
	0.01	7	$13(11) \cdot 10^{-5}$	0.728(95)	-	-	$-8(15) \cdot 10^{-5}$	0.763(227)

Fig.1

Fig. 2a

Fig. 2b

Fig. 3