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A GRAND UNIFIED THEORY BASED ON $SU(n,1)$ -MINIMAL SUPERGRAVITY

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ABSTRACT

We discuss a grand unified theory in the framework of $SU(n,1)$ minimal supergravity with the Planck mass as the only input mass scale.

$M_W \sim m_{3/2}$ is fixed by radiative corrections to be naturally $\ll M_{Pl}$. Due to the particular form of explicit soft supersymmetry breaking a light singlet can be used to obtain naturally light Higgs doublets and for a new mechanism for radiative $SU(2) \times U(1)$ breaking. The low energy particle spectrum is very restricted with $m_{3/2} \sim 10^4$ GeV.

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1. - INTRODUCTION

An outstanding problem in supergravity theories is a natural explanation of the magnitude of the mass splitting ΔM , which breaks supersymmetry in the effective low energy theory. Often one has $\Delta M \sim m_{3/2}$, the gravitino mass. Having no other fundamental scale, ΔM should be expected to be related to the natural scale $M_{Pl} = 2.4 \cdot 10^{18}$ GeV of gravity. However, ΔM has normally to be of the order of the weak scale and is an unexplained small scale ($\Delta M \sim 0(10^{-16})M_{Pl}$), which is put in by hand in supergravity models with canonical kinetic energies¹⁾ (we call these models canonical).

There is, however, also the possibility to couple matter to supergravity by replacing all terms in the Lagrangian of a globally supersymmetric renormalizable model by their locally supersymmetric counterpart²⁾. The analogous procedure in gauging internal symmetries is called minimal substitution, which is why we call the resulting models minimal.

In the special case of a minimal model with one complex scalar field and additional reflexion invariance of the potential, a vanishing potential with a sliding VEV of the scalar field and a sliding gravitino mass had been found to emerge³⁾. Requiring the result $V \equiv 0$, this model was independently derived⁴⁾ in disguise and used as a hidden sector in combination with a canonical sector^{5),6)}.

Especially, use was made of the sliding gravitino mass, which can be fixed by radiative corrections at a scale which is naturally much smaller than M_{Pl} . It was also observed that the kinetic energy of the sliding field was invariant under $SU(1,1)$ transformations and that the requirement that also the potential be invariant under $SU(1,1)$ yielded the vanishing potential. So the ad hoc assumption $V \equiv 0$ could be replaced by a symmetry requirement of the potential⁴⁾⁻⁶⁾. Unfortunately, one does not know how to build acceptable models with vanishing tree potentials, and moreover $SU(1,1)$ is no symmetry of the action - it is broken by the gravitino mass. So it is hard to draw conclusions from these symmetry arguments.

Actually, it turns out that one does not need any of these symmetry assumptions: the scale of supersymmetry breaking slides in minimal models irrespective of the symmetries of the potential (if the cosmological constant vanishes). Moreover, all scales slide; a scale $M_x \sim m^{1/3} M_{Pl}^{2/3}$ - to be identified with the scale of gauge unification - and the scale $m_{3/2}$ - about the weak scale - appear automatically in minimal models⁷⁾. Supersymmetry is broken by (F-type) mass

splitting among the scalars with $\Delta M^2 \sim m_{3/2}^2$ ^{7),8)}. This breaking is so soft that radiative corrections to the vacuum energy never exceed ΔM^4 ⁹⁾ and that light gauge singlet fields can be used for the sliding singlet mechanism ^{10),11)} yielding a new mechanism of radiative $SU(2) \times U(1)$ breaking ^{9),13)} without running into the problems occurring in canonical models ¹⁴⁾.

The particle spectrum of minimal models below the weak scale turns out to be tightly restricted because of the special supersymmetry breaking at work. It contains only one free parameter (which can be taken to be the Higgs mass) which is not yet fixed experimentally. For a range of this parameter the model is consistent with present day experiments and cosmological constraints. The gravitino mass is about 10^4 GeV ¹³⁾.

The kinetic energies of the complex scalar fields are invariant under non-linear transformations which form the group $SU(n,1)$ ^{15),7)} (n is the number of complex scalar fields). This group is the distorted version of the invariance group $U(n) \times C^n$ of canonical kinetic energies which results if gravity is switched off - in both cases one is free to choose the origin in the space of scalar fields and to perform rotations. A non-compact symmetry group of the kinetic energy is therefore not uncommon, nor is it a strong additional assumption: the $SU(n,1)$ transformations emerge naturally in minimal models. [Readers who are familiar with the conformal approach to supergravity can trace the non-compactness of the symmetry group also to a wrong sign kinetic energy of a compensator field ¹⁶⁾].

Minimal models differ from no-scale models ¹⁵⁾ by the supersymmetry breaking which is fed to the observable low energy sector. Ellis et al. ¹⁵⁾ restrict the set of variables which enter the superpotential and can then show that the breakdown of supersymmetry cannot make itself felt in the observable sector unless one introduces non-minimal and non-canonical kinetic energies for the gauge fields which then yield explicit gaugino masses. In particular Ellis et al. ¹⁵⁾ observe that these features are realized in higher extended ($N > 4$) supergravity. But $N > 4$ supergravity does not only justify non-canonical kinetic energies of the gauge fields, but also rules out the introduction of the much-wanted matter fields. So we prefer to justify our approach as the simplest possible extension of $N = 1$ supersymmetry and keep the covariant extension minimal. We introduce all variables into the superpotential and obtain (F-type) supersymmetry breaking in the observed sector. It should not come as a surprise that the resulting low

energy model then differs considerably from the no-scale model even though both models are of closely related origin.

In Section 2 we present an SU(5) model which makes use of the SU(n,1) structure. The mechanism for SU(2) $\times$ U(1) breaking and the natural generation of hierarchies is discussed in Section 3. Section 4 contains the low energy particle spectrum; issues connected with grand unification can be found in Section 5, followed by the conclusions in Section 6.

2. - THE MODEL

Minimal N = 1 supergravity theories are characterized through a kinetic function

$$K(z, z^*) = 1 - \frac{|z|^2}{3} \quad (1)$$

as a result of which the matter kinetic energies are invariant under non-linear (non-compact) SU(n,1)/SU(n) $\times$ U(1) transformations^{7),15)}. The scalar tree potential is given by^{3),17)}

$$V = K^{-2} \left\{ \sum |g_i|^2 - 3 \left| g - \frac{1}{3} z^i g_i \right|^2 \right\} + V_D \quad (2)$$

where V_D denotes the D-terms due to gauge interactions, $g(z_i)$ is the superpotential depending on the fields z_i , $i = 1, \dots, n$ and we have put $M_{Pl} = 2.4 \cdot 10^{18}$ GeV = 1.

Whenever supersymmetry is spontaneously broken by the VEV of the F-component of a chiral superfield y and the cosmological constant vanishes, the VEV of the scalar component of y is never fixed at tree level^{7),*)}. As a consequence the normalization of the kinetic energies and the couplings in the superpotential, which depend on $\langle y \rangle$, are not fixed at tree level.

In order to obtain the variation of the physical parameters of the theory with $\langle y \rangle$, we use non-linear transformations to redefine the field y such that its VEV vanishes⁷⁾

*) Subsequently we will denote chiral superfields and their scalar component by the same letter, the fermionic component will obtain a twiddle.

$$y' = \frac{y + \sqrt{3} \bar{a}}{1 + \frac{\bar{a}^* y}{\sqrt{3}}} \quad z' = z \frac{\sqrt{1 - |\bar{a}|^2}}{1 + \frac{\bar{a}^* y}{\sqrt{3}}} \quad \bar{a} = \frac{\langle y' \rangle}{\sqrt{3}} \quad (3)$$

If the superpotential is chosen such that it leads to spontaneous supersymmetry breaking, it cannot be invariant under (3). It transforms, however, in a very simple way if we introduce the variables $y_{\pm} = y / \sqrt{3} M_{Pl} \pm 1$ ⁹⁾ and write g as a sum of terms of the form

$$\alpha_{ab} z^{3-a-b} y_+^a y_-^b \quad (4)$$

where the mass dimensions of the couplings α_{ab} are $\dim(\alpha_{ab}) = a+b$. For supersymmetry breaking to be fed to the observable sector, both y_+ and y_- have to appear in the superpotential.

After a transformation (3) with $a = -1 + 2\varepsilon^2$, $\varepsilon \ll 1$ (which will be of interest subsequently), the new superpotential will differ from the old one just by a rescaling of the couplings $\alpha_{ab} \rightarrow \varepsilon^{a-b} \alpha_{ab}$. Due to this simple transformation property, terms of the form (4) are the natural building blocks of the superpotential.

The model we want to discuss is based on an SU(5) grand unifying gauge symmetry. The superpotential is composed of the sectors $g_a - g_d$ as follows:

$$g_a = \mu y_+^3 + \frac{N}{2} y_- \hat{B}^2 + M_1 y_+ \hat{B} \hat{A} \quad (5.a)$$

$y_{\pm}$ is defined as above with y a gauge singlet field. This part of the superpotential will lead to spontaneous SUSY breaking, since we will find $\langle g \rangle = \langle g_a \rangle = \mu \neq 0$ (with a vanishing tree level cosmological constant). The fields $\hat{A}$ and $\hat{B}$, transforming both as 24 under SU(5), will be the source of explicit global supersymmetry breaking, as we will discuss below.

For the sector responsible for the SU(5) gauge symmetry breaking we will use

$$g_b = a X \Sigma^2 + M_2 y_+ X \Sigma \quad (5.b)$$

with X and Σ transforming also as a $\underline{24}$. We choose this sector such that $\langle g_b \rangle = 0$ even though $\langle \Sigma \rangle \neq 0$ in order to allow for a small $m_{3/2}$ and large M_X^* .

The sector involving the standard matter fields F_i (transforming as $\underline{5}$, the generation index i runs from 1 to 3), T_i (transforming as $\underline{10}$), H and H' (transforming as $\underline{5}$ and $\bar{\underline{5}}$) is given by

$$g_c = H' (b \Sigma + f S) H + t_{ij}^d F_i T_j H' + t_{ij}^u T_i T_j H \quad (5.c)$$

where we have added couplings of the Higgs fields to Σ and a singlet S in order to obtain naturally light $SU(2)$ doublet and heavy $SU(3)$ triplet components of H and H' (10, 11).

In order to allow for a new mechanism for radiative $SU(2) \times U(1)$ breaking via a singlet tadpole graph and to maintain $\langle \hat{B} \rangle = \langle \hat{A} \rangle = 0$ (VEVs of $\hat{B}$ or $\hat{A}$ would lead to $SU(5)$ non-invariant explicit supersymmetry breaking in the low energy theory), we have to introduce two more fields U and R (transforming as $\underline{24}$) and the couplings

$$g_d = c \hat{A}^2 U + M_3 y_+ U R + H' (d \hat{A} + e U) H \quad (5.d)$$

In (5.a)-(5.d) an obvious $SU(5)$ index contraction is understood, the mass parameters $\mu^{1/3}$, N , M_1 , M_2 and M_3 are all supposed to be of $O(M_{Pl})$ and $a-f$, t_{ij}^d and t_{ij}^u denote dimensionless Yukawa coupling constants.

Plugging (5.a)-(5.d) into the scalar potential (2), one finds for a wide range of parameters (at least for $\mu^{1/3}$ small enough) an absolute minimum of V with a vanishing cosmological constant for vanishing VEVs of all fields except for y , the VEV of which is undetermined, and except for Σ . The VEV of Σ could leave $SU(5)$ unbroken or break $SU(5)$ to $SU(4) \times U(1)$ or to $SU(3) \times SU(2) \times U(1)$; we could either choose the latter vacuum by hand or else extend our model with the help of additional singlet fields à la Ellis-Ibáñez-Ross¹²⁾ such that

*) Below we will find $m_{3/2} \sim 10^4$ GeV. With $m_{3/2} = \langle g \rangle / M_{Pl}^2$, the relation $\langle g \rangle \sim M_X^3$ would imply an uncomfortably small value for M_X . This can be avoided as we will show in Section 5.

temperature effects in the early Universe select the desired vacuum automatically.

Now we will anticipate the fact that radiative corrections will fix the VEV of y to be very close to $-\sqrt{3}$ and redefine it according to (3). As noted above this leads to a rescaling of the couplings in g as follows:

$$\mu \rightarrow \epsilon^3 \mu \quad N \rightarrow N/\epsilon \quad M_i \rightarrow \epsilon M_i \quad (6)$$

The dimensionless Yukawa couplings remain invariant. We will discuss later how the parameter ϵ (which now replaces the undetermined VEV of y) is fixed by minimizing the effective potential including radiative corrections.

After this rescaling, the desired VEV of Σ is given by

$$\langle \Sigma \rangle = \frac{\epsilon}{a} M_2 \begin{pmatrix} 2 & & & \\ & 2 & & \\ & & 2 & \\ & & & -3 \\ & & & & -3 \end{pmatrix} \quad (7)$$

Supersymmetry is spontaneously broken as is evident from the non-vanishing gravitino mass:

$$m_{3/2} = \frac{\langle g \rangle}{M_{Pl}^2} = \frac{\epsilon^3 \mu}{M_{Pl}^2} \quad (8)$$

Explicit supersymmetry breaking in the effective low energy theory results from the terms $|g_{1y}|^2 - 3|g - \frac{1}{3} z_i g_i|^2 = \frac{2N}{3M_{Pl}^2} \text{Re}\{\hat{B}^{\dagger*2} [3\epsilon^2 \mu y_+^2 + M_1 \hat{A} \hat{B} + M_2 \chi \Sigma + M_3 \chi R]\}$ in the scalar potential:

$$|g_{1y}|^2 - 3|g - \frac{1}{3} z_i g_i|^2 = \frac{2N}{3M_{Pl}^2} \text{Re}\{\hat{B}^{\dagger*2} [3\epsilon^2 \mu y_+^2 + M_1 \hat{A} \hat{B} + M_2 \chi \Sigma + M_3 \chi R]\} \quad (9)$$

After diagonalizing the A, B mass matrix to leading order in ϵ with the help of

$$\hat{A} = A - \epsilon^2 \frac{M_1}{N} B \quad \hat{B} = B + \epsilon^2 \frac{M_1}{N} A \quad (10)$$

we can set $B \equiv 0$, since it has a mass $N/\epsilon \gg M_{Pl}$ (which we choose as a cut-off for the quantum effects) and is an approximate mass eigenstate. Inserting (10) into (9) and neglecting dimensionless couplings proportional to ϵ , we find that (9) gives rise to explicit supersymmetry breaking of the form

$$V_{\text{Breat}} = \Delta M^2 \text{Re } A^2 \quad \Delta M^2 = 2 \frac{M_1^2 \mu}{N M_{Pl}^2} \epsilon^6 \quad (11)$$

Disregarding non-renormalizable interactions involving inverse powers of M_{Pl} , the complete theory now looks like one with global supersymmetry, softly broken by (11), and a superpotential, which is given by (5.a)-(5.d) after the modifications

$$y_+ \rightarrow \epsilon \quad \hat{A} \rightarrow A \quad \frac{N}{2} y_- \hat{B}^2 + M_1 y_+ \hat{A} \hat{B} \rightarrow \frac{1}{2} M_A A^2 \quad (12)$$

$$\text{with} \quad M_A = \epsilon^3 M_1^2 / N$$

This procedure is discussed in more detail in Ref. 9). Note that from (8), (11) and (12) we obtain

$$m := \frac{\Delta M^2}{M_A^2} = 2 m_{3/2} \quad (13)$$

ΔM , M_A and $m_{3/2}$ scale with ϵ^3 while the SU(5) scale (7) varies with ϵ .

3. - RADIATIVE CORRECTIONS AND THE EFFECTIVE POTENTIAL

Now we turn to the radiative corrections to the effective potential; again we refer the reader for further details of the subsequent analysis to Ref. 9), where a similar locally supersymmetric standard model has been investigated.

To one loop, the Higgs fields H and H' obtain negative masses² given by (to leading order in $\Delta M^2/M_A^2$)

$$= \frac{d^2}{40\pi^2} \frac{\Delta M^8}{M_A^6} (|H|^2 + |H'|^2) \quad (14)$$

A similar graph gives negative masses to the scalar U fields, which will, however, play no role since the U fields have a huge ($\epsilon M_3 \gg \Delta M$) supersymmetric mass.

Also at one loop the gaugino receive masses m_1 [where $i = 1, 2, 3$ denotes the U(1), SU(2) and SU(3) subgroups of SU(5)] given by

$$m_i = \frac{5\alpha_i}{4\pi} m \left(* \frac{5}{3} \right) \quad (15)$$

where the last factor 5/3 occurs only in the U(1) case.

At two loops all scalars (except for gauge singlets) obtain positive masses² mediated by the gauge interactions U(1), SU(2) and SU(3)¹⁸⁾:

$$\begin{aligned} \text{U(1):} & \quad \frac{6}{25} m_1^2 q_H^2 \\ \text{SU(2):} & \quad \frac{3}{10} m_2^2 \delta_2 \\ \text{SU(3):} & \quad \frac{8}{15} m_3^2 \delta_3 \end{aligned} \quad (16)$$

Here q_H denotes the hypercharge and $\delta_2(\delta_3)$ are 1 for SU(2)-doublets [SU(3)-triplets] or zero otherwise.

All radiative corrections discussed up to now are ultra-violet finite. With F-type supersymmetry breaking as in (11), one finds from the analysis of Girardello and Grisaru¹⁹⁾ that the only logarithmically divergent contributions to the effective potential in this case are proportional to the F-component of gauge singlet fields. We introduced the part g_d of the superpotential such that we can draw the three-loop graph of Fig. 1, which gives a divergent contribution proportional to F_S . After replacing F_S by $fH'H$ through its equation of motion, this contribution to the effective potential reads

$$k \cdot (16\pi^2)^{-3} d^2 c e f^2 \Delta M^2 (Re H'H) \ln(\epsilon^2 M_3^2 / \Lambda^2) \quad (17)$$

Here k stands for a Feynman parameter integral of $O(1)$ and statistical and group theoretical factors; due to our computation of the latter we find k likely to be of $O(10^{-3})$, but its precise value will play no role subsequently. Inside the logarithm occurs the mass ϵM_3 of the field U , the heaviest field participating in

Fig. 1. For the cut-off Λ^2 we insert the Planck mass M_{Pl}^* .

There is another three-loop contribution to the effective potential which is potentially important in spite of its finiteness because it involves the large top Yukawa coupling constant t_{33}^u (called h_t subsequently). It involves two outgoing scalar Higgs fields (of the type H) coupled to two top quarks or two top squarks. Since the top squarks have received two-loop masses as given by (16), these two types of diagrams do not cancel^{(11), (12)}. For a top squark mass m_{st} much smaller than M the net contribution to the Higgs mass can be estimated to be [up to $O(m_{st}^2/M_A^2)$]^{(11), (12)}:

$$- \frac{3}{4\pi^2} h_t^2 m_{st}^2 \left(\ln \frac{M_A}{m_{st}} \right) |H|^2 \quad (18)$$

Now we are ready to discuss the scalar potential including the leading radiative corrections. Its tree part is given by

$$V_{tree} = \sum |F_i|^2 + V_D + \Delta M^2 \text{Re} A^2 \quad (19)$$

where V_D denotes the D-terms and the F_i are given by

$$\begin{aligned} F_A &= M_A A + c \{A U\} + d H H' \\ F_U &= c A^2 + \varepsilon M_3 R + e H H' \\ F_R &= \varepsilon M_3 U \\ F_\Sigma &= a \{X, \Sigma\} + \varepsilon M_2 X + b H H' \\ F_X &= a \Sigma^2 + \varepsilon M_2 \Sigma \\ F_{H'} &= (c A + b \Sigma + e U + f S) H \\ F_H &= H' (c A + b \Sigma + e U + f S) \\ F_S &= f H' H \end{aligned} \quad (20)$$

*) In order to avoid a similar tadpole graph with the fields U or R as outgoing lines, these fields have to be gauge non-singlets.

where we have used an obvious 5×5 matrix notation with $H(H')$ a column (row) vector and $M|$ denotes the traceless part $M - 1/5 \text{tr } M$ for any 5×5 matrix M .

Now we will look for VEVs which minimize the potential. We include the corrections (16), (17) and (18) and assume $\Delta M^2 \ll M_A^2$ such that (14) becomes negligible compared with (16). Due to the same inequality A retains a vanishing VEV.

Clearly (17) and (18) can contribute a negative amount to the vacuum energy if their negative contribution to the H, H' mass overcomes the positive contribution from (16). This will happen first for the $SU(2)$ doublets contained in H and H' , which do not feel the contribution involving the large $SU(3)_c$ coupling constant from (16).

The VEV of the $SU(2)$ doublets in H and H' and the VEV (7) for Σ with the contributions $|F_H|^2$ and $|F_{H'}|^2$ to V_{tree} sets the sliding singlet mechanism^{10),11)} in motion: in order to ensure $F_H = F_{H'} = 0$, the field S assumes a VEV given by

$$\langle S \rangle = \frac{3b}{af} \epsilon M_2 \quad (21)$$

which renders the Higgs doublets massless compared with the GUT scale $\sim \epsilon/a M_2$, but which induces a mass $5b \epsilon M_2/a$ for the coloured Higgs triplets. Minimizing V_{tree} for $\langle H'H \rangle \neq 0$, one finds that the VEVs of H and H' will be given by [up to $SU(2)$ rotations]

$$\langle H \rangle^T = (0, 0, 0, 0, h(1+\delta)) \quad H' = \pm (0, 0, 0, 0, h(1-\delta)) \quad (22)$$

with an obvious embedding of $SU(2)$ into $SU(5)$ according to the VEV of Σ (7) and with the relative sign between H and H' such that (17) becomes negative. With these VEVs for H and H' the fields A and R will assume small VEVs [and the VEV of Σ will slightly deviate from (7)] in order to minimize V_{tree} ; the resulting contribution of these VEVs to the effective potential as a function of h will, however, be small compared with the contributions we take into account [see the corresponding discussion in Ref. 9)].

The only remaining contribution from V_{tree} will be due to $|F_S|^2$; including (15)-(19) and the D-terms we end up with a potential of the form

$$V_{eff}(h, \delta, m) = f^2 h^4 (1 - \delta^2)^2 + p (1 + \delta^2) h^2 m^2 +$$

$$+ q m^2 f^2 h^2 (1 - \delta^2) \ln \frac{m^2}{M_{Pl}^2} + 2(g_1^2 + g_2^2) h^4 \delta^2 + r h^2 m^2 (1 + \delta)^2$$
(23)

where we have introduced the abbreviations

$$p = \frac{1}{4\pi^2} \left(\frac{25}{12} \alpha_1^2 + \frac{15}{4} \alpha_2^2 \right)$$

$$q = (16\pi^2)^{-3} \frac{k}{3} c d^2 e \frac{M_A^2}{\Delta M^2}$$
(24)

$$r = \frac{5}{8\pi^4} \alpha_3^2 h_t^2 \ln \frac{M_t}{m_{stop}}$$

which are ϵ -independent coefficients and replaced $\ln(\epsilon^2 M_3^2 / M_{Pl}^2)$ by $1/3 \ln(m^2 / M_{Pl}^2)$ in the third term. This potential has to be minimized with respect to h , δ and ϵ or equivalently, h , $\delta \cdot h$ and m . Neglecting the complications due to the non-vanishing δ and r for the moment, the potential has the form

$$V = f^2 h^4 + m^2 h^2 (p + q f^2 \ln m^2 / M_{Pl}^2)$$
(25)

which is minimal with respect to variations of h^2 and m^2 if

$$h^2 = \frac{q}{2} m^2$$

and

$$\ln \frac{m^2}{M_{Pl}^2} = -1 - \frac{p}{q f^2}$$
(26)

So a hierarchy $M_{Pl} = 0(10^{16}) \cdot m$ emerges if $p \sim 70 q f^2$ and h is comparable to m^9 .

We now include the effects from non-vanishing r and δ . The extremal condition for h (fixed $\delta \cdot h$ and m) allows us to express q in terms of the remaining parameters

$$9f^2 \ln \frac{m^2}{M_{PL}^2} = r(1+\delta) - \rho - 2\delta \frac{h^2}{m^2} (g_1^2 + g_2^2) - 2f^2 \frac{h^2}{m^2} (1-\delta^2) \quad (27)$$

The value of $h^2(1+\delta^2)$ is fixed experimentally by the W-boson mass

$$M_W^2 = g_2^2 h^2 (1+\delta^2) \quad (28)$$

Minimizing with respect to $\delta \cdot h$ determines δ and h by (27) and (28)

$$0 = 2\delta \left(\frac{g_1^2 + g_2^2}{g_2^2} M_W^2 + (p-r)m^2 \right) - r(1+\delta^2) \quad (29)$$

Finally, the potential has also to be minimal with respect to m ! The extremal condition can be solved for the coupling constant f

$$f^2 = - \frac{\delta^2}{1-\delta^2} (g_1^2 + g_2^2) + \frac{m^2}{2h^2(1-\delta^2)} \left\{ -\rho + r(1+\delta) + \right. \\ \left. + \frac{\ln \frac{m}{M_{PL}}}{\ln \frac{m}{M_{PL}} + \frac{1}{2}} \frac{1}{1-\delta^2} \left((p-r)(1+\delta^2) - 2r\delta \right) \right\} \quad (30)$$

In this form of the extremal conditions all parameters, which are essential for the light particle spectrum, are dependent only on the scale m and the experimental input which (at the weak scale) we take to be

$$\begin{aligned} M_W &= 82 \text{ GeV} \\ \sin^2 \theta_W &= 0.23 \\ \alpha_2 &= 0.034 \\ \alpha_3 &= 0.12 \\ m_{top} &= 40 \text{ GeV} \end{aligned} \quad (31)$$

To be more precise, the Yukawa coupling h_t , which enters the coefficient r (24), is related to the top mass by

$$h_t^2 = \frac{m_{top}^2}{h^2(1+\delta)^2} = g_2^2 \frac{m_{top}^2}{M_W^2} \frac{1+\delta^2}{(1+\delta)^2} \quad (32)$$

So, if one holds the top mass fixed, there is a dependence of the coefficient r on δ and (29) is no longer a quadratic equation for δ . One has also to take into account that the coupling constants h_t and α_3 which enter the coefficient r are to be evaluated at M_A rather than at M_W . This leads to

$$r = \frac{5}{8\pi^4} \frac{g_2^2 m_{top}^2}{M_W^2} \frac{1+\delta^2}{(1+\delta)^2} \frac{\alpha_3^2(M_W)}{\left(1 + \frac{3}{2\pi} \alpha_3(M_W) \ln \frac{M_A}{M_W}\right)^2} \ln \frac{M_A}{M_{stop}} \quad (24a)$$

We are quite particular about the value of the small parameter r because it is essential that r does not exceed a critical value. Note that for $r > p/2$ a VEV of H would occur even without the tadpole diagram, but then V_{eff} would be unbounded from below as a function of m^2 ($\delta = 1$). The reason is that the top contribution r to V_{eff} is ultra-violet finite and r does not depend logarithmically on m . Hence, if the negative contribution r from the top diagram to the mass² of H dominates the positive contribution p from the gauge interactions, this will hold true for arbitrarily large values of m . The tadpole graph, on the other hand, acts differently. It depends logarithmically on m and dominates the positive contribution p only for $\ln M_{Pl}^2/m^2$ sufficiently large, hence for $m \ll M_{Pl}$. So, for a hierarchy to emerge and for the potential to be bounded from below, it is essential that r be not too large. Inspecting the three-loop graphs which give the top contribution r , we find that the most important contributions to the integrals are due to virtual momenta of $O(M_A)$. Accordingly we choose M_A as the argument of the running coupling constant α_3 and h_t . Taking this into account, r never gets close to the critical value where the potential becomes unbounded in m^2 . Moreover, its dependence on M_A is negligible. If the top mass, however, was 55 GeV, a range $5.8 \text{ TeV} < m < 9.5 \text{ TeV}$ would become excluded and for $m_{top} = 65 \text{ GeV}$ the ranges of m allowed from stability requirements and from experimental constraints (see below) would be disjunctive. So for our minimal model, it is essential that the top quark is not too heavy (and the strong interactions not

too strong) in contrast to canonical models which, in their simplest versions, need heavy top quarks to trigger the breakdown of $SU(2) \times U(1)$.

Before we exhibit the light particle spectrum, we want to comment on some radiative corrections neglected so far. There are finite contributions to the effective potential linear and bilinear in the scalar S field which could destabilize the presently assumed vacuum. These are sufficiently small not to cause any trouble, as has been discussed in detail in the similar scenario in Ref. 9). We note at this point that problems associated with light singlets in sugra-GUTs with canonical kinetic energies^{1),14)} do not arise here. The F-type breaking is not as large as $m_{3/2} \cdot M_X$ and explicit supersymmetry breaking in the form of positive definite scalar masses² or gaugino masses [corresponding to dimensionless spurion fields in superspace¹⁹⁾] does not exist.

There are potentially dangerous (finite) diagrams linear in the D-component of the $U(1)$ vector superfield which could cause negative masses² for squarks and sleptons¹⁸⁾. However, since we have F-type splitting in an adjoint representation of $SU(5)$, these diagrams occur starting at the three-loop level only and are negligible¹⁸⁾.

There are contributions to the vacuum energy with no external fields at all, but with two F-type supersymmetry breaking insertions and hence ΔM^4 . Higher loop diagrams of this type will also depend logarithmically on VEVs like $\langle E \rangle$ and $\langle S \rangle$. We have to assume that these contributions to the vacuum energy are cancelled by some additional presently unknown (gravitational?) effects which we need anyhow in order to cancel the negative vacuum energy present at the minimum of V_{eff} in (24): in terms of ϵ and M_{Pl} , this vacuum energy is $O(\epsilon^{12} M_{Pl}^4) \sim \Delta M^4$ [see Eq. (11)], hence it is consistent to assume that a cancellation of the contributions mentioned before goes hand in hand with the cancellation of the cosmological constant⁹⁾.

4. - LIGHT MASSES

Let us now discuss the masses of the light particles. Quarks, leptons and W- and Z-bosons obtain their masses after the Higgs fields H and H' get their VEV just as in the usual standard model. The masses of the Higgs fields and the sliding singlet S can be obtained by expansion of the potential (19) and (23) around the VEVs (22).

A complex charged Higgs boson is eaten by the W-bosons which thereby acquires its mass. The remaining charged Higgs is approximately $(1+\delta^2)^{-1/2}((1+\delta)H_4 - (1-\delta)H_4^*)$; it is the scalar partner of the W-boson and degenerate in mass if supersymmetry is exact: we therefore denote it by sW . Its mass is computed as

$$m_{sW}^2 = M_W^2 + (\rho - r) m^2 \quad (33)$$

The complex neutral scalar field S acquires a mass

$$m_S = \sqrt{2} \frac{f}{g_2} M_W \quad (34)$$

One neutral scalar field $(m(1+\delta^2)^{-1/2}((1+\delta)H_5 - (1-\delta)H_5^*))$ is eaten by the Z-boson, the real part is the scalar partner of the Z-boson and denoted sZ analogously. In the mass matrix it mixes with the combination $(1/\sqrt{1+\delta^2})\text{Re}((1-\delta)H_5 + (1+\delta)H_5^*)$ which is the Higgs particle in the standard model and therefore called the standard Higgs or H for short. In the basis $\text{Re } H_5, \text{Re } H_5^*$ the mass² matrix has the matrix elements

$$\begin{aligned} M_{11}^2 &= (g_1^2 + g_2^2) h^2 \left(\delta + \frac{1}{2} (1+\delta)^2 \right) + f^2 h^2 (1-\delta)^2 + \left(\frac{\rho}{2} - r \right) m^2 \\ M_{12}^2 &= M_{21}^2 = -\rho m^2 + (2f^2 - 2g_2^2 - g_1^2) h^2 (1-\delta^2) - 2h^2 \delta^2 (g_1^2 + g_2^2) - r \delta m^2 \\ M_{22}^2 &= (g_1^2 + g_2^2) h^2 \left(-\delta + \frac{1}{2} (1-\delta)^2 \right) + f^2 h^2 (1+\delta)^2 + \frac{\rho}{2} m^2 \end{aligned} \quad (35)$$

The masses are obtained by numerical determination of the eigenvalues of this matrix. The mass of the remaining imaginary field, called $\text{Im } H$, follows as

$$M_{\text{Im } H}^2 = 2 \frac{f^2}{g_2^2} M_W^2 + (\rho - r) m^2 \quad (36)$$

The masses of squarks and sleptons are raised above the quark and lepton masses by the radiative corrections given in (16) and by D-terms which contribute for non-vanishing δ . Including this to first order in δ and neglecting the quark and lepton masses, the masses² of squarks and sleptons are given by

$$m_{sq,sl}^2 = \frac{6}{25} m_1^2 g_H^2 + \frac{3}{10} m_2^2 \delta^2 + \frac{8}{15} m_3^2 \delta^2 + 2\delta h^2 (g_1^2 g_H^2 - g_2^2 I_3) \quad (37)$$

with q_H , δ_2 and δ_3 as defined after (16) and m_1 given by (15) and (31). By now the masses of all light scalars are given.

The charged fermionic partners of the Higgs fields and the W-bosons form two Dirac fields. Because of supersymmetry breaking they are not degenerate with the W-boson and the sW scalar, but split into light and heavy charged Dirac fields. Consequently we call them $\tilde{W}_{1,h}$. Their masses are

$$m_{\tilde{W}_{1,h}} = \left(\frac{m_2^2}{4} + g_2^2 h^2 (1 - \delta^2) \right)^{1/2} \pm \frac{m_2}{2} \quad (38)$$

There are five neutral fermionic partners of B, W_3 , H_5 , H_5^c and S. $\tilde{S}$ together with the fermionic partner $\tilde{H}$ of the standard Higgs form two degenerate Majorana fermions which have the same mass as the scalar S (34)

$$m_{\tilde{S}, \tilde{H}} = \sqrt{2} \frac{f}{g_2} M_W \quad (39)$$

The three remaining neutral fermions are mixed by a mass matrix with the non-vanishing elements

$$\begin{aligned} M_{11} &= m_2 \\ M_{13} &= M_{31} = g_2 h (1 + \delta) \\ M_{22} &= m_1 \\ M_{23} &= M_{32} = g_1 h (1 - \delta) \end{aligned} \quad (40)$$

m_2 and m_1 are given by (15). The resulting masses are obtained by numerical evaluation of the eigenvalues of the mass matrix. Because supersymmetry is broken, the two Majorana fermions which are partners to the Z-boson and the sZ-scalar are not degenerate in mass but split into a light and a heavy component called $\tilde{Z}_{1,h}$. The third Majorana field is the photino, if supersymmetry is exact, or the bino, for large m . We denote it by $\tilde{\gamma}/\tilde{B}$.

By now all light masses are given and can be evaluated as functions of $m = 2m_{3/2}$. The result is plotted in Figs. 2a-c.

Let us consider the lightest charged partners of the standard particles, the scalar partner of the right handed electron se_R , and the light fermionic partner of the W-boson $\tilde{W}_1$. Neither of them have been observed at PEP or PETRA²⁰⁾, so their mass should be larger than 21 GeV. This restricts m to the interval

$$5.8 \text{ TeV} < m < 22 \text{ TeV} \quad (41)$$

Furthermore $\tilde{W}_1$ should be heavier than $\tilde{H}-\tilde{S}$, the lightest neutral partner into which it can decay. This has to be required since we do not observe charged relics from the big bang with masses below 1 TeV²¹⁾. Demanding $m_{\tilde{W}_1} > m_{\tilde{H}-\tilde{S}}$ decreases the upper bound on m

$$m < 19.4 \text{ TeV} \quad (42)$$

If the top mass or the strong coupling constant had been larger than the numerical values (31) which we used, this would have decreased $m_{\tilde{H}-\tilde{S}}$ and we would have been left with the bound (41). This bound restricts the gravitino mass to values which are consistent with cosmological constraints²²⁾ derived from the effect of a heavy gravitino on nucleosynthesis.

Note that the lightest charged partner is always lighter than 38 GeV, so e^+e^- machines will either confirm or rule out the considered class of models once they reach the energy 2×38 GeV necessary for pair production.

The mass of the standard Higgs $Re(H_5 + H_5^*)$ is fixed in terms of m . Turning the argument around, we could equally well use the Higgs mass as a remaining undetermined parameter of the low energy sector. So this sector contains no more parameters than the standard model, but quite a lot more particles. These are not excluded by present-day experiments, but some of them could be ruled out or (hopefully) confirmed in the near future.

We emphasize that this spectrum is independent of the GUT sector g_b and the sector g_d of the superpotential [the parameters of the latter only occur in q (24) which, in turn, is fixed through (27)]. The only important ingredients are the F-type supersymmetry breaking in a 24 of SU(5) and the fixing of the sliding mass scales by V_{eff} (30). Therefore, the spectrum will survive possible

modifications of the theory which involve the sectors beyond the minimal supersymmetric standard model.

In the Table we have given the masses of the supersymmetric partners of the standard particles for a gravitino mass of 3.5, 6.5 and 9.5 TeV. With these masses, the experimental signature of the model in the near future will most likely be restricted to $\tilde{W}_1$ production in W and Z decays as discussed in Ref. 23). This type of event still remains to be found; since in our model gluinos and squarks with masses < 200 GeV are incompatible with the PETRA limit²⁰⁾, these particles are not available in order to explain the unconventional $p\bar{p}$ collider events²⁴⁾.

One might worry whether the rate of pair annihilation of the neutral $\tilde{S}$ Dirac fermions (the lightest sparticle) is sufficiently large such that their number density does not conflict with the bound on the mass density of the Universe²¹⁾. However, in comparison with the similar case considered in Ref. 21) their annihilation rate is enhanced by a contribution involving the $H'SH$ vertex and the absence of p-wave suppression for Majorana fermions. Furthermore, $\langle H \rangle^T - \langle H' \rangle \neq 0$ as a result of the top contribution to the effective potential, so annihilation via a Z becomes possible.

5. - GRAND UNIFICATION SECTOR

Let us now put aside low energy physics and turn to grand unification. In order to compute the GUT scale $M_X \sim \langle \Sigma \rangle$ [with $\langle \Sigma \rangle$ as in (7)], we have to know the parameter ϵ which can be found from (8):

$$\epsilon = \frac{3}{7} \sqrt{\frac{m_{3/2} M_{Pl}^2}{\mu}} \quad (43)$$

Both in order to have a stable tree potential and in order to have $\Delta M^2/M_A^2 \ll 1$ (which we have used before), we should choose $\mu^{1/3} < M_{Pl}$ (without introducing a hierarchy by hand at this point, however); let us put, e.g., $\mu^{1/3} = M_{Pl}/10$. then, with $m_{3/2}$ restricted by (41), we find $\epsilon \sim 10^{-4}$. With $M_2 = M_{Pl}$ and a Yukawa coupling a of $10^{-2} - 10^{-3}$, the GUT scale $M_X \sim \langle \Sigma \rangle$ becomes 10^{16-17} GeV, not far from M_{Pl} itself, but leading to a still acceptable (one loop) value for $\sin^2 \theta_w$ of ~ 0.23 [and $\alpha_{QCD}(M_X) \sim 0.12$] and still consistent with constraints from proton decay data²⁵⁾.

We have not yet discussed the mass M_A of the 24 representation under SU(5) which carries the F-type supersymmetry breaking mass splitting ΔM , but practically no SU(5) breaking (and hence does not affect the computation of $\sin^2\theta_W$). Due to the unknown parameters μ , M_1 and N in (11) and (12), it is difficult to pin down ΔM and M_A themselves. We only have a lower bound on M_A coming from the requirement of perturbative grand unification. Demanding, e.g., $\alpha_{\text{GUT}}(M_X) < 0.2$ leads to $M_A > 10^6$ GeV (with M_X and $\sin^2\theta_W$ as before), which is easily fulfilled as it is seen with $\epsilon = 10^{-4}$ and $M_1 = M = M_{P\bar{L}}$, resulting in $M_A \sim 7 \cdot 10^6$ GeV and $\Delta M \sim 3 \cdot 10^5$ GeV. Let us finally remark that with $M_2 = M_3 = M_{P\bar{L}}$ the masses of U, R, X and Σ become $\sim 10^{14}$ GeV.

6. - CONCLUSIONS

We conclude by repeating the essential features of the model. The class of $N = 1$ supergravity theories considered here allows one to obtain a scale of supersymmetry breaking and $SU(2) \times U(1)$ breaking $\ll M_{P\bar{L}}$ naturally. The explicit supersymmetry breaking of the F-type in the effective low energy theory allows for the existence of a light gauge singlet field which plays an important role in the new mechanism of radiative $SU(2) \times U(1)$ breaking presented here. The same singlet field can be used for the elegant sliding singlet mechanism resulting in natural Higgs triplet/doublet mass splitting. Since the same effective potential fixes the scales of $SU(2) \times U(1)$ as well as supersymmetry breaking (which are undetermined at tree level), the resulting low energy particle spectrum is very constrained irrespective of many details of the model. This will make this class of theories easy to distinguish from others.

The gravitino mass is restricted to be $\sim 10^4$ GeV consistent with cosmological bounds²²⁾.

Besides its advantageous features the model presented here has unsatisfactory aspects: there are radiative contributions to the vacuum energy (roughly of the same order of magnitude as the ones taken into account) which we had to neglect together with gravitational radiative corrections. This problem might be related to the one of the cosmological constant to which the quantum effects discussed here still contribute an amount $\sim M_W^4$ (after starting with zero at tree level).

The unification sector introduces a multitude of fields to satisfy a small number of requirements: breakdown of SU(5), radiative generation of finite gaugino masses, and generation of the logarithmically divergent Mercedes star

diagram without unwanted competing diagrams. The superpotential is not natural insofar as many possible interactions are simply omitted. Hopefully, additional symmetry will teach us how to obtain a simpler and completely natural unification sector. As long as the breakdown of supersymmetry is fed to the low energy sector as discussed in the paper, one will, however, have difficulty in distinguishing experimentally between the proposed model and the more elegant ones which are, hopefully, to come.

Common to all these models are the experimental tests which they have to pass in the foreseeable future: it is an exciting realization that minimal supergravity models are either confirmed or ruled out if one is able to separate the predicted new physics from the background in collider experiments or, probably more conclusive, if e^+e^- machines reach 2×40 GeV energies.

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Table

Parameters and Particle Masses (in GeV)
for $m = 7, 13$ and 19 TeV

r	$1.3 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$
f	$4.2 \cdot 10^{-2}$	$8.3 \cdot 10^{-2}$	$12.8 \cdot 10^{-2}$
δ	$2.4 \cdot 10^{-2}$	$3.9 \cdot 10^{-2}$	$4.4 \cdot 10^{-2}$
Gravitino	3500	6500	9500
$\tilde{Z}_1$	62	46	36
$\tilde{Z}_h$	149	212	284
$\tilde{\gamma}/\tilde{B}$	55	97	137
Gluino	334	621	907
$\tilde{H}-\tilde{S}$	7.5	14.7	22.7
$\tilde{W}_1$	47	32	23.9
$\tilde{W}_h$	141	208	281
s_W	108	155	210
s_Z	117	162	215
s_q	244	453	663
se_L	52	96	141
se_R	25	44	64
H	7.9	15.8	24
S	7.5	14.7	22.7
$\text{Im } H$	71	132	195

FIGURE CAPTIONS

Fig. 1: Supergraph leading to a logarithmic divergence proportional to F_S .

Fig. 2: Particle masses as a function of the SUSY breaking scale $m = 2m_{3/2}$ (in GeV).

Fig. 1

Fig. 2a

Fig. 2b

Fig. 2c