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EFFECTS OF THE RANDOM NUMBER GENERATOR
ON COMPUTER SIMULATIONS

Giorgio Parisi
II Universita' di Roma, Tor Vergata
Roma

and

Federico Rapuano

CERN -- Geneva

A B S T R A C T

We have measured the susceptibility of a three-dimensional Ising system in a box of 24^3 size. Our results do not agree within four standard deviations with the previous result obtained with a special-purpose machine. The origin of the discrepancy is due, in our opinion, to the different random number.

The possibility of doing careful experiments on the Ising model has made a qualitative progress with the introduction of special-purpose machines [1]. These machines open up the possibility of a careful determination of critical exponents, by Monte Carlo [2], using finite-size scaling techniques. Unfortunately the results of [1] have been disappointing because of the absence of an early onset of the asymptotic regime as a function of the size L . In particular the data could be fitted with an asymptotic expansion in powers of L^{-1} only for large L (i.e. $L > 30$) and some structure in the data was observed at $L=24$. As stressed by the authors of [1], the random number generator is a possible source of systematic errors (only pseudo-random numbers can be generated by computer) and one can only hope that "...the deviations from randomness are sufficiently complex and obscure that they do not affect our results" [1].

Motivated by this situation, we have set up a Monte Carlo program for the 3-dimensional Ising model with helicoidal boundary conditions as in [1]. We have measured the expectation value $\langle M^2 \rangle$, M being the total magnetization, and the renormalized coupling constant g_R defined as

$$g_R = \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2} - 3$$

Our results at the critical temperature $\beta = .221650$ and the corresponding results of [1] are shown in tables 1a and 1b for $L=8$ and $L=24$. The errors are unbiased statistical errors. We have done the simulation on eight systems in parallel. The first 40,000 iterations have been discarded, and the total length of the run was about 900,000 iterations for each system, corresponding to about 2 months of VAX-11/750 CPU time. The data for $\langle M^2 \rangle$ and g_R are shown in fig 1 with the data of [1]. We have a good agreement at $L=8$, while at $L=24$ our result for $\langle M^2 \rangle$ is four standard deviations out and for g_R is two standard deviations out. In our opinion, the only reasonable source of this discrepancy is the different random number generator used for the calculation. Before describing the algorithm, we note that our point is on a smooth interpolation of the results obtained on smaller lattices, suggesting that the early onset of asymptotic scaling may be related to the random number used. The algorithm that is used in both experiments is based on the feedback register random number method [3]

$$R(i) = R(i - \Delta_1) + R(i - \Delta_2)$$

In [1], R was a single bit and the sum was done modulo 2 ($\Delta_1=97$, $\Delta_2=127$). In our case R was a 32 bit fixed-point number in the range 0-1 and the sum was done modulo 1 ($\Delta_1=24$, $\Delta_2=57$). Although two random numbers may be uncorrelated, in both methods the correlation of three numbers is very strong. We partially alleviated this problem by using as the actual random number

$$R_e(i) = \text{EOR}(R(i), R(i-61))$$

where the EOR is the bitwise exclusive-or [4]. We can only hope that this trick hides the order-three correlation in the sequence and that our method produces a less correlated string than in [1]. Our result strongly suggests that any high accuracy machine should be equipped with different random number algorithms, to be eventually mixed in order to test the independence of the result from the algorithm. Generally speaking, we note that an algorithm like the congruence method, as used for example in the Vax/VMS RAN function, which has relatively strong short-range correlations, but does not have long-range correlations, should be effective in computing universal quantities like critical exponents as this kind of effect could be reabsorbed in a redefinition of the Hamiltonian.

REFERENCES

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g_r		
size	our data	ref [1]
8	-1.447(2)	-1.4469(3)
24	-1.414(3)	-1.4206(7)

Table 1a
Renormalized coupling constant. Only the values of ref[1] for which
a comparison was possible are shown.

$\chi \cdot L^{-1.96}$		
size	our data	ref [1]
8	1.4942(40)	1.4919(8)
24	1.5398(43)	1.5567(10)

Table 1b
Same as table 1a for the magnetic susceptibility.