CERN-TH.4143/85

LIGHT HIGGS BOSONS AND SUPERSYMMETRY

John Ellis, K. Enqvist and D.V. Nanopoulos
CERN - Geneva

and

S. Ritz
University of Wisconsin, Madison

ABSTRACT

QCD radiative corrections cause a large reduction in the $T \rightarrow H^0 + \gamma$ decay rate which clouds the interpretation of experimental upper limits on this decay. Electroweak radiative corrections make an important contribution to the lightest Higgs mass in models with supersymmetry broken at low energies. Calculating the Higgs mass and the T decay branching ratio incorporating these two effects, we find that a light supersymmetric Higgs may be lurking just beyond the sensitivity of present experiments on T radiative decays. This Higgs boson may be produced in association with the next-to-lightest Higgs in Z^0 decays at a large rate and with a distinctive signature.

CERN-TH.4143
April 1985

Many particle physicists now place the mechanism of spontaneous gauge symmetry breaking at the top of their agenda of problems to be solved. In particular, searches for the putative Higgs boson or bosons H are now becoming¹⁾ very topical. Experimentally, searches²⁾ for monochromatic photons in quarkonium decay are now approaching the sensitivity required to detect $T \rightarrow H + \gamma$ ³⁾ in the Standard Model, and the opportunities for extensive Higgs searches at the SLC and LEP are fast approaching. Theoretically, ideas which make elementary Higgses respectable by stabilizing their masses using supersymmetry⁴⁾ enable one to make increasingly specific predictions for their masses, couplings and production rates. In this paper we re-examine the phenomenology of light neutral Higgs bosons in general, and of those expected in supersymmetric theories in particular.

First we reconsider the radiative decay $T \rightarrow H + \gamma$. Two approaches suggest large reductions in the branching ratio for this decay mode $B(T \rightarrow H + \gamma)$, namely a calculation⁵⁾ of perturbative QCD radiative corrections and estimates of effects due to mixing⁶⁾ with P-wave bottomonium states. We show how these two effects can be combined phenomenologically without double counting, and present numerical estimates of the total reduction in $B(T \rightarrow H + \gamma)$. Typically we find that $B(T \rightarrow H + \gamma)$ is reduced by a factor $O(2)$, indicating that the ongoing searches for monochromatic photons in T decay will have to become somewhat more sensitive before $m_H < m_T$ can be excluded.

Next we reconsider the mass to be expected for the lightest neutral Higgs in supersymmetric models which contain squarks and/or gluinos light enough to be detected^{7),8)} at the CERN $p\bar{p}$ collider. Radiative corrections⁹⁾ to the light Higgs mass are often larger than its tree-level value, reflecting the closet Coleman-Weinberg nature of this neutral Higgs scalar H . Its mass is generally < 10 GeV, making it vulnerable to early exposure in T decays.

Finally, we point out that in many supersymmetric models^{9),10)} of the type introduced above, the second lightest neutral Higgs is a pseudoscalar closet axion which is light enough to be produced in Z^0 decay, and the decay $Z^0 \rightarrow a + H$ (closet axion + closet Coleman-Weinberg) is likely to have a large branching ratio. Such decays would have a clear event signature: $a \rightarrow b\bar{b}$ jets accompanied by $H \rightarrow c\bar{c}$ or more distinctively $H \rightarrow \tau^+\tau^-$.

We start with our re-evaluation of the branching ratio for $T \rightarrow H + \gamma$. Vysotsky⁵⁾ has calculated the perturbative QCD radiative corrections to $\Gamma(T \rightarrow H + \gamma)$ and found them to be large:

$$\frac{\Gamma(\Upsilon \rightarrow H + \gamma)}{\Gamma_0(\Upsilon \rightarrow H + \gamma)} = 1 - \frac{4\alpha_s}{3\pi} a(K) \quad (1)$$

with

$$R_0(\Upsilon \rightarrow H + \gamma) \equiv \frac{\Gamma_0(\Upsilon \rightarrow H + \gamma)}{\Gamma_0(\Upsilon \rightarrow e^+e^-)} = \frac{G_F m_q^2}{\sqrt{2}\pi\alpha} (1 - K) \quad (2)$$

and $K \equiv m_H^2 / m_T^2$, with $a(K)$ a mathematically non-trivial function which takes values between 10 (for $K = 0$), 15 (for $K = 0.87$) and 20 (for $K = 0.94$). The function $a(K)$ exhibits a singularity as $K \rightarrow 1$ which corresponds to $k_\gamma \rightarrow 0$ in $T \rightarrow H + \gamma$ decay:

$$a(K) \xrightarrow{K \rightarrow 1} a_s(K) \equiv \frac{4\pi}{3\sqrt{1-K}} \quad (3)$$

We interpret this singularity as a first order manifestation of binding effects. Note that $a_s(K)$ is not negligible even at $K = 0$: $a_s(K) = 4.19$. There is also a known correction¹¹⁾ to $\Gamma(T \rightarrow e^+e^-)$:

$$\frac{\Gamma(\Upsilon \rightarrow e^+e^-)}{\Gamma_0(\Upsilon \rightarrow e^+e^-)} = 1 - \frac{16\alpha_s}{3\pi} \quad (4)$$

so that the overall correction to $R(T \rightarrow H + \gamma) \equiv \Gamma(T \rightarrow H + \gamma) / \Gamma(T \rightarrow e^+e^-)$ can be written in the forms

$$\frac{R(\Upsilon \rightarrow H + \gamma)}{R_0(\Upsilon \rightarrow H + \gamma)} = 1 - \frac{\alpha_s}{\pi} \left(\frac{4}{3} a(K) - \frac{16}{3} \right) \quad (5a)$$

or

$$\frac{1 - \frac{4\alpha_s}{3\pi} a(K)}{1 - \frac{16\alpha_s}{3\pi}} \quad (5b)$$

which are in principle equivalent at leading order in α_s . In practice, formulae (5a) and (5b) exhibit a significant numerical difference which reflects higher order effects.

Several authors have recently evaluated the effects of mixing with P-wave bottomonia on $\Gamma(T \rightarrow H + \gamma)$. They find a wave-function model-dependent correction

$$\frac{\Gamma(\Upsilon \rightarrow H + \gamma)}{\Gamma_0(\Upsilon \rightarrow H + \gamma)} = 1 + \mathcal{O}(1/k_\gamma) \quad (6)$$

for large k_γ . There is no singularity in $\Gamma(T \rightarrow H + \gamma)$ as $k_\gamma \rightarrow 0$, since a full treatment of binding effects smooths out the rate as $K \rightarrow 1$.

The perturbative and mixing corrections to $\Gamma(T \rightarrow H + \gamma)$ share the common features that they are largest when $K \rightarrow 0(1)$ ($k_\gamma \rightarrow 0$), and both have a piece $\propto 1/k_\gamma$ (at small k_γ for perturbative QCD, at large k_γ for mixing). These $1/k_\gamma$ pieces have consistent normalizations within the theoretical uncertainties. We wish to construct a phenomenological form including all these corrections to $R(T \rightarrow H + \gamma)$ which does not double-count. We do this in two alternative ways. One is (A) to use (5a) for $m_H < \bar{m}$, and for $m_H > \bar{m}$ use a phenomenological parametrization of the functional form of the mixing corrections⁶⁾, normalized so that $R(T \rightarrow H + \gamma)$ is continuous at $m_H = \bar{m}$. The other method (B) is to use (5b) for $m_H < \bar{m}$ and to use for $m_H > \bar{m}$ the same phenomenological parametrization of the functional form of the mixing corrections, but now normalized so that $\Gamma(T \rightarrow H + \gamma)$ ^{1),6)} is continuous at $m_H = \bar{m}$, and then divided by the $T \rightarrow e^+e^-$ correction factor (4).

These are two alternative patch-ups which seem to avoid double-counting and are equivalent to leading order in α_s . While they both give continuous curves for $R(T \rightarrow H + \gamma)$, their derivatives exhibit an unsightly jump at m_0 which we remove by Gaussian smoothing. Our final phenomenological corrections to $R(T \rightarrow H + \gamma)$ are insensitive to $\bar{m}$ in the range of 5 to 7 GeV. Fig. 1 shows the leading order

and the corrected ratios $R(T \rightarrow H + \gamma) \equiv \Gamma(T \rightarrow H + \gamma)/\Gamma(T \rightarrow e^+e^-)$ calculated using prescriptions (A) and (B) described above. We see that they give similar results, and that both reduce the expected $R(T \rightarrow H + \gamma)$ by a factor $O(2)$. Under these circumstances, we cannot exclude the possibility that higher order corrections might also be important and suppress $T \rightarrow H + \gamma$ decay even further. However, Fig. 1 means that monochromatic photon searches²⁾ must be at least twice as sensitive as was suggested by the zeroth order formula (2) before light Higgses can be observed or excluded in $T \rightarrow H + \gamma$ decay.

After the above model-independent analysis, we now turn to the model-dependent predictions for the light Higgs masses in supersymmetric theories. The effective scalar potential for neutral Higgses in such models is

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 [H_1 H_2 + \text{h.c.}] + \left(\frac{g_2^2 + g'^2}{8} \right) [|H_1|^2 - |H_2|^2]^2 + \delta V \quad (7)$$

where δV contains the radiative corrections⁹⁾. The physical spectrum contains¹²⁾ two neutrals $H_{P,K}$ ^{*} with masses

$$m_{H_{P,K}}^2 = \frac{1}{2} \left[(m_1^2 + m_2^2 + m_z^2) \pm \sqrt{(m_1^2 + m_2^2 + m_z^2)^2 - 4(m_1^2 + m_2^2)m_z^2 \cos^2 2\theta} \right] \quad (8)$$

at the tree level and one with mass

$$m_a^2 = 2m_3^2 / \sin 2\theta \quad (9)$$

where $\tan \theta \equiv \langle 0 | H_2 | 0 \rangle / \langle 0 | H_1 | 0 \rangle$ and

$$\cos^2 2\theta = 1 - \frac{4m_3^4}{(m_1^2 + m_2^2)^2} \quad (10)$$

*) The subscript P stands for painava (heavy in Finnish), while K stands for kevyt (light in Finnish).

We work in the context of minimal supergravity models in which radiative corrections due to a heavy t quark drive weak gauge symmetry breaking⁹⁾, for which $\langle 0 | H_2 | \rangle \sim \langle 0 | H_1 | 0 \rangle$ and $\cos 2\theta \ll 1$, so that $m_{H_K}^2$ (8) is small and corrections to $m_{H_K}^2$ become important. In this framework

$$m_1^2 - m_2^2 = \frac{3r^2}{1+6r^2} [m_Q^2 + m_U^2 + m_{\tilde{L}}^2 + m_0^2 A^2] \quad (11a)$$

$$m_1^2 + m_2^2 = 2m_4^2 + 2m_{\tilde{L}}^2 - \frac{3r^2}{1+6r^2} [m_Q^2 + m_U^2 + m_{\tilde{L}}^2 + m_0^2 A^2] \quad (11b)$$

$$m_3^2 = B m_4 m_0 \quad (11c)$$

where

$$m_4 = \frac{1}{2} \frac{B m_0}{\sin 2\theta} + \frac{1}{2} \left\{ \frac{B^2 m_0^2}{\sin^2 2\theta} - \frac{4}{1+6r^2} \left[m_{\tilde{L}}^2 - \frac{3r^2}{2} (m_Q^2 + m_U^2 + m_0^2 A^2 - 3m_{\tilde{L}}^2) \right] \right\}^{1/2} \quad (11d)$$

The parameter r^2 is related to the top quark mass by

$$r^2 = \frac{(m_t/m_W)^2}{N - 6(m_t/m_W)^2} \quad (11e)$$

where N is a parameter which is calculable from the renormalization group (see Table 1), and the spin-zero sparticle masses

$$m_{Q,U,\tilde{L}}^2 = m_0^2 (1 + C_{Q,U,L} \xi^2) \quad (11f)$$

with the coefficients $C_{Q,U,L}$ given in Table 1. The parameter m_0 is a model-dependent scalar mass which may (or may not) be identified with the gravitino

mass $m_{3/2}$, $A = A(\mu)$ is a model-dependent parameter expected to be $O(1)$ with $A(m_X) \equiv A_X = 3$ a theoretically favoured value. We take $B = A - 1$ before renormalization, and after renormalization

$$\begin{aligned} A &= (A_X + C_A \xi) / (1 + br^2) \\ B &= (A_X - 1 + C_B \xi) - 3r^2 A \end{aligned} \quad (11g)$$

where C_A and C_B are given in Table 1, and ξ is the primordial gaugino/scalar mass ratio $m_{1/2}/m_0$ which determines the gluino mass

$$m_{\tilde{g}} \approx \frac{\alpha_3(\mu)}{\alpha_3(M_X)} \xi m_0 \approx 2.8 \xi m_0 \quad (11h)$$

Our general strategy for solving the Equations (11) and hence evaluating the Higgs boson masses (8,9) is (i) assume values of m_t and $m_{\tilde{q}}$ motivated by experiment, and supplement these assumptions with a suitable (likely) choice of the dimensionless parameters A_X, ξ . Now $r, M_0, B, m_{\tilde{\chi}_L}$ and hence m_1^2, m_2^2 are determined and we can (ii) solve (10), (11c) and (11d) iteratively to extract m_3, m_4 and θ . Finally we can (iii) calculate $m_{H_{P,K}}^2$ (8) and m_a (9). We know from previous experience¹⁰⁾ that consistent solutions for $m_t = O(40)$ GeV and $m_{\tilde{q}} = O(60)$ GeV exist for $(\xi, A_X) \sim (0.7, -2.5)$ and $(1, 3)$. Figure 2 shows the dependence of m_{H_K} on m_t for fixed $m_{\tilde{q}}, \xi, A_X$ and on $m_{\tilde{q}}$ for fixed m_t, ξ, A_X : typically m_{H_K} is a few GeV¹²⁾. Figure 3 shows the dependence of m_a on ξ and A_X for fixed $m_t = 40$ GeV and $m_{\tilde{q}} = 60$ GeV: often $m_a \lesssim O(m_Z)$. We have found that in the region of interest, m_a is insensitive to the value of m_t but increases approximately linearly with $m_{\tilde{q}}$ at fixed ξ, A_X .

So far we have only discussed the tree level contributions to Higgs scalar masses, but the smallness of m_{H_K} (8) in Fig. 2 suggests that radiative corrections⁹⁾ from δV in Eq. (7) may be important, although they are not important for m_{H_P} or m_a . They contribute⁹⁾

$$\begin{aligned} \delta m_H^2 = & - \frac{6\alpha_2}{4\pi} (M_2^2 + m_4^2 - M_2 m_4 - \frac{1}{2} B m_0 m_4) \\ & - \frac{6\alpha_1}{20\pi} (M_1^2 + m_4^2 - M_1 m_4 - \frac{1}{2} B m_0 m_4) \\ & + \frac{3h_t^2}{(4\pi)^2} \left[(C_Q + C_U - 2C_L) \xi^2 m_0^2 - 2(m_4^2 - B m_0 m_4) + (A m_0 - m_4)^2 \right] \end{aligned} \quad (12)$$

where M_2 (M_1) is the mass of the SU(2) [$U(1)_Y$] gaugino: $M_1 = \alpha_1(\mu)/\alpha_1(m_X) \xi m_0$ and $h_t = g_2(m_t/m_W)$ is the top quark Yukawa coupling. This expression is clearly more important for larger m_q as seen in Fig. 2, which also exhibits the dependence of δm_H^2 on m_t . The total light Higgs mass is

$$m_H^2 = m_{H_K}^2 + \delta m_H^2 \quad (13)$$

which is also plotted in Fig. 2. Clearly the radiative corrections δm_H^2 (12) make the dominant contribution to m_H^2 in most of the domain of interest, making it a closet Coleman-Weinberg Higgs boson.

It is this total light Higgs mass m_H (13) which is plotted below m_a (9) in Fig. 3, as a function of ξ and A_X for fixed $m_t = 40$ GeV and $m_q = 60$ GeV. It is clear from Figs. 2 and 3 that in general m_H is a few GeV, so that the closet Coleman-Weinberg Higgs boson is accessible in $T \rightarrow H + \gamma$ decay. Because θ given by Eq. (10) is close to $\pi/4$, it has similar couplings to quarks and leptons as does a conventional minimal Weinberg-Salam Higgs boson¹³⁾:

$$g_{H\bar{q}_{2/3} q_{2/3}} = g_{H\bar{q}_{2/3} q_{2/3}} \Big|_{WS} \times \frac{\cos \alpha}{\sin \theta} \quad (14)$$

$$g_{H\bar{q}_{-1/3} q_{-1/3}}, g_{H\bar{l}l} = g_{H\bar{q}_{-1/3} q_{-1/3}}, g_{H\bar{l}l} \Big|_{WS} \times \frac{\sin \alpha}{\cos \theta}$$

where α is fixed by¹³⁾

$$\tan 2\alpha = \tan 2\theta \left(\frac{m_a^2 + m_z^2}{m_a^2 - m_z^2} \right) \quad (15)$$

and $\alpha \sim \theta \sim 45^\circ + 0(2^\circ)$ in the domain of parameter space of interest in this paper. Thus the rate for $T \rightarrow H + \gamma$ and the H decay modes are expected to be almost indistinguishable from those of a conventional light Weinberg-Salam Higgs boson. The model-independent analysis of $T \rightarrow H + \gamma$ decay earlier in this paper therefore suggests that the experimental noose may be tightening around the neck of this closet Coleman-Weinberg Higgs: see Fig. 1 where the range of m_H from Fig. 3 is indicated. Note also that toponium $\theta \rightarrow H + \gamma$ and $\theta \rightarrow a + \gamma$ could both be observable if $m_\theta \sim 80$ GeV.

Finally we turn to $Z^0 \rightarrow a + H$ decay. The relevant decay vertex is¹³⁾

$$g_{Z^0 a H} = g_2 \frac{\cos(\alpha - \theta)}{2 \cos \theta_W} (p_a + p_H)^\mu \quad (16)$$

which has no unpleasant suppression factor if $\alpha \sim \theta \sim \pi/4$ as we expect. The vertex (16) gives a cross-section

$$\sigma(s) = \frac{\pi \alpha^2}{3s} \lambda^{3/2} \left(1, \frac{m_{H,K}^2}{s}, \frac{m_a^2}{s} \right) \times \frac{[(1 - 4 \sin^2 \theta_W)^2 + 1] \cos^2(\alpha - \theta)}{64 \sin^4 \theta_W \cos^4 \theta_W} \left[\frac{s^2}{(s - m_z^2)^2 + m_z^2 \Gamma^2} \right] \quad (17)$$

where λ is the conventional phase space function. Figure 4 exhibits the cross-section (17) at its peak, modified by initial state bremsstrahlung corrections¹⁴⁾. Rather than use the input theoretical parameters m_0 and $m_{\frac{1}{2}} = \xi m_0$, we have labelled the axes in Fig. 4 by the physical observables m_q (11f) and m_g (11h). The contours correspond (from right to left) to $\sigma = 10^{-3}$, 5×10^{-3} , 10^{-2} , 5×10^{-2} , 10^{-1} and 5×10^{-1} nb: for comparison, $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 1.0$ nb at the Z^0 peak. We see that if the squarks and gluinos have similar masses

and are light enough to be produced at the CERN $p\bar{p}$ collider, the Z^0 may well have an observably large branching ratio into $a + H$. For the larger values of m_a which occur when $m_q \gg m_g$, the maximum of $\sigma(e^+e^- \rightarrow Z^0 \rightarrow a + H)$ does not occur on the Z^0 peak itself, but at some higher centre-of-mass energy. Table 2 shows for representative choices of m_g and m_q the corresponding values of m_H and m_a , $R(T \rightarrow H + \gamma)$ calculated in leading order (L) and in higher order using method (B), and the radiatively corrected σ and $\sqrt{s}$ at peak.

Associated production of $a + H$ in Z^0 decay or elsewhere should have a distinctive experimental signature. The closet axion a is heavy enough to decay into $b\bar{b}$ (of interest to precision vertex detectors) but not heavy enough to decay into $t\bar{t}$, while the light closet Coleman-Weinberg Higgs mostly decays into $c\bar{c}$ or $\tau^+\tau^-$ (vertex detectors again, also with a distinct topology). A final state containing a larger mass $b\bar{b}$ jet pair and a lower mass $\tau^+\tau^-$ pair should be easy to find.

We have seen in this paper that a light neutral Higgs may lurk just beyond the sensitivity of present $T \rightarrow H + \gamma$ searches, that a light Higgs is expected in supersymmetric models with a light sparticle spectrum accessible to the CERN $p\bar{p}$ collider, and that such models suggest a large $Z^0 \rightarrow a + H$ decay branching ratio with a distinctive signature. Perhaps the mechanism of spontaneous gauge symmetry breaking will soon be elucidated.

ACKNOWLEDGEMENTS

We would like to thank Haimo Zoernig for checking some of the calculations in this paper. One of us (K.E.) gratefully acknowledges the financial support of the Academy of Finland, and one of us (S.R.) thanks the CERN Theoretical Physics Division for its hospitality.

C_Q	6.65
C_U	6.23
C_D	6.18
C_L	0.52
C_E	0.15
C_A	3.84
C_B	0.59
C_H	2.55
C_4	1.40
N	23.49

Table 1: Values of renormalization parameters

The values for various parameters renormalized at m_W entering into calculation of the low-energy mass spectrum [(for definitions, see Ref. 9)]. These values have been obtained by taking $\alpha_{em}(M_W) = 1/127.5$, $\alpha_3(M_W) = 0.122$ and $\sin^2\theta_W(M_W) = 0.22$ with $m_Z = 93.8$ GeV.

$m_{\tilde{g}}$	$m_{\tilde{q}}$	m_H	m_a	R(T $\rightarrow$ H + γ) ($\times 10^{-3}$)		σ_{peak} (nb)	$\sqrt{s}$ at peak (GeV)
				(L)	(B)		
40	40	3.5	49	6.9	3.4	0.35	94.1
	50	4.7	90	6.1	2.9	-	-
	60	5.5	121	5.4	2.5	-	-
	70	6.0	148	4.8	2.2	-	-
50	50	4.7	61	6.1	2.9	0.17	94.1
	60	5.8	105	5.0	2.3	-	-
	70	6.5	137	4.2	1.8	-	-
60	60	5.9	74	4.9	2.2	0.046	94.2
	70	7.0	118	3.6	1.5	-	-
70	70	7.2	86	3.4	1.3	8.3×10^{-4}	96.6

Table 2: Predictions for different $m_{\tilde{g}}$, $m_{\tilde{q}}$ values

All values are calculated with $A_X = 3$, $m_t = 40$ GeV. In cases where σ_{peak} and $\sqrt{s}$ at the peak are not tabulated, the peak cross-section occurs above $\sqrt{s} = 150$ GeV and $\sigma_{\text{peak}} < 10^{-3}$ nb. The angle factors (14, 15) which increase R(T $\rightarrow$ H + γ) by 2 to 7% are not included in the above Table.

REFERENCES

- 1) J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106 (1976) 292;
L. Okun, Proc. of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, ed. W. Pfeil, Universität Bonn (1981), p. 1118.
- 2) CUSB Collaboration, S. Youssef et al., Phys. Lett. 139B (1984) 332;
P. Franzini, Talk presented at the Fifth Moriond Workshop on Heavy Quarks, Flavour Mixing and CP-violation, January 1985;
Argus collaboration, M. Albrecht et al., DESY preprint 85-014 (1985).
- 3) F.A. Wilczek, Phys. Rev. Lett. 39 (1977) 1304.
- 4) For recent reviews, see:
J. Ellis, CERN preprint TH.3802 (1984);
D.V. Nanopoulos, CERN preprint TH.3995 (1984);
H.P. Nilles, Phys. Rep. C110 (1984) 2.
- 5) M.I. Vysotsky, Phys. Lett. 97B (1980) 159.
- 6) H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B16 (1979) 493;
J. Ellis, M.K. Gaillard, D.V. Nanopoulos and C.T. Sachrajda, Phys. Lett. 83B (1979) 339;
J. Pantaleone, M. Peskin and S.-H. Tye, SLAC preprint PUB-3439 (1984);
J. Polchinski, S.R. Sharpe and T. Barnes, Harvard preprint HUTP-84/A064 (1984);
J.P. Jackson and J.L. Rosner, CERN preprint TH.3992 (1984);
H.J. Lipkin, Phys. Lett. 151B (1985) 155;
W. Bernreuther and W. Wetzel, Univ. Heidelberg preprint HD-THEP-85-2 (1985).
- 7) J. Ellis and H. Kowalski, Phys. Lett. 142B (1984) 441; Nucl. Phys. B246 (1984) 189; CERN preprint TH.4072 (1984).
- 8) E. Reya and D.P. Roy, Phys. Lett. 141B (1984) 442; Phys. Rev. Lett. 52 (1984) 881; Dortmund preprint (1984).
- 9) J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K.A. Tamvakis, Phys. Lett. 115B (1983) 275;
C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quiros, Nucl. Phys. B226 (1984) 438.
- 10) J. Ellis and M. Sher, Phys. Lett. 148B (1984) 309;
L.E. Ibañez, C. Lopez and C. Muñoz, CERN preprint TH.4071 (1984);
K. Enqvist, A.B. Lahanas and D.V. Nanopoulos, CERN preprint TH.4095 (1984).
- 11) R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt, Phys. Lett. 97B (1979) 499.
- 12) K. Inoue et al., Prog. Theor. Phys. 68 (1982) 927; *ibid.* 71 (1984) 413;
M. Drees, M. Glück and K. Grassie, Univ. Dortmund preprint DO-TH85/4.
- 13) see also:
N. Deshpande, X. Tata and D. Dicus, Phys. Rev. D29 (1984) 1527;
H.E. Haber and J.F. Gunion, SLAC preprint SLAC-PUB-3404 (1984).
- 14) F.A. Berends and R. Kleiss, Leiden University preprint (1984).

FIGURE CAPTIONS

Fig. 1 $R(T \rightarrow H + \gamma) \equiv \Gamma(T \rightarrow H + \gamma)/\Gamma(T \rightarrow e^+e^-)$ plotted as a function of m_H , calculated using (L) the leading order formula (2), and incorporating first order radiative and binding corrections according to the prescriptions (A) and (B) described in the text. The difference between these two latter curves is a measure of higher order certainties. Also shown is the preferred range of light supersymmetric Higgs masses.

Fig. 2 Tree level (m_{H_K}) and radiative correction (δm_H) contributions to the total light Higgs mass m_H . Values are shown for typical¹⁰⁾ values of $(\xi, A_X) = (0.7, -2.5)$ and $(1.0, +3.0)$, as functions of m_t (for $m_{\tilde{q}}$ fixed at 60 GeV) and of $m_{\tilde{q}}$ (for m_t fixed at 40 GeV).

Fig. 3 Values of the closet axion and light Higgs masses for typical values of $m_t = 40$ GeV, $m_{\tilde{q}} = 60$ GeV as functions of ξ and A_X in the preferred domains of Ref. 10).

Fig. 4 Contours of the radiatively corrected¹⁴⁾ cross-section (17) for $e^+e^- \rightarrow a + H$ at its peak for ranges of values of $m_{\tilde{g}}$ and $m_{\tilde{q}}$ and two selected values of A_X : see also Table 2. We use $m_{Z^0} = 93.8$ GeV and $\Gamma_{Z^0} = 3.0$ GeV. The contours from right to left are for $\sigma_{\text{peak}} = 10^{-3}, 5 \times 10^{-3}, 10^{-2}, 5 \times 10^{-2}, 10^{-1}$ and 5×10^{-1} nb respectively.

Fig. 1

Fig. 2

Fig. 3

Fig. 4