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g-2 IN SPONTANEOUSLY BROKEN SUPERGRAVITY *)

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ABSTRACT

We calculate the one-loop corrections to the anomalous magnetic moment of a lepton in (spontaneously) broken $N = 1$ supergravity. The result is in general divergent. However, it can be made finite by requiring a massless photino or an involved relation among different couplings. In particular, if the Kähler metric G_n^m is constant, this relation is trivially satisfied and the one-loop calculation is finite. In this case, we use the finite part to set an order of magnitude lower limit on the gravitino mass.

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Recently, it was shown explicitly that the anomalous magnetic moment of a lepton $(g-2)_\ell$ is zero at one loop in unbroken supergravity¹⁾. This result has been confirmed using supergraphs²⁾ and also component fields³⁾. The absence of the anomalous magnetic moment term was not a surprise because supersymmetry forbids such a term⁴⁾. In fact, supersymmetry can be used to explain why the graviton corrections themselves are finite⁵⁾, once it is realized that the gravitino corrections must be finite due to an effective chiral symmetry³⁾.

In this paper we calculate the one-loop corrections to $(g-2)_\ell$ in spontaneously broken $N = 1$ supergravity (in the general non-minimal case). The result is finite^{*}) if the photino mass is zero or if a more involved relation among different couplings is satisfied. In that case the corresponding finite part can be used to set an order of magnitude lower bound on the gravitino mass⁷⁾, assuming that the non-renormalizability of the theory does not make such a limit meaningless.

We first derive the necessary vertices for spontaneously broken supergravity. We then study the one-loop finiteness of $(g-2)_\ell$. Finally, a lower limit on the gravitino mass is obtained.

For definiteness we will concentrate on the simplest case, that with three chiral, one vector and one Weyl (super)multiplets. The three chiral multiplets will have charges 0, Q_ℓ and $-Q_\ell$, corresponding to the Goldstino χ_1 and the two chiralities of a charged lepton $\chi_{2,3}$ respectively, and their scalar partners $z_{1,2,3}$. The vector multiplet will contain the photon A_μ and the photino λ , whereas the graviton $e_{a\mu}$ and the gravitino ψ_μ will be in the Weyl multiplet. All our results can be extended and applied to a more realistic case with extra matter.

The general $N = 1$ supergravity Lagrangian for such a system is well known⁸⁾. For calculating the corrections to $(g-2)_\ell$ to order $\kappa^2 = 8\pi G_N$ where G_N is Newton's constant, we must consider the following kinetic, mass and interaction terms [the κ dependence is not explicit and a linear supersymmetry transformation parameter was used⁸⁾]

^{*}) A similar calculation but with explicit breaking of supergravity has been done⁶⁾. There, explicit supersymmetry breaking mass terms were introduced. The result is finite only for a massless photino.

$$\mathcal{L}_K = \langle G_n^m \rangle D_\mu z_m D^\mu z^{*n} - \frac{1}{4} \text{Re} \langle f \rangle F_{\mu\nu} F^{\mu\nu} + \{ \langle G_j^k \rangle \bar{\chi}_{Lk} \not{D} \chi_R^j - \frac{1}{4} \text{Re} \langle f \rangle \bar{\lambda} \not{D} \lambda - \frac{1}{4} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\eta \varepsilon^{\mu\nu\rho\eta} + \text{h.c.} \}, \quad (1.a)$$

$$\begin{aligned} \mathcal{L}_M = & \frac{1}{2} \langle e^{-G} \left(G^{11} \frac{G_1}{G^1} + G_{11} \frac{G^1}{G_1} - G^{11} G_1^{-11} G_{11} + G^1 G_{11}^{-11} G_1 + G^1_{11} \right) \rangle z_1 z^{*1} \\ & + \frac{1}{2} \langle e^{-G} \left(G^{111} G_1^{-11} G_1 + 2 G^{11} G_1^{-111} G_1 + 2 G^{11} + G^1 G_1^{-111} G_1^1 + G^1 G_1^{-1111} G_1 \right) \rangle z_1 z_1 \\ & + \frac{1}{2} \langle e^{-G} \left(G^{\ell k} G_\ell^{-11} G_{pj} + G^{\ell k} G_\ell^{-11} G_1 + G^1 G_\ell^{-11k} G_{1j} + G^1 G_{1j}^{-11k} G_1 + G_j^k \right) \rangle z_k z^{*j} \\ & + \frac{1}{2} \langle e^{-G} \left(G^{1kj} G_1^{-11} G_1 + G^{\ell k} G_\ell^{-11j} G_1 + G^{\ell j} G_\ell^{-11k} G_1 + G^1 G_1^{-11kj} G_1 + G^1 G_1^{-11k} G_\ell^j \right. \\ & \left. + 2 G^{kj} \right) \rangle z_k z_j + \langle e^{-\frac{G}{2}} (G^{kj} - G^1 G_1^{-11} G_1^{kj}) \rangle \bar{\chi}_{Lk} \chi_{Lj} + \\ & \frac{1}{4} \langle f^1 e^{-\frac{G}{2}} G_1 G_1^{-11} \rangle \bar{\lambda}_L \lambda_L + \langle e^{-\frac{G}{2}} \rangle \bar{\Psi}_{\mu R} \varepsilon^{\mu\nu} \Psi_{\nu R} + \text{h.c.}, \end{aligned} \quad (1.b)$$

$$\begin{aligned} \mathcal{L}_I = & -\frac{1}{4} \text{Re} (\langle f^1 \rangle z_1) F_{\mu\nu} F^{\mu\nu} + \frac{i}{4} \text{Im} (\langle f^1 \rangle z_1) F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & + \{ (\langle G_j^{k1} \rangle z_1 + \langle G_{j1}^k \rangle z^{*1}) \bar{\chi}_{Lk} \not{D} \chi_R^j - (\langle G_j^{k1} \rangle + \frac{1}{2} \langle G_j^k G^1 \rangle) \bar{\chi}_{Lk} \not{D} z_1 \chi_R^j \\ & + (\langle e^{-\frac{G}{2}} [-\frac{1}{2} (G^{kj} - G^1 G_1^{-11} G_1^{kj}) + \frac{G^{kj1}}{G^1} + \frac{G_{1j}^k}{G_1} - G_1^{-11} G_1^{kj1}] G^1 \rangle z_1 \\ & + \langle e^{-\frac{G}{2}} [-\frac{1}{2} (G^{kj} - G^1 G_1^{-11} G_1^{kj}) - G_{11}^{-11} G_1^{kj} \frac{G^1}{G_1} - G_1^{-11} G_{11}^{kj} \frac{G^1}{G_1}] G_1 \rangle z^{*1}) \\ & \bar{\chi}_{Lk} \chi_{Lj} - \bar{\Psi}_R \gamma (\langle e^{-\frac{G}{2}} G^{kj} \rangle z_j + \langle e^{-\frac{G}{2}} G_j^k \rangle z^{*j}) \chi_{Lk} \\ & - 2ig \langle G_\ell^k \rangle T_j^\ell z^{*j} \bar{\lambda}_L \chi_{Lk} + \frac{1}{4} \text{Re} \langle f \rangle \bar{\lambda} \gamma^\mu \varepsilon^{\rho\eta} \Psi_\mu F_{\rho\eta} \\ & - \langle G_k^j \rangle \bar{\Psi}_{\mu L} \not{D} z^{*k} \gamma^\mu \chi_{Lj} + \text{h.c.} \}, \end{aligned} \quad (1.c)$$

where $m, n = 1, 2, 3$ and $k, j = 2, 3$ and $\langle \rangle$ means the value at the minimum. We assume that supergravity is spontaneously broken but charge is conserved. The cosmological constant is also assumed to vanish at the minimum. The gauge invariant functions G and f in (1) are real and analytic respectively, but otherwise arbitrary. Without loss of generality we can take $\langle G_n^m \rangle = -\frac{1}{2}\delta_n^m$, and $\text{Re} \langle f \rangle = 1$, which assures canonical kinetic terms. The different masses are in general arbitrary: the gravitino mass $m_{3/2} = \langle e^{-G/2} \rangle$, the photino mass $m_\lambda = -\frac{1}{2}\langle f^1 e^{-G/2} G_1 G_I^{-11} \rangle$ (in the minimal case $\langle f^1 \rangle = 0$ and $m_\lambda = 0$) and the lepton mass $m_\lambda = -2\langle e^{-G/2} (G^{23} - G^1 G_I^{-11} G_I^{23}) \rangle$. The masses of the sleptons and the neutral scalars result from diagonalizing the corresponding mass matrices in (1.b). The interactions, apart from the fermion-scalar-gravitino Yukawa term

$$- \bar{\Psi}_R \gamma \left(\langle e^{-G/2} G^{kj} \rangle z_j + \langle e^{-G/2} G^k_j \rangle z^{*j} \right) \chi_{Lk}, \quad (2)$$

and new interactions involving the neutral scalar partners of the Goldstino, are the same as in the unbroken case.

The fermion-scalar-gravitino Yukawa term for unbroken supergravity corresponds to $\langle e^{-G/2} G^{23} \rangle = -\frac{1}{2}m_\lambda$ and $\langle e^{-G/2} G^k_j \rangle = 0$ in (2), which together with arbitrary $m_{3/2}$, m_λ and slepton masses implies explicit breaking of supergravity^{6),7)}. In that case $(g-2)_\lambda$ is finite only for $m_\lambda = 0$. In general, for spontaneously broken supergravity $\langle e^{-G/2} G^{23} \rangle = -\frac{1}{2}m$, with $m \neq m_\lambda$, and $\langle e^{-G/2} G^k_j \rangle = m_{3/2} \langle G^k_j \rangle$. Then the contributions to $(g-2)_\lambda$ not involving the neutral scalars are finite only if $m_\lambda = 0$ ($\langle f^1 \rangle = 0$) or $m = m_\lambda$ ($\langle G_I^{23} \rangle = 0$). The term proportional to $m_{3/2}$ in Eq. (2) cancels the divergent contribution of the case with explicit breaking⁶⁾ regardless of the photino mass. However, the new mass parameter m when different from m_λ generates a new divergent term unless the photino is kept massless. On the other hand, the new contributions to $(g-2)_\lambda$ involving the neutral scalars are divergent (see below) unless

$$\langle f^1 (G_{11}^{-11} G_1^{23} + G_1^{-11} G_{11}^{23}) G^1 \rangle = 0, \quad (3)$$

which requires a massless photino ($\langle f^1 \rangle = 0$) or that the Yukawa coupling $z^{*1} \bar{\chi}_{Lk} \chi_{Lj}$ be only proportional to the lepton mass ($m' = 2 \langle e^{-G/2} (G_I^{-11} G_I^{kj} +$

$G_I^{-11} G_I^{kj} \frac{G^1}{G_1} > 0$ [see Eq. (1.c)]^{*}). Let us make explicit the results of our calculation before discussing further the conditions for a finite $(g-2)_\lambda$.

The one-loop corrections to $(g-2)_\lambda$ are of order g^2 when a photon or a photino is interchanged and of order κ^2 when a graviton, gravitino or neutral scalar is interchanged. The former have been calculated by several authors⁹⁾ in the case of broken supersymmetry (massive photino). We are interested in the order κ^2 corrections. The graviton contributions are the same as in the unbroken supergravity case¹⁾ as gravity is still exact. Then, we have to evaluate only the new corrections due to a massive gravitino and to the neutral scalars. There are five diagrams involving the gravitino and three diagrams involving the neutral scalars (see Fig. 1). The corresponding Feynman rules are given in Fig. 2 where we have changed to standard conventions^{6),10)}, and for convenience a factor i has been absorbed in the photino $\tilde{\gamma}$ and gravitino $\tilde{g}$ fields. We have also used the fact that the cosmological constant vanishes at the minimum, $\langle G^1 G_1 \rangle = 3/2$. $\tilde{\ell}_1 = (\cos\alpha z_2 + \sin\alpha z_3^*)/\sqrt{2}$ and $\tilde{\ell}_2 = (-\sin\alpha z_2 + \cos\alpha z_3^*)/\sqrt{2}$ are the two mass eigenstate sleptons, with masses $m_{\tilde{\ell}_{1,2}}$ and couplings $a_1 = -b_2 = \cos\alpha + \sin\alpha$, $b_1 = a_2 = \cos\alpha - \sin\alpha$, whereas $\tilde{s}_1 = \cos\beta x - \sin\beta y$ and $\tilde{s}_2 = \sin\beta x + \cos\beta y$ ($z_1 = x + iy$) are the two neutral mass eigenstate scalars, with masses $m_{\tilde{s}_{1,2}}$ and couplings

$$c_1 = d_2 = \cos\beta, \quad d_1 = -c_2 = -\sin\beta,$$

$$\frac{r_1}{\cos\beta} = \frac{r_2}{\sin\beta} = h_1 + h_2,$$

$$\frac{s_1}{\cos\beta} = \frac{t_1}{-\sin\beta} = \frac{s_2}{\sin\beta} = \frac{t_2}{\cos\beta} = h_1 - h_2,$$

$$\frac{u_1}{-\sin\beta} = \frac{u_2}{\cos\beta} = -\frac{1}{2}\sqrt{\frac{3}{2}} + h_1 + h_2,$$

$$\frac{v_1}{\cos\beta} = \frac{v_2}{\sin\beta} = \sqrt{\frac{3}{2}} (m_\ell - m' - m''),$$

$$\frac{w_1}{-\sin\beta} = \frac{w_2}{\cos\beta} = \sqrt{\frac{3}{2}} (m' - m''),$$

^{*}) We have assumed for simplicity, as in the following, that all the expectation values in Eq. (1) are real. In general, the constraint will be on the real part.

where

$$h_1 = \langle G_2^{21} \rangle, \quad h_2 = \langle G_3^{31} \rangle, \\ m'' = -2 \langle e^{-\frac{G}{2}} \left(\frac{G^{231}}{G^4} + \frac{G^{23}}{G_1} - G_1^{-11} G_1^{231} \right) \rangle.$$

The Table collects the divergent part of the different diagrams contributing to $(g-2)_\ell^*)$ at order κ^2 (see Fig. 1), where $\mu_\ell, \mu_{\tilde{\ell}_{1,2}}, \mu, \mu_\lambda, \mu'$ are the ratios of the corresponding masses $m_\ell, m_{\tilde{\ell}_{1,2}}, m, m_\lambda, m'$ to the gravitino mass $m_{3/2}$ and dimensional reduction¹¹⁾ has been used. $(g-2)_\ell$ is finite when the total sum in the Table cancels,

$$m_\lambda^2 (m_\ell - m) - 3 m_\lambda m' m_{3/2} = 0. \quad (4)$$

This requires $m_\lambda = 0$ or $m_\lambda(m_\ell - m) = 3m'm_{3/2}$. Thus $(g-2)_\ell$ is in general divergent, as expected from the non-renormalizability of supergravity. It is interesting, however, given the one loop finiteness of those contributions to $(g-2)_\ell$ in unbroken supergravity to discuss under what conditions they are finite in the spontaneously broken case. The infinite value of the anomalous magnetic moment of a lepton in the general case seems to prove that the symmetry protecting the unbroken case was a one loop accident, and in this sense an effective symmetry. In general, (4) seems not to be related to the breaking of any particular effective symmetry. If we concentrate on the five diagrams involving the gravitino we see, however, that they are finite if $m_\lambda^2 = 0$ or $m = m_\ell$. The latter is what one may expect from the spontaneous breaking of the effective chiral symmetry which makes the unbroken case finite³⁾. The former may indicate that there is another chiral symmetry associated with the photino which is also relevant to the unbroken case, but we have not been able to identify it. If we look to the three new contributions involving the neutral scalars we see that they are finite if $m_\lambda m' = 0$. This may indicate that there is another effective chiral symmetry in the corresponding Lagrangian related to the analyticity of f (only the Yukawa coupling involving z^{*1}, m' , appears and does it in relation to m_λ), but we have not been able to identify it either. Our discussion should apply to more realistic minimal¹²⁾ and non-minimal¹³⁾ models.

*) By contribution to the anomalous magnetic moment we always mean to $a \equiv (g-2)/2$ as in Ref. 1).

Asking for one-loop finiteness independently of the photino mass requires $m = m_{\tilde{\chi}}$ and $m' = 0$ [see Eq. (4)], which is equivalent to

$$\langle G_{\tilde{1}}^{23} \rangle = \langle G_{\tilde{1}\tilde{1}}^{23} \rangle = 0.$$

This is the case for any model with a constant Kähler metric G_n^m , which in turn is satisfied by many popular models. In what follows we will stick to this particular finite case to obtain an order of magnitude lower bound on the gravitino mass. The finite part of $(g-2)_{\tilde{\chi}}$ is regularization scheme-dependent, as one expects from the non-renormalizability of supergravity. We use dimensional reduction¹¹⁾ to evaluate it because it preserves supersymmetry at the order we are interested in, thereby hoping to obtain a correct order of magnitude estimate. The total finite result is in that case (see Figs. 1 and 2 and other definitions in the text)

$$\begin{aligned} a_{\tilde{\chi}} &= a_{\tilde{g}} + a_{\tilde{\chi}} ; \\ a_{\tilde{g}} &= \frac{G_n m_{\tilde{\chi}}^2}{6\pi} \sum_{k=\tilde{l}_{1,2}} \int_0^1 dt \left\{ \left[A_1^{(k)} + \frac{A_2}{D^{(k)}} + A_3^{(k)} \ln D^{(k)} + \right. \right. \\ &\quad \left. \frac{1}{\mu_{\tilde{\chi}}^2 - 1} \left([(B_1 + B_2 D_{\tilde{\chi}}^{(k)} + B_3 D_{\tilde{\chi}}^{(k)2}) \ln D_{\tilde{\chi}}^{(k)}] - [D_{\tilde{\chi}}^{(k)} \rightarrow D^{(k)}] \right) \right] \\ &\quad \left. + (-1)^k \frac{2 \sin 2\alpha}{\mu_{\tilde{\chi}}} [A, B \rightarrow A', B'] \right\} , \end{aligned} \quad (5)$$

$$D^{(k)} = \mu_{\tilde{\chi}}^2 t^2 + (\mu_k^2 - \mu_{\tilde{\chi}}^2 - 1)t + 1, \quad D_{\tilde{\chi}}^{(k)} = \mu_{\tilde{\chi}}^2 t^2 + (\mu_k^2 - \mu_{\tilde{\chi}}^2 - \mu_{\tilde{\chi}}^2)t + \mu_{\tilde{\chi}}^2,$$

$$A_1^{(k)} = \frac{9}{20} \mu_k^2 - \frac{9}{10} \mu_{\tilde{\chi}}^2 - \frac{1}{4} \mu_{\tilde{\chi}}^2 - \frac{5}{3} \mu_{\tilde{\chi}} + \frac{233}{60},$$

$$A_2 = -\mu_{\tilde{\chi}}^4 t^7 + 5\mu_{\tilde{\chi}}^4 t^6 - 2\mu_{\tilde{\chi}}^2 (4\mu_{\tilde{\chi}}^2 - 1)t^5 + 4\mu_{\tilde{\chi}}^2 (\mu_{\tilde{\chi}}^2 - 1)t^4 + (6\mu_{\tilde{\chi}}^2 - 1)t^3 - (4\mu_{\tilde{\chi}}^2 + 1)t^2 + 2t,$$

$$A_3^{(k)} = -30\mu_{\tilde{\chi}}^2 t^5 + 10(-2\mu_k^2 + 9\mu_{\tilde{\chi}}^2 + 2)t^4 + 4(9\mu_k^2 - 20\mu_{\tilde{\chi}}^2 - 12)t^3 + 3(-4\mu_k^2 + 6\mu_{\tilde{\chi}}^2 + 14)t^2 - 12t,$$

$$B_1 = -2\mu_{\tilde{\chi}}^4 t^5 + 4\mu_{\tilde{\chi}}^4 t^4 + 2\mu_{\tilde{\chi}}^2 (1 - 2\mu_{\tilde{\chi}})t^3 + 2\mu_{\tilde{\chi}}^2 (-2 + 3\mu_{\tilde{\chi}})t^2 + 4\mu_{\tilde{\chi}} t - 6\mu_{\tilde{\chi}},$$

$$B_2 = -16 \mu_\ell^2 t^3 + 18 \mu_\ell^2 t^2 + 6(-2\mu_\lambda + 1)t + 4\mu_\lambda - 2,$$

$$B_3 = -12t + 3,$$

$$A_1^{(k)} = -\frac{7}{12} \mu_\ell^2 + \frac{1}{2} \mu_\ell^2 \mu_\lambda + \frac{59}{60} \mu_\ell^2 - \frac{5}{12} \mu_\ell^2 \mu_\lambda + \frac{1}{2} \mu_\lambda^3 - \frac{4}{3} \mu_\lambda^2 + \frac{1}{12},$$

$$A_2' = \mu_\ell^4 t^6 - 4 \mu_\ell^4 t^5 + \mu_\ell^2 (5 \mu_\ell^2 - 2) t^4 + \mu_\ell^2 (-2 \mu_\ell^2 + 5) t^3 + (-5 \mu_\ell^2 + 1) t^2 + (2 \mu_\ell^2 - 1) t,$$

$$A_3^{(k)} = 20 \mu_\ell^2 t^4 + 4(3 \mu_\ell^2 - 13 \mu_\ell^2 - 3) t^3 + 6(-3 \mu_\ell^2 + 6 \mu_\ell^2 + 4) t^2 + (2 \mu_\ell^2 - 3 \mu_\ell^2 - 12) t - 1,$$

$$B_1' = \mu_\ell^4 (2 + \mu_\lambda) t^4 - \mu_\ell^4 (3 + 2 \mu_\lambda) t^3 - \mu_\ell^2 (2 + \mu_\lambda) t^2 + \mu_\ell^2 (3 + 2 \mu_\lambda) t,$$

$$B_2' = 4 \mu_\ell^2 (3 + \mu_\lambda) t^2 - \mu_\ell^2 (9 + 2 \mu_\lambda) t - 4,$$

$$B_3' = 6,$$

$$a_{\tilde{S}} = \frac{G_N m_\ell^2}{\pi} \int_0^1 dt \left\{ \left[\left(1 - \frac{m''}{m_\ell}\right) (E_1 + \frac{E_2}{D} + E_3 \ln D) + \frac{m_\lambda}{m_{\tilde{S}_2}} (\overline{F}_1 + \frac{\overline{F}_2}{D} + \overline{F}_3 \ln D) \right] + [\sin^2 \beta \rightarrow \cos^2 \beta, m_{\tilde{S}_1}^2 \rightarrow m_{\tilde{S}_2}^2] \right\}, \quad (5)$$

cont.

$$D = t^2 - \frac{m_{\tilde{S}_1}^2}{m_\ell^2} (t-1),$$

$$E_1 = -\frac{1}{4} \sin^2 \beta,$$

$$E_2 = -\frac{3}{4} \sin^2 \beta t^4 - \frac{3}{2} \left(\cos^2 \beta - \frac{m''}{m_\ell} \right) t^3 + 3 \cos^2 \beta \left(1 - \frac{m''}{m_\ell} \right) t^2,$$

$$E_3 = \frac{3}{4} \sin^2 \beta (-3t^2 + 2t),$$

$$\overline{F}_1 = -\frac{1}{6} \sin^2 \beta + 2 \cos^2 \beta \left(1 - \frac{m''}{m_\ell} \right) + \frac{1}{2} \sin^2 \beta \frac{m_{\tilde{S}_1}^2}{m_\ell^2} \left(\frac{1}{2} - \ln \frac{m_{\tilde{S}_1}^2}{m_\ell^2} \right),$$

$$\overline{F}_2 = \frac{2}{3} (\sin^2 \beta - 2 \cos^2 \beta) \left(1 - \frac{m''}{m_\ell} \right) \frac{m_{\tilde{S}_1}^2}{m_\ell^2} (t^2 - 3t + 2),$$

$$\overline{F}_3 = \frac{3}{2} \sin^2 \beta t^2 + [\sin^2 \beta - 2 \cos^2 \beta \left(1 - \frac{m''}{m_\ell} \right) - 2 \sin^2 \beta \frac{m''}{m_\ell} - \sin^2 \beta \frac{m_{\tilde{S}_1}^2}{m_\ell^2}] t + \sin^2 \beta \frac{m_{\tilde{S}_1}^2}{m_\ell^2}.$$

We find Eq. (5) the most convenient way of giving the final result, with the remaining integrals all being elementary. In fact Eq. (5) can be adapted to more

realistic models. That expression includes two very different types of contributions, those involving the gravitino $a_{\tilde{g}}$ [Figs. 1(a), (b), (c)] and those involving the neutral scalars $a_{\tilde{g}}$ [Figs. 1(d), (e)]. The latter, in particular that from diagram (d) in Fig. 1, include the standard neutral scalar contribution to $(g-2)_\ell$. Their size is the standard one for $\kappa m'' = -2\sqrt{2/3} \kappa < e^{-G/2} G^{231} > \sim G_F^{1/2} m_\lambda$ (where G_F is Fermi's constant) or larger for larger m'' (or m_λ). They must be discussed in specific models. We will concentrate on the other contributions $a_{\tilde{g}}$ which really involve the gravitino to obtain a lower bound on its mass.

In general

$$a_{\tilde{g}} \sim \frac{G_N m_\ell^2}{6\pi} \simeq 4 \times 10^{-42} \left(\frac{m_\ell}{106 \text{ MeV}} \right)^2, \quad (6)$$

i.e., very small compared with the possible discrepancy between the measured and the predicted anomalous magnetic moment of a lepton, for instance, of a muon¹⁴⁾

$$\Delta a \simeq 2 \times 10^{-8}. \quad (7)$$

This turns out to be the most restrictive case due to the m_ℓ dependence of $a_{\tilde{g}}$. However, these gravitational corrections may be phenomenologically relevant if some mass ratios are large. The most interesting case is that of a very light gravitino⁷⁾. In that case $\mu_{\tilde{\chi}_{1,2}}, \mu_\ell, \mu_\lambda \gg 1$ and

$$a_{\tilde{g}} \simeq \frac{G_N m_\ell^2}{6\pi} \sum_{k=\tilde{\ell}_{1,2}} \left[\frac{1}{6} + \frac{\mu_\lambda^2}{\mu_k^2 - \mu_\lambda^2} \left(\frac{\mu_\lambda^2}{\mu_k^2 - \mu_\lambda^2} \ln \frac{\mu_k^2}{\mu_\lambda^2} - 1 \right) + (-1)^k \sin 2\alpha \frac{\mu_\lambda}{\mu_\ell} \right] \mu_k^2. \quad (8)$$

We have also used the experimental fact that $\mu_{\tilde{\chi}_{1,2}} \gg \mu_\ell$. A lower bound on the gravitino mass is then obtained by requiring that (8) must at most saturate (7). This bound will increase with the limits on slepton masses as well as on the photino mass. Taking them of order 20 GeV,

$$m_{3/2} \gtrsim 2 \times 10^{-7} \text{ eV} .$$

This is at most an order of magnitude estimate.

One must be aware that in specific models not all the parameters entering in Eq. (5) are independent. In particular, the masses obey a definite sum rule according to the (super)Higgs mechanism^{8),15)}. Also, other phenomenological constraints, for instance that on the mixing angle α from the g^2 contributions to $(g-2)_\ell$ ⁹⁾, must be considered simultaneously.

We knew of a similar calculation¹⁶⁾ during the writing of this paper. Our results are different and so are our conclusions. The main discrepancy seems to be in an incomplete use of the (super)Higgs mechanism by the other authors. The specific Feynman rules (see Fig. 2) can be modified by arbitrarily choosing the non-linear terms of the supersymmetry transformation (we put them equal to zero). In particular, the Yukawa term in Eq. (2) can be transformed away, but then care must be taken to appropriately modify the fermion-scalar-gravitino coupling involving derivatives, taking into account the new vertices coming from the variation of the kinetic term of the gravitino. In this case, we expect our results to hold. Besides, as we stated above, the diagrams of Fig. 1(e) give a finite result if the Kähler metric G_n^m is constant, in contrast with the result of Ref. 16).

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TABLE

Divergent contributions to the anomalous magnetic moment of a lepton in units of $G_N m_\ell^2 / \pi(n-4)$

$$(a): -\frac{8}{9} - \frac{2}{9}\mu_\ell^2 + \frac{1}{9}(\mu_{\tilde{\ell}_1}^2 + \mu_{\tilde{\ell}_2}^2) + \frac{2}{3} \frac{\sin 2\alpha}{\mu_\ell} (\mu_{\tilde{\ell}_1}^2 - \mu_{\tilde{\ell}_2}^2)$$

$$(b): \frac{2}{3}\mu_\ell^2 - (\mu_{\tilde{\ell}_1}^2 + \mu_{\tilde{\ell}_2}^2) + \frac{2}{3} \frac{\mu}{\mu_\ell} (\mu_{\tilde{\ell}_1}^2 + \mu_{\tilde{\ell}_2}^2) - \frac{4}{3}\mu_\lambda^2 (1 - \frac{\mu}{\mu_\ell}) - \frac{4}{3} \frac{\sin 2\alpha}{\mu_\ell} (\mu_{\tilde{\ell}_1}^2 - \mu_{\tilde{\ell}_2}^2)$$

$$(c): \frac{8}{9} - \frac{4}{9}\mu_\ell^2 + \frac{8}{9}(\mu_{\tilde{\ell}_1}^2 + \mu_{\tilde{\ell}_2}^2) - \frac{2}{3} \frac{\mu}{\mu_\ell} (\mu_{\tilde{\ell}_1}^2 + \mu_{\tilde{\ell}_2}^2) + \frac{2}{3} \frac{\sin 2\alpha}{\mu_\ell} (\mu_{\tilde{\ell}_1}^2 - \mu_{\tilde{\ell}_2}^2)$$

$$(a)+(b)+(c): -\frac{4}{3}\mu_\lambda^2 (1 - \frac{\mu}{\mu_\ell})$$

$$(d): 0$$

$$(e): 4 \frac{\mu_\lambda \mu'}{\mu_\ell}$$

$$(d)+(e): 4 \frac{\mu_\lambda \mu'}{\mu_\ell}$$

$$(a)+(b)+(c)+(d)+(e): -\frac{4}{3} \mu_\lambda^2 (1 - \frac{\mu}{\mu_\ell}) + 4 \frac{\mu_\lambda \mu'}{\mu_\ell}$$

Fig. 1

	$\frac{i}{k^2 - m^2 + i\epsilon} \chi_{j, \tilde{s}_j}$
	$\frac{i(k+m_\lambda)}{k^2 - m_\lambda^2 + i\epsilon}$
	$\frac{i(k+m_{3/2})}{k^2 - m^2 + i\epsilon} \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2}{3} \frac{k_\mu k_\nu}{m^2_{3/2}} + \frac{1}{3m_{3/2}} (k_\mu \gamma_\nu - k_\nu \gamma_\mu) \right]$
	$\frac{-ig_{\mu\nu}}{k^2 + i\epsilon}$
	$\frac{i(k+m_\lambda)}{k^2 - m_\lambda^2 + i\epsilon}$
	$ieQ_\lambda (p+p')_\mu$
	$i \frac{\kappa}{2\sqrt{2}} [(a_j - b_j \gamma_5)(p+m) + (a_j + b_j \gamma_5)m_{3/2}] \gamma_\nu$
	$-i \frac{eQ_\lambda}{\sqrt{2}} (a_j - b_j \gamma_5)$
	$i \frac{\kappa}{4} [A, \gamma_\mu] \gamma_\nu$
	$i \frac{eQ_\lambda \kappa}{2\sqrt{2}} \gamma_\mu \gamma_\nu (a_j - b_j \gamma_5)$
	$ieQ_\lambda \gamma_\mu$
	$i\kappa(-2r_j \not{p} + 2s_j \not{p} \gamma_5 - it_j \not{p}' + iu_j \not{p}' \gamma_5 + v_j - iw_j \gamma_5)$
	$-i \sqrt{2/3} \frac{m_\lambda \kappa}{m^{3/2}} \left[c \left(p^\sigma q g_{\mu\nu} - p_\mu q_\nu \right) + d \epsilon_{\sigma\nu\rho\mu} p^\sigma q^\rho \right]$

Fig. 2