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COMMUTATORS IN AN ANOMALOUS NON-ABELIAN
CHIRAL GAUGE THEORY

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S.-G. Jo

*Department of Physics
Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.*

ABSTRACT

The fixed-time commutator of gauge group generators in an anomalous gauge theory is calculated up to one-loop level using the BJL limit. The result agrees with the cohomological prediction. A necessary ingredient is the observation that the commutator of $E_a(\mathbf{x})$ with itself has an anomaly which violates the Jacobi identity, indicating the existence of 3-cocycles.

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I. INTRODUCTION

Since it was suggested¹ that the infinitesimal 2-cocycle descending from the Chern-Pontryagin density is the anomalous fixed-time commutator² of gauge group generators G_a , a number of authors³⁻⁵ have tried to prove this. But so far, no perturbative proof has been completed. Especially in our previous work,⁵ we considered a gauge theory in which massless fermions couple chirally to an external gauge field. There $G_a(\mathbf{x}) = -(\mathbf{D} \cdot \mathbf{E})_a + \rho_a \equiv G_a^0(\mathbf{x}) + \rho_a(\mathbf{x})$, and we represented $\mathbf{E}_a(\mathbf{x})$, which is the negative momentum conjugate to $\mathbf{A}_a(\mathbf{x})$ in the full theory quantized in the Weyl gauge ($A_a^0 = 0$), by $i\delta/\delta A_a(\mathbf{x})$. Here, $\mathbf{D}$ is the covariant derivative and ρ_a is the fermion charge density. We observed that the canonical assumption does not give the commutator anomaly as a non-trivial 2-cocycle.

In this paper, we consider the full theory in which the gauge field is also quantized. Since the gauge theory is anomalous, it is not clear what kind of quantum theory emerges when the gauge degrees of freedom are quantized in the Weyl gauge. We do not dwell on this question of consistency, and remain content with the one-loop graphs which we need to calculate. Using the Bjorken-Johnson-Low (BJL) limit,⁶ we find that the anomalous term in the commutator of gauge group generators is indeed a non-trivial 2-cocycle. However, we find that in the $(1+3)$ -dimensional case, $\mathbf{E}_a$ cannot be represented canonically, since it does not commute with itself, indicating that there exists an "external" $U(1)$ field curvature in the space of vector potentials, as already seen in other examples.⁷

$$[E_a^i(\mathbf{x}), E_b^j(\mathbf{y})] = \frac{1}{24\pi^2} \epsilon^{ijk} \text{tr}(ab + ba) A^k \delta(\mathbf{x} - \mathbf{y}) . \quad (1.1)$$

[Generators T^a are simply represented by "a".] Moreover, this commutator does not satisfy the Jacobi identity, so that a 3-cocycle is present.⁸ Nevertheless, a four-fold Malcev identity⁹ holds.

In the previous paper,⁵ we have already found the commutator of the fermion charge density.

$$i[\rho_a(\mathbf{x}), \rho_b(\mathbf{y})] = f_{abc}\rho_c\delta(\mathbf{x}-\mathbf{y}) + J_{ab}(\mathbf{x},\mathbf{y}) \quad (1.2)$$

$$\begin{aligned} (1+3)d: J_{ab}(\mathbf{x},\mathbf{y}) = & -\frac{i}{8\pi^2}\epsilon^{ijk} \text{tr}(ab+ba)\partial_i A_j \partial_k \delta(\mathbf{x}-\mathbf{y}) \\ & + \frac{i}{24\pi^2}\epsilon^{ijk} \text{tr}(acdb-abdc)\{\partial_i A_j^c A_k^d \delta(\mathbf{x}-\mathbf{y}) \\ & - 2A_i^c \partial_j A_k^d \delta(\mathbf{x}-\mathbf{y}) - 3A_i^c(\mathbf{y})A_j^d(\mathbf{y})\partial_k \delta(\mathbf{x}-\mathbf{y})\} \\ & - \frac{i}{16\pi^2}\epsilon^{ijk} \text{tr}(adbc-acbd)\partial_i (A_j^c A_k^d) \delta(\mathbf{x}-\mathbf{y}) \\ & - \frac{i}{16\pi^2}\epsilon^{ijk} \{ \text{tr}(ba-ab)cde + 2\text{tr}(acb-bca)de \} A_i^c A_j^d A_k^e \delta(\mathbf{x}-\mathbf{y}) \end{aligned} \quad (1.3a)$$

$$(1+1)d: J_{ab}(\mathbf{x},\mathbf{y}) = -\frac{1}{2\pi}\text{tr} ab \partial_1 \delta(\mathbf{x}-\mathbf{y}) + \frac{1}{4\pi} \text{tr}(ab-ba)(A_0+A_1)\delta(\mathbf{x}-\mathbf{y}) \quad (1.3b)$$

Here, we calculate the commutator of $(\mathbf{D} \cdot \mathbf{E})_a$ with itself and with ρ_a using the BJL limit. Combining all of these we obtain the anomalous term $G_{ab}(\mathbf{x},\mathbf{y})$.

$$G_{ab}(\mathbf{x},\mathbf{y}) \equiv i[G_a(\mathbf{x}), G_b(\mathbf{y})] - f_{abc}G_c\delta(\mathbf{x}-\mathbf{y}) \quad (1.4)$$

$$\begin{aligned} & = G_{ab}^0(\mathbf{x},\mathbf{y}) + J_{ab}(\mathbf{x},\mathbf{y}) - i[(\mathbf{D} \cdot \mathbf{E})_a(\mathbf{x}), \rho_b(\mathbf{y})] - i[\rho_a(\mathbf{x}), (\mathbf{D} \cdot \mathbf{E})_b(\mathbf{y})] \\ & \quad G_{ab}^0(\mathbf{x},\mathbf{y}) \equiv i[(\mathbf{D} \cdot \mathbf{E})_a(\mathbf{x}), (\mathbf{D} \cdot \mathbf{E})_b(\mathbf{y})] + f_{abc}(\mathbf{D} \cdot \mathbf{E})_c\delta(\mathbf{x}-\mathbf{y}) \end{aligned} \quad (1.5)$$

We observe that the pure Yang-Mills part may also develop an anomalous term $G_{ab}^0(\mathbf{x},\mathbf{y})$, which would not arise if $\mathbf{E}_a(\mathbf{x})$ is represented by $i\delta/\delta\mathbf{A}_a(\mathbf{x})$.

II. EVALUATION OF $[(\mathbf{D} \cdot \mathbf{E})_a(\mathbf{x}), (\mathbf{D} \cdot \mathbf{E})_b(\mathbf{y})]$ AND $[(\mathbf{D} \cdot \mathbf{E})_a(\mathbf{x}), \rho_b(\mathbf{y})]$

We consider the $(1+3)$ dimensional theory in the Weyl gauge. We neglect the vector part which we believe is anomaly free. The gauge boson propagator $D_{\mu\nu}^{ab}(p)$ in this gauge is non-vanishing only in its spatial components, where it reads $D_{ij}^{ab}(p) = \frac{i}{p^2} \left(\delta_{ij} - \frac{p_i p_j}{p_0^2} \right) \delta_{ab}$. Furthermore, the second term of $D_{ij}^{ab}(p)$ does not contribute in our application of the BJL limit, so we use the simpler form $D_{ij}^{ab}(p) = \frac{i\delta_{ij}}{p^2} \delta_{ab}$. The $(1+1)$ -dimensional case is handled similarly.

(A) Evaluation of $G_{ab}^0(\mathbf{x}, \mathbf{y})$. The BJL limit reads

$$\begin{aligned} & \int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle 0 | G_{ab}^0(\mathbf{x}, 0) | \beta \rangle \\ &= \lim_{p_0 \rightarrow \infty} p_0 \int d^4x e^{ipx} \langle 0 | T G_a^0(x) G_b^0(0) | \beta \rangle - f_{abc} \langle 0 | G_c^0(0) | \beta \rangle \end{aligned} \quad (2.1)$$

with any polynomial in p_0 on the right hand side neglected. The contributions to $G_{ab}^0(\mathbf{x}, \mathbf{y})$ come from the diagrams in Fig. 1. The first order term, represented by diagram 1-a, can be obtained by choosing $|\beta\rangle$ to be a one-gluon state, $|\beta\rangle = |\gamma_c(\epsilon, q)\rangle \equiv |C\rangle$. From the above formula, we get

$$\begin{aligned} & \int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle 0 | G_{ab}^0(\mathbf{x}, 0) | C \rangle \\ &= \lim_{p_0 \rightarrow \infty} (-i)^3 p_0 i p_0 (-i p_i) \epsilon_k \frac{-i}{p^2} \Pi_{acb}^{ijk}(p, q) (-i)(p+q)_0 i(p+q)_j \frac{-i}{(p+q)^2} \end{aligned} \quad (2.2)$$

where $\Pi_{abc}^{\alpha\beta\gamma}(p, q)$ is the symmetrized 3-point function. Note that in a Feynman integral representation, a shift of integration variables gives different results for $\Pi_{abc}^{\alpha\beta\gamma}(p, q)$, but such ambiguities are completely eliminated by requiring the correct Bose symmetry of $\Pi_{abc}^{\alpha\beta\gamma}(p, q)$.¹⁰ When p_0 goes to infinity, $\Pi_{abc}^{\alpha\beta\gamma}(p, q)$ behaves as

$$\begin{aligned} \lim_{p_0 \rightarrow \infty} \Pi_{abc}^{\alpha\beta\gamma}(p, q) &= \text{tr}(ab + ba) c \left[\frac{1}{16\pi^2} \left\{ \epsilon^{\alpha\beta\gamma\epsilon} p_\epsilon + (g^{\alpha 0} \epsilon^{0\gamma\beta\epsilon} + g^{\gamma 0} \epsilon^{0\alpha\beta\epsilon}) q_\epsilon \right\} \right. \\ &\quad \left. - \frac{1}{48\pi^2} \epsilon^{\alpha\beta\gamma\epsilon} (p - q)_\epsilon \right] + \mathcal{O}\left(\frac{1}{p_0}\right) \end{aligned} \quad (2.3)$$

Therefore, we get

$$\int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | G_{ab}^0(\mathbf{x}, 0) | C \rangle = -\frac{i}{24\pi^2} \epsilon^{ijk} q_i \epsilon_j p_k \text{tr}\{a, b\} c \quad (2.4a)$$

which implies

$$G_{ab}^0(\mathbf{x}, \mathbf{y}) (1^{\text{st}} \text{ order}) = -\frac{i}{24\pi^2} \epsilon^{ijk} \text{tr}\{a, b\} \partial_i A_j \partial_k \delta(\mathbf{x} - \mathbf{y}) \quad (2.4b)$$

The second order contribution, corresponding to the diagrams 1-b, can be obtained

by taking $|\beta\rangle$ to be a two gluon state, $|\beta\rangle = |\gamma_c(\epsilon, q) \gamma_{c'}(\epsilon', q')\rangle \equiv |C, C'\rangle$ We get

$$\begin{aligned} & \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | G_{ab}^0(\mathbf{x}, 0) | C, C' \rangle \\ &= \lim_{p_0 \rightarrow \infty} (-i)^3 p_0 \left[\left\{ f_{ace} \epsilon_i i(p+q)_0 \frac{-i}{(p+q)^2} \epsilon'_k \Pi_{ec'b}^{ikj}(p+q, q') \frac{-i}{(p+q+q')^2} i(p+q+q')_j \right. \right. \\ & \quad \times (-i)(p+q+q')_0 + (-ip_i) i p_0 \frac{-i}{p^2} \epsilon'_k \epsilon_j f_{bce} \Pi_{ac'e}^{ikj}(p, q) \frac{-i}{(p+q)^2} (-i)(p+q)_0 \left. \right\} + \left(\begin{array}{c} c \leftrightarrow c' \\ \epsilon \leftrightarrow \epsilon' \\ q \leftrightarrow q' \end{array} \right) \right] \quad (2.5) \end{aligned}$$

Using the formula (2.3), we get

$$\begin{aligned} G_{ab}^0(\mathbf{x}, \mathbf{y}) (2^{\text{nd}} \text{ order}) &= -\frac{i}{24\pi^2} \epsilon^{ijk} \left\{ 2 \text{tr}\{a, b\} A_i(\mathbf{y}) A_j(\mathbf{y}) \partial_k \delta(\mathbf{x} - \mathbf{y}) \right. \\ & \quad \left. - \text{tr}\{a, b\} \partial_i (A_j A_k) \delta(\mathbf{x} - \mathbf{y}) - 2 \text{tr} a \partial_i (A_j b A_k) \delta(\mathbf{x} - \mathbf{y}) \right\} \quad (2.6) \end{aligned}$$

By taking $|\beta\rangle = |\gamma_c(\epsilon, q) \gamma_{c'}(\epsilon', q') \gamma_{c''}(\epsilon'', q'')\rangle \equiv |C, C', C''\rangle$, we obtain the contribution from diagram 1-c.

$$\begin{aligned} & \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | G_{ab}^0(\mathbf{x}, 0) | C, C', C'' \rangle \\ &= \lim_{p_0 \rightarrow \infty} (-i)^3 p_0 \left\{ f_{acd} \epsilon_i i(p+q)_0 \frac{-i}{(p+q)^2} \epsilon'_k \Pi_{dc'e}^{ikj}(p+q, q') f_{bc''e} \right. \\ & \quad \times (-i)(p+q+q')_0 \frac{-i}{(p+q+q')^2} \epsilon''_j + (C, C', C'' \text{ permutations}) \left. \right\} \quad (2.7) \end{aligned}$$

Again, using the formula (2.3) in order to evaluate the limit, we get the third order term in $G_{ab}^0(\mathbf{x}, (\mathbf{y}))$.

$$G_{ab}^0(\mathbf{x}, \mathbf{y}) (3^{\text{rd}} \text{ order}) = -\frac{i}{24\pi^2} \epsilon^{ijk} \left\{ 3 \text{tr} (aA_i b - bA_i a) A_j A_k - \text{tr} [a, b] A_i A_j A_k \right\} \delta(\mathbf{x} - \mathbf{y}) \quad (2.8)$$

No other one-loop diagrams contribute, hence combining all of the above, we get

$$\begin{aligned} G_{ab}^0(\mathbf{x}, \mathbf{y}) &= -\frac{i}{24\pi^2} \epsilon^{ijk} \text{tr} \{a, b\} \partial_i A_j \partial_k \delta(\mathbf{x} - \mathbf{y}) \\ &\quad - \frac{i}{12\pi^2} \epsilon^{ijk} \text{tr} \{a, b\} A_i(\mathbf{y}) A_j(\mathbf{y}) \partial_k \delta(\mathbf{x} - \mathbf{y}) \\ &\quad + \frac{i}{24\pi^2} \epsilon^{ijk} \left\{ \text{tr} \{a, b\} \partial_i (A_j A_k) + 2 \text{tr} a \partial_i (A_j b A_k) \right\} \delta(\mathbf{x} - \mathbf{y}) \\ &\quad - \frac{i}{24\pi^2} \epsilon^{ijk} \left\{ 3 \text{tr} (aA_i b - bA_i a) A_j A_k - \text{tr} [a, b] A_i A_j A_k \right\} \delta(\mathbf{x} - \mathbf{y}) \\ &= iD_i^{ac}(\mathbf{x}) D_j^{bd}(\mathbf{y}) \left\{ -\frac{1}{24\pi^2} \epsilon^{ijk} \text{tr} \{c, d\} A_k \delta(\mathbf{x} - \mathbf{y}) \right\} \end{aligned} \quad (2.9)$$

From this calculation, we recognize that all the contributions to $G_{ab}^0(\mathbf{x}, \mathbf{y})$ arise from the anomalous behavior of the equal time commutator of $E_a^i(\mathbf{x})$ with itself, which is represented by Fig. 2. This commutator can also be computed from the term linear in p_0 embedded in the seagull of the fermion current-current time ordered product.

$$i[E_a^i(\mathbf{x}), E_b^j(0)] = \lim_{p_0 \rightarrow \infty} \frac{1}{p_0} \int dt e^{ip_0 t} \tau_{ab}^{ij}(x, 0) \quad (2.10)$$

where⁵

$$\begin{aligned} \tau_{ab}^{\mu\nu}(x, y) &= \frac{1}{16\pi^2} \text{tr} \{a, b\} (\partial_\alpha A_\beta + A_\alpha A_\beta) (g^{\mu 0} \epsilon^{0\nu\alpha\beta} + g^{\nu 0} \epsilon^{0\mu\alpha\beta}) \delta(x - y) \\ &\quad - \frac{1}{48\pi^2} \text{tr} \{a, b\} \epsilon^{\mu\nu\alpha\beta} (A_\alpha(x) + A_\alpha(y)) \partial_\beta \delta(x - y) + \frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} a A_\alpha b A_\beta \delta(x - y) \end{aligned} \quad (2.11)$$

Taking the appropriate components leads to the formula (1.1).

In the (1+1)-dimensional case, there are no diagrams contributing to $G_{ab}^0(\mathbf{x}, \mathbf{y})$.

Therefore, $G_{ab}^0(\mathbf{x}, \mathbf{y})$ vanishes, as does the commutator of $E_a(\mathbf{x})$ with itself.

B. Evaluation of $[G_a^0(\mathbf{x}), \rho_b(\mathbf{y})]$.

Relevant diagrams for this commutator are shown in Fig. 3. We see that

$$[G_a^0(\mathbf{x}), \rho_b(\mathbf{y})] = -\mathbf{D}^{ac}(\mathbf{x}) \cdot [\mathbf{E}_c(\mathbf{x}), \rho_b(\mathbf{y})] \quad (2.12)$$

where $[E_c^i(\mathbf{x}), \rho_b(\mathbf{y})]$ is represented by the diagrams in Fig. 4. Let us evaluate this. First

$$\int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle 0 | i[E_a^i(\mathbf{x}), \rho_b(0)] | C \rangle = \lim_{p_0 \rightarrow \infty} (-i)^2 p_0 (-i) p_0 \frac{-i}{p^2} \epsilon_j \Pi_{acb}^{ij0}(p, q) \quad (2.13)$$

and

$$\int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle 0 | i[E_a^i(\mathbf{x}), \rho_b(0)] | C, C' \rangle = \lim_{p_0 \rightarrow \infty} (-i)^3 p_0 (-i) p_0 \frac{-i}{p^2} \epsilon_j \epsilon'_k \Pi_{acc'b}^{ijk0}(p, q, q') \quad (2.14)$$

where $\Pi_{abcd}^{\alpha\beta\mu\nu}(p, q, \ell)$ is the symmetrized 4-point function whose high energy behavior is

$$\begin{aligned} \lim_{p_0 \rightarrow \infty} \Pi_{abcd}^{\alpha\beta\mu\nu}(p, q, \ell) = & -\frac{1}{16\pi^2} \text{tr} \{a, d\} [b, c] (g^{\nu 0} \epsilon^{0\alpha\beta\mu} + g^{\alpha 0} \epsilon^{0\nu\beta\mu}) \\ & + \frac{1}{16\pi^2} \text{tr} (acdb - abdc) \epsilon^{\alpha\beta\mu\nu} + \mathcal{O}\left(\frac{1}{p_0}\right) \end{aligned} \quad (2.15)$$

Using formula (2.3) and (2.15), we get

$$\begin{aligned} [E_a^i(\mathbf{x}), \rho_b(\mathbf{y})] = & -\frac{1}{24\pi^2} \epsilon^{ijk} \text{tr} \{a, b\} A_j(\mathbf{y}) \partial_k \delta(\mathbf{x} - \mathbf{y}) - \frac{1}{12\pi^2} \epsilon^{ijk} \text{tr} \{a, b\} \partial_j A_k \delta(\mathbf{x} - \mathbf{y}) \\ & - \frac{1}{16\pi^2} \epsilon^{ijk} [\text{tr} \{a, b\} A_j A_k - \text{tr} a A_j b A_k] \delta(\mathbf{x} - \mathbf{y}) \end{aligned} \quad (2.16)$$

Note that this can be extracted with the BJL limit from the $(i0)$ component of the seagull.

$$[E_a^i(\mathbf{x}), \rho_b(\mathbf{y})] = - \lim_{p_0 \rightarrow \infty} \int dt e^{ip_0 t} \tau_{ab}^{i0}(x, 0) \quad (2.17)$$

Similarly in $(1+1)d$, we find

$$[E_a(\mathbf{x}), \rho_b(\mathbf{y})] = \frac{i}{4\pi} \text{tr } ab \delta(\mathbf{x} - \mathbf{y}) \quad (2.18)$$

Combining all of these, we get

$$\begin{aligned} (1+3)d: G_{ab}(\mathbf{x}, \mathbf{y}) = & -\frac{i}{48\pi^2} \epsilon^{ijk} \left\{ \text{tr } a \partial_i (A_j b A_k) + \text{tr } (ab - ba) (\partial_i A_j A_k + A_i \partial_j A_k) \right\} \delta(\mathbf{x} - \mathbf{y}) \\ & - \frac{i}{48\pi^2} \epsilon^{ijk} \text{tr } (ab - ba) A_i A_j A_k \delta(\mathbf{x} - \mathbf{y}) \end{aligned} \quad (2.19a)$$

$$(1+1)d: G_{ab}(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi} \text{tr } (ab - ba) (A_0 - A_1) \delta(\mathbf{x} - \mathbf{y}) \quad (2.19b)$$

These coincide, apart from trivial terms, with the appropriate 2-cocycles obtained by the descent procedure.⁵

III. Discussion

Let us observe that the 3-dimensional E_a^i commutator relations (1.1) do not satisfy the Jacobi identity. Defining the Jacobian by $J(A, B, C) \equiv [[A, B]C] + [[B, C], A] + [[C, A], B]$, we find a non-zero result.

$$J(E_a^i(\mathbf{x}), E_b^j(\mathbf{y}), E_c^k(\mathbf{z})) = -\frac{i}{8\pi^2} \epsilon^{ijk} \text{tr } \{a, b\} c \delta(\mathbf{x} - \mathbf{y}) \delta(\mathbf{x} - \mathbf{z}) \quad (3.1)$$

This implies that E_a^i cannot be represented by a linear operator. Since there are no anomalies in commutators with A_a^i , the algebra which closes has the following structure.

$$\begin{aligned} [A_a^i(\mathbf{x}), A_b^j(\mathbf{y})] &= 0 \\ [E_a^i(\mathbf{x}), A_b^j(\mathbf{y})] &= i \delta^{ij} \delta_{ab} \delta(\mathbf{x} - \mathbf{y}) \\ [E_a^i(\mathbf{x}), E_b^j(\mathbf{y})] &= \frac{1}{24\pi^2} \epsilon^{ijk} \text{tr } \{a, b\} A^k \delta(\mathbf{x} - \mathbf{y}) \end{aligned} \quad (3.2)$$

This is not a Lie algebra, owing to the failure of the Jacobi identity. It is natural to check whether this algebra satisfied the Malcev condition. A Malcev algebra generalizes a Lie algebra by relaxing the Jacobi identity. Instead, the requirement of the Malcev identity is imposed. The Malcev identity, a fourth order relation, reads⁹

$$[[A, B], [A, C]] = [[A, B], C], A] + [[B, C], A], A] + [[C, A], A], B] \quad (3.3a)$$

or equivalently

$$J(A, B, [A, C]) = [J(A, B, C), A] \quad (3.3b)$$

It is easy to check that the given commutation relation indeed satisfies the Malcev identity. Thus, the algebra is the quantum theoretic version of the quantum mechanical magnetic algebra,¹¹ of velocities $\mathbf{v}$ and coordinates $\mathbf{r}$ in the presence of non-vanishing constant magnetic charge density.

The Jacobian does not have the same form as the 3-cocycle arising from the descent from the 6-dimensional Chern-Pontryagin density. That 3-cocycle is a 2-form, while ours is a 0-form. However, we believe that the Jacobian is related to some quantity descending from the Chern-Pontryagin density. Finally, note that in the 3-dimensional case also $J(E_a^i(\mathbf{x}), E_b^j(\mathbf{y}), \rho_c(\mathbf{z}))$ is non-zero, which is another indication that $[\mathbf{E}_a(\mathbf{x}), \mathcal{O}(\mathbf{A})]$ cannot be represented by $i\delta\mathcal{O}(\mathbf{A})/\delta\mathbf{A}_a(\mathbf{x})$.

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FIGURE CAPTIONS

Figure 1

Diagrams contributing to the anomalous term in the commutator $i[G_a^0(\mathbf{x}), G_b^0(0)]$.

Figure 2

Diagram for the commutator $i[E_a^i(\mathbf{x}), E_b^j(0)]$.

Figure 3

Diagrams for the commutator $i[G_a^0(\mathbf{x}), j_b^0(0)]$.

Figure 4

Diagrams for the commutator $i[E_a^i(\mathbf{x}), j_b^0(0)]$.

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Diagram (1-a) shows a triangle loop with a wavy line entering from the top and two wavy lines exiting from the bottom vertices. The external momenta are labeled $\vec{\nabla} \cdot \vec{E}_a^{(x)}$ and $\vec{\nabla} \cdot \vec{E}_b^{(o)}$.

Diagram (1-b) shows two triangle loops. The left loop has a wavy line entering from the top and a wavy line exiting from the bottom-left vertex, with external momenta $-f_{ade} \vec{A}_d \cdot \vec{E}_e^{(x)}$ and $\vec{\nabla} \cdot \vec{E}_b^{(o)}$. The right loop has a wavy line entering from the top and a wavy line exiting from the bottom-right vertex, with external momenta $\vec{\nabla} \cdot \vec{E}_a^{(x)}$ and $-f_{bd'e'} \vec{E}_d \cdot \vec{A}_{e'}^{(o)}$.

(1-a) (1-b)

Diagram (1-c) shows a triangle loop with a wavy line entering from the top and two wavy lines exiting from the bottom vertices. The external momenta are labeled $f_{ade} \vec{A}_d \cdot \vec{E}_e^{(x)}$ and $f_{bd'e'} \vec{E}_e \cdot \vec{A}_{d'}^{(o)}$.

(1-c)

(Fig. 1)

Diagram (Fig. 2) shows a triangle loop with a wavy line entering from the top and two wavy lines exiting from the bottom vertices. The external momenta are labeled $E_a^i(x)$ and $E_b^i(o)$.

(Fig. 2)

$$-\vec{\nabla} \cdot \vec{E}_a(x)$$

$$(3-a)$$

$$f_{ade} \vec{A}_d \cdot \vec{E}_a(x)$$

$$(3-b)$$

$$-\vec{\nabla} \cdot \vec{E}_a(x)$$

$$f_{ade} \vec{A}_d \cdot \vec{E}_a(x)$$

$$(3-c)$$

(Fig. 3)

$$E_a^i(x)$$

$$E_a^i(x)$$

(Fig. 4)