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STRINGS AND QCD

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A B S T R A C T

We point out that the occurrence of a large-distance "Coulomb" term in the static quark-antiquark potential is related to tachyons in the underlying string model. Thus the tachyon contents determine the coefficient of the "Coulomb" term. For a scalar string, one has Lüscher's result $-(d-2)\pi/24R$, for a Neveu-Schwarz string one obtains $-(d-2)\pi/16R$, whereas for the Ramond string (and strings without tachyons) the "Coulomb" term vanishes. Monte Carlo data for QCD prefers a "Coulomb" term. This is not inconsistent with the tachyonic origin of this term, since the large-distance behaviour of string models is consistent even with tachyons. Also, critical dimensions are not necessary at large distances.

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It is an interesting question to investigate whether QCD corresponds to a string theory, and if so, what string theory reproduces QCD. An important tool in this kind of investigation is the observation by Lüscher¹⁾ that the heavy quark-antiquark potential $V(R)$ has a "universal" Coulomb term,

$$V(R) = \sigma R - \frac{\pi}{24} \frac{d-2}{R} + O\left(\frac{1}{R^2}\right), \quad (1)$$

where σ is the string tension. Equation (1) follows from Lüscher's observation¹⁾ that at large distances the effective string theory becomes

$$S_{\text{eff}} \xrightarrow{\text{large distances}} \text{const.} \int d^2\xi \partial_a x_\perp \partial_a x_\perp, \quad (2)$$

where $x_\perp$ represents the transverse displacement of the string with fixed end points. It is important to realize that in Ref. 1) there is an implicit assumption, namely that $x_\perp$ is the only relevant dynamical variable at large distances. Thus, one should rather consider Eq. (1) as describing a universality class of string models.

In Polyakov's approach²⁾, the metric field is also a dynamical variable. Although this approach has not been solved, there exists a saddle point expansion³⁾ which indicates that in general Polyakov's model does not reproduce the Coulomb term in Eq. (1). This may of course be due to a failure of the saddle point calculation, but it is clear in general that Polyakov's string does not belong to the universality class considered by Lüscher.

In this paper we shall present arguments that the Neveu-Schwarz, the Ramond, and the superstring do not fall into the universality class described by Eq. (2). Instead we argue that the "Coulomb" term in $V(R)$ depends critically on the degrees of freedom needed in addition to $x_\perp$. This opens up the interesting possibility of trying to decide on the basis of Monte Carlo data for QCD which degrees of freedom are contained in the "QCD string".

We also find that the notorious troubles (tachyons and critical dimensions) with ordinary strings are asymptotically disappearing at large distances. In other words, if one looks at a string model which is in a strict sense inconsistent in four dimensions, it turns out that at large distances it becomes a consistent model. This implies that a priori the large-distance behaviour of QCD can be described by a string model which contains tachyons and which is only Lorentz (or rotational) invariant in higher than four dimensions. Of course, if this is the case it is evident that such a description must break down at some

distance, since QCD does not contain tachyons and is Lorentz (and rotational) invariant in two, three and four dimensions.

On the basis of present-day Monte Carlo data for QCD, it is likely that there exists a long-distance "Coulomb" term. We shall see that, at least for simple string models, this "Coulomb" term is related to the occurrence of a tachyon at short distances. Thus, from what was said above, it appears very unlikely that QCD is described globally (i.e., for all distances), by a simple string model, although we cannot exclude that this possibility exists in more complicated models à la Polyakov.

We shall now derive the results mentioned above. Some time ago it was shown by Alvarez⁴⁾ that the behaviour (1) of $V(R)$ could be understood as coming from the expression

$$V(R) = \sigma \sqrt{R^2 - R_c^2}, \quad R_c^2 = \frac{\pi(d-2)}{12\sigma}. \quad (3)$$

This result was obtained⁴⁾ in a saddle point expansion around $d = \infty$. However, Arvis⁵⁾ showed that Eq. (3) holds in all dimensions for the Nambu-Goto action.

Intuitively, Eq. (3) can be understood in the following way: in ordinary dual models with free ends for the string (normal derivative of x_μ vanishes at the end points) the equations of motion are solved by

$$x^i(\sigma, \tau) = x^i + 2\pi\alpha' p^i \tau + i \sum_{n=1}^{\infty} \frac{a_n^i}{n} \cos n\sigma e^{-in\tau}. \quad (4)$$

This leads to the mass formula

$$\alpha' p_0^2 = \alpha' (p^i)^2 + \mu^2. \quad (5)$$

For the lowest mass state, μ^2 is the contribution from the zero-point fluctuations⁶⁾ of the harmonic oscillators,

$$\mu^2 = \frac{d-2}{2} \sum_{n=1}^{\infty} n = -\frac{d-2}{24}, \quad (6)$$

corresponding to a tachyon. With boundary conditions appropriate for a string with ends fixed at $x_1 = 0$ and $x_1 = R_1$, $R^2 = R_1^2$, Eq. (4) is replaced by⁵⁾:

$$\begin{aligned}
 x^i(\sigma, \tau) &= \frac{R^i}{\pi} \sigma + \sum_{n=1}^{\infty} \frac{a_n^i}{n} \sin n\sigma e^{-in\tau}, \\
 x^0(\sigma, \tau) &= x^0 + 2\pi\alpha' p^0 \tau + i \sum_{n=1}^{\infty} \frac{a_n^0}{n} \cos n\sigma e^{-in\tau}. \quad (7)
 \end{aligned}$$

Thus, relative to Eq. (4), the space-part of the momenta have been replaced by R^i/π and τ by σ . Since there is a large amount of symmetry between σ and τ , the energy formula (5) turns out in an analogous way to be⁵⁾

$$\alpha' V^2 = \alpha' \left(\frac{R^i}{2\pi\alpha'} \right)^2 + \mu^2, \quad V = p^0 = \text{the energy}, \quad (8)$$

where μ^2 is again the contribution from the zero-point fluctuations of the harmonic oscillators. Thus μ^2 is given by Eq. (6). Taking the square root, we see that one recovers the Alvarez formula (for all d) Eq. (3) from Eqs. (6) and (8). Thus, Eq. (3) can be understood as the standard mass formula of the Nambu-Goto string with the unusual fixed boundaries.

The above arguments show that the "Coulomb" term arises from the zero-point fluctuations. This, however, means that the coefficient of the long-distance "Coulomb" term is directly related to the tachyon mass! Of course, this also means that Eq. (3) cannot represent the QCD potential down to distances of the order R_c , where the potential $V(R)$ becomes imaginary (due to the tachyon).

Arvis studied⁵⁾ rotational invariance of the model. For the rotation generators L^{ij} he obtained various commutation relations, one of which contains an anomaly, namely

$$\begin{aligned}
 [L^{1i}, L^{1j}] &= -i L^{ij} + \frac{4}{(p^0 + R^1/\pi)^2} \sum_{m=1}^{\infty} \left\{ \left(\alpha_0 - \frac{d-2}{24} \right) \frac{1}{m} \right. \\
 &\quad \left. + m \left(\frac{d-2}{24} - 1 \right) \right\} (a_{-m}^i a_m^j - a_{-m}^j a_m^i), \quad (9)
 \end{aligned}$$

where the special role of $i = 1$ is due to a gauge choice. It is seen that the theory is only consistent in 26 dimensions (and with intercept $\alpha_0 = 1$), as is to be expected. However, one important feature emerges from the "mass" formula (3) or (8): the coefficient of the anomaly behaves like

$$4 / (p_0 + R^1/\pi)^2 \rightarrow 16 \pi^2 \alpha'^2 / R^2 \quad (10)$$

for $R \rightarrow \infty$. Thus, we see that the anomaly disappears asymptotically for any dimension (and for any intercept). The model is asymptotically consistent!

We now turn to the Neveu-Schwarz and the Ramond models. Here one could attempt to do a functional integral in order to obtain the heavy quark-antiquark potential, in analogy with calculations of the partition function⁷⁾ in dual models. The domain could then be taken as a rectangle with sides R and T . However, it is not clear how to define the Neveu-Schwarz and the Ramond models in this case. On two opposite sides, one has of course the usual boundary conditions for these two models. With four sides the situation is less clear. We have therefore decided to generalize Arvis method⁵⁾ to the supersymmetric case. Our arguments presumably lead to the same results as the functional integral method, but we do not have any proof of this.

Because of the "mass formula" interpretation of the Alvarez-Arvis formula (3), it is clear what one should expect on the basis of the known zero-point fluctuations in the Neveu-Schwarz and Ramond models: R_C^2 should be $3/2$ times the scalar value in the N-S case, and R_C^2 should vanish in the Ramond case. Thus, in the latter case, the "Coulomb"-term should vanish!

The supersymmetric strings (N-S and Ramond) are, e.g., defined by the action

$$S = \text{const.} \int \left[\frac{1}{2} (\partial_\alpha x)^2 + \frac{i}{2} \psi \not{\partial} \psi \right] d^2 \xi \quad (11)$$

with the supplementary conditions

$$\partial_\mu x \partial_\nu x - \frac{1}{2} \eta_{\mu\nu} (\partial_\rho x)^2 + \frac{i}{4} \psi (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi = 0,$$

$$\partial_\nu x \gamma^\nu \gamma_\mu \psi = 0. \quad (12)$$

The supplementary conditions can be derived from the complete action of Deser and Zumino⁸⁾ and Brink, Di Vecchia and Howe⁹⁾. It is also easy to generalize Lüscher's argument¹⁾ in such a way that Eq. (11) (or the complete action^{8),9)} become the effective large distance action for a universality class of string models with scalar fields x and a two-dimensional Majorana spinor $\psi(\sigma, \tau)$, which is also a Minkowski vector. The main point in this argument is that Eq. (11) contains the minimal number of derivatives, and hence all other conceivable terms will die out faster at large distances.

The equations of motion

$$\square x = 0 \quad \text{and} \quad \not\partial \psi = 0 \quad (13)$$

with the boundary conditions

$$\begin{aligned} x^i(0, \tau) &= 0, \quad x^i(\pi, \tau) = R^i, \\ \psi_1^i(0, \tau) &= \psi_2^i(0, \tau), \\ \psi_1^i(\pi, \tau) &= \begin{cases} \psi_2^i(\pi, \tau) & (\text{Ramond}) \\ -\psi_2^i(\pi, \tau) & (\text{Neveu-Schwarz}) \end{cases} \end{aligned} \quad (14)$$

have the solutions (7) as well as

$$\begin{aligned} \psi_1^i(\sigma, \tau) &= \sum_{r=-\infty}^{\infty} b_r^i e^{-ir(\tau-\sigma)}, \\ \psi_2^i(\sigma, \tau) &= \sum_{r=-\infty}^{\infty} b_r^i e^{-ir(\tau+\sigma)}, \end{aligned} \quad (15)$$

where r is an integer in the Ramond case and a half-integer in the Neveu-Schwarz case. In the above equations, i runs from 1 to $d-1$. Because of the supersymmetry transformation

$$\delta x = i \bar{\alpha} \psi \quad (16)$$

it is seen from the boundary conditions for ψ in Eq. (14) that ^{*}) $\delta x^1 = 0$ on the boundary. Thus a supersymmetry transformation leaves the boundary conditions for x in Eq. (14) invariant.

Following Arvis⁵⁾ we choose a gauge where

$$\begin{aligned} \partial_0 x^1 + \partial_1 x^0 &= 0, \quad \partial_0 x^0 + \partial_1 x^1 = p^0 + \frac{R^1}{\pi}, \\ x^0 &= 0. \end{aligned} \quad (17)$$

^{*}) One should choose α such that it has the opposite boundary conditions to the ψ 's. Thus, for $\sigma = 0$ one should have $\alpha_1(0, \tau) = -\alpha_2(0, \tau)$. Similarly, $\alpha_1(\pi, \tau) = \mp \alpha_2(\pi, \tau)$. With these choices it is easily seen that $\psi_1^1(0, \tau) - \psi_2^1(0, \tau)$ and $\psi_1^1(0, \tau) \mp \psi_2^1(\pi, \tau)$ are invariant under supersymmetry transformations.

This corresponds to taking $a_n^0 + a_n^1 = 0$, and (17) is therefore the closest one can get to the light-cone gauge. The latter is not a possible choice because Eq. (7) involves $\sin n\sigma$ for x^1 but $\cos n\sigma$ for x^0 . Furthermore we impose the gauge condition^{*})

$$\psi^0 + \psi^1 = 0. \quad (18)$$

We shall now find the energy. Since we have identified x^μ with the position of the string, the time-component of the displacement operator is given by

$$\frac{1}{2\pi\alpha'} \int_0^\pi \partial_0 x^0(\sigma) d\sigma = p_0. \quad (19)$$

We shall identify this with the energy. We do not have a proof that this agrees with what one would obtain for the Wilson loop (in the time $\rightarrow \infty$ limit) from a functional integration. Thus, our results are valid modulo the assumption that (19) agrees with some functional integration (where it is not clear to the author how to specify the boundary conditions for the Neveu-Schwarz and Ramond models for a rectangle).

We can now use Eqs. (7), (15) and (19) together with the first supplementary condition (12) to obtain ($2\pi\alpha' = 1$):

$$\begin{aligned} p_0^2 = & \frac{1}{\pi^2} R_1^2 + \frac{1}{\pi} \int_0^\pi d\sigma \left[(\partial_0 x^k)^2 + (\partial_1 x^k)^2 \right. \\ & \left. - i \psi_1^k \partial_1 \psi_1^k + i \psi_2^k \partial_1 \psi_2^k \right], \\ & k = 2, 3, \dots, d-1. \end{aligned} \quad (20)$$

This leads again to Eq. (3) by taking into account the appropriate zero-point fluctuations, i.e.,

$$p_0 = \alpha \sqrt{R^2 - R_c^2}, \quad (21)$$

with

^{*}) Logically one should first choose (18), and then impose (17) by a suitable conformal transformation [which leaves (18) invariant].

$$R_c^2 = \begin{cases} 0 & (\text{Ramond}) \\ \frac{(d-2)\pi}{8\sigma} & (\text{Neveu-Schwarz}). \end{cases} \quad (22)$$

Expanding Eq. (21),

$$P_0 = \sigma R - \frac{\sigma R_c^2}{2} \frac{1}{R} + O\left(\frac{1}{R^3}\right) \quad (23)$$

one sees that the coefficient of the long-distance "Coulomb" term depends on the dynamical structure of the various string models.

One can also check rotational invariance. The anomaly analogous to the one in Eq. (9) becomes

$$\begin{aligned} & \frac{4}{(p^0 + \frac{R^1}{\pi})^2} \left\{ \sum_{m=1}^{\infty} \left[\left(2\alpha_0 - \frac{d-2}{8} \right) \frac{1}{m} + \left(\frac{d-2}{8} - 1 \right) m \right] \right. \\ & \times (a_{-m}^i a_m^j - a_{-m}^j a_m^i) + \sum_{r=1/2}^{\infty} \left[\left(2\alpha_0 - \frac{d-2}{8} \right) \right. \\ & \left. \left. + 4r^2 \left(\frac{d-2}{8} - 1 \right) \right] (b_{-r}^i b_r^j - b_{-r}^j b_r^i) \right\}, \end{aligned} \quad (24)$$

which of course vanishes for $d = 10$ and $\alpha_0 = \frac{1}{2}$. However, since $p^0 \rightarrow R$ for $R \rightarrow \infty$, we see that the anomaly vanishes asymptotically like $1/R^2$ for $R \rightarrow \infty$. Thus, the Neveu-Schwarz model becomes consistent at large distances for any number of dimensions.

It should also be mentioned that from the "mass formula" interpretation of the Alvarez-Arvis Eq. (3), one expects that a dual model without tachyons leads to $R_c = 0$, i.e., it has no long-distance "Coulomb" term. Thus, the superstring of Green and Schwarz is not expected to have such a term.

We now turn to a comparison with QCD Monte Carlo data. Since the long-distance "Coulomb" term has the form $1/R$ in all dimensions, it was argued by Ambjørn et al.¹⁰⁾ that a comparison was more straightforward in three dimensions (where the genuine Coulomb term is of the form $\ln R$) than in four dimensions. The results obtained in Ref. 10) are in perfect agreement with the scalar string (2). We refer to Ref. 10) for a detailed discussion.

In four dimensions, the situation is not so clear, because the string "Coulomb" term and the genuine Coulomb term have the same functional form and

the same sign. One can argue that for distances of the order of a correlation length or larger, the genuine Coulomb term has disappeared. Although this argument should in principle be valid, it is not quite clear to what extent the correlation length is determined at present. Otto and Stack¹¹⁾ have obtained data for the SU(3) potential. Their analysis gives a result

$$0.25 \pm 0.02 \quad (25)$$

for the coefficient of the Coulomb term which is very close to $\pi/12$ (the scalar string). A similar conclusion has been obtained by Barkai, Moriarty and Rebbi¹²⁾. However, using a different type of fit, Flensburg and Peterson¹³⁾ obtained higher values, in the range

$$(0.30) - (0.35), \quad (26)$$

depending on the value of β . Thus, in four dimensions the situation is rather unclear. The values (26) do not agree with the scalar or the Neveu-Schwarz models, so perhaps QCD has selected a string which is at present unknown. However, it should be noticed that (25) as well as (26) are inconsistent with the Ramond string.

Thus, in conclusion it seems that some long-distance "Coulomb" term is needed to describe QCD in three and (less convincing perhaps) in four dimensions. At least for simple string models this implies that QCD has selected a string with tachyons. As we have demonstrated, this is consistent at large distances (irrespective of critical dimensions), but is of course impossible in a global sense. The "requirement" of a tachyon could be a sign there does not exist a string theory which reproduces QCD at all distances. This remark is of course only valid for simple string theories, and does not apply to more complicated cases, like the Polyakov string or the strings obtained by adding terms with higher derivatives à la Lüscher¹⁾. Furthermore, in QCD one does not really expect strings, but rather flux tubes. From this point of view, it is also clear that at short distances the string picture must break down.

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