N150(1985)

A 85-150

9

EFFECTIVE ACTION APPROACH TO SUPERSTRING THEORY

E.S.Fradkin and A.A.Tseytlin

Department of theoretical physics,

P.N.Lebedev Physical Institute,

Leninsky pr. 53, Moscow 117924 USSR

ABSTRACT

We define the quantum effective action for local fields corresponding to superstring excitation modes. It is given by a superstring theory path integral with a generalized superstring action containing couplings to background fields of D=10 super-Yang-Mills and supergravity multiplets. Dilaton couplings in a low-energy approximation for the effective action are shown to be the same as in corresponding D=10 supergravity actions.

CERN LIBRARIES, GENEVA

CM-P00067788

1. Superstring theory is presently considered as an interesting candidate for a consistent unification of all fundamental interactions [1]. To study consequences of this theory one needs a formalism which makes possible to formulate the string-theory ground-state problem and provides a systematic covariant method for computation of corresponding low-energy field-theory action. Such a formalism was recently suggested in [2] in the context of the Bose string theory. Here we consider generalization to the superstring case.

The approach of ref. [2] is based on an off shell effective action Γ for the infinite set of local fields corresponding to the modes of a free string spectrum. Γ accumulates the full quantum dynamics of the "second-quantized" string theory: it is a generating functional of all possible off-shell scattering amplitudes on arbitrary backgrounds (i.e. is a direct analog of the ordinary QFT effective action). In the case of the theory of interacting open and closed

Bose strings Γ is defined as follows [2]

$$\Gamma[\Phi, G_{\mu\nu}, A_{\mu\nu}, \mathcal{G}, \dots | \phi, A_{\mu}, \dots] = \sum_{\chi=1,0,-1,\dots} e^{\epsilon \chi} \sum_{M^2} \int_{\Sigma} de(t) dx^\mu(t) \int_{M^2} dg_{ab}(z) dx^\mu(z) \times \exp(-I_2 - I_1), \quad (1)$$

$$I_2 = \int_{M^2} d^2z \left\{ \sqrt{g} \Phi(x(z)) + \frac{1}{4\pi} \sqrt{g} R \mathcal{G}(x(z)) + \frac{1}{4\pi\alpha'} \sqrt{g} g^{ab} \partial_a x^\mu \partial_b x^\nu G_{\mu\nu}(x(z)) + i \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu A_{\mu\nu}(x(z)) + \dots \right\}, \quad (2)$$

$$I_1 = \int_{\Sigma} dt \left\{ e \phi(x(t)) + i \dot{x}^\mu A_\mu(x(t)) + \dots \right\} \quad (3)$$

Here: $x^\mu(z)$ define the embedding of a virtual string world sheet M^2 in a space-time $\mathcal{M}^D$ ($\mu, \nu = 1, \dots, D$); z^a ($a=1,2$) are coordinates on M^2 ; g_{ab} is a metric on M^2 ; R is the curvature scalar of g_{ab} ; Σ is the boundary of M^2 parametrized by t ; $e^2(t)$ is a metric on Σ . In (1) one sums over all possible virtual string world sheets M^2 - compact oriented 2-manifolds with boundary Σ , which topologically are a disc with arbitrary numbers of holes (n) and handles (k) ($\chi = 1-n-2k$ is the Euler number of M^2). In each topological class one first fixes the boundary, integrates over surfaces with given boundary conditions ($x^\mu(z)|_{\Sigma} = x^\mu(t)$, $g_{ab} \dot{z}^a(t) \dot{z}^b(t) = e^2(t)$)

and then sums over all boundary curves. The fields in (2) ((3)) correspond to the modes of the closed (open) Bose string spectrum: Φ is the "ground-state" scalar, φ is the "dilaton", $G_{\mu\nu}$ is the metric ($G_{\mu\nu} - \delta_{\mu\nu}$ is the "graviton"), $A_{\mu\nu}$ is the antisymmetric tensor, etc. (ϕ is the open string "ground-state" scalar, A_μ is the vector, etc.)^{1/}. Dots in (2), (3) stand for terms containing higher spin tensor fields, e.g. $\sqrt{g} g^{ab} g^{cd} \partial_\mu x^\mu \partial_\nu x^\nu \partial_\rho x^\rho \partial_\sigma x^\sigma C_{\mu\nu\rho\sigma}$ in (2) and $e^{-1} \dot{x}^\mu \dot{x}^\nu C_{\mu\nu}$ in (3). The constant σ in (1) is related to the dimensionless coupling constant of open strings $g_0 \sim e^{-\frac{1}{2}\sigma}$. In the case of a theory of non-oriented open and closed strings one is to sum over all compact non-oriented 2-manifolds $M^{2,2/}$ which topologically are a disc with holes, handles and Möbius strip insertions (thus one is also to omit in (2) all terms containing ε^{ab} -symbol). In a closed string theory case one is to sum over closed M^2 .

2. To generalize (1) to the case of superstring theories of type I (non-oriented open and closed strings with N=2, D=10 supersymmetry) or type II (oriented closed strings with N=2, D=10 supersymmetry) we are to generalize (2) and (3), constructing their supersymmetric extensions (and also are to integrate in (1) over Grassman variables θ). Superstring spectra [1] are known to start with the massless multiplets corresponding to the fields of D=10 supersymmetric theories [4-7]^{3/}:

type I closed strings: $\{G_{\mu\nu}, \varphi | B_{\mu\nu}, \psi_\mu, \xi\}$ (N=1 supergravity) (4)

-
- 1/ one can also introduce an internal symmetry group G by assigning the fields corresponding to the open string sector to proper representations of G and path ordering along Σ .
 - 2/ this prescription automatically implies that there are just seven types of interaction of closed and open non-oriented virtual strings, cf. [3].
 - 3/ it is probably possible to understand the absence in superstring spectra of the "tachyon" scalars Φ and ϕ as non-existence of supersymmetric extensions of the corresponding terms in (2) and (3).

type I open strings: $\{A_\mu | \lambda\}$ (super Yang-Mills)

type IIA closed strings: $\{G_{\mu\nu}, A_{\mu\nu}, \varphi | B_\mu, B_{\mu\nu\rho}, \psi_\mu^\pm, \varphi^\pm\}$
(N=2 non-chiral supergravity)

type IIB closed strings: $\{G_{\mu\nu}, A_{\mu\nu}, \varphi | B, B_\mu, B_{\mu\nu\rho\sigma}^+, 2\psi_\mu^+, 2\varphi^+\}$
(N=2 chiral supergravity)

Here B are antisymmetric tensors, $B_{\mu\nu\rho\sigma}^+$ has self-dual generalized field strength and λ, ψ_μ and φ are Majorana-Weyl spinors ($\pm$ indicate chirality). The fields in (4) are separated into two groups: we shall see that the first group originates from the bosonic sector of superstring action and is already present in (2), while the second group couples to the fermionic sector. The supersymmetry algebras corresponding to the multiplets in (4) close only on the mass shell (on the super YM or SG field equations), i.e. no complete off-shell formulation of the D=10 supersymmetric theories is presently known [8,9]. This suggests impossibility to construct an off-shell supersymmetric coupling of superstring (or a boundary superparticle) to the fields of D=10 supermultiplets. That is why we shall adopt the following strategy: we shall construct the coupling using the condition that generalizations of (2) and (3) (and hence Γ) are to be super-invariant if the background fields are restricted on the classical mass shell; having defined the analog of (1) we shall ^{then} consider the arguments of Γ to be arbitrary. (Γ will thus be non-super-invariant off-shell). The resulting coupling in (2), (3) will of course be ambiguous in terms which vanish on the classical mass shell (we shall use the "minimal" prescription simply omitting all such terms). This ambiguity hopefully does not influence the physical consequences (e.g. the structure of the true ground state) predicted by Γ . 4/

4/ this situation is clearly analogous to that in an ordinary field theory with an open gauge algebra, $[\delta_1, \delta_2]\phi = \delta_1\phi + \Omega(\delta S/\delta\phi)$ (Ω is some antisymmetric operator, S is a classical action). Consideration of $[\delta_1, \delta_2]\Gamma$ shows that an arbitrary functional Γ can be invariant only if it is an arbitrary function of S or if ϕ is restricted to satisfy $\delta S/\delta\phi = 0$. For example, if we neglect the auxiliary field in N=1, D=4 super Yang-Mills theory then the cor-

One can in principle maintain the full supersymmetry of L in 8 transverse dimensions, using the string theory light-cone gauge and assuming that all background fields have only transverse components being non-trivial (and depending only on transverse coordinates). To generalize Γ computed for this particular background to a full covariant effective action one is then to invoke the condition of D=10 Lorentz invariance. This procedure, however, has an obvious drawback of missing terms containing the $\varepsilon^{\mu_1 \dots \mu_8}$ -symbol. Yet this approach may have some advantages over the covariant one (e.g. it is applicable, at least at the level of effective equations, to type IIB superstring theory).

3. Our starting point is the covariant superstring action [10]

$$I = -\frac{1}{4\pi\alpha'} \int d^2z \mathcal{L}, \quad \mathcal{L} = \sqrt{g} g^{ab} (\partial_a x^\mu - i \bar{\theta} \gamma^\mu \partial_a \theta) (\partial_b x^\mu - i \bar{\theta} \gamma^\mu \partial_b \theta) + 2i \varepsilon^{ab} \partial_a x^\mu (\bar{\theta}^1 \gamma^\mu \partial_b \theta^1 - \bar{\theta}^2 \gamma^\mu \partial_b \theta^2) - 2 \varepsilon^{ab} \bar{\theta}^1 \gamma^\mu \partial_a \theta^1 \bar{\theta}^2 \gamma^\mu \partial_b \theta^2 \quad (5)$$

Here θ^A ($A=1,2$) are two D=10 Majorana-Weyl spinors and $\{\gamma_\mu, \gamma_\nu\} = -\eta_{\mu\nu}$, $\eta_{\mu\nu} = (- + \dots +)$. Introducing 2-dimensional Dirac matrices

$(\rho_{\hat{a}})_{AB}$ ($\hat{a} = 1,2$; "hatted" indices correspond to the tangent-space basis): $\rho^{\hat{0}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\rho^{\hat{1}} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, $\rho^{\hat{2}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\rho^{\hat{a}} \rho^{\hat{b}} = -g^{\hat{a}\hat{b}} + \varepsilon^{\hat{a}\hat{b}} \rho^{\hat{3}} / \sqrt{g}$, $g = -\det g_{\hat{a}\hat{b}}$, $\varepsilon^{\hat{0}\hat{1}} = 1$, $\eta_{\hat{a}\hat{b}} = (- +)$, we can rewrite (5) in the following simple form

$$\mathcal{L} = \sqrt{g} \left[g^{ab} \partial_a x^\mu \partial_b x^\mu + 2i \partial_a x^\mu \bar{\theta} \gamma^\mu \rho^{\hat{a}} \rho^{\hat{b}} \partial_b \theta - \bar{\theta} \gamma^\mu \partial_a \theta \bar{\theta} \gamma^\mu \rho^{\hat{a}} \rho^{\hat{b}} \partial_b \theta \right], \quad (6)$$

where the A-indices of θ^A are contracted with the help of $\rho^{\hat{a}}$. Note that (6) is Lorentz-invariant only if θ^A transforms as a couple of 2-scalars and not as a 2-spinor ($\bar{\theta} = \theta^T \gamma^0$ does not include $\rho^{\hat{0}}$), i.e. the use of $\rho^{\hat{a}}$ in (6) is only a book-keeping device. The action (5) or (6) is invariant under the rigid N=2, D=10 supersymmetry

$$\delta \theta = \varepsilon, \quad \delta x^\mu = i \bar{\varepsilon} \gamma^\mu \theta, \quad \delta g^{ab} = 0, \quad \varepsilon = (\varepsilon^{\hat{a}}), \quad (7)$$

and also under the local "k-supersymmetry" [10]

responding effective action is non-supersymmetric (and gauge-dependent) off-shell.

$$\delta x^\mu = i \bar{\theta} \gamma^\mu \delta \theta, \quad \delta \theta = -i (\partial_a x^\mu - i \bar{\theta} \gamma^\mu \partial_a \theta) \gamma^\mu \rho^a \rho^b \kappa_b, \quad \delta g^{ab} = \dots \quad (8)$$

It is trivial to see that in the light-cone (l.c.) gauge

$$g_{ab} \sim \eta_{ab}, \quad x^+ \equiv \frac{x^0 + x^3}{\sqrt{2}} = a + p z^0, \quad \gamma^+ \theta = 0 \quad (9)$$

(6) reduces to the light-cone superstring action of ref. [11]

$$\mathcal{L} = \partial_a x^i \partial_a x^i - i \alpha' \bar{S} \gamma^\mu \rho^a \partial_a S, \quad i=1, \dots, 8, \quad (10)$$

where $S \equiv \sqrt{\frac{2p}{\alpha'}} \theta$, $\bar{S} = S^\tau \gamma^0 \rho^\delta$ and so S^A in (10) is a 2-dimensional spinor.

To construct the coupling of the open string to the fields of N=1, D=10 super YM theory (A_μ, λ) we are to find a supersymmetric generalization of the vector coupling $\dot{x}^\mu A_\mu(x)$ in (3) (analogous problem in D=4 was discussed in [12]). We assume the following

D=10 N=1 supersymmetry transformation laws

$$\delta \theta = \epsilon, \quad \delta x^\mu = i \bar{\epsilon} \gamma^\mu \theta, \quad \delta A_\mu = i \bar{\epsilon} \gamma_\mu \lambda, \quad \delta \lambda = -\frac{1}{2} \gamma^{\mu\nu} F_{\mu\nu} \epsilon \quad (11)$$

($\theta^1 = \theta^2 = \theta(t)$ at the boundary). The result can be written in a superfield form

$$I_1 = \int dt \dot{Z}^\mu \mathcal{A}_\mu(Z), \quad Z(t) = \{x^\mu, \theta\} \quad (12)$$

The condition that the coupling (12) does not spoil k-supersymmetry of the free supersymmetric "boundary" particle [13] ($\delta x^\mu = i \bar{\theta} \gamma^\mu \delta \theta$, $\delta \theta = (\dot{x}^\mu - i \bar{\theta} \gamma^\mu \theta) \gamma^\mu \kappa$, $\delta \epsilon = \dots$) implies (cf. [14]) that the vector potential superfield $\mathcal{A}_\mu$ should satisfy the known constraint ($\tilde{F}_{\alpha\beta} = 0$)

[8], reducing the independent components of $\mathcal{A}_\mu$ to A_μ and λ subject to the super-YM equations of motion. In the abelian case

$$\mathcal{A}_\mu = A_\mu + \frac{i}{2} \bar{\theta} \gamma_\mu \rho^\nu \theta \partial_\nu A_\nu + \dots, \quad \mathcal{A}_\alpha = i (\gamma_\mu \theta)_\alpha \left\{ \frac{i}{4} \bar{\theta} \gamma_\mu \rho^\nu \theta \partial_\nu A_\nu + \dots \right\} \quad (13)$$

$$I_1 = \int dt \left\{ \dot{x}^\mu \sum_{n=0}^8 \frac{1}{(2n)!} (i \bar{\theta} \gamma_\mu \rho^\nu \theta \partial_\nu)^n (A_\nu - \frac{1}{2n+1} i \bar{\theta} \gamma_\nu \lambda) + \frac{1}{3} \bar{\theta} \gamma_\mu \theta \bar{\theta} \gamma_\mu \lambda \right. \\ \left. + i \bar{\theta} \gamma^\mu \theta \sum_{n=1}^8 \frac{1}{(2n+2)(2n-1)!} (i \bar{\theta} \gamma_\mu \rho^\nu \theta \partial_\nu)^n (A_\nu - \frac{2n+2}{2n(2n+3)} i \bar{\theta} \gamma_\nu \lambda) \right\} \quad (14)$$

$$= \int dt \left\{ \dot{x}^\mu A_\mu - \frac{i}{4} \bar{\theta} \gamma^{\mu\nu} \rho^\sigma \theta F_{\mu\nu} \dot{x}_\sigma + i \bar{\lambda} \gamma_\mu \theta \dot{x}^\mu + \frac{1}{6} \partial_\mu \bar{\lambda} \gamma_\nu \theta \bar{\theta} \gamma^{\mu\nu} \rho^\sigma \theta \dot{x}_\sigma - \frac{1}{48} \bar{\theta} \gamma_{\rho\mu\nu} \theta \bar{\theta} \gamma_{\sigma\lambda} \theta \partial_\mu F_{\lambda\sigma} \dot{x}^\rho + \dots \right\} \quad (15)$$

The result (14) is in agreement with the covariant vertices for emission of an on-shell vector or spinor found in [15].^{5/} Specializing

to the l.c. gauge (9) one observes that only the terms given explicitly in (15) survive and that they reproduce the l.c. vertices of ref.

[16]. The non-abelian generalization of (13)-(15) is obtained by replacing $\partial_\rho \rightarrow D_\rho$, $\partial_{[\rho} A_{\mu]} \rightarrow \frac{1}{2} F_{\rho\mu}$ and introducing additional nonlinear terms like $(\bar{\lambda}\lambda)^n$ (cf. [12]).

Next let us generalize (6) to include coupling ^{to the} background fields of N=1 or N=2 D=10 supergravity. The coupling can be made consistent (i.e. preserving the supersymmetries (7), (8)) only if the fields satisfy the supergravity equations of motion. The N=2 D=10 supersymmetric generalization of the $G_{\mu\nu}$ - and $A_{\mu\nu}$ - terms in (2) is as follows (θ^1 and θ^2 have the same (opposite) chirality in type IIB (IIA) theory)

$$\mathcal{L} = \sqrt{g} \left\{ g^{ab} \partial_a x^\mu \partial_b x^\nu G_{\mu\nu} + \frac{1}{\sqrt{g}} \varepsilon^{ab} \partial_a x^\mu \partial_b x^\nu \bar{A}_{\mu\nu} + 2i \partial_a x^\mu \bar{\theta} \gamma_\mu \rho^a \rho^b \tilde{D}_b \theta + \bar{\theta} \gamma^\mu \tilde{D}_\mu \theta \bar{\theta} \gamma_\mu \rho^a \rho^b \tilde{D}_b \theta + \right. \\ \left. \frac{1}{64} \tilde{R}_{\lambda\mu\nu\rho} \bar{\theta} \gamma^{\lambda\mu} \rho^a \rho^b (1+\rho_3) \theta \bar{\theta} \gamma^{\nu\rho} \rho^c \rho^d (1-\rho_3) \theta \partial_a x^\mu \partial_b x^\nu \partial_c x^\rho \partial_d x^\sigma + \dots \right\} \quad (16)$$

Here: $\bar{A}_{\mu\nu} \equiv 4\pi\alpha' A_{\mu\nu}$, $\tilde{R}^\lambda{}_{\mu\nu\rho} = \partial_\nu \tilde{\Gamma}^\lambda{}_{\mu\rho} - \dots$, $\tilde{\Gamma}^\lambda{}_{\mu\rho} = \Gamma^\lambda{}_{\mu\rho}(G) + \frac{1}{2} \bar{F}^\lambda{}_{\mu\rho}$, $\bar{F}_{\lambda\mu\rho} = 3 \partial_{[\lambda} \bar{A}_{\mu\rho]}$, $\tilde{D}_a = \partial_a - \frac{1}{4} \gamma^{\hat{\rho}\hat{\sigma}} \tilde{\omega}^{\hat{\rho}\hat{\sigma}}{}_a \partial_{\hat{\rho}} x^\mu$, $\tilde{\omega}_{\dots} = \omega_{\dots} + \frac{1}{2} \bar{F}_{\dots} \rho_3$, $\omega_{\mu\nu\rho}$ is the Lorentz connection. The highest-order (θ^{32}) term in

(16) contains 16 derivatives of $G_{\mu\nu}$ and $A_{\mu\nu}$. Choosing the l.c. gauge (9) and assuming the fields to be non-trivial only in 8 transverse dimensions (so that $\mathcal{M}^{10} = \mathbb{R}^{1,1} \times \mathcal{M}^8$) we get from (16)

$$\mathcal{L}_{l.c.}^{(G,A)} = \partial_a x^\mu \partial_b x^\nu G_{\mu\nu} + \varepsilon^{ab} \partial_a x^\mu \partial_b x^\nu \bar{A}_{\mu\nu} - i\alpha' \bar{S} \gamma^\mu \rho^a \tilde{D}_a S \\ + \frac{\alpha'^2}{256} \tilde{R}_{ijkl} \bar{S} \gamma^{ij} \rho^a (1+\rho_3) S \bar{S} \gamma^{kl} \rho^b (1-\rho_3) S + \dots, \quad \gamma^{ij} \equiv \gamma^{ij} \gamma^\mu. \quad (17)$$

The counterpart of (16) for type I superstring theory is obtained by

putting $A_{\mu\nu} = 0$. Then the S^4 -term in (17) takes the form:

$$\frac{\alpha'^2}{128} \tilde{R}_{ijkl} \bar{S} \gamma^{ij} \rho^a S \bar{S} \gamma^{kl} \rho_a S. \text{ Linearizing (17) with respect to } \tilde{H}_{ij} = H_{ij} + \bar{A}_{ij} \\ (H_{ij} = G_{ij} - \delta_{ij}) \text{ and going on shell one can reproduce the result for}$$

5/ $\bar{\theta} \gamma^{\mu\nu} \rho \theta$ is the only combination that can appear in the expansion of a superfield. To find the coefficients in (14) one is to solve recurrent relations following from the condition of E- and κ -supersymmetry. Note that the Grassman nature of θ makes it possible to truncate the infinite series appearing in analogous expression in [15].

the l.c. vertices for emission of a graviton or antisymmetric tensor given in [1] (the correct dilaton emission vertex is obtained after the replacement $G_{ij} \rightarrow G_{ij} \exp\left(\frac{\varphi}{D-2}\right)$ [2]). Eq. (17) is interesting to compare with the generalized G -model action of ref. [17] also containing the first two bosonic terms but depending on the fermionic 2-spinor - 8-vector variable ψ^i (while S in (17) is a 2-spinor - 8-spinor).^{6/} Note that to find backgrounds corresponding to conformally-invariant 2-dimensional theories (i.e. candidates for superstring theory vacuum configurations) one must generally study the renormalization properties of the non-standard theory (16) or (17) and not of the G -model (cf. [18]).

To determine the superstring couplings of the other Bose fields in (4) one may use (16) and impose the rigid $D=10$ supersymmetry relating the fields in supergravity multiplet. It is simpler, however, to start with the known on-shell l.c. vertices [1], to guess their off-shell generalizations and then to check the super-invariance. Assuming that all the B...-tensors in (4) have only transverse components being non-zero we find in the l.c. gauge

$$\mathcal{L}_{l.c.}^{(B)} = \frac{i}{2\sqrt{\alpha'}} \left\{ \bar{S} \gamma^{(i} \Gamma_{\Lambda} \gamma^{j)} (1-\rho_3) S [-\partial_a x^i \partial_a x^j + \frac{i}{12} \alpha' \bar{S} \gamma^{ni} \rho^a S \partial_a x^j \partial_n \right. \\ \left. - \frac{\alpha'^2}{576} \bar{S} \gamma^{ni} \rho^a S \bar{S} \gamma^{mj} \rho^a S \partial_n \partial_m] B_{\Lambda} + \right. \quad (18)$$

$$+ \bar{S} \gamma^{[i} \Gamma_{\Lambda} \gamma^{j]} (1-\rho_3) S [\varepsilon^{ab} \partial_a x^i \partial_b x^j + \frac{i}{12} \alpha' \bar{S} \gamma^{ni} \rho^a S \partial_a x^j \partial_n \\ \left. - \frac{\alpha'^2}{576} \bar{S} \gamma^{ni} \rho^a S \bar{S} \gamma^{mj} \rho_a \rho_3 S \partial_n \partial_m] B_{\Lambda} \right\} + \dots, \quad (19)$$

$$\Gamma_{\Lambda} B_{\Lambda} = \{ \gamma^{\Lambda} B, \gamma^i B_i, \gamma^{ij} B_{ij}, \gamma^{ijk} B_{ijk}, \hat{\gamma}^{ijkl} B_{ijkl}^+, \hat{\gamma}^{ijkl} B_{ijkl}^- \}, \quad \hat{\gamma}^{ijkl} = \gamma^{[i} \gamma^j \gamma^k \gamma^{l]} = \frac{1}{4!} \varepsilon_{ijkl} \gamma^5$$

The indices $i, j, \dots$ are contracted with the help of G_{ij} and

$\gamma_i \gamma_j = G_{ij}$. Dots in (18) stand for higher-derivative terms which do not contribute to the l.c. vertices. A covariant generalization

6/ for some particular backgrounds both actions can be put in one-to-one correspondence, cf. [18]

of (18) can be obtained using the rule: $\alpha' \rho^\alpha \gamma^\mu \rightarrow \rho^\alpha \gamma^\mu \partial_\alpha x^\mu$
 so that the covariant expression will look like: $\bar{\theta} \dots \theta \partial x \partial x \partial x B \dots ?!$

Next we are to consider the fermionic couplings. The corresponding l.c. vertices [1] contain the fermions in the combination $\psi_i + \gamma_i \zeta$ (ψ_i is the on-shell gravitino field, $\gamma_i \psi_i = 0$). Hence we encounter the problem of separation of the spinor ζ from the gravitino ψ_μ in an off-shell expression. This problem is solved in the same way as the analogous problem in the bosonic sector (where we had to separate the dilaton φ from the metric $G_{\mu\nu}$ [2]), i.e. by introducing two independent groups of couplings:

$(\dots) \zeta + (\dots) (\psi_\mu + \gamma_\mu \zeta)$. The D=10 supersymmetry implies that the $R \sqrt{g} \varphi$ -term in (2) is to be supplemented by $R \sqrt{g} \{ \bar{\theta} \zeta + \bar{\theta} \dots \theta \partial \varphi + \dots \}$

The l.c. gravitino coupling reproducing on-shell the l.c. gravitino

omission vertex is

$$\mathcal{L}_{l.c.}^{(\psi)} = \alpha'^{-1} \left[- \partial_\beta x^i \partial_\alpha x^j - \frac{i}{8} \alpha' (\bar{S} \gamma^{jm} \rho_\alpha S \partial_\beta x^i + \frac{1}{3} \bar{S} \gamma^{im} \rho_\beta S \partial_\alpha x^j) \partial_m \right. \\ \left. + \frac{\alpha'^2}{192} \bar{S} \gamma^{im} \rho_\beta S \bar{S} \gamma^{jn} \rho_\alpha S \partial_m \partial_n \right] \bar{S} \gamma_i \rho^\alpha \rho^\beta \psi_j + \dots \quad (20)$$

(∂_n are covariant derivatives acting on ψ_j). A covariant counterpart of (20) is third-order in ∂x : $\bar{\theta} \dots \psi \partial x \partial x \partial x$ (the couplings

which are at least fourth-order in ∂x correspond to higher spin massive supermultiplets, cf. (2)). Finally one can in principle "glue" together various pieces in superstring-supergravity interaction action imposing the condition of on-shell invariance under the standard supergravity transformation laws.

4. Having determined the type of couplings generalizing (2), (3) we can find the corresponding low-energy effective action. Consider first the case of closed type IIA superstring theory. The general structure of the effective action is then analogous to that in the Bose string case [2]

7/ a covariant extension of (18) is probably impossible in the case of N=2 chiral theory because no covariant analog of the coupling $\hat{\gamma}^{ijkl} B_{ijkl}$ (i.e. of $\hat{\gamma}$) exists. Hence it is not clear how to define a covariant effective action for type IIB superstring theory (see [1,7] for analogous conclusions).

$$\Gamma[G, \varphi, A_{\mu\nu}] \simeq -\frac{1}{k^2} \int d^D x \sqrt{G} e^{-2\varphi} \left\{ \mathcal{R} + 4(\partial_\mu \varphi)^2 - \frac{1}{12} \bar{F}_{\mu\nu\rho}^2 + \dots \right\}^{K \sim g_c \alpha'^{\frac{D-4}{4}}} \quad (21)$$

where $\mathcal{R}$ is the curvature of $G_{\mu\nu}$, $\bar{F}_{\mu\nu\rho} = 3 \partial_{[\mu} \bar{A}_{\nu\rho]}$, $g_c \sim e^{-6}$ is the closed string theory coupling constant and D is assumed to be a critical one ($D=10$ for superstrings). After the Weyl rescaling $G_{\mu\nu} \rightarrow G_{\mu\nu} \exp\left(\frac{4\varphi}{D-2}\right)$ (21) coincides with the corresponding part of $N=2$, $D=10$ supergravity action [6] (a remarkable fact which is difficult to prove within the approach based on on-shell amplitudes is that the string theory predicts just the right form of coupling of φ to $A_{\mu\nu}$ [2]). To see how other bosonic couplings of $N=2$ SG can appear from superstring theory suppose that after integration over θ (21)

$$\Gamma \simeq -\frac{1}{k^2} \int d^D x e^{-2\varphi} \left\{ \sqrt{G} \left[\mathcal{R} + 4(\partial_\mu \varphi)^2 - \frac{1}{12} \bar{F}_{\mu\nu\rho}^2 + a_1 \hat{F}_{\mu\nu}^2 + a_2 \hat{F}_{\mu\nu\lambda\rho}^2 \right] + a_3 \varepsilon^{\mu_1 \dots \mu_{10}} \hat{F}_{\mu_1 \dots \mu_4} \hat{F}_{\mu_5 \dots \mu_8} \bar{A}_{\mu_9 \mu_{10}} + \dots \right\} \quad (22)$$

where dots stand for $\partial_\mu \varphi$ -dependent terms and $\hat{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $\hat{F}_{\mu\nu\rho\sigma} = 4 \partial_{[\mu} B_{\nu\rho\sigma]} + 8 B_{[\mu} \bar{F}_{\nu\rho\sigma]}$, $a_i = \text{const}$.

Then making the rescaling ($D=10$): $G_{\mu\nu} \rightarrow \tilde{G}_{\mu\nu} e^{\varphi/2}$, $B_\mu \rightarrow \tilde{B}_\mu e^\varphi$, $B_{\mu\nu\rho} \rightarrow \tilde{B}_{\mu\nu\rho} e^\varphi$, we get from (22)

$$\Gamma \simeq -\frac{1}{k^2} \int d^D x \sqrt{\tilde{G}} \left\{ \tilde{\mathcal{R}} - \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{12} \tilde{F}_{\mu\nu\rho}^2 e^{-\varphi} + a_1 \tilde{F}_{\mu\nu}^2 e^{\frac{3}{2}\varphi} + a_2 \tilde{F}_{\mu\nu\rho\sigma}^2 e^{\varphi/2} + a_3 \varepsilon^{\mu_1 \dots \mu_{10}} \tilde{F}_{\mu_1 \dots \mu_4} \tilde{F}_{\mu_5 \dots \mu_8} \bar{A}_{\mu_9 \mu_{10}} + \dots \right\} \quad (23)$$

These couplings of φ are exactly the same as present in $N=2$, $D=10$ non-chiral SG [6]. The reason why we need to make the rescaling of B_μ and $B_{\mu\nu\rho}$ and not of $A_{\mu\nu}$ is probably related to the fact ^{that} B 's couple to the fermionic sector of the superstring action (18) (where also $G_{\mu\nu}$ is present, contracting the indices).

In the case of non-oriented type I superstrings $A_{\mu\nu}$ is absent in (16) and (21). As follows from (18), (19) the $N=1$, $D=10$ SG anti-symmetric tensor $B_{\mu\nu}$ couples to the $\bar{\theta}\theta$ -sector of the string action (this point was misunderstood in [18]). It is natural to anticipate that integration over θ will produce the term $\sim F_{\mu\nu\rho}^2(B)$ in the square brackets in (21). The square brackets in (21) will also

receive contribution $\sim \text{tr } F_{\mu\nu}^2(A)$ of the vector coupling in (15). Then after $G_{\mu\nu} \rightarrow G_{\mu\nu} e^{g/2}$ we will get the action ($\mathcal{L} \sim \mathcal{R} - \frac{1}{2}(\partial\varphi)^2 + F_{\mu\nu}^2 e^{-\varphi} + t_2 F_{\mu\nu}^2 e^{-g/2} + \dots$) having the same structure as the (part of) N=1 SG - super YM action [5].^{8/}

On-shell supersymmetry of Γ implies that (21) or (23) will be supplemented by other (fermionic) terms present in supergravity actions. It is possible to prove (using the l.c. gauge and considering only transverse components of the fields) that the standard kinetic terms for ψ_μ and φ do appear in (23). (It is not clear at present how to integrate over θ in a covariant gauge in view of the absence of a "free" bilinear in θ term in (5) or (6)). Thus Γ will start with the SG (plus super YM) action. Higher order terms in Γ ($\sim \mathcal{R}^2 + \dots$) will not be invariant under the standard supersymmetry transformations, i.e. will not be supersymmetric off-shell. (Γ is also unlikely to be invariant under a simple modification of the classical supersymmetry transformations suggested in ref. [18]). Yet the resulting effective action may not be useless. Suppose, e.g., that the quantum vacuum solution determined by extremizing Γ coincides with a classical solution depending on a (classically arbitrary) parameter r (e.g. a scale of compact 6-space). Then computing Γ on a classical solution we get an unambiguous function $f(r)$ the extremum of which determines the value of r (e.g. to be $\sqrt{\alpha'} \exp(1/g_c^2)$).

8/ an interesting consequence of (18), (19) is that type I superstr-

ing theory seems to prefer the formulation of N=1, D=10 SG with a 2-tensor $B_{\mu\nu}$ [5] and not the dual formulation with a 6-tensor $B_{\mu_1 \dots \mu_6}$ [19]: though we can substitute B_{ij} for $E_{ij\kappa_1 \dots \kappa_6} B_{\kappa_1 \dots \kappa_6}$ in (18), (19) no D=10 covariant extension of the coupling $\gamma^\mu E_{ij\kappa_1 \dots \kappa_6} B_{\kappa_1 \dots \kappa_6}$ seems to exist. Note that a covariant version with the 6-tensor could be well-come in view of the result [20] that the low-energy action for type I theory depends on $B_{\mu\nu}$ through $H = dB + \omega_Y + a \omega_L$, where ω 's are Chern-Simons forms for the YM and Lorentz connections. The duality transformation $B_{\mu\nu} \rightarrow B_{\mu_1 \dots \mu_6}$ transforms the term $H^2 e^{-\varphi}$ into $e^{2\varphi} F_{\mu\nu}^2 + [t_2(F_{\mu\nu} F_{\alpha\beta}) + t_2(R_{\mu\nu} R_{\alpha\beta})] B_{\mu_1 \dots \mu_6} \epsilon^{\mu_1 \dots \mu_6}$, so that $B_{\mu_1 \dots \mu_6}$ unlike $B_{\mu\nu}$ need not transform under the Lorentz and YM gauge groups.

We finish with a remark that it may be possible to study the problem of (YM, Lorentz and supersymmetry) anomalies in chiral superstring theories working directly with the interacting superstring action (augmented by boundary terms) discussed in this paper. Cancellation of string theory anomalies may be possible to understand as cancellation of anomalies (a priori present due to chiral θ -variables) in corresponding 2-dimensional field theory defined on particular 2-spaces related to string theory loop processes.

References

1. J.H.Schwarz, Phys.Reports 89 (1982) 223;
M.B.Green, Surveys in H.E.Physics 3(1983) 127; Caltech prepr.
CALT-68-1219
2. E.S.Fradkin and A.A.Tseytlin, Zh.Exsp.Teor.Fiz.Pis.Red. 41(1985) 169; Lebedev Inst.preprint N261(1984); Phys.Lett., to be published
3. M.B.Green and J.H.Schwarz, Phys.Lett. 151B (1985) 21
4. F.Gliozzi, J.Scherk and D.Olive, Nucl.Phys. B122 (1977) 253
L.Brink, J.Scherk and J.H.Schwarz, Nucl.Phys. B121 (1977) 77
5. A.N.Chamseddine, Nucl.Phys. B185 (1981) 403
E.Bergshoeff, M.de Roo, B.de Wit and P.van Nieuwenhuizen, Nucl.
Phys. B195 (1982) 97
G.Chapline and N.Manton, Phys.Lett. 120B (1982) 105
6. I.C.G. Campbell and P.C.West, Nucl.Phys. B243 (1984) 112
F.Giani and M.Pernici, Phys.Rev. D30 (1984) 325
7. M.B.Green and J.H.Schwarz, Phys.Lett 109B (1982) 444
J.H.Schwarz and P.C.West, Phys.Lett. 126B (1983) 301
J.H.Schwarz, Nucl.Phys. B226 (1983) 269
8. B.E.W.Nilsson, Goteborg preprint 81-6 Feb. 1981
J.Wess, Princeton preprint (1981)
9. B.E.W.Nilsson, Nucl.Phys. B188 (1981) 176
10. M.B.Green and J.H.Schwarz, Phys.Lett. 136B (1984) 367
11. M.B.Green and J.H.Schwarz, Phys.Lett. 109B (1982) 444
12. J.L.Gervais, M.T.Jaekel and A.Neveu, Nucl.Phys. B155 (1979) 75
13. W.Siegel, Phys.Lett. 128B (1983) 397
14. L.Lusanna and B.Milewski, Nucl.Phys. B247 (1984) 396
15. M.B.Green and J.H.Schwarz, Nucl.Phys. B243 (1984) 285
16. M.B.Green and J.H.Schwarz, Nucl.Phys. B198 (1982) 252
17. T.L. Curtright and C.K.Zachos, Phys.Rev.Lett. 53 (1984) 1790
18. P.Candelas, G.T.Horowitz, A.Strominger and E.Witten, Princeton
preprint, 1984
19. A.H.Chamseddine, Phys.Rev. D24 (1981) 3065
20. M.B.Green and J.H.Schwarz, Phys.Lett. 149B (1984) 117;
Caltech preprint CALT-68-1224 (1984)