A Potential Model for Quark Confinement

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In the absence of a fundamental understanding of confinement in terms of QCD, a number of models of confinement have emerged in past years.¹ One of these popular models is the static potential model of interacting quarks and gluons. In this model confinement is achieved by introducing a static confining potential which contains the one-gluon-exchange (OGE) for small quark separations and a QCD motivated large distance part. If used in a second order relativistic wave equation, to describe the spectra of multiquark hadrons, these potential models lead to the confinement of color-singlet configurations. In nature, however, only colored configurations should be confined. We discuss a relativistic potential description which avoids the above drawbacks of the conventional potential models. The wave equation which we use to describe a two-quark system is obtained from the Dirac equation. We apply it to a study of heavy quark spectra where spin-dependent effects are less important. In this case the wave equation simplifies considerably, though it contains the important relativistic effects for these systems. Later we will include spin-dependent effects to study lighter meson spectra. However, it is not clear if a pure $q\bar{q}$ description of light mesons is adequate, particularly for the pion. The absence of spin-dependent effects allows for an easier treatment of the center-of-mass-motion which is a crucial problem in most potential and bag model descriptions.

To describe the motion of quarks in hadrons, we assume that the quarks obey the Dirac equation with some scalar (S) and vector (V) potentials which we will specify below. Then the stationary Dirac equation for the i th quark is

Abstract

A static quark potential model obtained from a relativistic wave-equation is considered. The long-range part of the quadratic terms is suppressed by a glueball exchange mechanism and compatibility with the meson spectra is shown.

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$$(E_1 - V(r))\psi_1(\vec{r}) = (-i\vec{\alpha}\cdot\vec{\nabla}_1 + \beta(m_1 + S(r)))\psi_1(\vec{r}) . \quad (1)$$

Such an ansatz is used e.g. in the Regensburg scalar confinement model,² where the confining potential is assumed to transform as a scalar (no vector part is included). Equation (1) does not allow to separate the relative and center-of-mass (CM) variables. This problem can be treated in the quadratic version of the Dirac equation, known as the Kramer equation.³ For the case of a scalar and vector potential the Kramer equation does not decouple in any representation. We write it in the familiar low energy representation

$$\frac{\vec{p}_1^2}{2m_1} \begin{pmatrix} \phi_1(\vec{r}) \\ \chi_1(\vec{r}) \end{pmatrix} = \begin{pmatrix} D_1 & -i\vec{\sigma}_1 \cdot \frac{\vec{\nabla}_1}{2m_1} (V-S) \\ -i\vec{\sigma}_1 \cdot \frac{\vec{\nabla}_1}{2m_1} (V+S) & D_1 \end{pmatrix} \begin{pmatrix} \phi_1(\vec{r}) \\ \chi_1(\vec{r}) \end{pmatrix} \quad (2)$$

where ϕ_1 and χ_1 denote the upper and lower components of ψ_1 respectively.

The differential operator D_1 is

$$D_1 = -\frac{\nabla_1^2}{2m_1} + \frac{E_1}{m_1} V(r) + S(r) - \frac{V^2(r)}{2m_1} + \frac{S^2(r)}{2m_1}$$

and

$$E_1^2 = \vec{p}_1^2 + m_1^2$$

Eq. (2) is a four-component spinor equation.

Note that the gradient in the non-diagonal parts of Eq. (2) acts only on the potential and not on the wave function. The Laplace operator acts only on the wave function, so that the equation has exactly the differential structure of the Schrödinger equation. Nonetheless, in the presence of both scalar (S) and vector (V) potentials, large and small components of ψ_1 cannot be decoupled in a simple way. If we consider Eq. (2) as a two-channel Schrödinger equation (with the two channels

corresponding to ϕ_1 and χ_1 , we can apply the standard multi-channel formalism. In a multi-channel formalism it is always possible to explicitly eliminate the coupled channels by the introduction of an effective potential for a single channel. In Eq. (2) the coupling between the two channels introduces a non-local spin-dependent interaction into the effective interaction, which determines the upper (lower) component $\phi_1(\chi_1)$. Like the Schrödinger equation, Eq. (2) has a number of different applications. The simplest is that of a single-particle description of quark states in hadrons. In this case the potential only depends on the relative distance of the i th quark from the center of the hadron. In our treatment we go beyond the single particle models (which are known to give spurious center-of-mass motion), and describe the hadronic states by two-body interactions between quarks. In this case the Schrödinger-like structure of Eq. (2) is essential, because it allows to separate the relative and CM motion. In the case of a two-body $q\bar{q}$ system, Eq. (2) is very similar to the second-order two-body relativistic wave-equation of Todorov.⁴

In potential model calculations it is generally assumed that the confinement of the quarks results from a linearly increasing scalar potential while the short-range part of the interaction between the quarks is described by the vector one-gluon exchange, which also gives spin-dependent contribution to the interaction. The central part of the one-gluon exchange (OGE) potential is

$$V_G(r) = \vec{r}_1 \cdot \vec{r}_2 \frac{\alpha_S(r)}{r} \quad (3)$$

where $\vec{r}_1$ and $\vec{r}_2$ are the usual color matrices. The running coupling constant is given by⁵

$$\alpha_S(r) = \lambda_0 \frac{\lambda r - 1}{\ln(\lambda r)}$$

where λ_0 is a known constant and λ is related to QCD scale parameter. We describe the confining part of the potential by a simple linear function which for heavy quarks is suggested by lattice calculations⁶

$$S(r) = \vec{p}_1 \cdot \vec{p}_2 (ar - c) \quad (4)$$

The slope 'a' of the linear potential is determined to be about $a =$

0.16 GeV². So far we have simply described the quark potential model

which is widely applied in the literature.⁷ Let us consider the implications, which follow from the application of the above potentials in a relativistic second-order wave equation (2). In this case the wave equation has the general form

$$\frac{k^2}{2\mu} \phi(r) = \left(-\frac{\nabla^2}{2\mu} + U(r) \right) \phi(r) \quad (5)$$

with $\sqrt{S} = \sqrt{k^2 + m_1^2} + \sqrt{k^2 + m_2^2}$ and $\mu = m_1 m_2 / (m_1 + m_2)$. The total potential $U(r)$ is given by

$$U(r) = \frac{\varepsilon}{\mu} V(r) + S(r) - \frac{V^2(r) - S^2(r)}{2\mu} \quad (6)$$

$$\varepsilon = (E_1 E_2 + k^2) / \sqrt{S}.$$

If the above scalar confining potential given by Eq. (4) is used in Eq. (6), it gives rise to a contribution

$$\frac{1}{2\mu} (\vec{p}_1 \cdot \vec{p}_2)^2 a^2 r^2.$$

For a $\bar{q}q$ -system, one finds

$$(\vec{p}_1 \cdot \vec{p}_2)^2 = \frac{2}{9} - \frac{7}{6} \vec{p}_1 \cdot \vec{p}_2. \quad (7)$$

If we consider a system of two colorless hadrons and describe the hadron-hadron interaction as a sum of two-body interactions of the above form,

the summation of the terms proportional to $\vec{p}_1 \cdot \vec{p}_2$ cancel but the first term on the right hand side of Eq. (7) gives a non-vanishing contribution, which has a confining behavior. Therefore, the description of the multi-quark system with two-body scalar confining potentials leads to the confinement of hadrons, which is unrealistic. An obvious argument to avoid this complication is to limit the application of two-body confining potentials to two-body systems only. It has been discussed in the framework of the MIT bag model that the confining interaction in a three-heavy quark system is a pure three-body force.⁷ However, Carlson et al.⁸ argue that such a three-body interaction can be approximated by a sum of two-body interactions. Therefore, an effective two-body quark-quark interaction can give a good description of the properties of hadrons. Now a linear confining potential gives rise to a Van der Waals force, which for large relative distances falls off as r^{-4} .⁹ It is known that such a force is unrealistically strong for the nucleon-nucleon interaction and conflicts with the well-known one-pion exchange tail. The term quadratic in OGE is given by

$$V_g^2(r) = (\vec{p}_1 \cdot \vec{p}_2)^2 \frac{\alpha_S^2(r)}{2\mu r^2} \quad (8)$$

and this falls off even slower than the Van der Waals force. Note that $V_g^2(r)$ describes a part of the two-gluon exchange potential valid for small distances. At larger distances multi-gluon exchange becomes important. This is supported by the fact that for large distances perturbative QCD breaks down. Thus, one expects phenomenologically that at the large distances mutual interaction of two exchanged gluons confines them to a glueball state. The gluon-gluon interaction gives a

mass to the glueball, which in lattice calculations is estimated to be about one or two GeV.¹⁰ A possible candidate for the glueball state is considered to be the recently discovered $\chi(1400)$.

The effect of mutual interaction of gluons is that the two-gluon exchange potential at large distances will have an exponential drop-off $\exp(-m_g r)$, where m_g is the glueball mass. The same arguments can be applied to the Van der Waals problem and provides a cutoff for QCD contributions to hadron-hadron interaction. We thus modify the quadratic interaction as below

$$-\frac{1}{2\mu}(V^2-S^2) + -\frac{1}{2\mu}(V^2-S^2)e^{-m_g r} \quad (9)$$

where we take $m_g = 1$ GeV. Compared to an attempt by Bhaduri et al.¹⁰ our approach has the apparent advantage that in the limit of vanishingly small interquark separations we obtain the correct perturbative result. This modification has no effect in the asymptotic freedom region while it avoids conflict between theory and experiment in the confinement region. For small r we can consider the expansion of $\exp(-m_g r)$ in powers of the glueball mass as an effective expansion in terms of the QCD coupling constant. Although such a modification of the two-gluon exchange interaction could arise from a fundamental theory, we consider it as a phenomenological description though motivated by the basic process of glueball formation. The situation is, however, unclear for the quadratic scalar potential because the scalar interaction itself is not well understood. In this case we adopt the same procedure as for the quadratic vector part to avoid the introduction of new parameters.

We have calculated the spectra for $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ states based upon Eq. (5) and (6) with the quadratic terms in the potential modified as

in Eq. (9). The gluon mass (m_g) as stated above is taken to be 1 GeV. In Table I we give a list of the parameter values in our model. The only parameter which changes for different quark families is the quark mass, which is fitted to get the mass of the lowest state. In Table II we give our results for charmonium ($c\bar{c}$) states and compare them with the results of Crater et al.¹¹ Table III contains the results for bottomium ($b\bar{b}$) states and Table IV contains the results for toponium ($t\bar{t}$) states. We chose the mass of top quark so as to fit mass of $1\ 3S_1$ state with that of Godfrey and Isgur.¹² As can be seen results are very good.

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Table I

Parameters of the potential and their values.

$m_g = 1.0$ GeV
$a = 0.16$ GeV ²
$c = 0.36$ GeV
$\lambda_0 = 0.685$
$\lambda = 0.732$ GeV
$m_c = 1.496$ GeV
$m_b = 4.880$ GeV
$m_t = 34.9$ GeV

Table II

Masses for the $c\bar{c}$ -system.

$c\bar{c}$ -state	Exp mass (GeV)	This work	Crater et al. ¹¹ $m_c = 1.574$ GeV
$\psi(1\ ^3S_1)$	3.095	3.095 (fit)	3.123
$\psi(2\ ^3S_1)$	3.685	3.687	3.664
$\psi(3\ ^3S_1)$	4.030	4.110	4.076
$\psi(4\ ^3S_1)$	4.415 [†]	4.447	—
$\chi(1\ ^3P_1)$	3.510	3.507	3.482
$\psi(1\ ^3D_1)$	3.770	3.813	3.768
$\psi(2\ ^3D_1)$	4.159	4.195	4.147
$\psi(3\ ^3D_1)$	4.415 [†]	4.517	4.485

[†]Experimentally $\psi(4\ ^3S_1)$ and $\psi(3\ ^3D_1)$ seem to be degenerate.