SOLENOID OPTICS FOR SLOW ATOMIC BEAMS

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ABSTRACT

We present analytic calculations and track-tracing simulations for a polarized atomic beam obtained by spin-flip extraction of magnetically stored hydrogen atoms from a superfluid helium-coated cell. Conceptual designs are discussed for high-intensity polarized atomic-beam sources based on this principle. The source would feature pulse characteristics ideal for injection into synchrotrons, and might allow the polarized beam intensity to be raised to the space-charge limit.

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1. INTRODUCTION

In high-energy physics the continuing trend towards collider experiments is a strong incentive to develop high-intensity polarized proton ion sources that are suitable for injection into synchrotrons. The new methods of fast [1] and slow [2] resonance crossing in strong-focusing machines, and promising techniques of resonance suppression in large machines by means of 'snakes' [3], may totally remove the traditional technical obstacles which hinder the acceleration of polarized beams. However, a high source intensity would greatly facilitate the setting-up of the resonance-crossing and suppression devices, because the beam polarization measurement relies on rather small asymmetries or small cross-sections [4]. Also, users of intense unpolarized primary and secondary beams would not be excluded during acceleration periods with polarized beams if the space-charge limited intensity could be reached. The highest priority technical development would therefore seem to be the high-intensity polarized ion source, for which new techniques will be required.

A emerging new promising technique in pulsed polarized atomic- and ion-beam sources is based on the use of stable atomic hydrogen. The stability conditions for atomic hydrogen were discovered in 1980 [5, 6]. At 0.5 K temperature and a 5 T field the magnetic compression of paramagnetic atoms results in a high density of electron-spin polarized hydrogen atoms, if the wall recombination is reduced by a coating of superfluid ^4He . The ^4He has an extraordinarily low adsorption energy of $\epsilon_a = 1$ K for atomic hydrogen; this and the repulsive interaction for the triplet pairs makes it possible to reach high gas-phase density before the surface recombination gives rise to instability. The available densities are around 10^{17} cm^{-3} at 5 T and 0.3 K [7]. This should be compared with the best polarized atomic-beam densities around $10^{11}-10^{12} \text{ cm}^{-3}$ [8, 9]. With a storage volume of 10^3 cm^3 , an extraction efficiency of 10^{-3} would be sufficient to improve the present polarized atomic beams by many orders of magnitude.

There are two promising extraction schemes for the atoms stored in the magnetic bottle: spin-flip extraction [10], or direct ionization in the storage

cell [11]. Here we shall concentrate on the spin-flip extraction, which leads to a monochromatic, magnetically accelerated and focused polarized beam. Direct ionization would require the stored atoms to be nuclear-spin polarized, which is made possible by selective recombination [12] or by applying microwave transitions.

The polarized atomic-beam sources have many potential applications, in addition to the ion sources. Among these we should mention the following:

- i) atomic-beam polarized targets for nuclear and high-energy physics experiments, particularly in storage rings and colliders (these targets may also be necessary in beam polarimeters, or in beam polarizers);
- ii) polarized 2s hydrogen-beam sources for accurate parity non-conservation measurements: excitation from the ground state to the 2s state should be developed; the proposed schemes include resonant two-photon processes, a Lyman β lamp, and electron beams;
- iii) polarized electrons, which could possibly be obtained from the H $\downarrow$ beam if an efficient photodetachment can be made, using resonant laser multiphoton excitation to a state from which a non-resonant flashlamp can bring the electron to the continuum;
- iv) metrology (frequency standards): here one could possibly profit from the slow velocity and monochromaticity of the atoms, which should reduce the Doppler broadening.

In the above applications, particularly the ion sources of synchrotron injectors, advantage can be taken of the ability to pulse the microwave extraction.

The monochromaticity of the atoms could allow their recirculation in a magnetic ring made of solenoids and quadrupole and hexapole magnets. The storing of the atoms could further enhance the applications listed above.

The purpose of this work is to find field configurations that would be favourable for the magnetic focusing elements in the atomic-beam extraction and transport. Furthermore, we shall try to find solenoid field geometries that

could lead to linear optics, profiting from the monochromaticity of the magnetically accelerated beam.

2. FORCE ON SLOW MAGNETIC DIPOLE IN MAGNETIC FIELD

The geometry of the system studied in this work is based on the compensated solenoid used to produce a high magnetic field for the stabilization of atomic hydrogen. A schematic view of the apparatus is shown in fig. 1.

The magnetic energy of a free hydrogen atom with a magnetic dipole moment $\vec{\mu}$ in a magnetic field B is given by

$$U = \vec{\mu} \cdot \vec{B} . \quad (1a)$$

As the spin of a slow atom follows closely the direction of the field, eq. (1) reduces to

$$U = \mu |\vec{B}| , \quad (1b)$$

where μ is now the maximum projection of $\vec{\mu}$ on the axis parallel to the field; this depends on the hyperfine state of the atom and the strength of the field. The field dependence of U for the four hyperfine states of H is shown schematically in fig. 2. The force exerted on the atom by an inhomogeneous high field can be expressed as

$$F = -\nabla|U| \approx -\mu \nabla|\vec{B}| . \quad (2)$$

Only the two higher hyperfine states are considered here, as the potential energy in the field will only allow atoms in such states to exit from the apparatus; the force is directed towards the lower field.

In the high fields that are being studied, it can be approximated that the magnetic dipole moment is constant and equal to the Bohr magneton. A small difference, however, should be observable for the c and d hyperfine states, whose hyperfine energy is shown in fig. 2.

3. MOTION OF A MAGNETIC DIPOLE IN THE SOLENOID FIELD

3.1 Equation of motion

The force described by eq. (2) gives rise to a motion which can be conveniently calculated by means of Lagrangian mechanics. This method is necessary because of the desire to include also the angular motion around the solenoid axis. The initial angular velocities are non-negligible owing to the large extraction volumes.

The Lagrangian

$$L = T - U$$

(where T denotes the kinematic energy and U the potential energy) is written in the following way in cylindrical coordinates (r, ϕ , z):

$$L = (m/2)(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2) - U(r, z) . \quad (3)$$

This gives the equations of motion

$$\begin{aligned} p_r &= m\dot{r} \\ \dot{p}_r &= mr\dot{\phi}^2 - \partial U/\partial r \\ p_\phi &= mr^2\dot{\phi} \\ \dot{p}_\phi &= 0 \\ p_z &= m\dot{z} \\ \dot{p}_z &= -\partial U/\partial z . \end{aligned} \quad (4)$$

The angular momentum is described by p_ϕ and is a constant of motion because of the cylindrical symmetry of the solenoid field. Angular velocity can therefore be expressed as a function of radius

$$\dot{\phi} = k/r^2 = r_0^2\dot{\phi}_0/r^2 , \quad (5)$$

where r_0 is the initial radius and ϕ_0 the initial angular velocity.

Inserting the angular velocity in eqs. (4), we arrive at two equations of motion:

$$\begin{aligned}\ddot{r} &= k^2/r^3 - (\mu/m)(\nabla|\bar{B}|)_r, \\ \ddot{z} &= -(\mu/m)(\nabla|\bar{B}|)_z,\end{aligned}\tag{6}$$

where m is the mass of the hydrogen atom.

3.2 Evaluation of solenoid fields

These equations can be elaborated by studying the gradient of the absolute value of the field. In cylindrical coordinates it is written

$$\begin{aligned}\nabla|\bar{B}| &= \frac{1}{|\bar{B}|} \left\{ \left[B_r \left(\frac{\partial B_r}{\partial r} \right) + B_z \left(\frac{\partial B_z}{\partial r} \right) \right] \bar{r}^0 \right. \\ &\quad \left. + \left[B_r \left(\frac{\partial B_r}{\partial z} \right) + B_z \left(\frac{\partial B_z}{\partial z} \right) \right] \bar{z}^0 \right\} = (\nabla|\bar{B}|)_r \bar{r}^0 + (\nabla|\bar{B}|)_z \bar{z}^0.\end{aligned}\tag{7}$$

The first part of this expression is associated with the radial acceleration and the latter with the force along the axis. The radial expression can be simplified by noting that the Maxwell equation for a currentless region $\nabla \times \bar{B} = 0$ gives, in cylindrical coordinates, $\partial B_z/\partial r = \partial B_r/\partial z$, and thus

$$(\nabla|\bar{B}|)_r = \partial B_r/\partial z + (B_r/B_z)(\partial B_r/\partial r).\tag{8}$$

Here it was also taken into account that near the axis $B_r \simeq 0 \rightarrow |\bar{B}| \simeq B_z$.

Callaghan and Maslen [13] have derived exact analytical solutions for the magnetic field components of a field of a finite solenoid with infinitely thin walls. They also give accurate approximations for the components near the solenoid axis. These are given by

$$B_r(r, z) = \frac{\mu_0 ni}{4} \int_{\xi_-}^{\xi_+} \frac{a^2 r}{(\xi^2 + a^2)^{3/2}} d\xi = \frac{A_0 B_0}{a^2} \frac{r}{2} G(z), \quad 0 < r < a, \quad (9)$$

$$B_z(r, z) = \frac{\mu_0 ni}{2} \int_{\xi_-}^{\xi_+} \frac{\xi}{\sqrt{(\xi^2 + a^2)}} d\xi = \frac{A_0 B_0}{a} H(z), \quad 0 \leq r < a, \quad (10)$$

where μ_0 = vacuum permeability,

n = number of turns per unit length,

i = current in each filament,

a = coil radius,

$\xi_{\pm} = z \pm L/2$,

$A_0 = \sqrt{[(L/2)^2 + a^2]}$,

$B_0 = (\mu_0 ni/2)(L/A_0)$,

$G(z) = \int_{\xi_-}^{\xi_+} \{(a/L)[(\xi/a)^2 + 1]^{-3/2}\} d\xi$,

$H(z) = \int_{\xi_-}^{\xi_+} \{(\xi/L)[(\xi/a)^2 + 1]^{-1/2}\} d\xi$.

It can be seen that B_r is linear in r ; thus eq. (8) becomes

$$(\nabla|\vec{B}|)_r = \partial B_r / \partial z + (1/r)(B_r^2/B_z) . \quad (11)$$

By inserting the expressions (9) and (10) into eq. (11), we get

$$(\nabla|\vec{B}|) = A_0 B_0 (r/2a^3) [F(z) + G^2(z)/2H(z)] , \quad (12)$$

where

$$F(z) = a \partial G(z) / \partial z = \int_{\xi_-}^{\xi_+} \frac{(-3\xi/L)}{[(\xi/a)^2 + 1]^{5/2}} d\xi .$$

From the equation of motion (6) it can be seen that a focusing effect requires $(\nabla|\vec{B}|)_r$ to be positive and superior to the centrifugal term. By plotting eq. (12) we can see that the radial gradient has a positive maximum along the z axis, followed by a slightly defocusing part outside the solenoid.

4. INTEGRATION OF TRACKS

The behaviour of hydrogen atoms in the solenoid field was studied with a simulation program which integrated the equations of motion (6) in the following form:

$$\begin{aligned}\dot{r} &= v_r, \\ \dot{v}_r &= k/r^3 - (\mu/m)(\nabla|\vec{B}|)_r, \\ \dot{z} &= v_z, \\ \dot{v}_z &= -(\mu/m)(\nabla|\vec{B}|)_z.\end{aligned}\tag{13}$$

The integration was performed numerically with a CERN Library subroutine utilizing the Merson method. The algorithm is able to adjust the integration step length so as to keep the accuracy within limits. This feature was necessary in order to deal with tracks penetrating close to the solenoid axis, where the centrifugal contribution to radial acceleration has a strong dependence on r.

The magnetic field required as an input for the track-tracing program was calculated by a CERN Library program, POISCR [14]. The local gradients were calculated by a spline interpolation for each point separately.

The tracks started from a circular plane perpendicular to the z axis. The initial position was defined by r_0 , which was given discrete values that corresponded to the annuli of equal areas around the axis. The initial velocities were defined by the angles φ and θ and the magnitude v_0 .

In a population of tracks, the assignment of parameters φ and θ was made to correspond to an isotropic velocity distribution, whereas v_0 was assigned according to the Maxwell-Boltzmann distribution. The acceptance of the system, i.e. the maximum value of θ , was determined experimentally.

It is possible to introduce skimmers along the axis to limit the beam dimensions. The resulting intensity distribution can be studied on target planes placed at arbitrary positions perpendicular to the axis. The distributions are normalized to the percentage of the atoms.

5. CHECK OF TRACK INTEGRATION

The atom tracks given by the integration program were verified by using the approximation formulas (9) and (10) to obtain a focal length for atoms leaving parallel to the axis at a small radius, and comparing this with the point where the integrated track crosses the solenoid axis.

For fast atoms, the final radial velocity v_r picked up in the field is small compared with the initial velocity v_0 . Thus the integration of a_r , the radial acceleration, can be performed at constant radius. The focal length is obtained from

$$L_f = r(v_z/v_r) , \quad (14)$$

where the radial speed v_r is calculated from eq. (6) with $k = 0$ as

$$v_r = (\mu/m) \int_0^\infty [(\nabla|B|)_r/v_z(z)] dz . \quad (15)$$

For small v_r , the v_z is obtained as a sum of the initial velocity and the one gained in the field, $v_z = \sqrt{(v_0^2 + v_m^2)}$.

Now the focal length due to a half-solenoid can be expressed as

$$L_f^{hs} = 2a/[1 + (v_0/v)^2] I_{hs}^{-1} , \quad (16)$$

where the integral I_{hs} is

$$I_{hs} = \int_0^{\infty} \frac{a}{r} \frac{\nabla |\vec{B}|}{B_0} r \frac{v_m}{v_z(z)} dz . \quad (17)$$

Using the field approximation for the thin coil [eqs. (9) and (10)], valid close to the axis, we obtain

$$I_{hs} = \int_0^{\infty} \frac{\frac{A_0}{2a} [F(q) + \frac{G^2(q)}{2H(q)}]}{\left[1 + \left(\frac{v_0}{v_m} \right)^2 - \frac{A_0}{a} H(q) \right]^{1/2}} dq , \quad (18)$$

where $q = z/a$.

Figure 3 presents two plots of focal lengths of hydrogen atoms, leaving parallel to the solenoid axis at an initial radius of 8.8 mm, as a function of v_0/v_m . Both plots were calculated on the basis of a thin solenoid of a length $L = 176$ mm, $a = 75$ mm. One of them is obtained by track-tracing calculations and the other by numerical integration of eq. (16).

The figure shows that outside the solenoid the results from track tracing are in good agreement with those from the analytic calculations. The difference inside the solenoid is due to the fact that slow atoms cross the axis before reaching the defocusing part of the field; this defocusing is, however, taken into account in the analytic expression.

The accuracy of the integration method in the track tracing was verified also by the conservation of the total energy.

6. RESULTS OF TRACK TRACING

The results of track tracing displayed here are based on a storage solenoid with a length of 250 mm and an inner diameter of 80 mm. At both ends, the solenoid thickness is increased to produce a homogeneous volume in the centre. The homogeneity was calculated to be $\Delta B/B \approx 2 \times 10^{-4}$ over a cylindrical volume of 50 mm length and 10 mm diameter. Such an homogeneous volume is necessary for the microwave flipping of the spin.

As the homogeneous volume does not contribute to the focusing or acceleration of the atoms, the area where the tracks were to start was shifted by 25 mm along the solenoid axis, to the end of the homogeneous volume.

Figure 4 depicts the tracks of spin-flipped atoms at three different points of the Maxwell-Boltzmann velocity distribution at a temperature 0.3 K. The tracks are chosen to be representative of a circular starting area of radius ~ 6 mm. The distribution over the azimuthal angle is uniform; over the initial radius it is based on rings of equal area. The polar angles of 0° , 5° , 10° and 20° are displayed.

The divergent field at the mouth of the solenoid gives a clear focusing effect. However, many of the tracks leaving at 10° terminate at the inner wall of the refrigerator, which was defined to be at $r = 20$ mm. The acceptance was thus approximated at 15° .

6.2 Storage solenoid with focusing coil

A considerable gain in the acceptance was achieved by introducing a smaller solenoid with an opposite current at the opening of the stabilization solenoid. This 'funnel' coil was dimensioned to drop the superposed field to zero at its own centre point. The radial component of the field gradient was thereby greatly increased between the starting area and the funnel centre, giving an improved focusing effect.

The tracks in fig. 5 were calculated for a focusing coil that produces a 2.1 T field at its centre. The field originating from the main solenoid at the same point is 2.7 T. Three different points of the Maxwell-Boltzmann distribution are again displayed^{*)}.

The figure shows clearly the improved focusing when compared with fig. 4. The acceptance is increased from 15° to 20° , corresponding to an 80% increase in

^{*)} The geometry of the combination has been included in fig. 1. The gradient pair coil appearing in the figure was not, however, used in these calculations.

the number of atoms. Also, the waist of the beam has been made thinner. It was also seen that, changing the current of the 'focusing' coil, images of different sizes of the source area could be produced.

6.3 Short solenoid lens

The image of the source can be reproduced at a more distant point in the system if appropriate lenses are provided. We studied the focusing properties of a gradient coil pair, composed of two similar short solenoids with opposite currents. The geometry of the pair is displayed in fig. 1.

Figure 6 depicts the results of a track tracing where atoms accelerated by a 5 T magnetic field enter into the gradient pair. The focal lengths thus obtained for different initial radii and current densities are given in table 1.

7. CONCLUSIONS

The results of the track-tracing runs presented in the previous section can be evaluated in terms of intensity distributions. The intensities are presented as percentages of all the atoms with initial velocity vectors within a cone limited by the acceptance of the system.

The intensities of the atomic beam displayed in fig. 4 are given as a function of radius at three different points on the solenoid axis. The intensity close to the axis is seen to peak between $z = 135$ mm and $z = 312$ mm.

For the storage solenoid equipped with a focusing coil, as displayed by fig. 5, the intensities are plotted for only the innermost circle of the sampling targets, as a function of distance along the solenoid axis. The four different histograms correspond to different focusing-coil central field strengths. The value used in fig. 5 has been given a relative value of 1.

In addition to the greater number of atoms due to the increased acceptance, the system displays higher intensities along the axis. The top values are around

70%. The distribution peaks closer to the focusing coil as the current of the coil is increased.

A gradient coil pair can be seen to give focal lengths which can lead to a compact design for a focusing system. The focusing effect is produced over a large range of initial radii, and its strength can be adjusted by changing the current in the lens. This allows one to zoom the image of the source to an ionizer, for example.

The additional benefit of solenoid lenses is that their acceptance can be made large. The field of a gradient coil pair drops quickly, which facilitates magnetic shielding. The final design of a solenoid lens should include the iron shielding in the field calculations.

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Table 1

The focal lengths of hydrogen atoms
passing through a gradient pair coil

r_0 (mm)	Current density	$6.75 \times 10^7 \text{ A/m}^2$	$1.35 \times 10^8 \text{ A/m}^2$	$2.7 \times 10^8 \text{ A/m}^2$
1.2		166	83	41
2.5		328	163	81
4.9		472	235	125
8.9		575	286	140
15.5		621	310	157
23.7		610	300	146
32.2		522	257	123

Figure captions

- Fig. 1 : Configuration of the hydrogen storage and focusing solenoids.
- Fig. 2 : The magnetic energy U as a function of $|B|$ for the hyperfine states of H.
- Fig. 3 : Results of the calculation of the focal length L_f^{hs} of a half-solenoid, based on two approaches: an approximate analytical result, and a track-tracing simulation.
- Fig. 4a : Tracks of spin-flipped hydrogen atoms in the field of the storage solenoid at three initial velocities, corresponding to $0.5 \times$, $1.0 \times$, and $1.5 \times$ the most probable velocity at 0.3 K .
- Fig. 4b : Flux distribution as a function of radius at three distances along z . The fluxes are given relative to the flux at the starting area, and are based on 1000 tracks involving Maxwell-Boltzmann start velocity distribution and variation of the initial radius polar angle and azimuthal angle.
- Fig. 5a : Tracks of spin-flipped hydrogen atoms in the field of the storage solenoid and a focusing solenoid at three initial velocities, corresponding to $0.5 \times$, $1 \times$, and $1.5 \times$ the most probable velocity at 0.3 K .
- Fig. 5b : Flux through an orifice of $\emptyset 16 \text{ mm}$ as a function of z , basing of on 1000 tracks in the same field as fig. 5a; distributions track starts as fig. 4b.
- Fig. 6 : Tracks of hydrogen atoms entering into a gradient coil pair parallel to the z axis.

Fig. 1

Fig. 2

Fig. 3

(a)

Fig. 4a

Fig. 4b

a

$$v_0 = 0.5\bar{v}$$

$$v_0 = 1.0\bar{v}$$

$$v_0 = 1.5\bar{v}$$

Fig. 5a

Fig. 5b

Fig. 6