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SOME NONLINEAR PROCESSES RELEVANT TO THE BEAT WAVE ACCELERATOR

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ABSTRACT

The beat wave accelerator¹ depends on the generation of a large amplitude plasma wave with a phase velocity close to the velocity of light c . The plasma wave (ω_p, k_p) is generated by beating colinear laser beams (ω_1, k_1) and (ω_2, k_2) with $\omega_p = \omega_1 - \omega_2$, $k_p = k_1 - k_2$. Since the process involves both large amplitude transverse and longitudinal waves, various nonlinear instabilities associated with either wave may occur. The object of this article is to discuss some of the processes that may compete with the beat wave generation listing their threshold and growth rate.

INTRODUCTION

The beat wave process is very similar to four-wave stimulated Raman forward scattering. Stimulated Raman scattering is normally considered as a three wave process where the incident transverse laser beam decays into another transverse wave and a Langmuir wave. In an underdense plasma where $\omega_{1,2} \gg \omega_{pe}$, ω_{pe} is the plasma frequency, the forward Raman scattering becomes important. For phase matching the wavenumber k_p is much smaller than the laser wave number $k_{1,2}$, under these conditions we must consider an up-shifted or anti-Stokes transverse component as well as the down shifted Stokes component, since both are resonant with the laser wave and the Langmuir wave. The instability then becomes a "four wave" process with the incident laser wave (ω_1, k_1) decaying into a Stokes wave $(\omega_1 - \omega_p, k_1 - k_p)$ and an anti-Stokes wave $(\omega_1 + \omega_p, k_1 + k_p)$ and a Langmuir wave (ω_p, k_p) . Stimulated Raman backscattering or stimulated Compton scattering for $k_p \lambda_{De} \approx 1$, where λ_{De} is the Debye length, can also occur, this instability scatters the incident laser light out of the plasma. The above instabilities operate on the electron plasma time scales ω_{pe}^{-1} . On longer time scales such as the ion plasma time scale ω_{pi}^{-1} other instabilities such as stimulated Brillouin scattering, self-modulation and self-focusing or

filamentation due to ponderomotive force of the incident laser beam will compete with the beat wave process. In the case of stimulated Brillouin scattering the laser light will be scattered out of the plasma. Self-modulation will cause the incident laser light to increase its bandwidth while filamentation will break the beam up into a number of smaller beams. With the intense short laser pulses proposed for the beat wave scheme the electron quiver velocity or oscillating velocity v_{osc} in the laser field is sufficiently large that the electrons become weakly relativistic. This results in relativistic nonlinearities due to the Lorentz factor producing self-focusing and filamentation as well as self-modulation of the incident beam. In regions of high intensity the laser frequency is lower due to the electron mass increase this reduces the phase velocity causing the phase front to curve such that more wave energy flows into the intense region making it even more intense.

The large amplitude Langmuir wave which constitutes the beat wave is also subject to nonlinear instabilities such as the decay instability and self-modulation as well as filamentation. The decay instability occurs on ion plasma time scales $\sim \omega_{pi}^{-1}$ this instability results in the generation of a lower frequency Langmuir wave and an ion-sound wave. Self-modulation due to the ponderomotive force also occurs on the ion plasma time scale. Self-modulation results in the collapse of the plasma wave leading to strong turbulence and the break-up of the coherent structure of the wave that is necessary for efficient acceleration. Self-modulation can also occur due to relativistic effects and takes place on a time scale faster than that due to the ponderomotive force.

STIMULATED RAMAN SCATTERING

The normal treatment of stimulated Raman scattering is a three wave process, however in the presence of a sufficiently large pump field $v_{osc}/c > (\omega_{pe}/\omega_1)^{1/2}$ the properties of the Langmuir wave is determined by the pump wave. This is regarded as the regime of strong coupling and the process is called the modified Raman scattering instability². The thresholds and growth rates for these two processes are given by²

(a) Stimulated Raman scattering

$$\text{Threshold: } (v_{osc}/c)^2 > \omega_{pe}^2 v_{ei}^2 / \omega_1^3$$

$$\text{Growth rate: } \gamma \approx (\omega_{pe} \omega_1)^{1/2} v_{osc} / 2c$$

(b) Modified Raman scattering

$$\text{Threshold: } v_{\text{osc}}/c > (\omega_{\text{pe}}/\omega_1)^{1/2}$$

$$\text{Growth rate: } \gamma \approx 3(v_{\text{osc}}\omega_{\text{pe}}/4c\omega_1)^{2/3}$$

v_{ei} is the electron - ion collision frequency

In the above cases $k_p = 2k_1$ for maximum growth rate.

For strongly damped plasma waves ie $k_p\lambda_{\text{De}} \gg 1$ the normal three wave stimulated Raman scattering becomes stimulated Compton scattering where the laser beam scatters off free electrons with a threshold and growth rate given by³.

$$\text{Threshold: } (v_{\text{osc}}/c)^2 > v_{\text{ei}} \omega_{\text{pe}}^2 k_p^2 \lambda_{\text{De}}^2 / 2\omega_1^3$$

$$\text{Growth rate: } \gamma \approx 0.2 (\omega_{\text{pe}} v_{\text{osc}}/v_{\text{Te}})^2 / \omega_1$$

The modified Raman scattering instability can also operate even if $k_p\lambda_{\text{De}} = 1$.

STIMULATED RAMAN FORWARD SCATTERING

This instability occurs with k_p, k_1 , with the short wavelength pump wave (ω_1, k_1) being modulated by the long wavelength electrostatic wave resulting in the generation of high frequency transverse sidebands and bunching of the plasma particles. This is the single pump analogue of the beat wave process with growth rate and threshold given by⁴

$$\text{Threshold: } (v_{\text{osc}}/c)^2 > 4v_{\text{ei}}/\omega_1$$

$$\text{Growth Rate: } \gamma = \omega_{\text{pe}}^2 v_{\text{osc}}^2 / 2\sqrt{2}\omega_1 c^2$$

This "four-wave" modulational instability results in the broadening of the laser frequency and generation of broad-band plasma oscillations. The above instability corresponds to k_p, k_1 for intermediate cases where $k_p \cdot k_1 = k_p k_1 \cos\theta$ the process becomes a three wave resonant side-scatter instability with the following resonance condition for forward side scatter

$$k_p = k_1 \cos\theta - (k_1^2 \cos^2\theta - 2\omega_{\text{pe}} \omega_1 / c^2)^{1/2}$$

this results in a scattered signal with a maximum growth rate at an angle $\theta_s = \sin^{-1} (2\omega_{pe}/\omega_1)^{1/2}$ the threshold and growth rate corresponding to this angle are⁴

$$\text{Threshold: } v_{osc}/c > v_{ei}/2\omega_1$$

$$\text{Growth rate: } \gamma \approx \omega_{pe} v_{osc} \sqrt{2} c$$

This side-scatter instability results in the laser light being scattered out of the interaction column, this instability has a rather fast growth rate and will effect the efficiency of the beat wave generation. One way to overcome the instability is to increase θ_s by increasing ω_{pe}/ω_1 and reduce the beam diameter this will limit the number of e-folding lengths and hence help to quench the instability.

STIMULATED BRILLOUIN SCATTERING

Stimulated Brillouin scattering results in loss of laser power by scattering off ion-modes. The instability occurs on ion plasma time scales ω_{pi}^{-1} and will be important only for long pulse lengths. As with stimulated Raman scattering, stimulated Brillouin scattering has a number of different regimes of operation, the simplest being three wave backscattering with $k_s = 2k_1$ where k_s is the wavenumber of the ion-sound wave. The threshold and growth rate for this case are given by³

$$\text{Threshold: } v_{osc}/v_{Te} > (8 v_{ei} v_s / \omega_1 \omega_s)^{1/2}$$

$$\text{Growth rate: } \gamma \approx (v_{osc} \omega_{pe} / 4 v_{Te}) (\omega_s / \omega_1)^{1/2}$$

For sufficiently strong pumps $v_{osc}/c > (\omega_s / \omega_1)^{1/2}$ the frequency shift of the ion-sound wave can be equal to or greater than their eigenfrequency, under these conditions we have the modified stimulated Brillouin instability³ with threshold and growth rate given by

$$\text{Threshold: } v_{osc}/c > (k_s \lambda_{De})^{3/2} (\omega_{pi} / \omega_1)^{1/2}$$

$$\text{Growth rate: } \gamma \approx (2 v_{osc} \omega_{pi} / c \omega_1)^{2/3} \omega_1$$

again this reflects the $|E|^{3/2}$ dependence of the pump wave amplitude for the growth rate for large pump values.

For a plasma with equal electron and ion temperatures ($T_e < T_i$), the ion-sound wave is heavily Landau damped and scattering of the incident transverse wave off the ions into another transverse wave occurs with a grow rate given by ^{5,6}

$$\gamma \approx 0.2 \omega_{pi}^2 v_{osc}^2 / (\omega_1 v_{Te} k^2 \lambda_{De}^2)^2$$

FOUR-WAVE PROCESSES: SELF-MODULATION AND FILAMENTATION

Self-modulation and filamentation of the incident laser beam can occur as a result of the ponderomotive force⁷ or the relativistic term in the Lorentz factor for the electron mass⁸. The ponderomotive force effects occur on the ion plasma period time ω_{pi}^{-1} scales and will only contribute to beam break up and frequency broadening long after the acceleration phase has taken place. On the other hand relativistic effects only involve the electrons and will therefore take place on a much faster time scale $\sim \omega_{pe}^{-1}$ and could therefore disrupt the accelerating mode of the beat wave. For the ponderomotive force driving term only filamentation can occur in an underdense plasma⁷, since the density perturbation (ie the ion sound-wave) has got to travel at the group velocity of the laser beams for modulation to occur. The threshold and growth rate for filamentation due to ponderomotive effects are given by⁷

$$\text{Threshold: } v_{osc}/v_{Te} > 2(v_{ei}/\omega_1)^{1/2}$$

$$\text{Growth rate: } \gamma \approx \omega_{pi} v_{osc} / \sqrt{2} c$$

For the relativistic mechanism both filamentation and self-modulation will take place with a threshold and growth rate given by⁸

$$\text{Threshold: } v_{osc}/c > (2v_{ei}/\omega_1)^{1/2}$$

$$\text{Growth rate: } \gamma \sim q(\omega_{pe} v_{osc} / 2\omega_1 c)^2 \omega_1$$

where q is a numerical factor ≈ 0.5 in an underdense plasma with plane polarized waves⁸.

Filamentation and self-modulation due to the relativistic term have the same threshold and growth rate but the wave number for modulation k is different. For self-modulation $k \parallel k_1$, $k = \omega_1 v_{osc}/2c^2$ for maximum growth, and for filamentation $k \perp \omega_{pe} v_{osc}/\sqrt{2}c^2$ smaller by the factor ω_{pe}/ω_1 than the k for self-modulation.

NONLINEAR BEAT WAVE PROCESSES

We now turn our attention to nonlinear processes associated with the large amplitude beat wave, these include processes such as three wave decay where the large amplitude Langmuir wave decays into another Langmuir wave and an ion-sound wave and modulational type instabilities which lead to the development of strong turbulence and cavity formation. This results in complete brake-up of the plasma beat wave increasing its bandwidth, destroying the conditions required for coherent acceleration.

THE DECAY INSTABILITY

This was one of the first nonlinear wave processes to be studied, it involves the decay of a Langmuir wave into another Langmuir wave and an ion-sound wave. For Langmuir waves with large amplitude

$$v_{osc}/v_{Te} > \omega_s/\omega_{pe}$$

the instability develops into the modified decay where the frequency shift due to the pump wave exceeds the natural frequency of oscillation of the ion-sound wave. For equal electron and ion temperatures the ion-sound waves are heavily Landau damped and the process goes over to nonlinear Landau damping. For small pump waves with

$$k_p \lambda_{De} > (4/\gamma_s)(m_e/m_i)^{1/2}$$

where γ_s is the ratio of specific heats, we get the resonant backscatter decay instability with threshold and growth rates given by

$$\text{Threshold: } v_{osc}/v_{Te} > 4(v_{ei} v_s/\omega_p \omega_s)^{1/2}$$

$$\text{Growth rate: } \gamma = \omega_{pi} v_{osc}/2 v_{Te}$$

$$\text{for } k \lambda_{De} \approx (4/\gamma_s)(m_e/m_i)^{1/2}$$

where ν_s is the ion-sound damping coefficient note that u_{osc} is the oscillating velocity of the electron in the plasma wave.

For large pump fields, the frequency of the ion-sound wave is determined by the pump under these conditions we have the modified decay instability with threshold and growth rate given by

$$\text{Threshold: } u_{osc}^2 / u_{Te}^2 > 0.4 k_p \lambda_{De} m_e / m_i$$

$$\text{Growth rate: } \gamma \approx \sqrt{3} (u_{osc}^2 k_p^2 \omega_{pi}^2 / 2)^{1/3}$$

For $k_p \lambda_{De} < (m_e / m_i)^{1/2} / \gamma_s$, resonant decay into normal modes is no longer possible and we have instead modulational instabilities and the oscillating two stream instability⁹. These instabilities are due to the ponderomotive force and have similar thresholds and growth rates given by¹⁰

$$\text{Threshold: } u_{osc} / u_{Te} > (4 \nu_e / \omega_p)^{1/2}$$

$$\text{Growth Rate: } \gamma \approx \omega_p u_{osc}^2 / 8 u_{Te}^2$$

The above processes act on the ion-plasma time scale. However, for the beat wave relativistic effects become important and can also give rise to modulational type instabilities which act on a much faster time scale than the ponderomotive force mechanism, the threshold and growth rate for relativistic self-modulation are given by⁴.

$$\text{Threshold: } u_{osc} / c > (8 \nu_{ei} / 3 \omega_p)^{1/2}$$

$$\text{Growthrate: } \gamma \approx 3 u_{osc}^2 \omega_p / 32 c^2$$

CONCLUSIONS

In this article various nonlinear processes have been listed which could be important in the beat wave accelerator scheme. The list is by no means complete, giving only a flavour of the many processes possible. The list illustrates clearly the need to understand more fully the different processes to successfully generate a beat wave which will accelerate particles to high energies.

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