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EFFECT OF REACTIVE FEEDBACK ON THE TRANSVERSE MODE COUPLING
INSTABILITY USING THE FEW-PARTICLES MODEL AND SIMULATION

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Summary

For short bunches in electron storage rings the onset of the transverse mode coupling instability usually occurs when the frequency shift of the coherent dipole mode (mode zero) equals around three quarters of the synchrotron frequency. A reactive feedback system has been proposed which maintains the coherent dipole frequency constant and may thereby delay the onset of instability. The influence of this feedback system on the threshold of instability is studied using matrix eigenvalue analysis (few particles model) and a computer simulation code. This analysis shows that for a single localised transverse impedance, the reactive feedback system increases significantly the threshold for instability, provided the betatron phase shift between the kicker and the impedance is properly chosen. Results are also shown for the more realistic case of many localised sources of transverse impedance.

1. Introduction

A reactive feedback system has been proposed¹ for LEP so as to maintain the coherent betatron frequency constant and thus increase the threshold for the transverse mode coupling instability.^{2,3,4,5} The effect of this feedback system was first examined by Ruth⁶ using a two-particle model and continuous motion. This implied distributed wakefields and a distributed feedback system. Later Zotter⁸ modified the original two-particle model so as to study the effect of a single localised wakefield and a localised feedback system. In this model the betatron phase advance between the pick-up and kicker was fixed at multiples of π and the phase advance between the kicker and the single wakefield was zero. Myers⁹ then extended this model to allow

- i) more particles; up to five were studied
- ii) variable phase advance between the kicker and the wakefield, and
- iii) many wakefields per turn.

Very recently Ruggiero¹⁰ examined the transverse mode coupling instability without feedback using an infinite number of particles (Vlasov equation) and a single localised wakefield. Before this Ruth¹¹ had studied (using an infinite number of particles and distributed wakes and feedback) the effect of reactive feedback on the instability and found results in good agreement with his two-particle analysis⁶.

In all models involving feedback it has been assumed that the reactive feedback kick would be applied on the same turn. In practice this implies that the electrical signal detected at the pick-up would somehow 'catch up' with the bunch before the bunch arrived at the kicker. This is possible in small rings but extremely difficult in LEP.

In parallel with the few-particles studies a computer simulation¹² was developed which examined many superparticles and a localised realistic wakefield but without feedback. In fact it was with this simulation that the "coherent synchro-betatron resonances" were first observed. Another computer simulation¹³ was also developed which allowed many localised wakefields separated in betatron phase and many feedback systems. In particular this simulation allowed study of a realistic feedback system where the kick is applied on the next turn.

2. Extension of the discrete few-particles model

Figure 1 shows a schematic representation of type A feedback. Each particle is identified by its position in the vertical normalized phase space (z,y)

FIG 1 Type A Feedback

where

$$y = \beta z' - \beta' z/2 \quad (1)$$

Hence for n particles a column vector U [where the transpose of U is given by

$$U^T = (z_1, y_1, z_2, y_2, \dots, z_n, y_n)] \quad (2)$$

represents the state of the system of particles.

It has been shown⁷ that the single turn matrix for feedback type A with wakefields is

$$S = (L_P K + K) L_{WP} W (L_{BW} W)^{N_W - 1} L_{KW}$$

where K is the kick matrix and for two particles for example is

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sigma & 0 & \sigma & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 \end{pmatrix} \quad \text{where } \sigma = \frac{\beta_k g}{n} \quad (3)$$

and N_W is the number of wakes per turn.

The wakefield matrix W in this equation changes with the longitudinal position of the particles. For example when there are two particles equally spaced in longitudinal phase space there are two wakefields W_1 and W_2 corresponding to particle one leading particle two and vice versa, i.e.

$$W_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad W_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

where α is the wake strength parameter.

Generally for more than two particles there are $2n$ wake possibilities if the particles are equally spaced longitudinally. These wakes are readily obtained by inspection of the linearised longitudinal motion.

It may be seen that the matrix which must be inspected for stability is

$$S_T = S_1^M S_2^M \dots S_p^M \quad (5)$$

where $Q_s = 1/pM$, S_p is the single turn matrix corresponding to wakefield p and p is the number of different wakes depending on the number of particles.

For stability the eigenvalues of S_T are evaluated for a given machine configuration and as a function of α . When any of the amplitudes of the eigenvalues exceed unity this value of α is unstable. By incrementing α , stability plots can be obtained.

Figure 2 shows the schematic of type B feedback. In this case it is simpler to start at the pick-up to calculate the single turn matrix. The same technique applied in the previous section gives the single turn matrix for type B feedback.

FIG 2 Type B Feedback

$$S_B = LKP \left[L_{WK}(W L_{BW})^{N_W-1} W L_{PW} + K \right] \quad (6)$$

As before the stability is evaluated by substituting various forms of S_B in equation (5) and evaluating the eigenvalues as a function of α .

3. Simulation Program

This simulation technique has been described elsewhere¹³. The form of the transverse wake potential was that derived by Brandt¹⁴ for the LEP cavities. For all results shown here the longitudinal wake potential was set to zero.

The program was also modified so as to find stability limits, rather than simply output relevant parameters as a function of time.

4. Discussion of Results

4.1 One wake (RF) per turn

Figure 3 shows typical results for three particles with varying strengths of feedback (type A). It is clear that as the feedback strength is increased the threshold for stability increases for certain tune values and diminishes for others. When the feedback gain is set to produce a feedback tune shift equal to Q_s (lowermost plots), the tune ranges where the threshold current is enhanced are greatly reduced.

The conditions for Fig. 3 were repeated but with feedback type B. Surprisingly the results are identical to those of Fig. 3 indicating that the type of feedback has little or no influence on the stability curves.

Figure 4 shows the results of simulation runs for the threshold current dependence on tune for three feedback gain situations (type B feedback) corresponding to feedback tune shifts (ΔQ_{FB}) of zero, $Q_s/2$ and $2Q_s/3$. From these plots it is clear that for certain tune values the threshold current is enhanced by applying feedback. However for gains exceeding $\Delta Q_{FB} = 2Q_s/3$

the permissible tune range is greatly reduced.

It has previously been shown⁹ using the few-particles model and a single wakefield that the phase advance between the feedback kicker and the wake has a strong influence on the threshold for instability. The simulation program was run to confirm this result. The code was modified so as to maintain the machine tune and the phase advance between pick-up and kicker constant whilst varying the phase advance from the wake to the kicker. Figure 5 shows the results of these runs, which are in good agreement with those of the few-particles model⁹.

This dependence obviously poses a severe problem for a wakefield situation such as LEP where there are 640 RF cells spread out in betatron phase, as well as several hundred other transverse impedances spread throughout the machine circumference (bellows,

Fig. 3

separator plates, injection tanks etc.). There is clearly no optimum phase between a single kicker (or even several kickers) and a multitude of transverse impedances.

Fig. 4

4.2 Many wakes per turn

Here an attempt is made to study the realistic case of wakefields in LEP. To do this properly would require many hundred wakefields occurring at random phase throughout the machine circumference. Since this is not possible in either the few-particles model or the simulation (because of computer limitations) an approximation to the realistic wake situation is made. This involves placing many (between three and maximum of around 30) wakes (N_W) equally distributed in betatron phase, i.e. the tune advance between wakes is

$$\Delta Q_{BW} = 1.0/N_W$$

The few-particles model (without feedback) was used with three, ten and 20 wakefields. The results for these three cases were absolutely identical. The

Fig. 5

uppermost plot of Fig. 6 shows the stability threshold as a function of machine tune. There is absolutely no tune dependence.

Figure 6 also shows that as the gain of the feedback is increased from zero the tune structure (or 'coherent synchro-betatron' resonances) reappear. This is not yet properly understood but is probably due to the approximation of using a finite (and small) number of wakes equally spread in betatron phase.

Fig. 6

It can also be seen that the enhancement of the threshold current (due to the feedback) is significantly reduced in comparison with the single wakefield case (due to the feedback). This is easily understood by examination of Fig. 5.

Figure 7 shows the threshold current dependence on machine tune for three RF stations (equally spaced at $2\pi/3$ in betatron phase) using the simulation program. As in Fig. 6 with zero gain there is no tune dependence. Increasing the gain to produce a feedback tune shift of $Q_s/2$ produces a slight enhancement of the threshold current. The maximum enhancement is around 30%; this again is consistent with a simple interpretation of the results of Fig. 5. Increasing the gain further to give $\Delta Q_{FB} = Q_s$ produces no further enhancement of the threshold current. On the contrary over most of the tune range the threshold current is drastically reduced.

5. Conclusions

The following gives a summary of the main results presented in previous sections.

(i) One wakefield per turn

The threshold for instability is strongly dependent on the betatron tune value. The reactive

Fig. 7

feedback produces an enhancement of the threshold for a range in tune values. However as the gain is increased the range in tune diminishes rapidly.

There is no apparent difference in the threshold for instability between feedback type A (where there are no wakes between the pick-up and the kicker) and feedback type B (where there is a wakefield between pick-up and kicker).

The enhancement of the threshold current is strongly influenced by the betatron phase shift between the wakefield and the kicker.

ii) Many wakefields equally spaced over one betatron wavelength

The maximum enhancement of the threshold current is greatly reduced when many wakefields (equally spaced over one betatron wavelength) are assumed. With many wakefields the kicker cannot be positioned for optimum enhancement for each wakefield.

These results raise the question, why does Ruth's model⁶ predict an enhancement of between a factor of two and four for a distributed wakefield? One possible explanation is in the fact that Ruth's model implied a continuous feedback system as well as a continuous wakefield. It therefore seems likely that the betatron phase shift between each element of the feedback and each element of wakefield is optimum producing an effect similar to a single wake plus feedback. In this case the enhancement of the threshold current is maximised. Unfortunately such a feedback system is unrealistic. Nevertheless future work will attempt to approximate the required distributed feedback system by a set of localised ones.

References

- 1) S. Myers, LEP Note 436 (Feb. 1983).
- 2) R.D. Kohaupt, DESY Report N-80/19 (1980).
- 3) R. Talman, CERN-ISR-TH/81-7 (1981).
- 4) R. Talman, Nucl. Instr. and Methods 193, p. 423 (1982).
- 5) A. Chao, SLAC-PUB-2946 (pp. 77-83) and in Proc. Second Summer School on High Energy Particle Accelerators, SLAC, Stanford, CA, August 1982.
- 6) R. Ruth, CERN-LEP-TH/83-22 (1983).
- 7) S. Myers, LEP Note 523, December 1984.
- 8) B. Zotter, LEP Note 497 (May 1984).
- 9) S. Myers, LEP Note 498 (May 1984).
- 10) F. Ruggiero, CERN-LEP-TH/84-21 (Nov. 1984).
- 11) R. Ruth, XIIth Internat. Conf. on High Energy Accelerators, Fermilab, August 1983 (pp. 389-392).
- 12) D. Brandt, LEP Note 444, May 1983, also presented at XIIth Internat. Conf. on High Energy Accelerators, Fermilab, Aug. 1983.
- 13) S. Myers and J. Vancraeynest, CERN-LEP-RF/84-13 (Sept. 1984).
- 14) D. Brandt and B. Zotter, CERN-LEP-TH/84-2, Jan. 1984 [also in ref. 12].
- 15) D. Brandt, private communication, Nov. 1984.