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Supersymmetry in Anti de Sitter Space

Christopher J.C. Burges* and Daniel Z. Freedman*

Department of Mathematics and

Center for Theoretical Physics

Laboratory for Nuclear Science

Massachusetts Institute of Technology

Cambridge, MA 02139, U.S.A.

and

S. Davis and G.W. Gibbons

Department of Applied Mathematics and Theoretical Physics

University of Cambridge, Silver Street, Cambridge

CB3 9EW, United Kingdom

Abstract

We consider the Wess-Zumino model in a background Anti-de Sitter spacetime in four dimensions $(AdS)_4$. We show that the naive generators of the $O(3, 2)$ isometry group, obtained by the Noether method, must be improved by adding surface terms. The improved generators have a manifestly positive definite energy density, are conserved, and have vanishing expectation value in a supersymmetric vacuum. The naive generators enjoy none of these properties. Propagators for scalar and spinor fields in $(AdS)_4$ are presented, and a simple derivation of the trace anomaly is given.

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I. Introduction.

In a supersymmetric field theory in Minkowski space, it is a striking fact that supersymmetry remains unbroken if and only if the vacuum energy, the vacuum expectation value of the Hamiltonian, vanishes to all finite orders of perturbation theory [1,2]. Spontaneous breakdown of supersymmetry implies that the vacuum energy is positive, and conversely. However, for a class of supergravity theories in $d = 4$, and for most Kaluza-Klein supergravity theories in $d > 4$, the relevant space-time background is Anti-de Sitter space. It is natural, therefore, to discuss the relation between vacuum energy and spontaneous breakdown of supersymmetry in AdS . This question has been addressed for two dimensional supersymmetry in recent previous work [3], and there is also additional literature [4-10]. In this paper we concentrate on four dimensions, and study the Wess Zumino model generalised to Anti-de Sitter space.

In a fixed AdS background of $d = 2$ or 4 dimensions, supersymmetric field theories are invariant under the supergroup $OSp(N, d)$. For $N = 1$ we have

$$\begin{aligned}\{Q_\alpha, \bar{Q}_\beta\} &= (\gamma^a M_{ad} + i\sigma^{ab} M_{ab})_{\alpha\beta} \\ \{\gamma^a, \gamma^b\} &= 2\eta^{ab} \\ \frac{1}{4}[\gamma^a, \gamma^b] &= \sigma^{ab}\end{aligned}\tag{1.1}$$

where the Q_α are the spinor charges and M_{AB} the generators of the spacetime group $O(d-1, 2)$. In our notation the indices $A, B, \dots$ range from 0 to d , while $a, b, \dots$ (and $\mu, \nu, \dots$) range from 0 to $d-1$.

By taking a suitable trace of (1.1) we see that the energy operator, $H = M_{0d}$, is formally positive, and its expectation value in a supersymmetric vacuum would be expected to vanish. However, the classical energy density, computed using the Noether method, is not positive definite, and can have arbitrary sign even in a supersymmetric vacuum. Furthermore a ζ -function calculation [4] of the self energy of free supermultiplets in $(AdS)_4$ yielded non-zero values (for $N < 5$), whereas in flat space there is an exact cancellation between boson and fermion contributions.

An alternative, direct method of calculation [6] for a free supermultiplet found zero for the self energy, thus indicating that the ζ -function regularisation method of [4] breaks supersymmetry. The work of ref. [3] supports this conclusion, for interacting fields in two dimensions, and further resolves the puzzle of the positivity of the energy density. In this paper we follow the approach used in ref. [3]. We define the spinor charge Q_α as the integral of the supercurrent, and then effectively calculate the anticommutator $\{Q_\alpha, \bar{Q}_\beta\}$, thus identifying the space-time generators M_{AB} as they appear in (1.1). The latter are found to differ from the “naive” M_{AB} by surface terms in their integral representation. The energy, defined in terms of the “improved” generators, is found to have a non-negative integrand, which vanishes only

in supersymmetric field configurations. Further, the improved energy density is proved to have vanishing expectation value in a supersymmetric vacuum to all orders in perturbation theory. The formal proof requires some supersymmetry Ward identities, and the conclusions are confirmed by several 1-loop calculations using manifestly supersymmetric regularization procedures.

Although the general method used for the $d = 4$ and $d = 2$ cases is essentially the same, we shall see that the $d = 4$ case is rather more involved. The most striking difference is that, whereas in two dimensions the conformal anomaly cancelled between the different members of the supermultiplet, in $d = 4$ the new surface terms themselves have anomalies which provide the essential cancellations.

II. The Geometry of $(AdS)_4$.

Let us begin with a quick review of the geometry of $(AdS)_4$ [11,12] which is the hyperboloid $\eta_{AB}y^Ay^B = a^{-2}$ embedded in $\mathbb{R}^5$ with Cartesian coordinates y^A and flat metric $\eta_{AB} = (+, -, -, -, +)$. One can introduce intrinsic coordinates t, ρ, x^i with $\rho^2 = x^i x^i = \vec{x} \cdot \vec{x}$ via

$$y^0 = \frac{1}{a} \sin t \sec \rho, \quad y^i = \frac{x^i}{a\rho} \tan \rho, \quad y^4 = -\frac{1}{a} \cos t \sec \rho \quad (2.1)$$

and the line element induced from the embedding then takes the form

$$ds^2 = (a \cos \rho)^{-2} \left[(dt)^2 - (\hat{x} \cdot d\vec{x})^2 - \left(\frac{\sin \rho}{\rho} \right)^2 (d\vec{x} \cdot d\vec{x} - (\hat{x} \cdot d\vec{x})^2) \right] \quad (2.2)$$

where $\hat{x}^i = x^i/\rho$. A useful alternative form is given in terms of the spherical coordinates ρ, θ, ϕ , defined by $x^1 = \rho \sin \theta \cos \phi$, $x^2 = \rho \sin \theta \sin \phi$, $x^3 = \rho \cos \theta$. In terms of these the line element is

$$ds^2 = (a \cos \rho)^{-2} [(dt)^2 - (d\rho)^2 - \sin^2 \rho ((d\theta)^2 + \sin^2 \theta (d\phi)^2)] \quad (2.3)$$

The hyperboloid is covered once if one takes $0 \leq \rho < \pi/2$ and $-\pi \leq t \leq \pi$, but to avoid closed timelike curves one passes to the covering space, defined by (2.3), with $-\infty < t < \infty$. The non-vanishing Christoffel symbols are $\Gamma_{tt}^\rho = \Gamma_{\rho\rho}^\rho = \Gamma_{\rho t}^t = -\Gamma_{\theta\theta}^\rho = \tan \rho$, $\Gamma_{\phi\phi}^\rho = -\tan \rho \sin^2 \theta$, $\Gamma_{\rho\theta}^\theta = \Gamma_{\rho\phi}^\phi = (\sin \rho \cos \rho)^{-1}$, $\Gamma_{\phi\phi}^\theta = -\sin \theta \cos \theta$ and $\Gamma_{\theta\phi}^\phi = \cot \theta$. With the choice of vierbein one-forms ($e^a \equiv V_\mu^a dx^\mu$) suggested by (2.2), namely

$$e^0 = (a \cos \rho)^{-1} dt$$

$$e^i = (a \cos \rho)^{-1} \left[\hat{x}^i \hat{x} \cdot d\vec{x} + \frac{\sin \rho}{\rho} (dx^i - \hat{x}^i \hat{x} \cdot d\vec{x}) \right] \quad (2.4)$$

the connection one-forms are given by

$$\begin{aligned}\omega^{0i} &= -\hat{x}^i \tan \rho dt \\ \omega^{ij} &= \rho^{-1}(\sec \rho - 1)(\hat{x}^i dx^j - \hat{x}^j dx^i)\end{aligned}\tag{2.5}$$

The curvature is $R_{\mu\nu\alpha\beta} = a^2(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$.

To formulate supersymmetric theories one needs Killing spinors $\epsilon(x)$, which satisfy

$$D_\mu \epsilon(x) = (\partial_\mu + \frac{1}{2}\omega_{\mu ab}\sigma^{ab})\epsilon(x) = -\frac{ia}{2}\gamma_\mu \epsilon(x)\tag{2.6}$$

There are four linearly independent solutions of this equation which may be written in the form $\epsilon(x) = S(x)\xi$ where ξ is any constant spinor and

$$S(x) = (\cos \rho)^{-\frac{1}{2}} [\cos \frac{1}{2}\rho + i\vec{\gamma} \cdot \hat{x} \sin \frac{1}{2}\rho] \exp\left(-i\gamma^0 \frac{t}{2}\right)\tag{2.7}$$

in the frame defined above. The γ -matrices in (2.7) are flat, i.e., $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$. S satisfies

$$\begin{aligned}\bar{S}(x) &\equiv \gamma^0 S^\dagger(x) \gamma^0 = S^{-1}(x) \\ \bar{S}(x) \gamma^\mu S(x) &= \gamma^a K_{a4}^\mu + i\sigma^{ab} K_{ab}^\mu\end{aligned}\tag{2.8}$$

where the K_{AB}^μ are the Killing vectors which generate the $O(3, 2)$ isometry group of the hyperboloid: specifically, with $K_{AB} \equiv K_{AB}^\mu \partial/\partial x^\mu$,

$$\begin{aligned}K_{04} &= \frac{\partial}{\partial t} \\ K_{i4} &= -\sin t \sin \rho \hat{x}^i \frac{\partial}{\partial t} + \cos t \left[\frac{\rho}{\sin \rho} \left(\frac{\partial}{\partial x^i} - \hat{x}^i \hat{x} \cdot \frac{\partial}{\partial \vec{x}} \right) + \cos \rho \hat{x}^i \hat{x} \cdot \frac{\partial}{\partial \vec{x}} \right] \\ K_{i0} &= -\cos t \sin \rho \hat{x}^i \frac{\partial}{\partial t} - \sin t \left[\frac{\rho}{\sin \rho} \left(\frac{\partial}{\partial x^i} - \hat{x}^i \hat{x} \cdot \frac{\partial}{\partial \vec{x}} \right) + \cos \rho \hat{x}^i \hat{x} \cdot \frac{\partial}{\partial \vec{x}} \right] \\ K_{ij} &= x^j \frac{\partial}{\partial x^i} - x^i \frac{\partial}{\partial x^j}\end{aligned}\tag{2.9}$$

III. The Model and its Conserved Currents

The Wess-Zumino model in anti-de Sitter space was first formulated [13] using superspace techniques. Here we simply work with the component fields $z(x)$, $\psi(x)$ and $F(x)$ which have transformation rules (with $L, R = \frac{1}{2}(1 \mp \gamma_5)$ and with Killing spinor parameter ϵ)

$$\delta z = \bar{\epsilon} L \psi$$

$$\begin{aligned}\delta L\psi &= L(-i\partial z + F)\epsilon \\ \delta F &= -\bar{\epsilon}(i\partial + a)L\psi\end{aligned}\tag{3.1}$$

These transformation rules satisfy (1.1). For a general superpotential $W(z)$, the following kinetic and interaction Lagrangians each transform into a total derivative under (3.1):

$$\begin{aligned}L_{KIN} &= \partial^\mu \bar{z} \partial_\mu z + \frac{i}{2} \bar{\psi} \not{\partial} \psi + \bar{F} F + a(\bar{z} F + z \bar{F}) + 3a^2 \bar{z} z \\ L_{INT} &= F W' + \bar{F} \bar{W}' + 3a(W + \bar{W}) - \frac{1}{2} \bar{\psi} (L W'' + R \bar{W}'') \psi\end{aligned}\tag{3.2}$$

where primes on W ($\bar{W}$) denote differentiation with respect to z ($\bar{z}$). The equations of motion are

$$\begin{aligned}F &= -\bar{W}'(\bar{z}) - az \\ \square z &= -2aF + \bar{F} \bar{W}'' - \frac{1}{2} \bar{\psi} R \psi \bar{W}''' \\ i \not{\partial} \psi &= (L W'' + R \bar{W}'') \psi\end{aligned}\tag{3.3}$$

The model and the entire analysis of this paper is easily extended to the case of multi-component fields z_i , ψ_i , and F_i .

Expressions for the generators of $O(3, 2)$ transformations and for the supersymmetry charges Q_α may be obtained by Noether's method (or, alternatively, by the methods of [14]), which yields

$$\begin{aligned}\bar{M}_{AB} &= \int d^3x \sqrt{-g} T_{AB}^0 K_{AB}^\nu \\ Q_\alpha &= \int d^3x \sqrt{-g} \bar{S}(x)_{\alpha\beta} J_\beta^0\end{aligned}\tag{3.4}$$

with stress tensor and supercurrent

$$\begin{aligned}T^{\mu\nu} &= \partial^\mu \bar{z} \partial^\nu z + \partial^\nu \bar{z} \partial^\mu z - g^{\mu\nu} (\partial_\alpha \bar{z} \partial^\alpha z - \bar{F} F \\ &\quad + 3a(a\bar{z}z + W + \bar{W})) + \frac{i}{4} \bar{\psi} (\gamma^\nu D^\mu + \gamma^\mu D^\nu) \psi\end{aligned}\tag{3.5}$$

$$J^\mu = L(\partial \bar{z} - iF) \gamma^\mu \psi + R(\partial z - i\bar{F}) \gamma^\mu \psi\tag{3.6}$$

As expected, the stress tensor satisfies the covariant conservation law $D^\mu T_{\mu\nu} = 0$, but the divergence of the supercurrent is $D_\mu J^\mu = -(ia/2) \gamma_\mu J^\mu$. The term on the right hand side balances the gradient of the Killing spinor (2.6), thus giving a conserved charge.

The contribution of the scalar potential

$$V(z, \bar{z}) = F \bar{F} - 3a(a\bar{z}z + W + \bar{W})\tag{3.7}$$

to the energy density is not necessarily positive. Further, it is clear from (3.1) that the vacuum expectation value $\langle F \rangle$ is the order parameter for supersymmetry breaking, and it is particularly striking that (3.7) has indefinite sign even for supersymmetric classical vacua where $F = 0$. It is exactly this problem of the lack of manifest positivity of the energy density that will be addressed below. It will turn out that modification of the Noether generators is required.

In any $O(3, 2)$ invariant vacuum state $\langle z(x) \rangle = \text{constant}$. Since $\langle \bar{\psi}\psi \rangle$ vanishes classically, the $O(3, 2)$ invariant classical states of the system correspond to critical points of the potential (3.7). The supersymmetry breaking pattern in particular models of this type has been discussed in [13]. The stability of the various classical vacua may be discussed using the small fluctuation criterion of [12]. The supersymmetric states in which we are particularly interested always satisfy this criterion, and their global stability will be one consequence of the arguments below.

IV. Improved $O(3, 2)$ Generators M_{AB}

The $O(3, 2)$ generators $\bar{M}_{AB}$ of (3.4), obtained by the Noether method, are not satisfactory. However, it is well known that the Noether currents can be modified by the addition of identically conserved terms so that we can consider generators of the form

$$M'_{AB} = \int d\Sigma_\mu (K_{\nu AB} T^{\mu\nu} + D_\nu A^{\mu\nu}) \quad (4.1)$$

where $A^{\mu\nu}$ is any anti-symmetric tensor (and $d\Sigma_\mu = (3!)^{-1} \tilde{\epsilon}_{\mu\nu\lambda\sigma} dx^\nu dx^\lambda dx^\sigma$ with $\tilde{\epsilon}_{0123} = \sqrt{-g}$). For example, we can choose

$$\begin{aligned} D_\nu A^{\mu\nu} &= \frac{1}{3} D_\nu [K_{AB}^\mu D^\nu(\bar{z}z) - K_{AB}^\nu D^\mu(\bar{z}z) - (D^\nu K_{AB}^\mu)(\bar{z}z)] \\ &= \frac{1}{3} K_{\nu AB} (g^{\mu\nu} \square - D^\mu D^\nu + R^{\mu\nu}) \bar{z}z \end{aligned} \quad (4.2)$$

In this case we have

$$M'_{AB} = \int d\Sigma_\mu K_{\nu AB} \Theta^{\mu\nu} \quad (4.3)$$

where

$$\Theta^{\mu\nu} = T^{\mu\nu} + \frac{1}{3} (g^{\mu\nu} \square - D^\mu D^\nu + R^{\mu\nu}) \bar{z}z \quad (4.4)$$

is the conformally improved stress tensor which is traceless for the special cases $W(z) = 0$ and $W(z) = \frac{1}{3} \lambda z^3$ of the Wess-Zumino model which are conformal and superconformal [13] invariant in $(AdS)_4$. However, for a general tensor $A^{\mu\nu}$, the additional term in (4.1) cannot be rewritten as in (4.3) as the contraction of the Killing vector with a modified symmetric stress tensor.

A second general remark which is relevant to the choice of $O(3, 2)$ generators is the following. The generators M'_{AB} of (4.1) are formally conserved (for any $A^{\mu\nu}$) because the integrands have vanishing covariant divergence. However, we must require that the generators are actually conserved in time, i.e., that the flux through the surface at spatial infinity

$$\lim_{\rho \rightarrow \pi/2} \sqrt{-g} (K_{\nu AB} T^{\rho\nu} + D_{\nu} A^{\rho\nu}) \quad (4.5)$$

vanishes (where the superscript ρ indicates the radial coordinate). This is a non-trivial matter in AdS field theory because particular boundary conditions must be imposed to ensure that the classical Cauchy problem is well defined [15]. It is already known [12] that the Noether generators $\overline{M}_{AB}$ do not have vanishing flux for the free massive model with $W = \frac{1}{2}\mu z^2$ when $|\mu| < \frac{1}{2}\alpha$, and one choice of $A^{\mu\nu}$ leading to conserved generators was suggested in [12]. In this paper we will find a different and more natural choice of $A^{\mu\nu}$, and conservation of the resulting generators M_{AB} will be checked using the boundary conditions of the free field mode functions found in [12]. In the interacting theory, one can write an iterative solution to the classical nonlinear equation of motion (3.3) using the free field propagators discussed below. This solution suggests that the boundary behavior in the nonlinear case is also determined by the free massive model with mass parameter μ given by the coefficient of the quadratic term in the superpotential $W(z)$.

Similar considerations apply to the spinor charge. The Q_{α} of the Noether method can be modified by adding identically conserved terms to the integrand. It is natural to consider the conformally improved supercurrent [16] which takes the following form in $(AdS)_4$:

$$J_{imp}^{\mu} = J^{\mu} + \frac{4}{3}\sigma^{\mu\nu}(D_{\nu} - \frac{1}{2}ia\gamma_{\nu})[(\bar{z}L + zR)\psi] \quad (4.6)$$

which gives a formally conserved charge for any superpotential $W(z)$ and which satisfies $\gamma_{\mu}J_{imp}^{\mu} = 0$ and $D_{\mu}J_{imp}^{\mu} = 0$ when $W = 0$ or $W = \frac{1}{3}\lambda z^3$. Thus we are led to study the two supercharges

$$Q_{\alpha} = \int d\Sigma_{\mu} \bar{S}(x)_{\alpha\beta} J_{\beta}^{\mu} \quad (4.7)$$

$$Q_{\alpha}^c = \int d\Sigma_{\mu} \bar{S}(x)_{\alpha\beta} J_{imp\beta}^{\mu} \quad (4.8)$$

and it is necessary to test whether the spatial integrals converge and the flux at spatial infinity vanishes. In Appendix A we show using the asymptotic behavior of the modes of the free massless theory that both integrals do converge and have vanishing flux. In fact, the additional surface term in (4.8) which comes from the second term in (4.6) vanishes at spatial infinity, so that the supercharges Q_{α} and Q_{α}^c actually coincide. Nevertheless it is convenient to treat the Noether current J^{μ} of (3.6) and the improved supercurrent J_{imp}^{μ} of (4.6) separately.

Our strategy to obtain the improved $O(3,2)$ generators is to calculate $\{Q_\alpha, \bar{Q}_\beta\}$ carefully and then evaluate the generators M_{AB} which actually appear in the algebra. To perform the calculation in a simple covariant manner, we calculate the variation of the supercurrent J^μ using the transformation rules (3.1) with Killing spinor parameter $\epsilon(x) = S(x)\xi$ (and for convenience also $\bar{\epsilon}'(x) = S(x)\xi'$). The canonical formalism then allows us to write

$$\delta_\epsilon \bar{\epsilon}' J^\mu = i[\bar{\xi} Q, \bar{\epsilon}' J^\mu] \quad (4.9)$$

$$\delta_\epsilon \int d\Sigma_\mu \bar{\epsilon}' J^\mu = i\bar{\xi}_\alpha \{Q_\alpha, \bar{Q}_\beta\} \xi'_\beta \quad (4.10)$$

Then using (1.1) the generator M_{AB} can be obtained from the local computation of $\delta_\epsilon \bar{\epsilon}' J^\mu$.

This computation involves repeated use of (2.6), some fierce Fierz transformations and the exchange properties of Majorana spinor bilinears. The result is

$$\begin{aligned} \delta_\epsilon \bar{\epsilon}' J^\mu = & i\{K_\nu T^{\mu\nu} - \frac{1}{\alpha} D_\nu [(D^\nu K^\mu)(a\bar{z}z + W + \bar{W})]\} \\ & + \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\lambda\sigma} D_\nu \{K_\sigma (\frac{1}{2}\bar{z}\bar{\partial}_\lambda z - \frac{i}{8}\bar{\psi}\gamma_5\gamma_\lambda\psi) \\ & + \frac{1}{\alpha} (D_\sigma K_\lambda)(W - \bar{W})\} \\ & + \frac{1}{4} D_\nu \{\hat{K}^\nu \bar{\psi}\gamma_5\gamma^\mu\psi - \hat{K}^\mu \bar{\psi}\gamma_5\gamma^\nu\psi\} \end{aligned} \quad (4.11)$$

where we have introduced the Killing vector $K^\mu = \bar{\epsilon}\gamma^\mu\epsilon'$ and the conformal Killing vector $\hat{K}^\mu = \bar{\epsilon}\gamma_5\gamma^\mu\epsilon'$. The identity $\epsilon^{\mu\nu\lambda\sigma} D_\nu D_\lambda K_\sigma = 0$ has also been used. Note that every term except the first in (4.11) is of the form $D_\nu A^{\mu\nu}$ which is consistent with our earlier discussion.

The improved supercurrent can be treated similarly, and we find for its variation the result

$$\begin{aligned} \delta_\epsilon \bar{\epsilon}' J_{imp}^\mu = & i\{K_\nu \Theta^{\mu\nu} - \frac{1}{\alpha} D_\nu [(D^\nu K^\mu)(W + \bar{W} - \frac{1}{3}zW' - \frac{1}{3}\bar{z}\bar{W}')] \} \\ & + \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\lambda\sigma} D_\nu \{ \frac{1}{\alpha} [K_\lambda \partial_\sigma (W - \bar{W}) + \frac{1}{3} (D_\sigma K_\lambda)(\bar{z}\bar{W}' - zW')] - \frac{i}{3} K_\lambda J_\sigma^{(5)} \} \\ & - \frac{1}{3} D_\nu \{ \hat{K}^\mu J_{(5)}^\nu - \hat{K}^\nu J_{(5)}^\mu \} \end{aligned} \quad (4.12)$$

where $J_\mu^{(5)} = -i\bar{z}\bar{\partial}_\mu z + \frac{1}{4}\bar{\psi}\gamma_5\gamma_\mu\psi$.

Both (4.11) and (4.12) are considerably more complicated than the corresponding expressions in the two dimensional case [3]. However, we have shown (see Appendix A) that the boundary conditions imply that all axial surface terms vanish at spatial infinity and can be discarded in the integral (4.10).

If we now insert (4.11) and (4.12) in (4.10) and use (2.8) to relate $K^\mu = \bar{\epsilon}\gamma^\mu\epsilon'$ to the Killing vectors K_{AB}^μ , we can identify two sets of improved generators

$$M_{AB} = \int d\Sigma_\mu \{ K_{AB\nu} T^{\mu\nu} - a^{-1} D_\nu [(D^\nu K_{AB}^\mu)(a z \bar{z} + W + \bar{W})] \} \quad (4.13)$$

$$M_{AB}^c = \int d\Sigma_\mu \{ K_{AB\nu} \Theta^{\mu\nu} - a^{-1} D_\nu [(D^\nu K_{AB}^\mu)(W + \bar{W} - \frac{1}{3} z W' - \frac{1}{3} \bar{z} \bar{W}')] \} \quad (4.14)$$

The convergence and conservation properties of these generators can now be studied using the supersymmetric scalar field boundary conditions. It is fairly straightforward to ascertain that

- (a) When the supersymmetric mass parameter satisfies $|\mu| < \frac{1}{2}a$ so that the “irregular” boundary conditions of [12] are required to maintain $OSp[1, 4]$ invariance, the second terms of (4.13) and (4.14) are necessary to achieve convergent and conserved generators.
- (b) For $|\mu| > \frac{1}{2}a$, when only the “regular” boundary conditions of [12] are applicable, the second terms of (4.13) and (4.14) are actually trivial surface terms and could be discarded for classical purposes. However, it is still required to keep these terms as contributions to the “densities” of the M_{AB} generators in order to make the positivity properties manifest, and in order to obtain vanishing vacuum expectation value at the quantum level.
- (c) The difference between M_{AB} and M_{AB}^c is itself a surface term which is trivial at spatial infinity for scalar fields which asymptotically approach the constant values of a supersymmetric configuration, i.e., $F = 0$. Thus the integrated charges M_{AB} and M_{AB}^c actually coincide in supersymmetric vacua. This is expected since they come from supercharges Q_α and Q_α^c which coincide. Nevertheless the integrands of (4.13) and (4.14) differ and it is useful to study both of them.

The calculations to support these statements are discussed in Appendix A. They were actually performed only for the energy operators M_{04} and M_{04}^c . However, since the analysis of [12] led to $O(3, 2)$ invariant results, the statements above must be true for all the generators. Finally, we note that it does not seem possible to rewrite the second term in (4.13) and (4.14) in the form $K_{AB\nu} S^{\mu\nu}$ where $S^{\mu\nu}$ is a second rank tensor.

V. Positive Energy

We now show that the energy operator M_{04} of (4.13) has a positive density. Using the general relation $\square K^\mu = -3a^2 K^\mu$ for the Killing vectors of $(AdS)_4$, and the simple form of K_{04}^μ in (2.9), M_{04} can be written as

$$M_{04} = \int d^3x \sqrt{-g} [T^0_0 + 3a(a z \bar{z} + W + \bar{W}) - a^{-1} g^{\rho\rho} \Gamma_{\rho 0}^0 \partial_\rho (a z \bar{z} + W + \bar{W})] \quad (5.1)$$

The second term in (5.1) cancels against a similar term in T^0_0 (see (3.5)), and the remaining terms combine to give

$$M_{04} = a^2 \int d^3x \sqrt{-g} \cos^2 \rho [\partial_0 \bar{z} \partial_0 z + \frac{1}{2} i \bar{\psi} \gamma_0 D_0 \psi + (\sin \rho)^{-2} (\partial_\theta \bar{z} \partial_\theta z + (\sin \theta)^{-2} \partial_\phi \bar{z} \partial_\phi z) + (\partial_\rho \bar{z} - a^{-1} \tan \rho \bar{F}) (\partial_\rho z - a^{-1} \tan \rho F) + a^{-2} \bar{F} F] \quad (5.2)$$

We see that the bosonic terms of the integrand are manifestly positive and vanish only if $\partial_\mu z = 0$ and $F = 0$, i.e., when the classical vacuum state is supersymmetric. Thus the energy density of the modified generator M_{04} of (4.13) has all the classical properties expected from the $OSp(1, 4)$ algebra and from previous experience with flat-space supersymmetric models.

$$\text{VI. } \langle M_{AB} \rangle = \langle M_{AB}^c \rangle = 0$$

The next step is to show that the improved $O(3, 2)$ generators have vanishing expectation value in a supersymmetric vacuum to all orders in perturbation theory. Actually, we will prove the stronger result that the vacuum expectation values of the integrands of (4.13) and (4.14) vanish.

The basic technique, which was previously used in [3], is to exploit supersymmetric “Ward identities” for suitably chosen composite local operators. If $F_\alpha(x)$ is a generic fermionic operator which is a local function of the elementary fields, then its supersymmetry variation can be computed using (3.1) and takes the form

$$\delta F_\alpha(x) = i B_{\alpha\beta}(x) S_{\beta\gamma}(x) \xi_\gamma = i [\bar{Q} \xi, F_\alpha(x)] \quad (6.1)$$

where $B_{\alpha\beta}$ is a bosonic composite operator which is a local Lorentz bispinor. From (6.1) one obtains

$$B_{\alpha\beta}(x) = -i \{F_\alpha(x), \bar{Q}_\gamma\} \bar{S}_{\gamma\beta}(x) \quad (6.2)$$

Thus $\langle B_{\alpha\beta}(x) \rangle = 0$ in a supersymmetric vacuum. Due to $O(3, 2)$ invariance, only the scalar part of $\langle B_{\alpha\beta}(x) \rangle$, namely its trace, contributes to the non-trivial Ward identities we need.

We now take the vacuum expectation value of (4.13) and use $\square K^\mu = -3a^2 K^\mu$ together with $\langle T^{\mu\nu} \rangle = \frac{1}{4} g^{\mu\nu} \langle T^\alpha_\alpha \rangle$ and $\langle \partial_\mu (a \bar{z} z + W + \bar{W}) \rangle = 0$ which follows from the maximal symmetry of anti de Sitter space. Thus we can write

$$\langle M_{AB} \rangle = \frac{1}{4} \int d\Sigma_\mu K^\mu_{AB} \langle T^\alpha_\alpha + 12a(a \bar{z} z + W + \bar{W}) \rangle \quad (6.3)$$

$$= \frac{1}{4} \int d\Sigma_\mu K^\mu_{AB} \langle -2\partial^\alpha \bar{z} \partial_\alpha z + \frac{1}{2} i \bar{\psi} \not{D} \psi + 4\bar{F} F \rangle \quad (6.4)$$

where, in the last line, we have evaluated T_α^α using (3.5). To show that the integrand of (6.4) vanishes in a supersymmetric vacuum, we compute the variation of the fermionic operator $L(-i(\not{\partial}z) + 2\bar{F})\psi$. The scalar part of the corresponding bispinor has expectation value

$$\langle -\partial^\alpha \bar{z} \partial_\alpha z + \frac{1}{4} i \bar{\psi} \not{D} \psi + 2\bar{F}F - \frac{3i}{8} D_\mu (\bar{\psi} \gamma_5 \gamma^\mu \psi) \rangle = 0 \quad (6.5)$$

Since the last term vanishes in any $O(3,2)$ invariant state, we see that the integrand of (6.4) vanishes.

We treat the conformally improved $O(3,2)$ generators somewhat differently. The vacuum expectation value of (4.14) can be written

$$\langle M_{AB}^c \rangle = \frac{1}{4} \int d\Sigma_\mu K_{AB}^\mu \langle \Theta_\alpha^\alpha + 12a(W + \bar{W} - \frac{1}{3}zW' - \frac{1}{3}\bar{z}\bar{W}') \rangle \quad (6.6)$$

For the conformally invariant superpotential $W(z) = \frac{1}{3}\lambda z^3$, the trace Θ_α^α vanishes classically and the second term also vanishes. Thus $\langle M_{AB}^c \rangle = 0$ classically. At this point we see the looming puzzle of the trace anomaly, and we will discuss this in detail in Section IX.

We now wish to show that the integrand of (6.6) vanishes as a consequence of supersymmetric Ward identities. For this purpose we evaluate the trace Θ_α^α using the equations of motion (3.3) and obtain

$$\begin{aligned} \Theta_\alpha^\alpha = & -12a(W + \bar{W} - \frac{1}{3}zW' - \frac{1}{3}\bar{z}\bar{W}') \\ & + (FzW'' - \frac{1}{2}\bar{\psi}L(zW'')'\psi + \bar{F}\bar{z}\bar{W}'' - \frac{1}{2}\bar{\psi}R(\bar{z}\bar{W}'')'\psi) \\ & - 2(FW' - \frac{1}{2}\bar{\psi}LW''\psi) - 2(\bar{F}\bar{W}' - \frac{1}{2}\bar{\psi}R\bar{W}''\psi) \end{aligned} \quad (6.7)$$

where the terms have been grouped with judicious foresight.

We now apply the transformation rules (3.1) repeatedly to any scalar operator $U(z)$ and obtain

$$\begin{aligned} \delta U &= \bar{\epsilon}L(U'\psi) \\ \delta L(U'\psi) &= L(-i\not{\partial}U + FU' - \frac{1}{2}U''(\bar{\psi}L\psi))\epsilon \\ \delta(FU' - \frac{1}{2}U''(\bar{\psi}L\psi)) &= \bar{\epsilon}(i\not{D} + a)LU'\psi. \end{aligned} \quad (6.8)$$

This shows that $U(z)$, $LU'(z)\psi$ and $U'F - \frac{1}{2}U''(\bar{\psi}L\psi)$ transform as components of a composite chiral multiplet. Let us now compare the second line of (6.8) with (6.1) and (6.2). For fermionic operator $F_\alpha = (LU'\psi)_\alpha$, we can identify the scalar part of the bispinor $B_{\alpha\beta}$ into which it transforms, and we can write

$$\langle FU' - \frac{1}{2}U''(\bar{\psi}L\psi) \rangle = 0 \quad (6.9)$$

in any supersymmetric vacuum. One may also derive the conjugate relation

$$\langle \bar{F} \bar{U}' - \frac{1}{2} \bar{U}'' (\bar{\psi} R \psi) \rangle = 0 \quad (6.10)$$

by similar methods applied to $\bar{U}(\bar{z})$.

We now see that the combination of operators in the last two lines of (6.7) has vanishing vacuum expectation value if (6.9) and (6.10) are applied with $U(z) = zW'(z)$ and $U(z) = W(z)$. Thus we derive

$$\langle \Theta_\alpha^\alpha \rangle = -12a \langle W + \bar{W} - \frac{1}{3} z W' - \frac{1}{3} \bar{z} \bar{W}' \rangle \quad (6.11)$$

in any supersymmetric vacuum, which implies that the integrand of (6.7) vanishes.

The proof that $\langle M_{ab}^c \rangle = 0$ required Ward identities with a natural chiral multiplet structure, as in the two-dimensional case [3]. To prove that $\langle M_{ab} \rangle = 0$ we needed (6.5) which is correct but less natural. The reason for the difference presumably lies in the simpler transformation properties of the conformally improved supercurrent and stress tensor, as already known in flat space [16].

We would like to mention two other Ward identities, obtained by the method of (6.1) and (6.2). From the variation of $F_\alpha = (L(i \not{\partial} z + \bar{F})\psi)_\alpha$ one finds

$$\langle \partial^\mu \bar{z} \partial_\mu z + \frac{i}{2} \bar{\psi} \not{\partial} \psi + \bar{F} F \rangle = 0 \quad (6.12)$$

which is certainly natural for the flat-space Wess-Zumino model. From the variation of $F_\alpha = (L\bar{z}\psi)_\alpha$, one derives

$$\langle \bar{z} F \rangle = - \langle a \bar{z} z + \bar{z} \bar{W}'(\bar{z}) \rangle = 0. \quad (6.13)$$

For the free theory $W(z) = 0$, and we are left with a positive definite operator with vanishing vacuum expectation value. We will look into this puzzle below.

Our proof that the densities of the improved $O(3,2)$ generators M_{AB} and M_{AB}^c have vanishing vacuum expectation value is formal in the sense that it relies on Ward identities for local composite operators, and such operators are intrinsically singular at short distances. It is therefore important to test our conclusions using regularized perturbation theory, particularly since the trace anomaly in (6.6) and the positivity puzzle of (6.13) are problematic. We will find in the next two sections and in Appendix B, that the conclusions of the formal argument are validated by 1-loop regularized calculations, essentially because the required Ward identities are valid for all values of the regularization parameters.

The final point that should be stressed here is that the extra surface terms in the improved generators M_{AB} and M_{AB}^c play a crucial role in the proof above. There is no reason why $\langle T_\alpha^\alpha \rangle$ or $\langle a \bar{z} z + W + \bar{W} \rangle$

should vanish separately; only the combination in (6.3) vanishes. Similar remarks apply to (6.6). Thus in general, neither the naive Noether generators $\overline{M}_{AB}$ of (3.4) nor the purely conformal M'_{AB} of (4.3) have vanishing vacuum expectation value.

VII. Propagators

Let us begin by discussing the propagators for a free scalar field $\phi(x)$ in an n -dimensional anti-de Sitter space. The action is

$$S = \frac{1}{2} \int d^n x \sqrt{-g} \{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \} \quad (7.1)$$

and the Feynman propagator $G_F(x, x') = -i \langle T A(x) A(x') \rangle$ satisfies

$$(\square_x + m^2) G_F(x, x') = -(-g)^{-1/2} \delta(x, x') \quad (7.2)$$

As discussed in [3] for $n = 2$, one can obtain the solutions by observing that the propagator depends only on the $O(n-1, 2)$ invariant variable

$$u = \frac{1}{2} a^2 (y^A - y'^A)^2 \quad (7.3)$$

which is one-half the square of the chordal distance between the points y^A and y'^B on the hyperboloid of unit scale. In terms of this variable, (7.2) becomes the hypergeometric equation (for $u \neq 0$). We find the propagators by choosing the solutions with the same asymptotic behavior as regular or irregular scalar mode functions (see [3] and [17]) for mass parameters m^2 , and we normalize by matching the short distance singularity of the scalar propagator in n -dimensional flat space. The result is

$$G_F(x, x') = \frac{-i a^{(n-2)}}{(4\pi)^{n/2}} \frac{\Gamma(\lambda) \Gamma(\lambda + 1 - \frac{1}{2}n)}{\Gamma(2\lambda - n + 2)} (-2u^{-1})^\lambda F(\lambda, \lambda + 1 - \frac{1}{2}n, 2\lambda - n + 2, 2u^{-1})$$

$$\lambda_\pm = \frac{n-1}{2} \pm \frac{1}{2} \sqrt{(n-1)^2 + 4m^2 a^{-2}} \quad (7.4)$$

The $\pm$ signs correspond to regular (irregular) boundary conditions. For $n = 4$ the scalar field transforms in the representation $D(\lambda_\pm, 0)$ of $O(3, 2)$. This propagator agrees with [3] for $n = 2$, and it agrees with [15] for $n = 4$. For general n it can be shown to agree with recently obtained expressions [9,10] after use of transformation formulae for the hypergeometric function. See [18] for mathematical conventions. The standard analyticity property of $F(a, b, c; z)$ corresponds to the Feynman or time ordered Green's function.

A special case of some interest is the conformally coupled scalar field with mass parameter

$$m^2 = -\frac{(n-2)}{4(n-1)} R = -\frac{n(n-2)}{4} a^2 \quad (7.5)$$

In this case the λ -parameter of the regular (and irregular) modes is $\lambda_+ = \frac{1}{2}n$ (and $\lambda_- = \frac{1}{2}n - 1$). A simple expression for the propagators can be obtained either from special case hypergeometric formulae or by direct solution of the differential equation (7.2). This gives

$$G_F(x, x') = \frac{-ia^{(n-2)}}{2(2\pi)^{\frac{n}{2}}} \Gamma\left(\frac{n-2}{2}\right) \left\{ \left(\frac{1}{-u}\right)^{\frac{n-2}{2}} \mp \left(\frac{1}{2-u}\right)^{\frac{n-2}{2}} \right\} \quad (7.6)$$

where $\mp$ correspond to regular (irregular) boundary conditions. A separate analysis is required for $n = 2$ and gives a propagator with logarithmic singularities at $u = 0, 2$. The transformation $u \rightarrow 2 - u$ corresponds to the mapping of x^μ or x'^μ to its antipodal point on the hyperboloid. The evident symmetry of the propagator under this antipodal map arises because of the reflective boundary conditions discussed in [15].

We now take the physical dimension $n = 4$, and write the propagator together with the first few terms of its asymptotic expansion as

$$\begin{aligned} G_F(x, x') &= \frac{-ia^2}{(4\pi)^2} \frac{\Gamma(\lambda)\Gamma(\lambda-1)}{\Gamma(2\lambda-2)} (-2u^{-1})^\lambda F(\lambda, \lambda-1, 2\lambda-2, 2u^{-1}) \\ &\xrightarrow{u \rightarrow 0} \frac{ia^2}{8\pi^2} \left\{ \frac{1}{u} + \frac{1}{2} \left(2 + \frac{m^2}{a^2}\right) \left[-\log\left(-\frac{1}{2}u\right) + h_0\right] \right. \\ &\quad \left. - \frac{1}{8}u \frac{m^2}{a^2} \left(2 + \frac{m^2}{a^2}\right) \left[-\log\left(-\frac{1}{2}u\right) + h_1\right] + O(u^2 \log u, u^2) \right\} \\ h_0 &= \psi(1) + \psi(2) - \psi(\lambda) - \psi(\lambda-2) \\ h_1 &= \psi(2) + \psi(3) - \psi(\lambda+1) - \psi(\lambda-3) \\ \lambda_\pm &= \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{m^2}{a^2}} \end{aligned} \quad (7.7)$$

In the Wess-Zumino model, the propagators for the fields $z = (A + iB)/\sqrt{2}$ and ψ are determined from the action for the free supermultiplet ($W = \frac{1}{2}\mu z^2$ in (3.2)) which is

$$\begin{aligned} S &= \frac{1}{2} \int d^4x \sqrt{-g} \{ (\partial_\mu A)^2 + (\partial_\mu B)^2 + i\bar{\psi} \not{D} \psi - \mu \bar{\psi} \psi \\ &\quad + (2a^2 + a\mu - \mu^2)A^2 + (2a^2 - a\mu - \mu^2)B^2 \} \end{aligned} \quad (7.8)$$

The distinct mass parameters of the A and B fields, which are required by $OSp(1, 4)$ invariance, are a nuisance for us, but there is a partially compensating simplification in the λ -parameters. At first glance we find

$$\begin{aligned} \lambda_{A\pm} &= \frac{3}{2} \pm \left| \frac{\mu}{a} - \frac{1}{2} \right| \\ \lambda_{B\pm} &= \frac{3}{2} \pm \left| \frac{\mu}{a} + \frac{1}{2} \right| \end{aligned}$$

However, the requirement that the particle states of the theory transform in an irreducible representation of $OSp(1,4)$ forces regular boundary conditions λ_{A+} and λ_{B+} if $|\mu| \geq \frac{1}{2}|a|$, and the choice of two regular-irregular combinations $\lambda_{A\pm}$ and $\lambda_{B\mp}$ if $|\mu| < \frac{1}{2}|a|$. We now choose

$$\begin{aligned}\lambda_A &= 1 + \frac{\mu}{a} \\ \lambda_B &= 2 + \frac{\mu}{a}\end{aligned}\tag{7.9}$$

which is compatible with the boundary conditions if $\mu > -\frac{1}{2}a$ and provides a very reasonable test in perturbation theory of the formal conclusions of the previous sections. This means that the propagators for the fields $A(x)$ and $B(x)$ are given by (7.7) with the λ parameters of (7.9) and the mass parameter $m_{A,B}^2 = \mu^2 \mp a\mu - 2a^2$.

The spinor propagator will also be required in our calculation. It is related to the scalar propagators by Ward identities which follow simply from the identities $\langle \delta(\psi(x)A(x')) \rangle = \langle \delta(\psi(x)B(x')) \rangle = 0$ which can be evaluated using the transformation rules (3.1) in the free theory. From this one obtains

$$\begin{aligned}\langle T\psi_\alpha(x)\bar{\psi}_\beta(x') \rangle &= [(i\partial_x + \mu + a) \langle TA(x)A(x') \rangle S(x)\bar{S}(x')]_{\alpha\beta} \\ &= [(i\partial_x + \mu - a) \langle TB(x)B(x') \rangle \gamma_5 S(x)\bar{S}(x')\gamma_5]_{\alpha\beta}\end{aligned}\tag{6.10}$$

From the first of these relations one can obtain, using (2.8), the interesting representation

$$\bar{S}(x) \langle T\psi(x)\bar{\psi}(x') \rangle S(x') = (ia\gamma^a K_{a4} - a\sigma^{ab}K_{ab} + \mu + a) \langle TA(x)A(x') \rangle\tag{7.11}$$

of the same type obtained for $d=2$ in [3]. The matrix $S(x)_{\gamma\delta}$ plays the role of a bispinor of parallel transport in (7.10) and (7.11), since its left index transforms under local Lorentz transformation at the point x and its right index transforms under local Lorentz transformations at the origin. Note that (7.10) and (7.11) correctly define the propagator for a spinor field of mass μ in $(AdS)_4$ whether or not the underlying theory is supersymmetric. One can verify directly from (7.10) that $(i\partial - \mu) \langle T\psi(x)\bar{\psi}(x') \rangle = -i(-g)^{-\frac{1}{2}}\delta(x, x')$.

VIII. Pauli-Villars Regularized Calculation of $\langle M_{AB} \rangle$ and $\langle \bar{z}F \rangle$

In this section we describe a calculation of the densities of the vacuum expectation values of the generators $\langle M_{AB}^c \rangle$ of (6.6) using Pauli-Villars regularization at 1-loop order. For a 1-loop calculation it is sufficient to consider a set of free supermultiplets A_i, B_i, ψ_i each described by the action (7.8) with mass parameter μ_i . The regularized vacuum expectation value of the stress tensor $\Theta_{\mu\nu}$ is defined by

$$\langle \Theta_{\mu\nu} \rangle_{reg} = \sum_i c_i \langle \Theta_{\mu\nu}(A_i, B_i, \psi_i) \rangle\tag{8.1}$$

where the contribution of each multiplet is calculated from (4.4) and (3.5) with $W(z_i) = \frac{1}{2}\mu z_i^2$. Since $\Theta_{\mu\nu}$ is an operator of dimension 4 one must impose the sum rules $\sum_i c_i (\mu_i)^p = 0$ for $p = 0, 1, 2, 3, 4$ to make $\langle \Theta_{\mu\nu} \rangle_{reg}$ finite. The trace calculation of (6.7) is valid after regularization and one obtains

$$\langle \Theta_\mu^\mu \rangle_{reg} = \sum_i c_i \langle \mu_i (\mu_i - a) A_i^2 + \mu_i (\mu_i + a) B_i^2 + \frac{1}{2} \mu_i \bar{\psi}_i \psi_i \rangle \quad (8.2)$$

Under the summation sign one can use the relation between the traced fermion and boson propagators which follows from (7.10) in the short distance limit, namely

$$-\frac{1}{4} \langle \bar{\psi}_i \psi_i \rangle = (\mu_i + a) \langle A_i^2 \rangle = (\mu_i - a) \langle B_i^2 \rangle \quad (8.3)$$

which gives

$$\langle \Theta_\mu^\mu \rangle_{reg} = -2a \sum_i c_i \mu_i \langle A_i^2 - B_i^2 \rangle \quad (8.4)$$

The second term in (6.6) must be regularized in the same manner. One has

$$\begin{aligned} 12a \langle W - \frac{1}{3} z W' + \bar{W} - \frac{1}{3} \bar{z} \bar{W}' \rangle_{reg} &= 2a \sum_i c_i \mu_i \langle z_i^2 + \bar{z}_i^2 \rangle \\ &= 2a \sum_i c_i \mu_i \langle A_i^2 - B_i^2 \rangle \end{aligned} \quad (8.5)$$

Thus the regularized integrand of (6.6) vanishes for all values of the regulator parameters c_i, μ_i . This proves that the densities of the $O(3, 2)$ generators $\langle M_{AB}^c \rangle$ vanish in an explicit one-loop calculation. This is the main result which we wish to prove in this section.

Let us now consider the Ward identity, (6.13), which becomes

$$2 \langle a \bar{z} z + \mu \bar{z}^2 \rangle = (a + \mu) \langle A^2 \rangle + (a - \mu) \langle B^2 \rangle = 0 \quad (8.6)$$

when $W = \frac{1}{2}\mu z^2$. To test this Ward identity we write Pauli-Villars regulated expressions

$$((a + \mu) \langle A^2 \rangle)_{reg} = \sum_i c_i (a + \mu_i) \langle A_i^2 \rangle \quad (8.7)$$

$$((a - \mu) \langle B^2 \rangle)_{reg} = \sum_i c_i (a - \mu_i) \langle B_i^2 \rangle \quad (8.8)$$

and require sum rules $\sum_{i \geq 0} c_i \mu_i^p = 0$ for $p = 0, 1, 2, 3$. We then temporarily introduce a non-zero separation between the points at which the operators are evaluated and use the short distance expansions (7.7) of the

A_i and B_i propagators. The singular terms in the zero separation limit then vanish because of the sum rules, and one can write (after setting $a = 1$ to simplify notation)

$$\begin{aligned} ((1 + \mu) < A^2 >)_{reg} &= -\frac{1}{16\pi^2} \sum_{i \geq 0} c_i (\mu_i^2 - 1) \mu_i h_0(A_i) \\ &= \frac{1}{16\pi^2} \sum_{i \geq 0} c_i (\mu_i^2 - 1) \mu_i (\psi(\mu_i + 1) + \psi(\mu_i - 1)) \end{aligned} \quad (8.9)$$

$$\begin{aligned} ((1 - \mu) < B^2 >)_{reg} &= \frac{1}{16\pi^2} \sum_{i \geq 0} c_i (\mu_i^2 - 1) \mu_i h_0(B_i) \\ &= -\frac{1}{16\pi^2} \sum_{i \geq 0} c_i (\mu_i^2 - 1) \mu_i (\psi(\mu_i + 2) + \psi(\mu_i)) \end{aligned} \quad (8.10)$$

After using the standard identity

$$\psi(z + 1) = \psi(z) + z^{-1} \quad (8.11)$$

and the sum rules again, we find

$$((1 + \mu) < A^2 >)_{reg} = \frac{1}{8\pi^2} \sum c_i (\mu_i^2 - 1) \mu_i \psi(\mu_i) \quad (8.12)$$

$$((1 - \mu) < B^2 >)_{reg} = -\frac{1}{8\pi^2} \sum c_i (\mu_i^2 - 1) \mu_i \psi(\mu_i) \quad (8.13)$$

Thus we see that the Ward identity (8.6) is valid after regularization. Note that (8.12) and (8.13) are well defined for zero physical mass $\mu = \mu_0 = 0$, but that these expressions diverge as regulator masses become large. Thus we must supply the definition of $< A^2 >_{ren}$ and $< B^2 >_{ren}$ by a renormalization prescription, and there is no contradiction if we choose equal and opposite values for these quantities so that the Ward identity $< \bar{z}z > = 0$ is satisfied.

IX. Trace Anomalies.

There is one puzzling aspect of the basic expression (6.6) for the vacuum expectation values of $O(3, 2)$ generators. For a conformally invariant superpotential, and, in particular, for a free massless supermultiplet ($W(z) = 0$), one expects a non-vanishing one-loop trace anomaly for $< \Theta^\alpha_\alpha >$ because of the curvature of the $(AdS)_4$ background. Since the superpotential vanishes, it is not clear why the vacuum energy density vanishes despite the analysis of the previous sections. In the two-dimensional case [3], there is a similar expression for the $O(2, 1)$ generators. However, the puzzle does not arise there because the net trace anomaly vanishes since the scalar and spinor anomalies are equal in magnitude and of opposite sign. In four dimensions the trace anomaly of the combination of fields $A(x)$, $B(x)$, $\psi(x)$ does not vanish, as we review below, and we must provide a different resolution of the anomaly puzzle.

The solution is based on (8.4) and (8.5) which show that the sum of the Pauli-Villars regularized expressions for the trace and the superpotential combination required in the vacuum energy density vanishes. The supersymmetric Ward identity (6.11) also requires the sum to vanish, and the Pauli-Villars calculation shows that there is no "supersymmetry anomaly." Therefore, if the renormalized value of the trace, namely $\langle \Theta_\mu^\mu \rangle_{ren}$, is given by the conventional anomaly we must assign a value to $\langle W - \frac{1}{3}zW' \rangle_{ren}$ such that the net contribution to the vacuum energy vanishes. Thus we deduce the presence of a new curved space superpotential anomaly. The heuristic arguments for the trace and superpotential anomalies are very similar. In both cases we have operators which vanish classically due to conformal invariance, but in both cases the contributions to Θ_μ^μ and $W - \frac{1}{3}zW'$ from Pauli-Villars regulator fields do not vanish and have non-vanishing vacuum expectation values.

Thus our basic assertion is that the trace anomaly must be cancelled by a superpotential anomaly, so that zero vacuum energy density as required by $OSp(1, 4)$ supersymmetry is obtained. This assertion relies only on the regulated expressions (8.4) and (8.5), and on the supersymmetry Ward identity (6.11) which shows that the cancellation of (8.4) and (8.5) is no accident. No further supporting calculations are required. However, we do wish to present some calculations to elucidate the relation between $\langle \Theta_\mu^\mu \rangle_{reg}$ of (8.4) and the standard trace anomaly. For that purpose we first review the conventional argument.

In a calculation of the trace anomaly on a general space-time by Pauli-Villars regularization, one starts from the actions

$$S_0 = \frac{1}{2} \int d^4x \sqrt{-g} [\partial_\mu \phi_i(x) \partial^\mu \phi_i(x) - (m_i^2 - \xi R(x)) \phi_i^2] \quad (9.1)$$

$$S_{1/2} = \frac{1}{2} \int d^4x \sqrt{-g} \bar{\psi}_i(x) (i \not{D} - \mu_i) \psi_i \quad (9.2)$$

for each component field. The physical field ($i = 0$) has mass $m_0 = 0$ or $\mu_0 = 0$. The regulated expressions for the trace of the (conformally improved) stress tensor are

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(0)} = \sum_i c_i m_i^2 \langle \phi_i^2 \rangle \quad (9.3)$$

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(1/2)} = \frac{1}{2} \sum_i c_i \mu_i \langle \bar{\psi}_i \psi_i \rangle \quad (9.4)$$

Since these are obtained from $\langle \Theta_{\mu\nu} \rangle$ which is a dimension 4 operator, one must impose Pauli-Villars sum rules $\sum c_i m_i^p = 0$ and $\sum c_i \mu_i^p = 0$ up to and including fourth powers. We will use these sum rules henceforth, frequently without explicit mention.

The next step is to use the (coincidence limits of) the a coefficients which occur in the Minakshisundaram-Pleijel-Schwinger-de Witt short distance expansion of the Hadamard two-point function to write the previous

expressions as

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(0)} = \frac{1}{16\pi^2} \sum_{i \geq 1} c_i \left\{ a_0^{(0)} m_i^4 \ln m_i^2 - a_1^{(0)} m_i^2 \ln m_i^2 + a_2^{(0)} \right\} \quad (9.5)$$

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(1/2)} = -\frac{1}{16\pi^2} \sum_{i \geq 1} c_i \left\{ a_0^{(1/2)} \mu_i^4 \ln \mu_i^2 - a_1^{(1/2)} \mu_i^2 \ln \mu_i^2 + a_2^{(1/2)} \right\} . \quad (9.6)$$

where terms which vanish as $m_i, \mu_i \rightarrow \infty$ have been dropped. The scalar and traced spinor a coefficients are

$$\begin{aligned} a_0^{(0)} &= \frac{1}{2} a_0^{(1/2)} = 1 \\ a_1^{(0)} &= \left(\xi - \frac{1}{6} \right) R \quad a_1^{(1/2)} = \frac{1}{6} R \\ a_2^{(0)} &= \frac{1}{180} \left\{ C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 + 90 \left(\xi - \frac{1}{6} \right) R^2 + (6 - 30\xi) \square R \right\} \\ a_2^{(1/2)} &= \frac{-1}{720} \left\{ 7 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + 22 (R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2) + 12 \square R \right\} \end{aligned} \quad (9.7)$$

The third step is to observe that as regulator masses become large the divergent a_0 and a_1 terms can be cancelled by adding local counter terms of the form $\int d^4x \sqrt{-g}$ and $\int d^4x \sqrt{-g} R$ to the effective action. The a_2 contribution cannot be cancelled by the addition of local terms in the effective action (except for the $\square R$ term), and it is interpreted as the renormalized anomalous trace

$$\langle \Theta_\mu^\mu \rangle_{ren}^{(0)} = -\frac{1}{16\pi^2} a_2^{(0)} \quad (9.8)$$

$$\langle \Theta_\mu^\mu \rangle_{ren}^{(1/2)} = +\frac{1}{16\pi^2} a_2^{(1/2)} \quad (9.9)$$

Note that $\sum_{i \geq 1} c_i = -c_0 = -1$.

This summary of the standard framework was gleaned from the Pauli-Villars calculation of the scalar trace anomaly of Vilenkin [19] and from work of Christensen [20]. Trace anomaly calculations are discussed extensively in [21], and the presentation there suggested the notation used above.

To evaluate the trace anomaly expected for our supersymmetric theory in $(AdS)_4$, we note that the scalar fields are purely conformal coupled at $\mu = 0$, so that $\xi = \frac{1}{6}$. Further, only the single curvature combination $R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 = -12a^2$ is non-vanishing. Thus we obtain

$$\langle \Theta_\mu^\mu \rangle_{ren}^{(0)} = \frac{1}{16\pi^2} \frac{a^4}{15} \quad (9.10)$$

$$\langle \Theta_\mu^\mu \rangle_{ren}^{(1/2)} = \frac{1}{16\pi^2} \frac{11a^4}{30} \quad (9.11)$$

$$\langle \Theta_\mu^\mu \rangle_{ren}^{(1/2)} + 2 \langle \Theta_\mu^\mu \rangle_{ren}^{(0)} = \frac{1}{32\pi^2} a^4 \quad (9.12)$$

The last equality gives the net anomaly of the A, B, ψ supermultiplet.

We now wish to show that it is a fairly straightforward matter to reproduce these results from the standpoint of free field theory on a fixed $(AdS)_4$ background using the scalar and spinor propagators of Sec. VII. To do this one must be careful to choose regulator mass parameters exactly as in the Lagrangians (9.1-2), so that one can correctly identify the counter term contributions from (9.5-6).

The spinor anomaly calculation is actually simpler and we present it first. We insert the representation (7.11) for the spinor propagator in (9.4). After tracing over spinor indices we obtain

$$\begin{aligned} \langle \Theta_\mu^\mu \rangle_{reg}^{(1/2)} &= -2 \sum c_i \mu_i (\mu_i + a) \langle A_i^2 \rangle \\ &= \frac{1}{8\pi^2} \sum c_i \mu_i^2 (\mu_i^2 - 1) h_0(A_i) \\ &= \frac{-1}{4\pi^2} \sum_{i \geq 1} c_i \mu_i^2 (\mu_i^2 - 1) \psi(\mu_i) \end{aligned} \quad (9.13)$$

where the short distance expansion (7.7) has been used, and we have set $a = 1$ for convenience. The sum rules and (8.11) were also used. Note that the physical field, with zero mass, does not contribute.

Next we use the asymptotic expansion of $\psi(x)$

$$\psi(z) \approx \ln z - (2z)^{-1} - \sum_{n=1}^m B_{2n} z^{-2n} / (2n) + O(z^{-2m-2}) \quad (9.14)$$

where the B_{2n} are the Bernoulli numbers, and we will need only $B_2 = 1/6$ and $B_4 = -1/30$. We insert them in (9.13) and drop terms which vanish because of the sum rules or vanish at large μ_i . The result is

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(1/2)} = \frac{-1}{16\pi^2} \sum_i c_i (2\mu_i^4 \ln \mu_i^2 - 2a^2 \mu_i^2 \ln \mu_i^2 + \frac{11}{30}) \quad (9.15)$$

This result may be compared directly with (9.6) and we find exact agreement with the spinor a coefficients of (9.7) and therefore with the standard anomaly (9.11).

Next we obtain the scalar anomaly, and it is again easier and more instructive to use the mass parameterizations of the supersymmetric action (7.8) initially and then reexpress the result using the parameters of (9.1). By a series of steps similar to that just described, we obtain

$$\begin{aligned} \langle \Theta_\mu^\mu \rangle_{reg} &= \sum c_i \mu_i (\mu_i - 1) \langle A_i^2 \rangle \\ &= \frac{-1}{16\pi^2} \sum c_i \mu_i^2 (\mu_i - 1)^2 h_0(A_i) \\ &= \frac{1}{8\pi^2} \sum_{i \geq 1} c_i \mu_i^2 (\mu_i - 1)^2 \psi(\mu_i) \end{aligned} \quad (9.16)$$

If the same steps are carried out for the pseudoscalar field, one obtains only a sign change due to the distinct mass parameters in (7.8). Thus we can proceed to write the joint result as

$$\begin{aligned}
\langle \Theta_\mu^\mu \rangle_{reg}^{(0)} &= \frac{1}{8\pi^2} \sum_{i \geq 1} c_i \mu_i^2 (\mu_i \mp 1)^2 \psi(\mu_i) \\
&= \frac{1}{8\pi^2} \sum_{i \geq 1} c_i \mu_i^2 (\mu_i \mp 1)^2 \left\{ \ln \mu_i - \frac{1}{2\mu_i} - \frac{1}{12\mu_i^2} + \frac{1}{120\mu_i^4} + \dots \right\} \\
&= \frac{1}{16\pi^2} \sum_{i \geq 1} c_i \left\{ \mu_i^2 (\mu_i \mp 1)^2 \ln \mu_i^2 - \frac{3}{20} \right\}
\end{aligned} \tag{9.17}$$

where the sum rules were used in the last step. To proceed further we must write $m_i^2 = \mu_i(\mu_i \mp 1)$ and $\ln \mu_i = \frac{1}{2}[\ln m_i^2 - \ln(1 \mp \mu_i^{-1})]$. We then use the power series for $\ln(1 \mp \mu_i^{-1})$ and the original sum rules to obtain the same result for both the scalar and pseudoscalar, namely,

$$\langle \Theta_\mu^\mu \rangle_{reg}^{(0)} = \frac{1}{16\pi^2} \sum_i c_i \left\{ m_i^4 \ln m_i^2 - \frac{1}{15} \right\} \tag{9.18}$$

Again this may be compared with the general result (9.5), and we find that the scalar a coefficients and the anomaly agree.

The last subject we wish to discuss is whether the value of the renormalized trace of the stress tensor given in (9.12) as the sum of the standard trace anomalies for the fields $A(x)$, $B(x)$, $\psi(x)$ is really required in the strict context of the supersymmetric field theory in $(AdS)_4$. We believe that it is not required. To illustrate this point let us consider in more detail the regulated expression of $\langle \Theta_\mu^\mu \rangle$ in the Wess-Zumino model in either (8.2) or (8.5). To calculate this one can either apply the same manipulations as in previous calculations to (8.4) or one can use (8.2) and simply sum the previous results in (9.15) and (9.17). The latter strategy is simpler, and we do it and obtain

$$\langle \Theta_\mu^\mu \rangle_{reg}^{Total} = \frac{-1}{16\pi^2} \sum_{i \geq 1} c_i \left\{ 4a^2 \mu_i^2 \ln \mu_i^2 - \frac{2}{3} a^4 \right\} \tag{9.19}$$

Neither the effective a_1 or a_2 coefficient of this expression agrees with (9.5-7), and they should not since the scalar regulator masses are not properly chosen. In this fixed background context, it seems to us that (9.19) should be interpreted as requiring the addition of a constant counter term to the superpotential of the theory, namely

$$\Delta W = \frac{-a}{96\pi^2} \sum_i c_i \mu_i^2 \ln \mu_i^2 + \{finite\} \tag{9.20}$$

This will make an additional contribution to Θ_μ^μ , as seen from (3.5) and (4.4), which will cancel the divergence in (9.19). This same counter term will also appear in the expression (6.13) in such a way that the vacuum

energy density is unchanged. The specification of the finite term in (9.20) requires a renormalization condition to define $\langle \Theta_\mu^\mu \rangle_{ren}^{Total}$. It appears that we can choose this value arbitrarily and that it is not logically necessary to choose the conventional value (9.12). The inhabitants of $(AdS)_4$ would not be required to invent quantum field theory on a general space time to account for observations in their world. A different way to determine the trace anomaly is to use the anomalous scaling behavior of the vacuum to vacuum amplitude. It is not clear to us at present how to apply this in $(AdS)_4$ because it is not straightforward to define the 1-loop functional determinant for the non-compact manifold.

X. Conclusion

In this paper we have studied the Wess Zumino model in an Anti de Sitter background and found that the naive generators of the spacetime isometries must be improved by a surface term. The improvement term was found by constructing the supercurrent and taking its variation under a supersymmetry transformation, thus effectively calculating the anticommutators in (1.1). We have studied the improved generators M_{AB} in detail and found that they have the expected properties: they are conserved, give a positive definite energy density, and have vanishing vacuum expectation value in a supersymmetric vacuum. In fact the improvement term was found to give a contribution which exactly cancelled the trace anomaly in the 1-loop calculation of $\langle M_{AB} \rangle$.

In [12] it was found that the energy functional for a massive scalar field in $(AdS)_4$ could be made positive and conserved, if the stress tensor was improved by a conformal term whose coefficient had to be a specifically chosen function of the mass. The argument could be seen to hold only for the free field case. Here, we have arrived at the improvement term in a more natural way, and the energy functional is positive and conserved for arbitrary potentials $W(z)$.

An arbitrary superpotential $W(z)$ may in general have several stationary points. Furthermore, some of these may correspond to supersymmetric vacua, and some to supersymmetry breaking vacua. The latter can still be stable in $(AdS)_4$ [12]. We have not considered the form of the AdS generators for non-supersymmetric vacua.

Finally, there is the interesting question of the relation of the fixed background models considered here to $N = 1$ supergravity models coupled to a scalar matter multiplet. It could be useful to know how the surface terms we have found arise in supergravity.

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XI. References.

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Appendix A.

In this Appendix we discuss the application of the boundary conditions at spatial infinity to determine the convergence and conservation properties of the charges Q_α and Q_α^c of (4.7-4.8) and M_{AB} and M_{AB}^c of (4.13-4.14). The relevant asymptotic behavior of the field configurations in $OSp(1, 4)$ supersymmetry was established in [12] only for the free massive theory (7.8). However, there is a simple heuristic argument that in a field theory with superpotential $W(z)$, the asymptotic behavior is determined by the mass parameter μ which occurs as $W(z) = \frac{1}{2}\mu z^2 + \dots$. To show this one writes a formal solution of the classical field equations of the theory in Yang-Feldman form [22]. The asymptotic behavior is then determined by the Green's function and thus by the behavior of the free massive mode functions. We now use the known boundary behavior of the scalar and spinor modes to study the following questions:

- 1) Properties of M_{AB} and M_{AB}^c of (4.13-4.14);
- 2) Proof that the axial vector terms in (4.11-4.12) vanish at spatial infinity;
- 3) Properties of the supercharges Q_α and Q_α^c .

1. Properties of M_{AB} and M_{AB}^c .

To analyze the boson contribution to the improved $SO(3, 2)$ generators, we use the fact [12], that the regular (irregular) mode functions have asymptotic behavior

$$\phi(t, \rho, \theta, \phi) \xrightarrow{\rho \rightarrow \frac{r}{2}} (\cos \rho)^{\lambda \pm} [1 + O((\cos \rho)^2)] \quad (A.1)$$

where $\lambda_{A\pm}$ and $\lambda_{B\pm}$ were given below (7.8). For $|\mu| > \frac{1}{2}a$, when regular modes are required for both $A(x)$ and $B(x)$, both λ parameters satisfy $\lambda > \frac{3}{2}$. For $|\mu| < \frac{1}{2}a$, a regular-irregular combination is required and we choose the λ parameters of (7.9).

For $W = \frac{1}{2}\mu z^2$ the generator (4.13) takes the form

$$M_{AB} = \int d\rho d\theta d\phi \sqrt{-g} \{ K_{AB\nu} T^{0\nu} - (2a)^{-1} D_\nu [(D^\nu K_{AB}^0)((a + \mu)A^2 + (a - \mu)B^2)] \} \quad (A.2)$$

If one takes the time derivative and uses the local conservation property of the integrand and Stokes' theorem, one obtains

$$\frac{d}{dt} M_{AB} = \lim_{\rho \rightarrow \frac{r}{2}} \int d\theta d\phi \sqrt{-g} \{ K_{AB\nu} T^{\rho\nu} - (2a)^{-1} D_\nu [(D^\nu K_{AB}^\rho)((a + \mu)A^2 + (a - \mu)B^2)] \} \quad (A.3)$$

where the superscript ρ denotes the radial coordinate. For regular modes it is already known [12] that the Noether part of M_{AB} is convergent and conserved, and it is easy to verify that if $\lambda > \frac{3}{2}$ the contribution of the extra surface term in (A2) and (A3) vanishes.

The situation is more delicate when irregular modes are required. It is known that the Noether part gives divergent energy and must be improved, and we now show that the extra surface term of (4.13) gives a convergent and conserved energy M_{04} . Convergence of the energy integral is readily seen from (5.2) with $F = -az - \mu\bar{z}$. One should note that a term which threatens to make the integral diverge exactly cancels in the quantity $\partial_\rho z - a^{-1} \tan \rho F$. The energy flux in (A3) can be expressed, using (2.9) and (3.5) and the Christoffel symbol Γ_{tt}^ρ , as (with $a = 1$)

$$Flux \sim (\cos \rho)^{-2} \{ \partial_t A (\partial_\rho + \tan \rho(1 + \mu))A + \partial_t B (\partial_\rho + \tan \rho(1 - \mu))B \}$$

After insertion of $A \sim (\cos \rho)^{1+\mu}$ one sees that the flux vanishes as $\rho \rightarrow \frac{\pi}{2}$ due to a neat cancellation between the Noether and improvement term contributions. Clearly the same situation obtains if one takes regular modes for A and irregular modes for B .

Let us now turn to the conformally improved charges M_{AB}^c of (4.31). Here it is already known [12] that for the massless case, $\mu = 0$, they are convergent and conserved. By similar arguments to those above, one could generalize this result to the case $\mu \neq 0$. However, one can kill two birds with one stone by examining the net difference between the surface terms of M_{AB}^c and M_{AB} , namely,

$$\begin{aligned} & K_\nu \frac{1}{3} (g^{\mu\nu} \square - D^\mu D^\nu + R^{\mu\nu}) \bar{z}z - a^{-1} D_\nu [(D^\nu K^\mu)(W + \bar{W} - \frac{1}{3} zW' - \frac{1}{3} \bar{z}\bar{W}')] \\ & + a^{-1} K_\nu [(D^\nu K^\mu)(W + \bar{W} + a\bar{z}z)] = \\ & = \frac{1}{3} D_\nu [(K^\mu D^\nu - K^\nu D^\mu) \bar{z}z - (D^\nu K^\mu) \bar{z}z] + a^{-1} D_\nu [(D^\nu K^\mu)(a\bar{z}z + \frac{1}{3}(zW' + \bar{z}\bar{W}'))] \end{aligned} \quad (A.4)$$

where K^μ is any Killing vector and where standard manipulations (see (4.2)) were used to express the conformal improvement as a surface term. Let us first assume that the scalar fields approach constant values at spatial infinity corresponding to a stationary point of the scalar potential $V(z, \bar{z})$ of (3.7). From the right side of (A4) one sees that the difference in the improvement terms approaches the constant value

$$\frac{1}{3} (\square K^\mu)(2a\bar{z}z + zW' + \bar{z}\bar{W}') = -\frac{1}{3} (\square K^\mu)(\bar{z}F + z\bar{F}) \quad (A.5)$$

which vanishes for any supersymmetric stationary point. If (A4) is integrated over a spacelike surface, then by Stokes' theorem the integral is determined by the radial limit

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \frac{1}{3} \left\{ (K^0 D^\rho - K^\rho D^0) \bar{z}z + a^{-1} (D^\rho K^0)(2a\bar{z}z + zW' + \bar{z}\bar{W}') \right\} \quad (A.6)$$

For the energy Killing vector K_{04} this is proportional to

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \sqrt{-g} (\cos \rho)^2 [\partial_\rho (\bar{z}z) - \tan \rho (\bar{z}F + z\bar{F})] \quad (A.7)$$

If one inserts scalar mode functions in this expression and takes $W = \frac{1}{2}\mu z^2$ then one finds that the limit vanishes trivially for regular modes and by cancellation for irregular modes. This establishes that the integrated charges M_{AB} and M_{AB}^c actually coincide in supersymmetric vacua.

2. Axial terms in the variation of the supercurrent.

We next show that the axial terms in (4.11) and (4.12) give vanishing contribution to the generators M_{AB} of (4.13) and (4.14). We perform the calculation for massless, free fields. The scalar and spinor mode functions are studied in detail in ref. [12]. Since our calculations require the explicit form of these functions, we reproduce them here. The modes form bases for representations $D(\omega_0, s)$ of $SO(3, 2)$. For massless scalar fields we have two independent complete sets,

$$D(1, 0) : \omega = 2k + \ell + 1 \quad k = 0, 1, 2, \dots \quad \ell = 0, 1, 2, \dots$$

$$\Phi_{\omega \ell m} = C_{k\ell} e^{-i\omega t} Y_{\ell m}(\theta, \phi) (\sin \rho)^\ell \cos \rho P_k^{(\rho+\frac{1}{2}, -\frac{1}{2})}(\cos 2\rho) \quad (A.8)$$

and

$$D(2, 0) : \omega = 2k + \ell + 2, \quad k = 0, 1, 2, \dots, \quad \ell = 0, 1, 2, \dots$$

$$\Phi_{\omega \ell m} = D_{k\ell} e^{-i\omega t} Y_{\ell m}(\theta, \phi) (\sin \rho)^\ell (\cos \rho)^2 P_k^{(\ell+\frac{1}{2}, \frac{1}{2})}(\cos 2\rho) \quad (A.9)$$

where the $P_k^{(m,n)}$ are Jacobi polynomials and $C_{k\ell}$, $D_{k\ell}$ are normalisation constants [12]. For massless spinors we have the basis sets

$$\chi_{\omega j m}^{(+)} = e^{-i\omega t} (\sin \rho)^{j-\frac{1}{2}} (\cos \rho)^{\frac{1}{2}} \begin{pmatrix} f_{\omega j}(\rho) y_{jm}^{(+)} \\ i g_{\omega j}(\rho) y_{jm}^{(-)} \end{pmatrix}$$

$$\chi_{\omega j m}^{(-)} = i\gamma^5 \chi_{\omega j m}^{(+)} = e^{-i\omega t} (\sin \rho)^{j-\frac{1}{2}} (\cos \rho)^{\frac{1}{2}} \begin{pmatrix} -g_{\omega j}(\rho) y_{jm}^{(-)} \\ i f_{\omega j}(\rho) y_{jm}^{(+)} \end{pmatrix} \quad (A.10)$$

with

$$\omega = n + j + 1, \quad n = 0, 1, 2, \dots$$

$$f_{\omega j}(\rho) = \left(\frac{i}{2}\right)^{j-\frac{1}{2}} N_{\omega j} \cos(\rho/2) P_n^{(j, j+1)}(\cos \rho)$$

$$g_{\omega j}(\rho) = \left(\frac{i}{2}\right)^{j-\frac{1}{2}} N_{\omega j} \sin(\rho/2) P_n^{(j+1, j)}(\cos \rho) \quad (A.11)$$

where $N_{\omega j}$ is a normalisation constant and the $y_{jm}^{(\pm)}$ are spinor spherical harmonics. These modes are combined to give two copies of the irreducible representation $D(3/2, 1/2)$ of $SO(3, 2)$, labelled by parity:

$$D(3/2, 1/2)^+ : \omega = n + j + 1 : n = 0, 1, 2, \dots : j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$$

$$\begin{aligned}
\chi^+ &= i\chi_{\omega jm}^{(+)} \quad \text{for even } n \\
&= \chi_{\omega jm}^{(-)} \quad \text{for odd } n
\end{aligned} \tag{A.12}$$

and

$$\begin{aligned}
D(3/2, 1/2)^- : \omega = n + j + 1, \quad n = 0, 1, 2, \dots, \quad j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \\
\chi^- &= i\chi_{\omega jm}^{(-)} \quad \text{for even } n \\
&= -\chi_{\omega jm}^{(+)} \quad \text{for odd } n
\end{aligned} \tag{A.13}$$

The bose and fermi mode functions of (A8)-(A13) must be assigned so that the theory describes particles in an infinite dimensional irreducible representation of $OSp(1, 4)$ [12,23]. With z written as $(\frac{1}{\sqrt{2}})(A+iB)$, where A is a scalar and B a pseudoscalar, one has two choices as to the sets of modes one uses [12]: $D(3/2, 1/2)^+$ for the spinor, and $D(1, 0)$ for A , $D(2, 0)$ for B , or $D(3/2, 1/2)^-$ for the spinor, $D(2, 0)$ for A , and $D(1, 0)$ for B . The modes so grouped transform into each other under supersymmetry transformations. The calculations below employ the first choice for the mode functions: one may check that the second choice gives the same results.

Consider now, for example, the term

$$\frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\lambda\sigma} D_\nu \{ K_\sigma \bar{z} \vec{\partial}_\lambda z \} \tag{A.14}$$

appearing in (4.11) and (4.12). (We drop overall numerical constants.) After expanding in the appropriate mode function and integrating using Stokes' theorem, in the coordinates defined in Section II, one finds the surface contribution

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \int d\theta d\phi \epsilon^{t\alpha\beta\rho} (K_\alpha \bar{z} \vec{\partial}_\beta z) \tag{A.15}$$

where t and ρ label coordinates. Since the terms in the integrand have asymptotic behavior $\bar{z} \vec{\partial}_\beta z \sim \cos^3 \rho$, $K_\alpha \sim \cos^{-2} \rho$, we see that this term vanishes. This fact can also be established quite easily for massive bosons.

This example serves to illustrate two important features of the behavior of these surface terms: first, the grouping of the modes required by supersymmetric boundary conditions is essential to their vanishing (had we taken both A and $B \sim D(1, 0)$, the integrand in (A15) would not have vanished), and second, the integrand actually vanishes pointwise on the boundary surface at infinity. It is not necessary to do the angular integration.

The fermionic terms in (4.11) and (4.12) require for their evaluation a rather more detailed examination. As an example we take

$$\frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\lambda\sigma} D_\nu (K_\sigma \bar{\psi} \gamma_5 \gamma_\lambda \psi) \tag{A.16}$$

After integration this becomes

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \int d\theta d\phi \epsilon^{\rho\theta\alpha\beta} K_\alpha \bar{\psi} \gamma_\beta \gamma^5 \psi \quad (\text{A.17})$$

Naively, $K_\alpha \sim \cos^{-2} \rho$, $\bar{\psi} \psi \sim \cos^3 \rho$ and $\gamma_\beta \sim \cos^{-1} \rho$, and we would expect the integrand in (A.17) to remain finite. However, due to cancellations the asymptotic falloff of the spinor bilinears $\bar{\psi} \gamma_\theta \gamma^5 \psi$ and $\bar{\psi} \gamma_\phi \gamma^5 \psi$ is actually faster than expected. Study of the asymptotic behavior of the spinor fields is greatly facilitated by noting that the spinor spherical harmonics obey

$$y^{(\pm)} = \vec{\sigma} \cdot \hat{x} y^{(\mp)} \quad (\text{A.18})$$

and that, at $\rho = \pi/2$, $f_{\omega j}(\rho) = (-1)^n g_{\omega j}(\rho)$, since the Jacobi polynomials satisfy [18] $P_n^{(a,b)}(x) = (-1)^n P_n^{(b,a)}(-x)$. Using these facts we find that a typical term in the expansion of $\bar{\psi} \gamma^i \gamma^5 \psi$ (where the γ^i 's are flat) is of the form

$$\hat{x}^i f_{\omega j}^* f_{\omega' j'} y_{jm}^{(+)\dagger} \vec{\sigma} \cdot \hat{x} y_{j'm'}^{(+)} \quad (\text{A.19})$$

when n and n' are both even or both odd (see (A.11)). In our choice of coordinate system, γ_θ and γ_ϕ have the explicit form

$$\gamma_\theta = \frac{\tan \rho}{a} (\gamma_1 \cos \theta \cos \phi + \gamma_2 \cos \theta \sin \phi - \gamma_3 \sin \theta) \equiv c_\theta^i \gamma^i \quad (\text{A.20})$$

$$\gamma_\phi = \frac{\tan \rho}{a} (-\gamma_1 \sin \theta \sin \phi + \gamma_2 \sin \theta \cos \phi) \equiv c_\phi^i \gamma^i \quad (\text{A.21})$$

The vanishing of (A.17) is established by noting that

$$c_\theta^i \hat{x}^i = c_\phi^i \hat{x}^i = 0 \quad (\text{A.22})$$

A similar argument holds when one of the pair (n, n') is even and one is odd.

The remaining fermionic terms involve the conformal Killing vector $\hat{K}^\mu$. Detailed knowledge of the fermionic modes is again essential in checking that the surface terms vanish. The calculations are again facilitated by noting that, asymptotically, the $D(3/2, 1/2)^+$ mode functions take the form

$$\psi = (\cos \rho)^{3/2} (1 + i \vec{\gamma} \cdot \hat{x}) \eta \quad (\text{A.23})$$

where η is a function of ρ , θ and ϕ , which depends on n , j and m , and does not diverge as $\rho \rightarrow \frac{\pi}{2}$. Finally, the projection operators in the Killing spinor matrices $S(x)$, $\bar{S}(x)$ play an essential role in giving extra factors of $\cos \rho$ when required.

3. Supercurrents.

We now check that the supercurrents (3.6) and (4.6) give conserved charges (see (4.7) and (4.8)). In fact, we will show that J^μ gives conserved charges, for the mode functions used above, and that for these functions (i.e., for the massless, free case) Q_α and Q_α^c actually coincide. (One may also show as a further check that Q_α^c is conserved.)

One may easily check that $D_\mu(\bar{S}J^\mu)$ vanishes. It remains to be shown that the supercharge flux at spatial infinity also vanishes. It has the form

$$\lim_{\rho \rightarrow \pi/2} \int d\theta d\phi \sqrt{-g} \bar{S} J^\rho \quad (A.24)$$

The integrand takes the form

$$\begin{aligned} & (\cos \rho)^{-\frac{3}{2}} (\cos(\rho/2) + i \vec{\gamma} \cdot \hat{x} \sin(\rho/2)) ((\not{\partial} + ia)A - i\gamma^5(\not{\partial} + ia)B) \times \\ & \times \cos \rho \vec{\gamma} \cdot \hat{x} (\cos \rho)^{3/2} (1 + i \vec{\gamma} \cdot \hat{x}) \eta \end{aligned} \quad (A.25)$$

with notation explained above. One expands $\not{\partial}$ in terms of the flat γ^a , and notes that, asymptotically, A falls as $\cos \rho$, B as $\cos^2 \rho$. Using properties of the projection operator, one finds that (A.25) vanishes as $\rho \rightarrow \pi/2$. Note that one needs the details of the scalar modes as well as the spinors, in order to evaluate the derivatives, and that the scalar and spinor modes must be grouped as $(A, B, \psi) \sim (D(1, 0), D(2, 0), D(\frac{3}{2}, \frac{1}{2})^+)$ or $(D(2, 0), D(1, 0), D(\frac{3}{2}, \frac{1}{2})^-)$ in order to obtain a vanishing result (see ref. [12]).

Finally, we outline the proof that Q_α and Q_α^c are the same. The difference in the supercurrents is (see (4.6))

$$\begin{aligned} \Delta J^\mu &= \frac{4}{3} \sigma^{\mu\nu} (D_\nu - \frac{ia}{2} \gamma_\nu) [(\bar{z}L + zR)] \psi \\ &= \frac{4}{3} (D_\nu + \frac{ia}{2} \gamma_\nu) \sigma^{\mu\nu} [(\bar{z}L + zR)] \psi \end{aligned} \quad (A.26)$$

Then using the Killing spinor property of S , and dropping overall factors, one finds

$$\begin{aligned} \Delta Q &\equiv \int d\Sigma_\mu \bar{S} \Delta J^\mu \\ &= \int d\Sigma_\mu D_\nu [\bar{S} \sigma^{\mu\nu} (\bar{z}L + zR) \psi] \\ &= \int d^3x \sqrt{-g} D_\nu [\bar{S} \sigma^{t\nu} (\bar{z}L + zR) \psi] \\ &= \lim_{\rho \rightarrow \frac{\pi}{2}} \int d\theta d\phi \sqrt{-g} \bar{S} \sigma^{t\rho} (\bar{z}L + zR) \psi \end{aligned} \quad (A.27)$$

where again t and ρ refer to coordinates. Once again, using the tricks described above, it is straightforward to show that the integrand vanishes in the free massless case.

Appendix B. Dimensional Regularization.

We considered several methods of regularization to test the supersymmetry Ward identities of Sec. VI. The Pauli-Villars method described in Sec. VIII proved to be a simple and reliable method, since it clearly maintained $OSp(1, 4)$ supersymmetry. It is also natural to consider a point-splitting procedure to regularize the vacuum expectation values of bilinear operators. This works for the Ward identities required for the zero vacuum energy proof in $(AdS)_2$, but it fails in $(AdS)_4$ because the propagators are too singular. For example, it is clear that (8.6) is not satisfied by the A and B propagators, even in the short distance limit. Another method of regularization is to differentiate singular expressions with respect to dimensionful parameters, such as the mass parameter μ . Indeed, it is straightforward to split points temporarily in (8.6), differentiate twice with respect to μ using the short distance expansion (7.7) and observe that the result does vanish. Then it appears that $\langle A^2 \rangle_{ren}$ and $\langle B^2 \rangle_{ren}$ can be defined as functions of μ so that (8.6) is satisfied.

The last regularization method studied was a version of dimensional regularization for $OSp(1, 4)$ supersymmetry. We wish to discuss this method briefly because it does give a very simple way to define the vacuum expectation values of bilinear operators in one-loop order such that the supersymmetry Ward identities are satisfied. We emphasize that we are talking about dimensional regularization, not dimensional reduction (we will use $\gamma^\mu \gamma_\mu = nI$). It is therefore probable that the method will break down beyond one loop order. The treatment we give will be rather cavalier for this reason. It is an open question whether one can develop a method of dimensional reduction applicable to AdS supersymmetry.

Let us start with the Lagrangian density

$$\begin{aligned} \mathcal{L} = & \partial_\mu \bar{z} \partial^\mu z + \frac{i}{2} \bar{\psi} \not{\partial} \psi + \bar{F} F + \frac{1}{2} (n-2) a (\bar{z} F + \bar{F} z) + \frac{1}{2} (n-1)(n-2) a^2 \bar{z} z \\ & + F W' + \bar{F} \bar{W}' - \frac{1}{2} \bar{\psi} (L W'' + R \bar{W}'') \psi + (n-1) a (W + \bar{W}) \end{aligned} \quad (B.1)$$

and the transformation rules

$$\begin{aligned} \delta z &= \bar{\epsilon} L \psi \\ \delta L \psi &= L(-i \not{\partial} z + F) \epsilon \\ \delta F &= -i \bar{\epsilon} \not{\partial} L \psi - \frac{1}{2} (n-2) a \bar{\epsilon} L \psi \end{aligned} \quad (B.2)$$

They were chosen to “interpolate” simply between the Lagrangian and field variations in the physical dimensions 2 and 4. One can easily show formally that the kinetic and interaction parts of (B.1) are invariant (up to a total divergence) under the transformation rules (B.2) except for one ambiguous term. This term is the trilinear spinor term which is normally shown to vanish by Fierz rearrangement. Here we simply impose $W'''(z) = 0$ so that the trilinear spinor term vanishes. This will be sufficient to regularize the zero separation two point functions in one loop order which we need to test the Ward identities. In the formal proof of invariance one also assumes that γ_5 anticommutes with all γ_μ and that $\gamma_\mu \gamma^\mu = nI$.

The solution of the auxiliary field equation for (B.1) is

$$F = -\frac{1}{2} (n-2) a z - \bar{W}'(\bar{z}) \quad (B.3)$$

which may be reinserted in (B.1) to give

$$\begin{aligned} \mathcal{L} = & \partial_\mu \bar{z} \partial^\mu z + \frac{1}{2} i \bar{\psi} \not{D} \psi + \frac{(n-2)n}{4} a^2 \bar{z} z - \frac{1}{2} (n-2) a (z W' + \bar{z} \bar{W}') \\ & - \bar{W}' W' + (n-1) a (W + \bar{W}) - \frac{1}{2} \bar{\psi} (L W'' + R \bar{W}'') \psi \end{aligned} \quad (B.4)$$

From now on we take $W = \frac{1}{2} \mu z^2$ which is the case of interest for 1-loop Ward identities.

One can now easily generalize the discussion of the boson propagators of Sec. VII and deduce that the correct propagators are obtained from (7.4) by taking (with $a = 1$ for convenience)

$$\begin{aligned} \lambda_A &= \frac{1}{2} n + \mu - 1 \\ \lambda_B &= \frac{1}{2} n + \mu \end{aligned} \quad (B.5)$$

so that

$$\begin{aligned} \langle T A(x) A(x') \rangle &= \frac{1}{(4\pi)^{n/2}} \frac{\Gamma(\mu) \Gamma(\frac{1}{2}n - 1 + \mu)}{\Gamma(2\mu)} (-2u^{-1})^{\frac{1}{2}n - 1 + \mu} F\left(\frac{1}{2}n - 1 + \mu, \mu; 2\mu; 2u^{-1}\right) \\ \langle T(B(x) B(x')) \rangle &= \\ &= \frac{1}{(4\pi)^{n/2}} \frac{\Gamma(1 + \mu) \Gamma(\frac{1}{2}n + \mu)}{\Gamma(2 + 2\mu)} (-2u^{-1})^{\frac{1}{2}n + \mu} F\left(\frac{1}{2}n + \mu, 1 + \mu; 2 + 2\mu, 2u^{-1}\right) \end{aligned} \quad (B.6)$$

We need expressions for these two point functions at zero separation. To do this we continue to the region $n < 2$ where there is no short distance singularity, and we then obtain the regularized values

$$\begin{aligned} \langle A^2 \rangle_{reg} &= \frac{1}{(4\pi)^{\frac{1}{2}n}} \Gamma\left(1 - \frac{1}{2}n\right) \frac{\Gamma(\frac{1}{2}n - 1 + \mu)}{\Gamma(1 - \frac{1}{2}n + \mu)} \\ \langle B^2 \rangle_{reg} &= \frac{1}{(4\pi)^{\frac{1}{2}n}} \Gamma\left(1 - \frac{1}{2}n\right) \frac{\Gamma(\frac{1}{2}n + \mu)}{\Gamma(2 - \frac{1}{2}n + \mu)} \\ &= \frac{(\mu + \frac{1}{2}(n-2))}{(\mu - \frac{1}{2}(n-2))} \langle 0 | A^2 | 0 \rangle_{reg} \end{aligned} \quad (B.7)$$

The n -dimensional analogue of the Ward identities (7.10) satisfied by the spinor propagator is

$$\begin{aligned} \langle T \psi(x) \bar{\psi}(x') \rangle &= (i \not{\partial} + \mu + \frac{1}{2}(n-2)) \langle T A(x) A(x') \rangle S(x) \bar{S}(x') \\ &= (i \not{\partial} + \mu - \frac{1}{2}(n-2)) \langle B(x) B(x') \rangle \gamma_5 S(x) \bar{S}(x') \gamma_5 \end{aligned} \quad (B.8)$$

At zero separation one obtains, after tracing and taking $\text{tr} I = 4$

$$\begin{aligned} -\frac{1}{4} \langle \bar{\psi} \psi \rangle_{reg} &= (\mu + \frac{1}{2}(n-2)a) \langle A^2 \rangle_{reg} \\ &= (\mu - \frac{1}{2}(n-2)a) \langle B^2 \rangle_{reg} \end{aligned} \quad (B.9)$$

It is gratifying that the relation between $\langle A^2 \rangle_{reg}$ and $\langle B^2 \rangle_{reg}$ required by (B.9) is actually satisfied by (B.7).

We are now ready to apply these results to the calculation of the densities of $\langle M_{AB}^c \rangle$ in one-loop order. However, we must first define a dimensional continuation of the improved generator M_{AB}^c of (4.14) and we take

$$M_{AB}^c = \int d\Sigma_\mu \left\{ K_{AB\nu} \Theta^{\mu\nu} - a^{-1} D_\nu [(D^\nu K_{AB}^\mu)(W + \bar{W} - \frac{n-2}{2(n-1)}(zW' + \bar{z}\bar{W}'))] \right\} \quad (B.10)$$

$$\begin{aligned} \Theta^{\mu\nu} = & \partial^\mu \bar{z} \partial^\nu z + \partial^\nu \bar{z} \partial^\mu z - g^{\mu\nu} (\partial_\alpha \bar{z} \partial^\alpha z - \bar{F} F + (n-1)(a\bar{z}z + W + \bar{W})) \\ & + \frac{i}{4} \bar{\psi} (\gamma^\nu D^\mu + \gamma^\mu D^\nu) \psi + \frac{(n-2)}{2(n-1)} (g^{\mu\nu} \square - D^\mu D^\nu + R^{\mu\nu}) \bar{z} z \end{aligned}$$

This expression is a reasonable interpolation between the improved *AdS* generators for the physical dimensions 2 and 4, and the coefficient of zW' in the surface term was chosen so that, as required, the surface term vanishes for the conformal invariant interaction $W = \lambda z^q$ where $q = 2(n-1)/(n-2)$. The vacuum expectation value of M_{AB}^c is

$$\langle M_{AB}^c \rangle = \int d\Sigma_\mu K_{AB}^\mu \left\{ \frac{1}{n} \langle \Theta_\nu^\nu \rangle + (n-1)a \langle (W + \bar{W} - \frac{1}{2} \frac{(n-2)}{(n-1)}(zW' + \bar{z}\bar{W}')) \rangle \right\} \quad (B.11)$$

since $\square K^\mu = -(n-1)a^2 K^\mu$. In the theory with $W = \frac{1}{2}\mu z^2$, the trace and the surface term contributions are

$$\langle \Theta_\nu^\nu \rangle = (\mu^2 - a\mu) \langle A^2 \rangle + (\mu^2 + a\mu) \langle B^2 \rangle + \frac{1}{2}\mu \langle \bar{\psi}\psi \rangle \quad (B.12)$$

$$\begin{aligned} \langle (W + \bar{W} - \frac{1}{2} \frac{(n-2)}{(n-1)}(zW' + \bar{z}\bar{W}')) \rangle &= \frac{1}{n-1} \mu \langle z^2 + \bar{z}^2 \rangle \\ &= \frac{1}{n-1} \mu \langle A^2 - B^2 \rangle \end{aligned} \quad (B.13)$$

If one inserts the regularized values of $\langle A^2 \rangle$, $\langle B^2 \rangle$ and $\langle \bar{\psi}\psi \rangle$ from (B.7) and (B.9) one finds after a short calculation that the integrand of (B.11) identically vanishes for all values of n . This shows that the dimensionally regularized vacuum energy density vanishes!

We may also apply dimensional regularization to the Ward identity (6.13), namely

$$\begin{aligned} \langle \bar{z} F \rangle &= - \langle \frac{1}{2}(n-2)a\bar{z}z + \mu\bar{z}^2 \rangle \\ &= -\frac{1}{2} \langle (\frac{1}{2}(n-2)a + \mu)A^2 + (\frac{1}{2}(n-2)a - \mu)B^2 \rangle \\ &= 0 \end{aligned} \quad (B.14)$$

One easily finds using (B.7) or (B.9) that this identity is satisfied, identically in n , by the regularized two-point functions. In particular, one finds that for $\mu = 0$, we have $\langle A^2 \rangle_{reg} = - \langle B^2 \rangle_{reg}$. This should not be surprising; there are other examples where dimensional regularization violates expected positivity properties [24].

We have also applied dimensional regularization to other Ward identities in the theory. They are all satisfied.

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Supersymmetry in Anti de Sitter Space

C.J.C. Burges, D.Z. Freedman, S. Davis and
G.W. Gibbons

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The statement made in reference 3, that the conformal anomaly cancels between the scalar and the spinor members of the scalar multiplet in two dimensions, is not correct. Similar arguments to those used in this paper show that the resultant non-zero anomaly is cancelled by a corresponding anomaly in the surface term.

Accordingly, in this paper the following modifications should be made: replace the last sentence in section 1 on page 3 with

"Further, the relation between the vacuum energy density and the trace anomaly is explored in detail here. We find that the new surface terms in the generators M_{AB} are themselves anomalous, and serve to cancel the conventional trace anomaly."

and the last two sentences in the first paragraph of section 9 on page 17 with

"In both two and four dimensions, the net trace anomaly of the fundamental fields does not vanish, and one might expect an anomalous contribution to the integrands of the spacetime generators M_{AB} . In this section we provide a resolution of this anomaly puzzle."