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STRINGS, TACHYONS AND DECONFINEMENT

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A B S T R A C T

We argue that the tachyons which exist in some string models are a signal of deconfinement. We compare the deconfinement temperature obtained from Monte Carlo calculations with some string models.

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It is well known that some string models contain tachyons. This is usually considered to be a very unhealthy feature of the models. Here we shall argue that the presence of tachyons is a signal of "deconfinement". Thus, models without tachyons (the Ramond model and superstrings) are not expected to have deconfinement.

As our starting point we mention that in a fundamental paper, Pisarski and Alvarez¹⁾ have studied the finite temperature properties of a general class of scalar string models. Their results can be summarized by the following formula for the temperature-dependent effective string tension

$$\sigma(T) \approx \sigma \sqrt{1 - (T/T_c)^2} \quad (1)$$

where T_c is the dual model Hagedorn temperature,

$$T_c^2 / \sigma \approx \frac{3}{\pi(d-2)} \quad (2)$$

Here d is the dimension of space and time. Equations (1) and (2) are at least valid¹⁾ for $d \rightarrow \infty$, but their range of validity is expected to be much more general (at least for the Nambu-Goto action), as we shall explain below.

From Eq. (1), we see that for $T \rightarrow T_c$ the effective string tension vanishes. This is certainly a strong indication for deconfinement. Of course, string models are based on strings as the fundamental dynamical objects, and hence cannot describe a quark-gluon plasma. Therefore, for $T > T_c$, Eq. (1) is meaningless. However, below we shall argue that if the string is given a finite transverse width, the new phase is indeed a deconfined state.

As pointed out in Ref. 1), the calculation of Eq. (1) is the same as the calculation of the static quark-antiquark potential $V(R)$ in the scalar dual models. The only difference is that in the "time"-direction involved in the calculation of Eq. (1), one has to impose periodic boundary conditions. The result for $V(R)$ was first obtained by Alvarez²⁾ (for time $\gg R$),

$$V(R) \approx \sigma(R)R, \quad (3)$$

$$\sigma(R) = \sigma \sqrt{1 - (R_c/R)^2}, \quad R_c^2 = \frac{\pi(d-2)}{12\sigma}. \quad (4)$$

Thus, the effective string tension behaves like in (1), and $\sigma(R)$ vanishes for

$R = R_c$. For $R < R_c$, Eq. (4) again becomes meaningless. From Eqs. (2) and (4), we see that T_c^2/σ and $1/\sigma R_c^2$ differ by a factor 4. This is due to the different boundary conditions.

Equation (3) was derived in the limit $d \rightarrow \infty$. However, Arvis has shown³⁾ that at least for the Nambu-Goto action, Eq. (3) is valid for all d . Recently we pointed out⁴⁾ that Arvis result³⁾ can be understood intuitively in a very simple way: in ordinary string models with free ends of the string (the normal derivative of x_μ vanishes at the end points) the equations of motion are solved by:

$$X^i(\sigma, \tau) = x^i + 2\pi\alpha' p^i \tau + i \sum_{n=1}^{\infty} \frac{a_n^i}{n} \cos n\sigma e^{-in\tau}, \quad (5)$$

with a similar formula for $x^0(\sigma, \tau)$. Equation (5) together with the subsidiary conditions lead to the mass formula

$$\alpha' p_0^2 = \alpha' \underline{p}^2 + \mu^2, \quad (6)$$

where (for the lowest mass state) μ^2 is the contribution from the zero point fluctuations

$$\mu^2 = \frac{d-2}{2} \sum_{n=1}^{\infty} n = - \frac{d-2}{24}. \quad (7)$$

Thus μ^2 corresponds to a tachyon. For a string with end fixed at $x_1 = 0$ and $x_1 = R_1$, $R^2 = R_1^2$, Eq. (5) is replaced by³⁾

$$X^i(\sigma, \tau) = \frac{R_1^i}{\pi} \sigma + \sum_{n=1}^{\infty} \frac{a_n^i}{n} \sin n\sigma e^{-in\tau},$$

$$X^0(\sigma, \tau) = x^0 + 2\pi\alpha' p^0 \tau + i \sum_{n=1}^{\infty} \frac{a_n^0}{n} \cos n\sigma e^{-in\tau}. \quad (8)$$

Thus, relative to Eq. (5), the spatial momenta p^i have been replaced by R^i/π and τ by σ . Due to the large amount of symmetry between σ and τ , the energy formula (6) turns out in an analogous way to be³⁾

$$\alpha' V^2 = \alpha' \left(\frac{R^i}{2\pi\alpha'} \right)^2 + \mu^2, \quad V = \text{the energy}, \quad (9)$$

where μ^2 is the contribution from the zero-point energy given by Eq. (7). Thus, μ^2 comes from the tachyon. Taking the square root of Eq. (9), one recovers Alvarez's formula (3) with

$$R_c^2 = -2\pi\mu^2/\sigma, \quad (10)$$

which is the same as Eq. (4). Thus we see that R_c is directly proportional to the tachyon mass.

As already mentioned, the calculation of the finite temperature corrections is the same as the calculation of $V(R)$, except for periodic boundary conditions in the "time"-direction. Thus, a calculation à la Arvis can be carried out in the finite temperature case, and we see that the existence of a T_c^2 is related to the existence of a tachyon, with $1/T_c$ being proportional to the tachyon mass^{*}). Also, Eq. (1) should be valid for all d for the Nambu-Goto action.

The properties of the theory near T_c were discussed by Pisarski and Alvarez¹⁾. Let $x_\perp$ be the vector describing the transverse fluctuations. For $T \rightarrow T_c$ it turns out¹⁾ that the entropy per unit length diverges like $(T_c - T)^{-\frac{1}{2}}$, whereas $\langle x_\perp^2 \rangle$ and $\langle (\partial_{t\perp} x_\perp)^2 \rangle$ are both finite. The spatial variation $\langle (\partial_{r\perp} x_\perp)^2 \rangle$ diverges as $(T_c - T)^{-\frac{1}{2}}$. Thus, the picture which results is that near T_c the length of the string increases. This costs energy. However, in thermal equilibrium this is compensated by an increase in the entropy. Thus, the surfaces which dominate the functional integral have a finite transverse extent but have an infinite area¹⁾. Therefore, the string is folded in a chaotic (large entropy) manner within a finite transverse extension.

The fact that $\sigma(T) \rightarrow 0$ for $T \rightarrow T_c$ is of course a strong indication of a deconfining phase transition. Also, the fact that $\sigma(R) \rightarrow 0$ for $R \rightarrow R_c$ (which arises in a similar manner) is such an indication. If the string model describes the large distance properties of QCD, we would expect, because of asymptotic freedom, that the string picture should break down at some distance, i.e., that $\sigma(R)$ should approach zero.

To see more clearly the physical nature of the phase transition, let us give the string a finite transverse width. Now, as $T \rightarrow T_c$ the string was folded in a chaotic manner. This means that there are so many foldings that the string touches itself in many places. If the string is a flux tube of the vortex

^{*}) The argument in Ref. 4) that for large R the critical dimension becomes irrelevant also applies here, except that it now means that at all small temperatures the critical dimension is irrelevant.

type⁵⁾, one can say that inside the vortex the Higgs fields ϕ approximately vanish, whereas outside $\phi \neq 0$. Now, in the region of the folded vortex it follows that ϕ nearly always vanish (approximately)^{*)}. Thus, for $T \rightarrow T_c$ there is a region of space (between the quark and the antiquark) where we have an unbroken symmetry,

i.e., we have the perturbative ground state (approximately). It is well known that in Higgs models the question of whether $\phi = 0$ or $\phi \neq 0$ is governed by the $\mu^2 \phi^2$ -term, where μ^2 is tachyonic if $\phi \neq 0$, i.e., if $|\phi|^2 \neq 0$ we have the usual string picture (except for the core of the vortex). If $\phi = 0$ in the ground state, μ^2 becomes an ordinary mass^{**)}. Thus we see a close analogy with the deconfinement transition, since if $\phi \approx 0$ a magnetic field between a north and a south pole can have a behaviour almost like an ordinary dipole field in the region of the folded vortex.

For a QCD flux tube, the situation is not so different. Outside the flux tube one has the QCD ground state, inside one has approximately (at short distances) the perturbative ground state. Hence the "folded region" of space is almost like a big bag, where the quarks can move approximately as free particles.

We thus see that if one gives the string a width it is extremely plausible that the phase transition is deconfining, and that the tachyon is a signal of this phase transition.

Next we shall investigate what one expects in the case of other strings than the scalar ones. From the arguments presented in Eqs. (5)-(10) one expects to obtain again the Pisarski-Alvarez formula (1) but with a different T_c , since the tachyon differ from the scalar case. The only thing one has to be careful about is the periodic boundary conditions. In this way one obtains

$$\frac{\sigma}{T_c^2} = \begin{cases} \frac{\pi(d-2)}{2} & (\text{Neveu-Schwarz}) \\ 0 & (\text{Ramond}) \end{cases} \quad (11)$$

The last result is due to the fact that there is no tachyon in the Ramond model.

*) This also follows from the facts that $\sigma(T) \rightarrow 0$ for $T \rightarrow T_c$ and⁵⁾ $\sigma(T) \propto |\phi|^2$. Thus, the effective $|\phi|^2$ must vanish.

***) In the core of the vortex, the seagull term $-A_\mu^2 |\phi|^2$ wins over $-\mu^2 |\phi|^2$. Thus, the effective μ^2 is positive.

For the Neveu-Schwarz string, T_c agrees with the Hagedorn temperature for that model. This is by itself a highly non-trivial check of the arguments, since the Hagedorn temperature arises in a completely different manner. For the Neveu-Schwarz string, the situation is thus very similar to the scalar string, and for the reasons mentioned above we again expect the physical significance of the tachyon to be that it signals a deconfining phase transition.

For the Ramond string, the situation is entirely different. There is no temperature where $\sigma(T)$ vanishes, and Eq. (1) just implies $\sigma(T) = \sigma$. Of course, this result is only expected to be valid when loop corrections are ignored, but it is unlikely that the tachyonic structure of Eq. (1) should emerge from these corrections. Thus, we do not expect that there is a deconfining transition. Although one expects that thermal fluctuations will cause the Ramond string to oscillate, we do not expect from Eq. (11) that it will be folded in a chaotic way. In spite of this, one does of course expect something dramatic to happen at high temperatures, since the Ramond string has a Hagedorn temperature (the same as for the Neveu-Schwarz string).

For superstrings⁶⁾, the situation is expected to be similar to the Ramond string, since there is no tachyon in the superstring theory. Thus, we do not expect a deconfining transition. Since superstrings are expected to represent a fundamental physical theory⁶⁾, the lack of "deconfinement" is a pleasant feature, since "deconfinement" in the language of a string theory implies that the theory breaks down at some temperature^{*}). Furthermore, since from physical arguments in the case of "deconfinement" we expect that a new phase (a plasma of various particles) exists, it will always be natural to think that the string theory is not a fundamental theory in this case. Thus, even though QCD is well described by a string theory at large distances⁷⁾, we do not expect the string theory to be fundamental because it contains tachyons^{4),7)}.

It is of interest to compare the predictions (2) and (11) with the Monte Carlo data for QCD, as was already done in Ref. 1) as far as Eq. (2) is concerned. Using the results of Ref. 8), one has, from Monte Carlo data

^{*}) To avoid misunderstandings it should be emphasized that the lack of "deconfinement" refers to the superstring itself. The superstring theory of course contains many other (less fundamental) strings⁶⁾, e.g., the QCD string, and these can have deconfinement.

$$T_c \approx 0.5 \sqrt{\sigma} \text{ for } SU(2) \text{ and } SU(3),$$
$$T_c \approx 0.4 \sqrt{\sigma} \text{ for } SU(64) \text{ and } SU(81). \quad (12)$$

Thus, the Ramond model seems to be ruled out with a large certainty. The scalar result (2) becomes in four dimensions^{*)}

$$T_c^{\text{scalar}} \approx 0.69 \sqrt{\sigma} \quad (13)$$

On the other hand, for the Neveu-Schwarz model one has from Eq. (11) in four dimensions

$$T_c^{\text{Neveu-Schwarz}} \approx 0.56 \sqrt{\sigma}. \quad (14)$$

Thus, considering $SU(N)$ QCD it seems that the Monte Carlo data in four dimensions are closest to the Neveu-Schwarz string. This is, of course, a rather interesting result if it is confirmed on larger lattices.

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^{*)} In Ref. 1) the formula for T_c corresponding to Eq. (2) has d instead of $d-2$ (our method clearly gives $d-2$, but in the limit $d \rightarrow \infty$ there is of course no difference between d and $d-2$). Therefore, the numerical value (13) differs from the value given in Ref. 1).

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