CERN-TH.4193/85

TEN-DIMENSIONAL SUPERSYMMETRY REQUIRES $E_8 \times E_8$ OR $SO(32)$

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A B S T R A C T

The $d=10$ Einstein-Yang-Mills supergravity Lagrangian is presented in the form $e/k^2 L(\phi, e, \dots)$, L being parameter independent, $\langle \phi \rangle = k^2/g^2 = \alpha'$ for a closed heterotic string. The scaling property of the action is shown to follow from the flat space canonical dimension transformations. The absence of Lorentz and Yang-Mills gauge anomalies is shown to be necessary but not sufficient for the absence of consistent gauge supersymmetry anomalies.

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1. At present, unified theories of fundamental interactions based on superstring theories attract much attention. The growing interest in these theories was stimulated by the discovery of Green and Schwarz¹⁾ of the fact that type I (unoriented closed and open) superstrings do not contain Yang-Mills and Lorentz anomalies if the gauge group is $SO(32)$. It was found also in Ref. 1) that in $d=10$ Einstein-Yang-Mills supergravity (EYMS), the above-mentioned anomalies cancel if the gauge group is $SO(32)$ or $E_8 \times E_8$ and suitable local interactions are added to the theory. More recently, a new type of superstring theory (closed, oriented), the "heterotic" theory, has been constructed by Gross, Harvey, Martinec and Rohm²⁾. This theory describes strings with $E_8 \times E_8$ and $SO(32)$ gauge groups and also has as a low-energy limit $d=10$ EYMS. The mass scale of the string theories is set by the string tension T . For type I open strings coupled to unoriented closed strings, there exists a relation¹⁾ between the gravitational and gauge coupling constant and the string tension of the type $k/g^2 \sim T$, the corresponding relation²⁾ for the heterotic string is $g^2/k^2 \sim T$. The field theory limit of string theories is connected with the limit $\alpha' = 1/(2\pi T) \rightarrow 0$.

The purpose of the present paper is to investigate the $d=10$ EYMS theory and its relation to superstring theories. We will write down the Lagrangian^{3),4)} in variables in which the dependence on the dimensional coupling constants k and g is especially simple. A step in this direction was already taken by Witten⁵⁾ who emphasized that the gauge coupling, g^2 , can be absorbed into the redefinition of the supergravity scalar field ϕ . We will show that in fact the proper redefinition of ϕ is such that the vacuum expectation value (vev) $\langle \phi \rangle$ is equal to k^2/g^2 , i.e., just this quantity which is proportional to α' of the closed heterotic string can be determined dynamically.

The scale invariance property of $d=10$ EYMS discussed by Witten⁶⁾ in the bosonic sector, will be presented for the whole theory and will be shown to follow from the invariance of the theory under canonical dimension transformation.

The structure of self-consistent one-loop anomalies of the low-energy limit of superstring theories is determined by the gauge algebra of $d=10$ EYMS. The analysis of this algebra makes it possible to establish the relation between Yang-Mills, Lorentz and supersymmetry anomalies.

2. The Lagrangian of $d=10$ Einstein-Maxwell supergravity was found in Ref. 3), its non-Abelian generalization, i.e., $d=10$ EYMS was found in Ref. 4). In both

Lagrangians, the normalization of fields was chosen in such a way that the coupling constants k and g enter the Lagrangians and also the supersymmetry transformation rules in an irregular way. It appears that the theory⁴⁾ can be represented in a form where there is the most simple dependence on gauge constants, namely the dependence on gravitational constant enters only through the factor $1/k^2$ in front of the whole Lagrangian and there is no other manifest dependence on gauge constants either in the Lagrangian or in the transformation rules, as happens in any pure supergravitational theory⁷⁾.

The Lagrangian takes the form

$$\mathcal{L} = \frac{e}{k^2} L = \frac{e}{k^2} [L_0 + \varphi^{1/2} L_{1/2} + \varphi^1 L_1 + \varphi^2 L_2] \quad (1)$$

and the supersymmetry transformation rules are:

$$\delta \Phi^i = \varphi^{-1} X_{-1}^i + \varphi^{-1/2} X_{-1/2}^i + X_0^i + \varphi^{1/2} X_{1/2}^i + \varphi^1 X_1^i \quad (2)$$

where $L_0, \dots, L_2$ are the functionals of Φ^i (all d=10 EYMS fields) and $X_{-1}^i, \dots, X_1^i$ depend on Φ^i and on the supertransformation parameter $\varepsilon(x)$. In (1) and (2) we have written down explicitly the terms with a dependence on different powers of the scalar field ϕ , or rather on the vev of ϕ *)

$$\varphi = \langle \varphi \rangle e^{\Phi} \quad (3)$$

where

$$\langle \varphi \rangle = \frac{k^2}{g^2} \quad (4)$$

and the dimensionless field Φ has zero vev. Taking into account that in the heterotic string²⁾ $\alpha' = k^2/g^2 = R^2$, where R is the radius of the sixteen compact dimensions 26-10, the Lagrangian (1) and the transformation rules (2) may be represented as follows:

*) The terms containing $\partial_\mu \phi / \phi = \partial_\mu \Phi$ enter L_0 , since the dependence on $\langle \phi \rangle = k^2/g^2$ is cancelled in $\partial_\mu \phi / \phi$.

$$\mathcal{L} = \frac{e}{k^2} \left[L_0 + (\alpha')^{1/2} e^{\frac{\Phi}{2}} L_{1/2} + \alpha' e^{\Phi} L_1 + \alpha'^2 e^{2\Phi} L_2 \right] \quad (1')$$

$$\delta\Phi^i = \alpha'^{-1/2} e^{-\Phi} \chi_{-1}^i + \alpha'^{-1/4} e^{-\frac{\Phi}{2}} \chi_{-1/2}^i + \chi_0^i + \alpha'^{1/4} e^{\frac{\Phi}{2}} \chi_{1/2}^i + \alpha' e^{\Phi} \chi_1^i \quad (2')$$

where

$$\begin{aligned} L_0 = & -\frac{1}{2} R - \frac{1}{2} \bar{\Psi}_\mu \Gamma^{\mu\rho} D_\rho \Psi_\mu - \bar{\lambda} \not{D} \lambda - \left(\frac{\partial_\mu \Phi}{\Phi} \right)^2 - \\ & - \bar{\Psi}_\mu \frac{\partial_\mu \Phi}{\Phi} \Gamma^\mu \lambda - \frac{1}{2} \text{Tr} \bar{\chi} D \chi + \{ \lambda^2 \psi^2 + \lambda \psi^3 + \\ & + \psi^4 + \chi^2 \psi^2 + \chi^4 \} \end{aligned} \quad (5)$$

The four-fermion terms are written down schematically, the explicit form is given in Refs. 3), 4), 8)

$$\begin{aligned} L_{1/2} = & -\frac{1}{4} \text{Tr} F_{\rho\sigma} \bar{\chi} \Gamma^\mu \Gamma^{\rho\sigma} \left(\Psi_\mu + \frac{1}{6} \Gamma_\mu \lambda \right) \\ L_1 = & -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{8} H_{\mu\nu\lambda} \left(\bar{\Psi}_\gamma \Gamma^{\gamma\mu\nu\lambda} \Psi_\delta + 6 \Psi^\mu \Gamma^\nu \Psi^\lambda - \right. \\ & \left. - 2 \bar{\Psi}_\gamma \Gamma^{\mu\nu\lambda} \Gamma^\gamma \lambda + \text{Tr} \bar{\chi} \Gamma^{\mu\nu\lambda} \chi \right) \\ L_2 = & -\frac{3}{2} H_{\mu\nu\rho}^2 \end{aligned} \quad (6)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad (7)$$

$$H_{\mu\nu\rho} = \partial_\mu a_{\nu\rho} - \frac{1}{30} \text{Tr} (A_\mu F_{\nu\rho}) - \frac{2}{3} A_\mu A_\nu A_\rho$$

The dimensions of fields and field strengths entering Eqs. (1) and (2) are as follows

$$\begin{aligned} \dim [\ell_\mu^{\tau}] &= 0, \quad \dim [\psi_\mu, \lambda, \chi] = -\frac{1}{2} \\ \dim [A_\mu^a, \tilde{\omega}_\mu^{\tau s}] &= -1, \quad \dim [\varphi^{-1}, a_{\mu\nu}, R_{\mu\nu}^{\tau s}, F_{\mu\nu}^a] = -2 \\ \dim H_{\mu\nu\rho} &= -3 \end{aligned} \quad (8)$$

Thus the classical d=10 EYMS exists for any gauge group G, and depends on the gravitational constant k and on the vev of scalar field $\langle\phi\rangle = k^2/g^2$. The generating functional of the theory in such a normalization has the most simple dependence on k^2 [as in any pure gravitational or supergravitational theory⁷⁾]

$$Z = \exp \left\{ i \sum_{\ell=0}^{\infty} (k^2)^{\ell-1} W^{(\ell)} \right\} \quad (9)$$

ℓ being the number of loops.

It is instructive to compare our equations (1)-(8) with the corresponding Chapline-Manton⁴⁾ equations and to observe in how many places the constants k and g now disappear. The reason for the simplicity obtained above is purely geometrical. We like to have the Yang-Mills form $D = d+A$ without the parameter g, so the dimension of A_μ is -1, and since $H = da + A \cdot F + AAA$ should also have no parameters inside, we see that the dimension of $H_{\mu\nu\lambda}$ must be -3.

However, the fact that all terms with spinor fields and all supersymmetry transformation rules also allow the corresponding redefinition resulting in a definite dependence of $\langle\phi\rangle$ on coupling constants, namely on the combination k^2/g^2 which is characteristic of the closed heterotic string α' , is an unexpected and non-trivial property of the theory.

The natural form of the Lagrangian (1) and (1') seems to indicate that it is the low-energy limit of the closed string theory with $\langle\phi\rangle = \alpha' = k^2/g^2$. Any higher derivative terms from the string theory will be added (at the tree level) either as additional terms in $L_0, \dots, L_2$ or will form a new structure of the type $\phi_{L_n}^n = (\alpha')^n e^{n\tilde{\Phi}} L_n$. The Green-Schwarz²⁾ modification of the three-form $H_{\mu\nu\lambda}$,

$$H = d\alpha - \frac{1}{30} \omega_{3r}^0 \rightarrow H = d\alpha - \frac{1}{30} \omega_{3r}^0 + \omega_{3L}^0 \quad (10)$$

is just an example of a modification of L_2 . The terms $R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta}$ proposed in Ref. 9) would modify L_1 , being added as partners of $F_{\mu\nu}^2$

$$\mathcal{L} = \frac{e}{k^2} \left(\dots + \varphi \left(\text{Tr} F_{\mu\nu}^2 - a (R_{\mu\nu}^{rs})^2 \right) + \dots \right) \quad (11)$$

where a is some numerical constant. Presumably also the supersymmetry transformation rules will be changed accordingly, i.e., the already existing $X_{-1}^1, \dots, X_1^1$ in Eqs. (2) and (2') might get additional terms, and also new powers of $\phi X_n^1 = (\alpha')^n e^{n\Phi} X_n^1$ may appear.

The whole generating functional associated with the low-energy heterotic closed string theory will take the form (9), where the ℓ -loop functional $W^{(\ell)}$ will be represented as some series in ϕ (or equivalently in α').

In the case of an open string interacting with a closed one, where $\alpha' = g^2/k$ ¹⁾ no such simple correlation between the field theory expansion in the powers of scalar field ϕ and string theory expansion in α' seems to exist.

3. The symmetry properties of the $d=10$ EYMS Lagrangian consist of five types of gauge transformations, namely the supersymmetry, general covariance, Lorentz, Yang-Mills and Abelian ones. The corresponding gauge algebra will be discussed later on in connection with gauge anomalies.

The $d=10$ EYMS Lagrangian (1) has also some scaling invariance. The scaling covariant property of the bosonic part of this Lagrangian was observed by Witten ⁶⁾. His statement in our notation is: under $g_{\mu\nu} \rightarrow t^2 g_{\mu\nu}$, $\phi \rightarrow t^2 \phi$ for $\phi_\mu = \lambda = \chi = 0$, $L \rightarrow t^{-2} L$. Also the argument that the classical scale invariance is an exact property of superstrings was given in Ref. 6).

We have investigated the scaling properties of the Lagrangian in Eq. (1) with all fermionic fields present and with our modified normalization of fields. The result is that the action $S = \int d^{10} x \mathcal{L}$ is scale invariant under the following scale transformations of fields and gravitational coupling constant

$$e_\mu^z \rightarrow t e_\mu^z = e_\mu^{z'}, \quad \varphi \rightarrow t^2 \varphi = \varphi' \quad (12)$$

$$\psi_\mu \rightarrow t^{1/2} \psi_\mu = \psi_\mu', \quad \lambda \rightarrow t^{-1/2} \lambda = \lambda', \quad \chi \rightarrow t^{-1/2} \chi = \chi'$$

$$k^2 \rightarrow t^0 k^2 = k^2', \quad dV = d^{10}x e \rightarrow t^{10} dV = dV' \quad (13)$$

and all connections ω_μ^{rs} , A_μ^a , $a_{\mu\nu}$ and all field strengths $R_{\mu\nu}^{rs}$, $F_{\mu\nu}^a$, $H_{\mu\nu\lambda}$ have zero global scale weights.

The above-mentioned additional higher-derivative terms ω_{3L} in H and $(R_{\mu\nu}^{rs})^2$ in L_1 do not violate the scaling property of the theory. In general it is likely that the field theory limit of the string theory will give the higher derivative terms in accordance with Eqs. (12) and (13), at least at the tree level, where the arguments about scale invariance of the string theory hold⁶⁾.

It is interesting that the one-loop order local interaction which was added by Green and Schwarz to cancel the remaining anomalies even after the gauge group is chosen to be $E_8 \times E_8$ or $SO(32)$, namely the terms^{1),10)}

$$S_c = \int 4 \left(\omega_{3L} - \frac{1}{30} \omega_{3r} \right) X_7 - 6 a X_8 \quad (14)$$

are also scale invariant in the sense of Eqs. (12) and (13) due to the independence of (14) on the gravitational constant, as well as on any scale non-invariant fields.

The scaling properties of the d=10 theory are related to the so-called "no-scale models"¹¹⁾ of d=4 N=1 EYMS which have interesting phenomenological properties. The difference between our treatment of the scaling symmetry and that of Witten, apart from extending the invariance property from bosonic to the fermionic and mixed bosonic fermionic parts of the Lagrangian, is as follows. After obtaining the Witten-type relation $L \rightarrow t^{-2} L$ we add also that since the only

place where the gravitational coupling constant appears is in front of L , we have

$$\frac{e}{k^2} L(\varphi, e, \lambda, \dots) = \frac{e'}{k'^2} t^2 L(\varphi', e', \lambda', \dots)$$

Making also the scale transformation of the coupling constant $k^2 \rightarrow t^8 k^2 = k'^2$ we get the invariance property of the action.

Moreover, it appears that the scale invariance property under the transformations (12) and (13) of actions of the type: $S = \int d^{10}x \mathcal{L}$, where $\mathcal{L}$ is defined in (1), or (14) is the consequence of the invariance of these actions under canonical dimension transformations acting in flat space. They are defined as follows (see also Table 1).

$$\begin{aligned} \dim [dV = dx^{x_1} \dots dx^{x_{10}}] &= 10, \quad \dim[k^2] = 8 \\ \dim [dx^\mu] &= 1, \quad \dim \left[\frac{\partial}{\partial x^\mu}, A_\mu^a, \omega_\mu^{st} \right] = -1 \\ \dim [\psi_\mu, \lambda, \chi] &= -\frac{1}{2} \\ \dim [\varphi^{-1}, a_{rs}, F_{rs}^a, R_{rs}{}^{tu}] &= -2 \\ \dim H_{rst} &= -3 \end{aligned} \tag{15}$$

Since the action can always be written in a flat space with a flat Lorentzian metric, the information about the transformation rules of the vielbein is not necessary.

However, if we consider the theory in curved x -space, we have the possibility of realizing the transformation (15) in two different ways.

The first one is to give a zero canonical dimension to the vielbein. We come immediately to the same dimensions of all objects in (15) in curved space, i.e., exactly to canonical dimensions, presented in (8) (see also Table 2).

The second possibility which leads to the scale invariance of the theory under the transformations (12) and (13), is the following. One defines the

global scale weight of the vielbein to be equal to 1, i.e., $e_{\mu}^r \rightarrow t e_{\mu}^r$. Using Eq. (15), we come to the scale invariance of the action under the transformations (12) and (13) (see also Table 3).

Thus any action of d=10 EYMS (the classical action, as well as any higher derivative actions) has the following important property: if it is written down in the flat space, do not depend on any dimensional parameters except k^2 , and is invariant under the flat space canonical dimension transformations (15), then it means that it is scale invariant when written in curved space with the global scale weights presented in (12) and (13). This is the analogue of the exact superstring scale invariance, discussed in Ref. 6).

4. Consider the general properties of consistent anomalies in supersymmetric theories. In the abstract De Witt's notations, we have the action $S[\Phi^i]$ invariant under the gauge transformations $\delta S = S_{,i} R_{\alpha}^i \xi^{\alpha}$. Usually the supersymmetric theories (in x-space) have the following gauge algebra¹²⁾

$$[R_{\alpha,j}^i, R_{\beta}^j] \xi^{\alpha} \xi^{\beta} = \xi^{\alpha} f_{\alpha\beta}^{\gamma} \xi^{\beta} R_{\gamma}^i + \xi^{\alpha} \eta_{\alpha\beta}^{ij} \xi^{\beta} S_{,j} \quad (16)$$

The last term in the left-hand side of Eq. (16) vanishes on-shell and is not essential for the anomalies. The Wess-Zumino consistency condition¹³⁾ follows from considering the commutator of two gauge transformations acting on the vacuum functional $W[\Phi^i]$

$$[S_{\alpha}, S_{\beta}'] W = \xi^{\alpha} f_{\alpha\beta}^{\gamma} \xi^{\beta} S_{\gamma} W \quad (17)$$

Denoting the anomaly by $G_{\alpha}[\Phi^i]$

$$S_{\alpha} W = \frac{\delta W}{\delta \Phi^i} R_{\alpha}^i \xi^{\alpha} \equiv G_{\alpha} \xi^{\alpha} \quad (18)$$

we get ^{*)}

$$\bar{\varepsilon}'^\beta \delta_\alpha G_\beta - \bar{\varepsilon}'^\alpha \delta'_\beta G_\alpha = \bar{\varepsilon}'^\alpha f_{\alpha\beta}^\gamma \bar{\varepsilon}'^\beta G_\gamma \quad (19)$$

In local supersymmetric theories Eq. (19) appears to be very restrictive despite the absence of the beautiful d+2 formal expressions for anomalies in dimension d known in the Yang-Mills theories as well as in gravitational theories with Lorentz anomalies. The main reason for this restriction lies in the fact that both Lorentz and Yang-Mills gauge symmetries are less fundamental than local supersymmetry. In fact, performing two local supersymmetric transformations with the parameters $\varepsilon_1, \varepsilon_2$ respectively, we produce the Lorentz transformation with the parameter $\omega_{12}^{mn} = \bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 \omega_\mu^{nm} + \dots$ and the Yang-Mills transformation with the parameter $\Lambda_{12}^a = \bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 A_\mu^a$. Just this property of the supersymmetric theories will give the necessary properties of all gauge anomalies which follow from the requirement of the preservation of supersymmetry, i.e., of the absence of supersymmetric gauge anomalies.

We will present now the most important part of the on-shell commutators of the theory (1) which will be used in what follows. The notations of Ref. 3) are used. The difference in the algebra which will be presented and those of Ref. 3) is as follows. For the Yang-Mills transformation of the two-form not the original ^{3),4)} equation is used, but that of Green-Schwarz ¹⁾, the difference being the field-dependent Maxwell transformation. Then we have ($\bar{\varepsilon}^\mu = \frac{1}{2} \bar{\varepsilon}_1 \gamma^\mu \varepsilon_2$)

$$\begin{aligned} [\delta^Q(\varepsilon_1), \delta^Q(\varepsilon_2)] = & \delta^{g.c.}(\bar{\varepsilon}^\mu) + \delta^Q(-\bar{\varepsilon}^\mu \psi_\mu + \dots) + \\ & + \delta^L(\bar{\varepsilon}^\mu \omega_\mu^{\alpha\beta} + \dots) + \delta^{Y.M.}(-\bar{\varepsilon}^\mu A_\mu^a) + \\ & + \delta^M(-\bar{\varepsilon}^\mu (a_{\nu\mu} - \text{Tr}(A_\mu A_\nu)) + \dots) \end{aligned} \quad (20)$$

*) In fact, the invariance of the off-shell one-loop vacuum functional exists only in the background gauges. In the non-background gauges, the corresponding Ward identities exist, however the anomalies are presumably gauge independent, so we use the simplest definition of G_α in Eq. (18), where W is the on-shell gauge independent part of the vacuum functional.

$$[\delta^Q(\varepsilon), \delta^{Y.M.}(\Lambda^a)] = 0 \quad (21)$$

$$[\delta^Q(\varepsilon), \delta^M(\Lambda_\mu)] = 0 \quad (22)$$

$$[\delta^Q(\varepsilon), \delta^L(\omega^{mn})] = \delta^Q\left(\frac{1}{2}\epsilon^{mn}\omega^{mn}\varepsilon\right) \quad (23)$$

$$[\delta^Q(\varepsilon), \delta^{g.c.}(\eta_\mu)] = \delta^Q\left(\frac{1}{2}\eta^\mu\partial_\mu\varepsilon\right) \quad (24)$$

The Yang-Mills and supersymmetry gauge transformations now commute, as distinct from Ref. 3), and the parameter of the Maxwell transformations in (20) get an addition of the type $\xi^\nu \text{Tr} A_\mu \cdot A_\nu$.

Consider now that part of the consistency condition (19) which is obtained using the commutator (20) of two supersymmetry transformations acting on the vacuum functional. We get (assuming for simplicity that general covariance and Maxwell anomalies are absent)

$$\begin{aligned} \bar{\varepsilon}_2 \delta_{\varepsilon_1} G_{\varepsilon_2}^Q - \bar{\varepsilon}_1 \delta_{\varepsilon_2} G_{\varepsilon_1}^Q &= (-\bar{\varepsilon}^\mu \psi_\mu + \dots) G_{-\bar{\varepsilon}^\mu \psi_\mu + \dots}^Q + \\ &+ (\bar{\varepsilon}^\mu \omega_\mu^{mn} + \dots) G_{\bar{\varepsilon}^\mu \omega_\mu^{mn} + \dots}^L + (-\bar{\varepsilon}^\mu A_\mu^a) G_{-\bar{\varepsilon}^\mu A_\mu^a}^{Y.M.} \end{aligned} \quad (25)$$

Let us require that supersymmetric gauge invariance has no anomalies, i.e.,

$$G_\varepsilon^Q = 0 \quad (26)$$

Using the expressions for Lorentz and Yang-Mills anomalies from Ref. 1) and 10) and substituting (26) into the consistency condition (25), we get

$$\begin{aligned}
 0 = & \int \omega_{10Y}^1 (\psi = -\bar{\psi} A_\mu, A, F) + \\
 & + \frac{2-496}{7560} \omega_{10L}^1 (\omega^{mn} = \bar{\psi} \gamma^m \psi + \dots, \omega, R) + \\
 & + 2 \left(\omega_{2L}^1 - \frac{1}{30} \omega_{2Y}^1 \right) X_8(F, R) + 4 \left(\text{tr} R^2 - \frac{1}{30} \text{Tr} F^2 \right) \times \\
 & \times X_6^1 (\psi = -\bar{\psi} A_\mu, \omega^{\mu\nu} = \bar{\psi} \gamma^{\mu\nu} \psi + \dots, A, \omega, F, R). \quad (27)
 \end{aligned}$$

In Eq. (27) all terms must be equal to zero since they are of different structure and no compensation is possible. When the gauge group is $E_8 \times E_8$ or $SO(32)$, the first two terms in (27), corresponding to Lorentz and Yang-Mills anomalies which cannot be cancelled by the addition of local interactions to the effective action, disappear¹⁾. Thus the necessary condition of preservation of supersymmetry is to take the Yang-Mills gauge group to be $E_8 \times E_8$ or $SO(32)$. However, the existence of remaining terms in (27) violates supersymmetry. The anomaly cancellation mechanism of Green and Schwarz [Eqs. (10) and (14)] concern only the bosonic part of the theory and it is not clear whether the resulting field theory can be made supersymmetric consistently. Also a doubt exists in connection with the completeness of the expression for the anomalies given in Refs. 1) and 10) which should be cancelled by local counterterms. In particular, only the anomalies of the diagrams with external Yang-Mills and gravitational fields were considered. However, the pentagon diagrams with one antisymmetric tensor external line, as well as the diagrams with one antisymmetric tensor line, connecting the two parts of one-particle reducible diagrams with external Yang-Mills or gravitational lines, were not checked although they also have closed fermion loops and are potentially dangerous. A supersymmetric calculation in accordance with Eq. (27) would require that the Green-Schwarz counterterms (14) appear in the theory automatically when calculating the above-mentioned diagrams. This statement is quite clear for the Yang-Mills anomalies. However, for the Lorentz anomalies which could be cancelled only when the Green-Schwarz three-form modification (10) is performed, it is not clear whether the original gauge algebra prediction (27) remains intact. This problem needs further investigation.

The information which could be obtained from the consistency conditions (19) about the supersymmetry anomalies in the case that Yang-Mills, general covariance and Lorentz anomalies are absent is the following:

$$\begin{aligned}
 \delta_{\epsilon_1} G_{\epsilon_2}^Q - \delta_{\epsilon_2} G_{\epsilon_1}^Q &= -\bar{\epsilon}_1 \psi_{\mu} + \dots G_{\bar{\epsilon}_1 \psi_{\mu} + \dots}^Q, \\
 \delta_{\Lambda^2}^{Y.M.} G_{\epsilon}^Q &= \delta_{\Lambda^2}^M G_{\epsilon}^Q = 0, \quad \epsilon^{\mu} \delta_{\gamma_{\mu}}^{g.c.} G_{\epsilon}^Q = \\
 &= \frac{1}{2} (\gamma^{\mu} \gamma_{\mu} \epsilon)_{\alpha} G_{\frac{1}{2} \gamma^{\mu} \gamma_{\mu} \epsilon}^Q, \quad \epsilon^{\mu} \delta_{\omega^{\mu\nu}}^L G_{\epsilon}^Q = \frac{1}{2} (\epsilon G^{\mu\nu} \omega^{\mu\nu})_{\alpha} G_{\frac{1}{2} \epsilon G^{\mu\nu} \omega^{\mu\nu}}^Q
 \end{aligned} \tag{28}$$

These equations describe the supersymmetry, Maxwell, Yang-Mills, general and Lorentz covariance properties of the supersymmetric anomaly G_{ϵ}^Q . There is no evident reason why G_{ϵ}^Q should be zero in Eq. (28), though the vanishing anomaly $G_{\epsilon} = 0$ does not contradict Eqs. (28).

Thus the main conclusion is the following. Despite the existence of a classical coupling of d=10 supergravity to the d=10 supersymmetric Yang-Mills theory with any gauge group, ten-dimensional supersymmetry can survive in the quantum theory only for the gauge groups $E_8 \times E_8$ and $SO(32)$.

NOTE ADDED

After this paper was written, we have seen the preprint by Dine and Seiberg¹⁴⁾ (D-S) "Couplings and Scales in Superstring Models". Equation (2) of that paper [without setting $M_g = k^2/g^2 = 1$] is obtained from Eq. (1) of the present paper by D-S change of variables which leads (in accordance with Table 3) to the following scale weights: $w[k^2] = +8$, $w[\phi] = +2$, all other weights in Table 3 are now equal to zero. Our Lagrangian (1), after this change of variables, $\mathcal{L}[e, \phi, \psi, \dots] = \tilde{\mathcal{L}}[\tilde{e}, \phi, \tilde{\psi}, \dots] = \tilde{e}(\phi^4/k^2)(-\frac{1}{2}R(\tilde{e}) + \dots)$ remains scale invariant. No powers of ϕ could appear in brackets () and in the transformation rules (after the rescaling of the supersymmetry parameter $\tilde{\epsilon} = \phi^{-\frac{1}{2}}\epsilon$ consistent with $w[dx^{\mu}] = w[\epsilon] = 0$) since now ϕ is the only non-zero scale-weight field. Using Eqs. (3) and (4), we present the d = 10 EYMS Lagrangian in the form $\tilde{\mathcal{L}} = \tilde{e}/\lambda^2 \times e^{4\Phi}(-\frac{1}{2}R(\tilde{e}) + \dots)$, the constant $\lambda = k^3/g^4$ being dimensionless.

ACKNOWLEDGEMENTS

It is a pleasure to thank D. Amati, M. Duff, S. Ferrara, P. Fré, M. Grisaru, M. Namazie, H. Nicolai, B. Nilsson and P.C. West for their interest in this work and for useful discussions. I am grateful for the hospitality extended to me at the CERN Theoretical Physics Division where this work was completed.

TABLE 1 :
Flat space canonical
dimensions

$dV=dx^{l_1} \dots dx^{l_0}$	+10
k^2	+ 8
ϕ	+ 2
dx^r	+ 1
ψ_r	- $\frac{1}{2}$
λ	- $\frac{1}{2}$
χ	- $\frac{1}{2}$
$\partial/\partial x^r$	- 1
A_r^a	- 1
ω_r^{tu}	- 1
a_{rs}	- 2
F_{rs}^a	- 2
R_{rs}^{tu}	- 2
H_{rst}	- 3

TABLE 2 :
Curved space canonical
dimensions

$dV=dx^{l_1} \dots dx^{l_0} e$	+10
k^2	+ 8
ϕ	+ 2
dx^μ	+ 1
e_μ^r	0
ψ_μ	- $\frac{1}{2}$
χ	- $\frac{1}{2}$
λ	- $\frac{1}{2}$
$\partial/\partial x^\mu$	- 1
A_μ^a	- 1
ω_μ^{tu}	- 1
$a_{\mu\nu}$	- 2
$F_{\mu\nu}^a$	- 2
$R_{\mu\nu}^{tu}$	- 2
$H_{\mu\nu\lambda}$	- 3

TABLE 3 :
Curved space scale
weights

dV	+10
k^2	+ 8
ϕ	+ 2
e_μ^r	+ 1
ψ_μ	+ $\frac{1}{2}$
dx^μ	0
$\partial/\partial x^\mu$	0
A_μ^a	0
ω_μ^{tu}	0
$a_{\mu\nu}$	0
$F_{\mu\nu}^a$	0
$R_{\mu\nu}^{tu}$	0
$H_{\mu\nu\lambda}$	0
λ	- $\frac{1}{2}$
χ	- $\frac{1}{2}$

R E F E R E N C E S

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