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TOP QUARK PRODUCTION AT LEP ENERGIES

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A B S T R A C T

Top quark pair production is analyzed for e^+e^- annihilation at LEP energies. γ -Z interference and the heavy mass of the quark cause the non-diagonal helicity density matrix elements of the produced quark to be sizeable. The detection of this feature through the angular distribution of the decay products is discussed. Coherence effects in the fragmentation process are also considered.

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INTRODUCTION

Heavy quarks are easily produced in e^+e^- annihilation, once energy threshold problems are overcome. The recent experimental signal¹⁾ of t quark production at the $p\bar{p}$ collider at CERN allows one to obtain an estimate of its mass, around 40 GeV. The next generation of electron-positron colliders (SLC and LEP) should thus produce a rich sample of "topped" particles ($T, T^*, \dots$). The gross features of topped particle production have already been extensively studied in the literature²⁾. In this paper, we want to stress that the heaviness of the top quark and the importance of interference effects between γ and Z^0 exchange amplitudes induce an interesting non-diagonal structure for the helicity density matrix of the produced heavy quark. We shall see that the observation of such an effect is likely to be possible through the measurement of the top weak decay product angular distribution. Another peculiar consequence of the top mass being around 40 GeV will be discussed at the end of this note. Coherence effects in the jet fragmentation may involve a production angle dependence of the fragmentation functions of a t quark into a topped hadron.

HEAVY QUARK PRODUCTION IN e^+e^- ANNIHILATION.

We are dealing with unpolarized lepton beams. The Born order (Fig. 1) expression for the helicity amplitudes reads (forgetting colour factors at this stage):

$$T_{\lambda\mu; \lambda_1\lambda_2}(e^+e^- \rightarrow t\bar{t}) = \frac{-ie^2 e_t}{s} \bar{v}(e^+, \lambda_2) \gamma^\mu u(e^-, \lambda_1) \bar{u}(t, \lambda) \gamma_\mu v(\bar{t}, \mu) \\ + \frac{ie^2}{s} D_Z \bar{v}(e^+, \lambda_2) \gamma^\mu (g_V + g_A \gamma_5) u(e^-, \lambda_1) \bar{u}(t, \lambda) \gamma_\mu (G_V^t + G_A^t \gamma_5) v(\bar{t}, \mu) \quad (1)$$

where s is the total c.m. squared energy, D_Z is the Z propagator, $D_Z = s/(s - m_Z^2 + im_Z \Gamma_Z)$ and e_t is the charge of the t quark in units of e (proton charge).

The various centre of mass helicity amplitudes then follow³⁾

$$T_{+-; +-} = ie^2 (1 + \cos\theta) \left[e_t - D_Z (G_V^t + G_A^t \frac{q}{E}) (g_V + g_A) \right] \\ T_{-+; -+} = ie^2 (1 + \cos\theta) \left[e_t - D_Z (G_V^t - G_A^t \frac{q}{E}) (g_V - g_A) \right]$$

$$\begin{aligned}
 T_{+-;-+} &= -ie^2(1-\cos\theta) \left[e_t - \mathcal{D}_z (G_V^t + G_A^t \frac{q}{E}) (g_V - g_A) \right] \\
 T_{-+;+-} &= -ie^2(1-\cos\theta) \left[e_t - \mathcal{D}_z (G_V^t - G_A^t \frac{q}{E}) (g_V + g_A) \right] \\
 T_{++;+-} &= -T_{--;+-} = ie^2 \sin\theta \frac{m_t}{E} \left[e_t - \mathcal{D}_z G_V^t (g_V + g_A) \right] \\
 T_{--;-+} &= -T_{++;-+} = -ie^2 \sin\theta \frac{m_t}{E} \left[e_t - \mathcal{D}_z G_V^t (g_V - g_A) \right]
 \end{aligned} \tag{2}$$

where $q = \sqrt{E^2 - m_t^2}$, $E = \sqrt{s}/2$. The weak coupling constants are given in the Weinberg-Salam model (with $\sin^2\theta_w = 1/4$) by:

$$g_V = 0 \quad ; \quad g_A = \frac{1}{\sqrt{3}} \quad ; \quad G_V^t = \frac{1}{3\sqrt{3}} \quad ; \quad G_A^t = \frac{-1}{\sqrt{3}} \quad . \tag{3}$$

We want here to focus on the four helicity flip amplitudes which vanish in the massless case but will lead to peculiar effects in the polarization properties of heavy quarks produced at SLC-LEP energies. The density matrix elements of the t quark read (in the unpolarized lepton beam case)

$$\begin{aligned}
 \rho_{--}^{++}(\theta) &= \frac{1}{N(\theta)} \left\{ (1+\cos^2\theta) \left[e_t^2 + |\mathcal{D}_z|^2 (G_V^t \pm G_A^t \frac{q}{E})^2 g_A^2 \right. \right. \\
 &\quad \left. \mp 4 \cos\theta e_t \operatorname{Re} \mathcal{D}_z (G_V^t \pm G_A^t \frac{q}{E}) g_A + \sin^2\theta \frac{m_t^2}{E^2} (e_t^2 + |\mathcal{D}_z|^2 G_V^{t2} g_A^2) \right\} \\
 \rho_{+-}(\theta) &= \rho_{-+}^*(\theta) = \frac{2}{N(\theta)} \sin\theta \frac{m_t}{E} g_A \left\{ 2e_t G_V^t \operatorname{Re} \mathcal{D}_z \right. \\
 &\quad \left. + ie_t \frac{q}{E} G_A^t \operatorname{Im} \mathcal{D}_z - \frac{q}{E} \cos\theta |\mathcal{D}_z|^2 G_A^t G_V^t g_A \right\}
 \end{aligned} \tag{4}$$

where $N(\theta)$ is fixed by the normalization

$$\operatorname{Tr} \rho(\theta) = 1 . \tag{5}$$

Let us notice that the non-diagonal elements of $\rho(t)$ are zero in the massless quark limit⁴⁾, which makes the corresponding hadronic quantities (for light vector mesons for instance) sensitive tests of the validity of the incoherent approach for hadronization⁵⁾.

The above results for $\rho(t)$ may be expressed in terms of the t quark polarization vector $\vec{P}_t = (P_S, P_N, P_L)$ using the well-known relation

$$\vec{P}_t = \langle \vec{\sigma} \rangle = \text{Tr}(\rho \vec{\sigma}) \quad (6)$$

where the σ 's are Pauli matrices. Then we have

$$\begin{aligned} P_L &= \rho_{++} - \rho_{--} \\ P_N &= -2 \text{Im} \rho_{+-} \\ P_S &= 2 \text{Re} \rho_{+-} \end{aligned} \quad (7)$$

for the longitudinal (P_L), normal to the scattering plane (P_N) and transverse in the scattering plane (P_S) polarizations of the top quark in its helicity rest frame.

In Fig. 2, we plot P_S , P_N and P_L as functions of the t quark c.m. production angle θ , for $\sqrt{s} = 100$ GeV; in Fig. 3 we present the values of the polarization vector components, integrated over θ , as functions of $\sqrt{s}$; we assume $m_t = 40$ GeV, $m_Z = 90$ GeV, $\Gamma_Z = 3$ GeV; in Fig. 4 we show the integrated value of P_S as a function of $\sqrt{s}$ for different values of m_t .

MEASURING THE HELICITY DENSITY MATRIX OF THE TOP QUARK.

Given this large degree of polarization at the quark level, we are now going to address the question of how to measure it. We will first consider the direct decay of the t quark into b , u , $\bar{d}$ quarks (respectively b , c , $\bar{s}$), Fig. 5, forming three hadronic jets. From a measurement of the jet momenta we know the momentum of the t quark, allowing us to transform into the t quark helicity rest frame. Furthermore, we assume that the b jet has been identified by its content on bottomed hadrons. Thus we are in the position to discriminate between t and $\bar{t}$ quarks.

The decay rate of the polarized t quark in its helicity rest frame is given by:

$$d\Gamma = \frac{1}{2m_t} \sum_{\lambda\lambda'\lambda_b\lambda_u\lambda_d} T_{\lambda_b\lambda_u\lambda_d;\lambda} T_{\lambda_b\lambda_u\lambda_d;\lambda'}^* \rho_{\lambda\lambda'}(t) dLips, \quad (8)$$

where $dLips$ stays for the usual Lorentz invariant phase space.

Calculating the helicity decay amplitudes $T_{\lambda_b\lambda_u\lambda_d;\lambda}(t \rightarrow b\bar{u}d)$ according to the standard model of electroweak interactions, neglecting all final quark masses and taking the W propagator as a constant, which we have checked to be a very good approximation, we get for the normalized decay rate

$$\frac{d\Gamma}{\Gamma} = \frac{12(8\pi)^3}{m_t^6} p_b \cdot p_u (p_t \cdot p_d - m_t a_t \cdot p_d) dLips \quad (9)$$

where $a_t = (0, \vec{p}_t)$ and the p_i 's ($i = b, u, d$) are the four momenta of the final quarks in the t quark helicity rest frame, $p_i = E_i (1, \vec{n}_i)$.

Integrating Eq. (9) over the u and d variables, we get for the angular distribution of the b quark as it emerges from the decay $t \rightarrow b\bar{u}d(bcs)$ in the helicity rest frame of t ,

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_b} \equiv W(\theta, \theta_b, \varphi_b) = \frac{1}{4\pi} \left(1 - \frac{1}{3} \vec{P}_t \cdot \vec{n}_b \right). \quad (10)$$

One may also think of making use of the semi-leptonic decay modes for one of the fermion, e.g.

$$t \rightarrow b \mu^+ \nu_\mu \quad \text{or} \quad b e^+ \nu_e,$$

while the other fermion decays hadronically

$$\bar{t} \rightarrow \bar{b} \bar{u} d \quad \text{or} \quad \bar{b} \bar{c} s.$$

Although suppressed by a factor 3 (due to colour) in rate, these decay modes offer a couple of advantages with respect to the purely hadronic weak mode discussed above. First, the rather high mean momentum of the lepton selects in a very efficient way $t\bar{t}$ production with respect to other fermion-antifermion channels. Second, the electric charge of this lepton signs the quark versus antiquark nature of its parent. The undetectability of the neutrino is not an obstacle to the determination of the parent quark helicity rest frame since it can straightforwardly be deduced from the reconstruction of the three hadronic jets coming from the other fermion. To decide which of the jets come from the t quark (versus

the $\bar{t}$ antiquark), one may use the invariant mass requirements, i.e.

$$M^2(\text{jet } 1 + \text{jet } 2 + \text{jet } 3) = m_t^2 \quad (11)$$

$$M^2(\text{jet } 4 + \text{lepton} + \text{missing momentum}) = m_t^2$$

with the energy conservation constraint:

$$E_{\text{jet } 1} + E_{\text{jet } 2} + E_{\text{jet } 3} = \sqrt{s}/2$$

$$E_{\text{jet } 4} + E_{\text{lepton}} + E_\nu = \sqrt{s}/2 \quad ,$$

where E_ν , the neutrino energy, is the magnitude of the missing momentum (needless to say, some Monte Carlo simulation is still needed to correct for undetected momenta). The decay angular distribution is obtained from the decay amplitudes $T(t \rightarrow b\nu\mu^+)$ which are deduced from $T(t \rightarrow b\bar{u})$ in a straightforward way, by carrying the phase space integration with respect to the b and ν momenta. Neglecting as before the masses of the decay products, one gets the familiar decay angle distribution for the lepton.

$$W = \frac{1}{4\pi} (1 + \vec{P}_t \cdot \vec{n}_\mu) \quad (12)$$

Eqs. (10) and (12) thus offer two ways of measuring $\vec{P}_t$, or $\rho(t)$, which would constitute a new severe test of the standard model of electro-weak interactions. It may also provide a supplementary determination of some of its parameters such as the top quark mass or its weak coupling constants G_V^t and G_A^t ⁶⁾.

COHERENT FRAGMENTATION OF THE t QUARK

In this section we are going to discuss another peculiarity of the t quark which again appears as a consequence of the vicinity of twice the t quark mass and the Z mass. As we will demonstrate below, coherence effects in the fragmentation reveal themselves in a production angle dependence of the fragmentation function quite contrary to the common incoherent approach.

As we have discussed previously⁵⁾, taking coherence effects into account requires the introduction of the joint helicity density matrix $\rho(t\bar{t})$ of the $t\bar{t}$ system as produced through e^+e^- annihilation

$$\rho_{\nu\mu\nu'\mu'}(t\bar{t}) = \frac{1}{N'} \sum_{\lambda_1\lambda_2} T_{\nu\mu\lambda_1\lambda_2}(e^-e^+ \rightarrow t\bar{t}) T_{\nu'\mu'\lambda_1\lambda_2}^*(e^-e^+ \rightarrow t\bar{t}) \quad (13)$$

The condition $\text{Tr} \rho = 1$ fixes again the normalization N' . The amplitudes T appearing here are those given in Eq. (2). The cross-section for the inclusive production of a topped hadron H carrying the fraction z of the t quark energy read

$$\frac{d\sigma(e^-e^+ \rightarrow HX)}{d\cos\theta dz} = \frac{d\hat{\sigma}}{d\cos\theta} D_t^H(\theta, z) \quad (14)$$

where, in the coherent picture, the fragmentation function is given by

$$D_t^H(\theta, z) = \sum_{\substack{\lambda\lambda_x\lambda \\ \nu\nu'\mu\mu'}} M_{\lambda\lambda_x\nu\mu}(t\bar{t} \rightarrow HX) \rho_{\nu\mu\nu'\mu'}(\theta) M_{\lambda\lambda_x\nu'\mu'}^* \quad (15)$$

This is just the inclusive cross-section for the production of H in $t\bar{t} \rightarrow HX$, starting with a $t\bar{t}$ system whose spin state is described by $\rho(t\bar{t})$. Since we neglect the transverse momentum of H with respect to the t quark, the amplitudes M only depend on z . The joint density matrix, on the other hand, is a function of θ . Therefore, the fragmentation function depends in general on both variables z and θ .

Taking into account parity conservation in the fragmentation process and rotational invariance (as a consequence of the neglect of transverse momentum) which requires for non-vanishing products of M 's appearing in Eq. (15) the condition

$$\lambda - \lambda_x = \nu - \mu = \nu' - \mu' \quad (16)$$

we get

$$\mathcal{D}_t^H(\theta, z) = \sum_{\lambda \lambda_X \lambda} \left\{ (1 - 2\rho_{++++}) |M_{\lambda \lambda_X+-}|^2 + 2\rho_{++++} [|M_{\lambda \lambda_X++}|^2 + \text{Re} M_{\lambda \lambda_X++} M_{\lambda \lambda_X--}^*] \right\} \quad (17)$$

Thus, we see that

$$\rho_{++++} = \frac{2}{N} \sin^2 \theta \frac{m_t^2}{E^2} [e_t^2 + |D_z|^2 G_V^{t^2} g_A^2] \quad (18)$$

controls the θ dependence of $\mathcal{D}_t^H$ in the coherent picture. This dependence simply reflects the fact that the polarization state of the $t\bar{t}$ system varies with θ .

Notice that for light quarks $\rho_{++++} \approx 0$, so that there is no θ dependence in the corresponding fragmentation functions. The same is true in the case of the incoherent fragmentation picture in which

$$\begin{aligned} \mathcal{D}_t^H &= \sum_{\lambda} \left\{ \rho_{++}(t, \theta) \mathcal{D}_{t++}^{H\lambda\lambda}(z) + \rho_{--}(t, \theta) \mathcal{D}_{t--}^{H\lambda\lambda}(z) \right\} \\ &= \sum_{\lambda} \mathcal{D}_{t++}^{H\lambda\lambda} (\rho_{++}(t, \theta) + \rho_{--}(t, \theta)) = \sum_{\lambda} \mathcal{D}_{t++}^{H\lambda\lambda}(z) \end{aligned} \quad (19)$$

In order to get an idea of how sizeable the difference between the coherent and the incoherent approach can be, we consider some simple examples. The production of t quarks at LEP or SLC energies will very likely result in low multiplicity jets, each containing a topped particle. Let us then consider the simplest cases in which we have the exclusive final states $H, X = T, T^*$ (or T^*, T) and $H, X = T, T$.

In the first case we get

$$\mathcal{D}_t^{T(T^*)} = (1 - 2\rho_{++++}) |M_{\substack{01-+ \\ (10)}}|^2 \quad (20)$$

while in the second one we have

$$\mathcal{D}_t^T = 4\rho_{++++} |M_{00++}|^2 \quad (21)$$

In Fig. 6 we plot the two quantities $1 - 2\rho_{++++}$ and $4\rho_{++++}$ as functions of θ , for $\sqrt{s} = 100$ GeV. We can then see how the t quark fragmentation functions vary

with Θ . Moreover, due to electroweak interference effects, there is also a large forward-backward asymmetry.

In Fig. 7 we plot the forward-backward asymmetries

$$A_{f-b}(\theta) = \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma(\theta) + d\sigma(\pi - \theta)} \quad (22)$$

as functions of Θ , at $\sqrt{s} = 100$ GeV, for the elementary process $e^-e^+ \rightarrow t\bar{t}$ and for the process $e^-e^+ \rightarrow T\bar{T}^*(T^*T)$, in the latter case the f-b asymmetry of the fragmentation function, Eq. (20), increases the elementary one. Notice instead, that the f-b asymmetry for the process $e^-e^+ \rightarrow T\bar{T}$ is zero, due to an exact compensation between the huge f-b asymmetries of the elementary process and of the fragmentation function, Eq. (21).

We believe that the possibility that coherence effects in the fragmentation process may manifest themselves in a production angle dependence of the fragmentation function D_t^H and in a forward-backward asymmetry different from that of the elementary reaction is very interesting and worth being tested experimentally.

SUMMARY

The production of heavy quarks ($m_q \approx \frac{1}{2}m_Z$) in e^+e^- annihilation at energies around 100 GeV, presents some unique features, due to the presence of both mass and electroweak interference effects.

The experimental indication of the existence of the top quark in that mass range makes its future production at LEP or SLC extremely interesting.

The top quarks result in being strongly polarized, with large components of the polarization vector both in the longitudinal direction and in the transverse direction in the scattering plane; there is also a smaller (up to ten per cent) polarization perpendicular to the scattering plane, which is sensitive to the imaginary part of the Z propagator^{*}). The measurement of such components,

* Imaginary parts which are not proportional to the width of the Z^0 may be obtained through loop effects in higher order calculations. They turn out to be very small⁷⁾.

feasible through the study of the top quark decay angular distribution, would provide a complete and severe test of the electroweak standard model including a determination of the mass of the t quark. Similar results are obtained when dealing with longitudinally polarized beams; in this case, the density matrix elements of the t quark differ somewhat from those given in Eq. 4.

Mass and electroweak effects also appear in a peculiar way when considering the coherent hadronization of the $t\bar{t}$ system; the usual fragmentation functions of the t quarks into topped final particles may acquire a dependence on the t quark production angle. Such a dependence also shows a strong forward-backward asymmetry which can drastically alter that of the elementary reaction, $e^+e^- \rightarrow t\bar{t}$. This is, of course, of extreme importance when analyzing data on final hadrons in order to test the standard model.

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FIGURE CAPTIONS

- Fig. 1: Born order diagram for $e^+e^- \rightarrow t\bar{t}$.
- Fig. 2: Plot of the polarization vector components of the t quark, P_L , P_S and P_N versus the c.m. production angle θ . We have used $\sin^2\theta_w = 1/4$, $m_t = 40$ GeV, $m_Z = 90$ GeV, $\Gamma_Z = 3$ GeV, $\sqrt{s} = 100$ GeV.
- Fig. 3: Plot of P_L , P_S and P_N (integrated over θ) as functions of $\sqrt{s}$. Same values for the other parameters as in Fig. 2.
- Fig. 4: Same as Fig. 3 for P_S and different values of the t quark mass.
- Fig. 5: Weak decay of the top quark.
- Fig. 6: Plots of $1-2\rho_{++++}(t\bar{t})$ and $4\rho_{++++}(t\bar{t})$ versus the production angle θ . Same values of the parameters as in Fig. 2.
- Fig. 7: Forward-backward asymmetries for the two processes $e^+e^- \rightarrow t\bar{t}$ and $e^+e^- \rightarrow T\bar{T}^*(T^*T)$. Parameters as in Fig. 2. Notice that the f-b asymmetry for the process $e^+e^- \rightarrow T\bar{T}$ is zero.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7