INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

KALUZA-KLEIN THEORIES AND THE SIGNATURE OF SPACE-TIME

I.Ya. Aref'eva

and

I.V. Volovich

1985 MIRAMARE TRIESTE

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

I. INTRODUCTION

Recently important results have been obtained in the construction of a unified theory of all known interactions [1]. These results were achieved within the framework of the Kaluza-Klein approach for superstrings and supergravity in which the effective four-dimensional theory is a theory of massless particles. Usually it is believed in this approach that extra coordinates are space-like, i.e. a d -dimensional manifold M^d is regarded as having the metric g_{MN} with signature $(-+++)$, i.e. $(1,d-1)$. This assumption leads to positive energy for matter fields in the four-dimensional zero-mass sector after compactification. However, it is desirable not to fix the signature in the d -dimensional space from the very beginning, but to obtain it in a dynamic way (i.e. as a solution of equations of motion) permitting in principle, the appearance of extra time-like variables. The hypothesis of the existence of extra time-like variables was suggested in Ref.[2]. In this paper we shall consider vacuum solutions and spontaneous compactification in Kaluza-Klein theories with extra time-like variables. In general, limiting the signature to $(1,d-1)$ (with one time-like variable) places great restrictions on the possible solutions of the d -dimensional equations of motion which may not be advantageous. In particular as was shown in Ref.[3] the signature $(7,3)$ appears naturally in the $N=1, d=10$ supergravity theory coupled to Yang-Mills matter from the solution to the equations of motion, which has the desirable property that the four-dimensional space-time has the Minkowski metric. In other words, in the ten-dimensional space with the $(7,3)$ signature we can solve the problem of the cosmological constant. Let us remember that in $(1,9)$ signature the compactification $M^{10} = M^4 \times B^6$ does not exist (a no go theorem of Ref.[4]) but there does exist a compactification $M^{10} = \text{adS}^3 \times B^7$ [5].

In this paper, we will explore what can be learnt about an effective four-dimensional theory extracted from the d -dimensional theory with extra time-like variables. In general, the extra time-like variables lead in the Kaluza-Klein approach to ghosts and to tachyons [6]. We shall point out the topological criteria for a compact manifold corresponding to time-like variables which avoid the appearance of massless ghosts in the effective four-dimensional theory.

We may consider different possibilities for interpretation of classical solutions in a $d > 4$ dimensional manifold with signature (t,s) , $t + s = d$ when $t > 1$.

The first point of view is that quantum theory is constructed in $(1,d-1)$ signature. The solutions in other signatures are used for calculating

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

KALUZA-KLEIN THEORIES AND THE SIGNATURE OF SPACE-TIME *

I.Ya. Aref'eva **

International Centre for Theoretical Physics, Trieste, Italy

and

I.V. Volovich

Steklov Mathematical Institute, Vavilov 42, Moscow,
GSP-I, 117966, USSR.

ABSTRACT

Higher-dimensional Kaluza-Klein theories with extra compactified time-like variables are considered. Topological criteria are presented for a compact manifold which prevent the appearance of massless ghosts in an effective four-dimensional theory. Some models are given in which these criteria hold. Among them is the bosonic sector of the low energy limit of the anomaly-free superstring theories. As a rule, the extra time-like variables lead to compactification with a compact hyperbolic manifold.

MIRAMARE - TRIESTE

June 1985

* Submitted for publication.

** On leave of absence from: Steklov Mathematical Institute, Vavilov 42, Moscow, GSP-I, 117966, USSR.

the functional integral in the stationary phase approximation. Our approach here is the same as that followed usually in dealing with Euclidean four-dimensional field theories. Another possibility is the following. Assuming that compactification of extra time-like dimensions is achieved, what are the implications of taking extra time-like dimensions seriously? At this point the problem of a physical interpretation of the theory with extra time dimensions arises. If extra time dimensions are not compactified, then in the d -dimensional theory causality is violated. But if extra time dimensions are compactified then along with the observable (macroscopic) time there are extra (microscopic) times and there is a reason to speak of macroscopic and microscopic causalities, respectively. *) To understand the physical meaning of compactified extra time-like dimensions in detail, we ought to investigate the initial Cauchy problem for ultra hyperbolic equations, for example with periodic boundary conditions for the extra time-like variables. Causality in Kaluza-Klein theories with ordinary signature was considered in Ref.[7].

Let us note that a change of the metric signature, generally speaking, changes the number of fermionic degrees of freedom [6,9] and the algebra of supersymmetry. So a Lagrangian which possesses the supersymmetry for metric signature (1,d-1) can have no supersymmetry in another signature.

There also exists the following possibility. If we assume the existence of extra time-like variables as well as extra space-like variables and if the theory possesses the ground state $M^d = M^4 \times M^r \times B^{d-r-l}$ with metric signatures (1,3), (1,r-1) and (0,d-r-l), respectively, then it is rather intriguing to think that two Universes exist each with its own time. These Universes are embedded in higher dimension world **. If we have such a vacuum solution, then it seems that it is necessary to modify the standard Kaluza-Klein approach. In this case it is unnatural to interrate the action over the internal coordinates. Instead, both Universes M^4 and M^r should be considered equal in rights and each Universe should have its own time. Note, incidentally, that the possibility of an observer existing in another Universe may apparently create a possibility for a new interpretation of quantum theory which is different from the ordinary Copenhagen interpretation and from Everett's interpretation as well.

*) The microscopic time may be appear as Zitterbewegung in the macroscopic time.

**) Let us note that this picture is obviously different from the picture of islands in the Universe with different metric signature embedded in four-dimensional space which was discussed in Ref.[9].

In this note we shall consider the following problem. Is it possible to obtain a satisfactory four-dimensional theory in the Kaluza-Klein approach with additional time-like dimensions? In the modern Kaluza-Klein approach it is usually assumed that the internal space is of a small size and therefore the four-dimensional massive states have masses in excess of the Planck mass. In this approximation, all observable particles are massless. Therefore we focus our attention on the massless sector and consider the problem of constructing a satisfactory four-dimensional massless sector (without ghosts).

II. RESTRICTIONS ON COMPACTIFICATION

Let us consider a manifold M^d having the metric g_{MN} with signature (d-3,3) which corresponds to the d-3 time-like dimensions. We shall consider the Einstein equations for the gravitation field interacting with matter fields

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN},$$

where T_{MN} is the stress tensor of the matter. We have also the corresponding equations for the matter fields. As we are now interested in vacuum solutions we consider only bosonic fields.

We would like to obtain the reduction $M^d = M^4 \times B^r$, where M^4 is a space-time whose metric has signature (-+++), and B^r is a compact manifold. We look for a vacuum solution in the form $g_{MN} = (g_{\mu\nu}(x), g_{mn}(y))$, where x^μ, y^m are the coordinates for the corresponding manifolds M^4 and B^r . The metric g_{mn} is negative definite.

We shall show that massless ghosts are absent in the four-dimensional theory if the following conditions for the manifold B are satisfied:

- 1) B has no Killing vector field at all;
- 2) If antisymmetric tensor fields of rank k are present, then it is necessary that all Betti numbers $b_{2k+1}(B)$ should be equal to zero for $2k + 1 \leq r$.

Indeed, it is well known that if B has the isometry group G then the theory may include massless Yang-Mills bosons with gauge group G . In the case of a negatively definite metric g_{mn} the Kaluza-Klein ansatz for the metric tensor [10,11]

III. HYPERBOLIC MANIFOLDS

We shall show that spontaneous dimensional reduction in a Kaluza-Klein theory with extra time-like variables implies, as a rule, compactification on manifolds with constant, negative curvature (hyperbolic manifolds)*. One should compare this with the possibility of the compactification on manifolds with positive curvature for space-like extra dimensions [6,13,12,11,14]. Suppose that the higher dimensional world is of the form $M^d = M^4 \times B^{d-4}$ with M^4 a four-dimensional anti-de Sitter or Minkowski with metric signature $(-+++)$ and B^r an r -dimensional manifold with metric signature $(---\dots)$. Let us also assume that the stress tensor T_{MN} for the vacuum solution has the form

$$T_{\mu\nu} = \gamma_1 g_{\mu\nu}, \quad T_{mn} = \gamma_2 g_{mn}.$$

This is a very strong restriction, but various ways of obtaining these equations have been found. The positivity of T_{00} gives us $\gamma_1 < 0$. According to Einstein's equations one gets

$$R_{\mu\nu} = \frac{(\gamma_1 - \gamma_2) - 2\gamma_1}{d-2} g_{\mu\nu}, \quad R_{mn} = \frac{4(\gamma_1 - \gamma_2) + 2\gamma_2}{d-2} g_{mn}.$$

If four-dimensional space-time is an adS-like or Ricci-flat, then for r -dimensional space we get $R_{mn} = C g_{mn}$ with a positive constant C . This means that the extra r -dimensional space can be a compact hyperbolic manifold. It is essential to point out that in this case extra dimensions are consistent with but do not imply compactification. (One can believe that the contributions to the functional integral from non-compact hyperbolic manifolds are suppressed.)

One can say more if $\gamma_1 - \gamma_2 < 0$ (this inequality is obtained, for example, for the Yang-Mills stress tensor when the ansatz for monopoles or an embedding of Yang-Mills connection in the spin connection is used). In the case, one can immediately notice that the r -dimensional space can always be a hyperbolic manifold. Usually, the hyperbolic manifolds are not considered in the Kaluza-Klein approach. Apparently, the reason for this lack of attention is that the simplest hyperbolic manifold is the Lobachevsky space (a realization of the Lobachevsky space is the mass shell hyperboloid in Minkowski space) which is non-compact. However, compact hyperbolic manifolds do exist, see Refs. [15,16].

A few facts about compact hyperbolic manifolds will be useful. Any small neighbourhood in a hyperbolic n -manifold L^n is isometric to a neigh-

* The use of a compact hyperbolic manifold as an extra space in Kaluza-Klein theories was suggested in Ref. [3].

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} + A_\mu^i A_\nu^j K^{mi} K^{nj} & A_\nu^i K^{mi} & 0 \\ A_\mu^i K^{mi} & g_{mn} & 0 \\ 0 & 0 & g_{mn} \end{pmatrix} \quad (1)$$

where the quantities K^{mi} are the Killing vector corresponding to the isometries of the metric and i runs over the dimension of the isometry group G gives rise to Yang-Mills ghosts. The claim is based on the observation that substituting (1) into the higher-dimensional action and integrating over the extra dimensional coordinates y , one obtains the four-dimensional action with a negative effective coupling constant in front of the kinetic energy term $F_{\mu\nu}^2$ for the gauge potential A_μ . In the case when B has no Killing vector fields at all the Kaluza-Klein mechanism for generation of gauge fields does not work. However, the condition 1) is a sufficient but not necessary condition because, as was mentioned in Ref. [11], the standard Kaluza-Klein ansatz for massless gauge fields is in general inconsistent with the higher dimensional field equations.

To illustrate the possibility of existence of a higher dimensional model with metric signature $(d-3,3)$ which implies a compactification $M^d \times M^4 \times B^{d-4}$ free from four-dimensional massless ghosts, let us consider the field equations of pure gravity in the eight-dimensional world with $R_{MN} = 0$. This theory admits a classical ground state solution (Minkowski space-time) $\times K3$ ($K3$ is Kummer's quartic surface in CP^3 with a Ricci flat metric and which has no Killing vectors). The effective four-dimensional massless theory describes an interaction of gravity with scalar fields and has no ghosts.

As for the second condition, we have in mind the simplest cases, where the higher dimensional gravity couples to the antisymmetric tensor field $A_{M_1 \dots M_\ell}$ through the Lagrangian $\mathcal{L} = e[R - \beta H^2]$, where $e^2 = \det[g_{MN}]$, $H = dA$. Then considering small fluctuations of the fields $A_{M_1 \dots M_\ell} = (A_{\mu_1 \dots \mu_\ell}, A_{\mu_1 \mu_2 \dots \mu_{\ell-1} n}, \dots, A_{n_1 \dots n_{\ell-1} n})$ about the ground state and performing a Fourier expansion of the terms of eigenfunctions of the Hodge de Rham operator one obtains (see Ref. [12]) a four-dimensional theory. The four-dimensional massless fields $A_{\mu_1 \dots \mu_k}(x)$ $k \leq \ell$ come from zero-modes of the Hodge de Rham operator on the internal space. If $(\ell - k)$ is odd then these fields are ghosts. These massless ghost fields do not appear in the case where the Hodge de Rham operator has no corresponding eigenfunctions, i.e., the corresponding Betti numbers are equal to zero.

bourhood in the n -dimensional Lobachevsky space H^n and the manifold itself is the quotient $L^n = H^n/\Gamma$ of a space H^n by a discrete group of motions Γ . A simple example of a compact hyperbolic two-manifold is the sphere with $g \geq 2$ handles, obtained by identifying sides of the regular (in the Lobachevsky metric) $4g$ -polygon. An example of a compact hyperbolic three-manifold is the dodecahedral space, obtained by identifying opposite faces of a dodecahedron after $3\pi/5$ right-handed rotations. The set of hyperbolic n -manifolds ($n \geq 3$) with any given volume is finite. The Mostow rigidity theorem [17] says that any invariant of the geometry of a hyperbolic n -manifold ($n \geq 3$) is actually an invariant of its homotopic type. So, for a fixed homotopic type, one gets a unique vacuum configuration. The Euler characteristic of a closed three-manifold is always 0 and the eta invariant [18] or the Chern-Simons invariant of oriented hyperbolic three-manifold, should probably be thought together with the volume as a characteristic of the vacuum solution.

The geodesics on the L^r space have stochastic behaviour and one can say that the extra-coordinate is spread all over the extra r -manifold. According to Bochner's theorem compact hyperbolic manifolds do not imply Killing vectors at all.

IV. MODELS

We now turn to a description of theories with extra time-like variables allowing compactification without massless ghosts.

1. Let us consider the coupling of seven-dimensional gravity and a Yang-Mills system with the Lagrangian

$$\mathcal{L} = e \left[R - \frac{1}{4g^2} F_{MN}^2 \right],$$

where

$$F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N]$$

and where the metric has signature $(-+++)$. In this case, we may obtain a solution of the form $adS^4 \times L^3$, where L^3 is a compact hyperbolic manifold. To obtain this solution we set the gauge field in the internal space equal to the spin connection, suitably embedded in the gauge group [19]. If we take L^3 equal to the hyperbolic space of the dodecahedron, then the first Betti number $b_1(L^3)$ is equal to zero and our conditions 1) and 2) are satisfied.

2. For the bosonic sector of the eleven-dimensional supergravity with signature $(-+++)$ using the Freund-Rubin ansatz [13] one can obtain a compactification $adS^7 \times L^7$, where L^7 is a compact hyperbolic manifold (or and Einstein space). The effective four-dimensional massless theory has no ghosts if the Betti numbers $b_1(L^7)$ and $b_3(L^7)$ are equal to zero.

3. Let us consider a compactification of d -dimensional gravity coupled to a 2-form A in a metric with a signature $(d-3,3)$ with the action as in Sec. II. The equation of motion for the H -field has the solution

$$H_{\mu\nu\sigma} = 0, \quad \hat{\mu}, \hat{\nu}, \hat{\sigma} = 0, 1, \dots, d-4$$

$$H_{mnk} = h\sqrt{g(y)} \epsilon_{mnk}, \quad m, n, k = d-2, d-1, d.$$

For this solution, the condition of positivity of T_{oo} is not satisfied and we get the equations

$$R_{\hat{\mu}\hat{\nu}} = C_1 \epsilon_{\hat{\mu}\hat{\nu}}, \quad R_{mn} = C_2 \epsilon_{mn}$$

with positive constant C_1 and negative constant C_2 .

This means that the theory admits the compactification $M^d = dS^{d-3} \times B^3$ as well as $M^d = dS^4 \times L^{d-7} \times B^3$, where B^3 is a three-manifold with constant positive curvature. However, we cannot take an arbitrary constant positive curvature manifold as a B -space because in the case the condition (1) does not hold. For example, we cannot consider the three-sphere as a B space because the three sphere has vector Killing fields. We also cannot take the quotient S^3/Γ with Γ acting freely on the S^3 -sphere as a B -space because S^3/Γ always has an isometry group S^1 and the condition 1) would be broken. To avoid this obstacle we ought to take for B the quotient S^3/Γ with a discrete group Γ acting on S^3 non-freely (i.e. with a fixed point). Then S^3/Γ is a finite volume manifold with singular points. Non-compact finite volume manifolds in the context of Kaluza-Klein theories were considered early in Refs. [20, 21].

4. Let us now show that extra time-like coordinates permit us to get a vacuum solution in the bosonic sector of the $d = 10$, $N = 1$ supergravity coupled with Yang-Mills theory [22]. This is the theory obtained from the Green-Schwarz bosonic Lagrangian

$$\mathcal{L} = e \left[\frac{R}{2k^2} - \frac{(V\varphi)^2}{k^2} - \frac{e^{-\varphi}}{4g^2} F^2 - \frac{3k^2 e^{-2\varphi}}{2g} H^2 \right] \quad (2)$$

$H = d\omega - \omega_{3Y} + \omega_{3L}$, where ω_{3Y} (ω_{3L}) is the Chern-Simons 3-form for the Yang-Mills connection (spin connection), and the notation is as Ref. [1]. In signatures (7,3) or (4,6) the matter field equations are satisfied by the following ansatz. Non-zero components of g_{MN} are split into three blocks $g_{MN} = (g_{\mu\nu}(x), g_{mn}(y), g_{ab}(z))$. The scalar field φ is constant $\varphi = \varphi_0$. Non-zero components of H and F are $H_{mnk} = h\sqrt{g}(y)\epsilon_{mnk}$, $h^2 = g^6 e^{3\varphi_0/2k}$, $F_{ab}^{ij} = R_{ab}^{ij}$, $\mu, \nu = 0, 1, 2, 3$; $m, n = 5, 6, 7$; $a, b = 8, 9, 10$. The Einstein equation yields the relations $R_{\mu\nu} = 0$, $R_{mn} = -(3g^2/4k^2)e^{\varphi_0} g_{mn}$, $R_{ab} = (g^2/k^2)e^{\varphi_0} g_{ab}$. These equations hold if we take $M^{10} = M^4 \times Q^3 \times L^3$ and $M^{10} = M^4 \times Q^3 \times S^3$ in the signature (7,3) and (4,6), respectively. Here M^4 is Minkowski space, $Q^3 = S^3/\Gamma$, with Γ a discrete subgroup of $SO(4)$ such that Q^3 has no Killing vectors at all (see Sec. IV), L^3 is a compact three-hyperbolic manifold. Note that a compactification with a flat space M^4 is possible due to the supersymmetry of the theory. If we consider a perturbation of the Lagrangian (2) for example if instead of the term $e^{-\varphi} F^2$ we substitute $e^{-q\varphi} F^2$ where q is a constant the M^4 cannot be a flat space.

For non-standard signature in the general case we do not have equality of the fermion and boson degrees of freedom and there is no supersymmetry. For example the Majorana-Weil spinors exist for signature (t,s) only if $s - t = 0 \pmod{8}$ (see Refs. [6,8]). In the ten-dimensional world this condition is satisfied, aside from the standard signature, for signature (-+++----) and in this case we can have supersymmetry. In this case a solution of the form $M^4 \times AdS^3 \times L^3$ exists. It is interesting to consider the possibility that here exists non-standard signatures as a possible mechanism for spontaneous breaking of supersymmetry.

5. Following Candellas et al. [1] we add a higher derivative term to the Lagrangian (2) and consider the following Lagrangian:

$$\mathcal{L} = e \left[\frac{R}{2k^2} - \frac{(\nabla\varphi)^2}{k^2} - \frac{e^{-\varphi}}{4g^2} (F^2 - \mathcal{R}^2) - \frac{3k^2 e^{-2\varphi}}{2g} \right], \quad \mathcal{R}^2 = R_{MNKL} R^{MNKL},$$

with arbitrary numerical coefficient γ . When $\gamma = 1/30$ for gauge group $E_8 \times E_8$ compactification on four-dimensional Minkowski space was described by Candellas et al. [1], where $M^{10} = M^4 \times K^6$ with K^6 a six-dimensional Calabi-Yao manifold. It is remarkable to note that the configuration is a vacuum solution also for the metric signature (7,3). Moreover, conditions (1) and (2) from Sec. II hold for this compactification and the zero mass bosonic sector is the same in both signatures (1,9) and (7,3).

When γ is not equal to $1/30$, the compactification $M^{10} = M^4 \times S^4 \times L^2$, $M^{10} = M^4 \times L^4 \times L^2$ exists for signature (1,9) and (5,5). In this case the matter field equations are trivially satisfied by the following background $F_{mn}^{ij} = R_{mn}^{ij}$, $m, n = 5, 6, 7, 8$; $i, j = 1, 2, 3, 4$; $F_{ab} = f\sqrt{g}(z)\epsilon_{ab}$, $a, b = 9, 10$.

In conclusion, we would like to note that of course many interesting problems are open. It would be interesting to consider more in detail the different possibilities for the physical interpretations of solutions with extra time-like variables described briefly in the Introduction. It is also necessary to consider the fermion sector in such theories. We have shown that one can obtain a good massless four-dimensional sector. It would be interesting to consider an interaction with massive tachyons. It seems that such interactions will be suppressed and the probability of production of tachyons will be very small, in particular, in superstring theory where there exists an effective ultraviolet cut-off. Another interesting problem would be to construct supersymmetric field models and superstrings in a high-dimensional world with extra time-like dimensions.

ACKNOWLEDGMENTS

The authors wish to thank V.A. Berezin, F.A. Bogomolov, V.P. Prolov, R.E. Kallosh, A.D. Linde and K. Stelle for helpful discussions. One of them (I.Ya.A.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where this work was completed. The author is also indebted to D. Gross, E. Sezgin and J. Strathdee for useful discussions.

REFERENCES

- [1] M.N. Green and J.H. Schwarz, Phys. Lett. 149B, 117 (1984);
E. Witten, Phys. Lett. 149B, 351 (1984);
P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Princeton University, preprint (1984);
D.J. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54, 46 (1985);
P.G.O. Freund, Santa Barbara University, preprint NSF-ITP-84-157 (1984);
S. Randjbar-Daemi, Abdus Salam, E. Sezgin and J. Strathdee, ICTP, Trieste, preprint IC/84/218 (1984);
for a recent review see:
I.Ya. Aref'eva and I.V. Volovich, Usp. Fiz. Nauk (to appear) (1985).
A.D. Sakharov, Zh. Eksp. Teor. Fiz. 87, 375 (1984).
- [2] I.Ya. Aref'eva and I.V. Volovich, Theor. Math. Phys. 64, 255 (1985).
- [3] D. Freedman, G. Gibbons and P. West, Phys. Lett. 124B, 491 (1983).
- [4] I.Ya. Aref'eva and I.V. Volovich, to appear in Phys. Lett.
- [5] J. Scherk in Recent Developments in Gravitation, Eds. M. Levy and S. Deser (Plenum Publ. Co., 1979).
- [6] S. Weinberg, Phys. Lett. 143B, 97 (1984).
- [7] C. Wetterich, Nucl. Phys. B211, 177 (1983).
- [8] A.D. Linde, Rep. Prog. Phys. 47, 925 (1984).
- [9] D. DeWitt in Relativity, Groups and Topology, (Gordon and Breach, New York, London, 1984);
E. Witten, Nucl. Phys. B186, 412 (1981).
- [10] M.J. Duff, B.E.W. Nilsson, C.N. Pope and N.P. Warner, Phys. Lett. 149B, 90 (1984).
- [11] M.J. Duff and C.N. Pope, in Supersymmetry and Supergravity 82, Eds. S. Ferrara, J.G. Taylor and P. van Nieuwenhuizen (World Scientific, Singapore 1983).
- [12] P.G.O. Freund and M.A. Rubin, Phys. Lett. 97B, 233 (1980).
- [13] J. Tosa, Phys. Rev. D30, 339 (1984).
- [14] W. Thurston, Bull. Amer. Math. Soc. 6, 357 (1982).
- [15] J. Milnor, Bull. Amer. Math. Soc. 6, 9 (1982).
- [16] G.D. Mostow, Ann. Math. Studies No.78 (1973), (Princeton University Press, Princeton, N.Y.).
- [17] M.F. Atiyah, V.K. Patodi and I.M. Singer, Proc. Cambridge Philos. Soc. 77, 43 (1975).
- [18] D.V. Volkov and V.I. Tkach, Pisma JETP 32, 681 (1980);
S. Randjbar-Daemi and R. Percacci, Phys. Lett. B117, 42 (1982).
- [19] C. Wetterich, Bern preprint BUTF-84/5; BUTF-84/6 (1984);
H. Nicolai and C. Wetterich, Phys. Lett. B150, 347 (1985).
- [20] M. Gell-Mann and B. Zwiebach, Phys. Lett. 147B, 111 (1984).
- [21] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys. B195, 97 (1982);
G.F. Chapline and N.S. Manton, Phys. Lett. 120B, 105 (1983).