CERN-TH.4191/85

MYSTERY OF THE $\delta(980)$

R.N. Cahn^{*)} and P.V. Landshoff⁺⁾

CERN - Geneva

A B S T R A C T

The apparent conflict between the dominance of the decay $\delta \rightarrow \eta\pi$ in $D \rightarrow \delta\pi$ and its absence in $\psi \rightarrow \delta\pi$ is analyzed. Explicit models are presented in which the nearby $K\bar{K}$ threshold plays an important rôle in resolving the conflict.

^{*)} Permanent address: Lawrence Berkeley Lab.

⁺⁾ Permanent address: DAMTP, Cambridge Univ.

CERN-TH.4191/85

June 1985

1. - INTRODUCTION

According to the 1984 edition of the Review of Particle Properties¹⁾, "the $\psi(1440)$ is largely coupled to the $\delta(980)\pi$ decay channel, although the lack of a signal in the $\eta\pi\pi$ system is a source of concern". This statement was based on data published²⁾ in 1983 by the Crystal Ball experiment at SPEAR; more recently, Mark III has improved its data and still finds no $\eta\pi\pi$ signal^{3),4)}. It is of some interest to resolve this mystery, not only in order to understand the δ but also because it may shed some light on the nature of the ψ - it has been suggested that it is either a glueball⁵⁾ or a radial excitation of a two-quark state⁶⁾.

In this paper, we do not solve the problem, but we explore some possible resolutions. In particular, we suggest the possibility that the relative frequency of the decay modes $\delta \rightarrow K\bar{K}$ and $\delta \rightarrow \eta\pi$ varies from reaction to reaction, depending on how the δ was produced. To test this, more experiment is needed to study the δ in various different reactions, not just in ψ decay but also in $D(1285)$ decay.

We begin in Section 2 by reviewing the available data on the δ . We will see that even if we confine attention to a single reaction, such as D decay, there are considerable quantitative uncertainties. Nevertheless, it is clear that there is a structure around 980 MeV which decays into $\eta\pi$, and a $K\bar{K}$ enhancement near threshold is seen in a variety of experiments. These two are commonly assumed to be associated with a single resonance.

Whatever structure is responsible for the $K\bar{K}$ enhancement, it must be strongly coupled in order to be seen at all since there is so little phase space. In Section 3 we review the constraints imposed by unitarity and analyticity on resonances near a threshold. A convenient formalism is that of the two-channel K-matrix. Usually, a resonance is associated with a single pole of the S-matrix, but when there is a threshold nearby for a strongly-coupled channel, things become rather more complicated. In particular, the decay of a resonance need no longer be independent of its production.

We apply the two-body amplitude calculated through the K-matrix to discuss the resonant two-body subsystem of the final state in D or ψ decay. We use standard final-state-interaction theory, though we emphasize the ambiguities in its application.

In Section 4 we use this formalism to explore several possible explanations of the data. The most striking experimental fact is that the δ seen in D decay goes primarily to $\eta\pi$, while that in ψ decay goes mainly to $K\bar{K}$. Such behaviour cannot occur in the case of a resonance represented by a single pole far above threshold. One remedy is to suppose that we have not one resonance, but two, one just below the $K\bar{K}$ threshold and one just above. It is easy to give a quantitative explanation of the data in this way. Alternatively, we show that the proximity of the $K\bar{K}$ threshold can substantially alter the single-resonance picture, so that the decay of the resonance depends on how it was formed. We can then explain the data if in addition we suppose that the D is composed mainly of non-strange quarks, while the ψ has a dominantly strange-quark content (or else its primary decay, before the resonating final-state interaction, prefers a strange-quark channel, as has been suggested in the glueball picture⁷⁾).

2. - REVIEW OF THE DATA

(i) $\psi(1440)$ decay

The $\psi(1440)$ is seen prominently in the radiative decays of the J/ψ , $J/\psi \rightarrow \gamma\psi$. It is a $J^{PC} = 0^{-+}$ isoscalar^{4),8)}, like the $\eta'(958)$. It cannot decay into two or three pions, but is allowed to go to $\eta\pi\pi$, $K\bar{K}\pi$ or $\rho\rho$. (Four pions are allowed, but not all S-wave.) The $\rho\rho$ decay is hard to observe because of the large width of the ρ , but it may be important: indeed, there is an indication that even the $\rho\gamma$ mode is significant^{4),9)}.

Because $J/\psi \rightarrow \gamma\psi$, the ψ has $C = +$. One of its observed decays is $K^+K^-\pi^0$, whence the K^+K^- system must have $C = +$ and so $I = 1$, and hence the ψ must have $I = 0$ (or two if one were to accept the possibility of an isotensor photon!). The decays $\psi \rightarrow K^{\pm}K_s^{\mp}\pi^{\mp}$ are also seen, with similar $K\bar{K}$ mass distribution to that seen in the $K^+K^-\pi^0$ mode and with rates consistent with $I = 0$ for the ψ . In each case there is a pronounced peak near $K\bar{K}$ threshold. From the angular distribution of the photon and the orientation of the ψ in the J/ψ decay, it has been convincingly demonstrated⁴⁾ that the ψ has spin 0; looking again at the $K\bar{K}\pi$ system we deduce that this implies $P = -1$ for the ψ . The spin of the $K\bar{K}$ system must be even; because it is so near threshold it is natural to assume that it is S-wave. So we are considering a $J^{PC} = 0^{++}$ isovector enhancement in the $K\bar{K}$ channel. (Note that an $s\bar{s}$ state would be an isoscalar.)

If this enhancement is caused by the $\delta(980)$, it might be expected to lead also to a peak in the $\eta\pi$ mass distribution in the decay $\iota \rightarrow \eta\pi\pi$. The Mark III experiment finds⁴⁾ no sign of such a peak, and gives the limits

$$\frac{B(\iota \rightarrow \eta\pi\pi)}{B(\iota \rightarrow K\bar{K}\pi)} < 0.26, \quad (2.1)$$

$$\frac{B(\iota \rightarrow \delta\pi) B(\delta \rightarrow \eta\pi)}{B(\iota \rightarrow K\bar{K}\pi)} < 0.14.$$

(ii) D(1285) decay

The D(1285) is well established to be a $J^{PC} = 1^{++}$ isosinglet. In the quark model, it is naturally interpreted as the 3P_1 isosinglet combination of $u\bar{u}$, $d\bar{d}$. Its branching ratio to $K\bar{K}\pi$ is only¹⁾ $11 \pm 3\%$, so presumably its $s\bar{s}$ content is small. About half its decays are to $\eta\pi\pi$, and there is a prominent peak in the $\eta\pi$ mass at about 980 MeV. Taking all the experiments together¹⁾,

$$\frac{B(D \rightarrow \eta\pi\pi)}{B(D \rightarrow K\bar{K}\pi)} > 4.5 \pm 1.0 \quad (2.2)$$

and about 3/4 of the $\eta\pi\pi$ mode is $\delta\pi$. So when the δ is produced from D decay,

$$\frac{B(\delta \rightarrow \eta\pi)}{B(\delta \rightarrow K\bar{K})} > 2.5 \quad (2.3)$$

The actual branching ratio is probably somewhat greater: we have supposed that all the $K\bar{K}\pi$ decay of the D is $\delta\pi$.

If we were to combine the information in (2.1) and (2.3), we should conclude that the fraction of $\iota \rightarrow K\bar{K}\pi$ decays that are via $\iota \rightarrow \delta\pi$ is less than 6%. This would mean that the prominent enhancement seen in Fig. 1 cannot be ascribed to the δ , if the δ is a single resonance of conventional type.

(iii) The $\eta\pi$ peak

The $\delta(980) \rightarrow \eta\pi$ peak is seen not only in D decay, but also¹⁰⁾ in $K^-p \rightarrow \Sigma(1385)\eta\pi$ and¹¹⁾ $\gamma p \rightarrow \eta\pi X$. The Review of Particle Properties¹⁾ quotes its mass as 983 ± 2 MeV and its width as 54 ± 7 MeV. The peak is consistent in shape

with a Breit-Wigner form. It must be said, however, that none of the experiments could detect a structure of width less than about 20 MeV, given their limited statistics and resolution.

(iv) The $K\bar{K}$ peak

The first observation¹²⁾ of the enhancement in the isovector $K\bar{K}$ channel near threshold was in kaon pairs produced in $\bar{p}p$ annihilation at rest. The peak was observed both in $\bar{p}p \rightarrow K\bar{K}\pi$ and $K\bar{K}3\pi$; in the latter case it seems¹³⁾ that the reaction went via what we now know as the ρ . The $K\bar{K}$ enhancement is seen also¹⁰⁾ in $K^-p \rightarrow \Sigma(1385)K\bar{K}$, as well as in D-decay¹⁴⁾. In each case, the peak rises very sharply from threshold, rising to a maximum within about 20 MeV of threshold, and this in spite of the very limited phase space available. The peak is not symmetrical but nevertheless, when phase space is taken into account, the amplitude could be of Breit-Wigner form.

3. - COUPLED-CHANNEL FORMALISM

To describe the two coupled channels, $\eta\pi$ and $K\bar{K}$, it is convenient to use a K-matrix formalism¹⁵⁾, since in this formalism it is easiest to impose unitarity. Consider the matrix T formed from the scattering amplitudes

$$\begin{aligned} T_{11} &: \eta\pi \rightarrow \eta\pi \\ T_{22} &: K\bar{K} \rightarrow K\bar{K} \\ T_{12} &: \eta\pi \rightarrow K\bar{K} \\ T_{21} &: K\bar{K} \rightarrow \eta\pi \end{aligned} \quad (3.1)$$

Introduce the phase-space matrix ρ , whose off-diagonal elements are zero and whose on-diagonal elements are

$$\begin{aligned} \rho_1 &= \left[\left(1 - \frac{(m_\eta + m_\pi)^2}{s} \right) \left(1 - \frac{(m_\eta - m_\pi)^2}{s} \right) \right]^{1/2} \\ \rho_2 &= \left[1 - \frac{4m_K^2}{s} \right]^{1/2} \end{aligned} \quad (3.2)$$

We neglect the mass difference between the charged and neutral kaons, though in this context, where we are concerned with a resonance so very near threshold,

this approximation may not be quite as good as it at first seems. Consider the s-wave isovector amplitudes. Neglecting the coupling to five or more pions (a smaller number is forbidden), unitarity requires

$$\underline{A} = \underline{T}^{-1} + i\underline{\rho} \quad (3.3)$$

to be a Hermitian matrix. By suitable choice of the phase of the state $|\bar{K}\bar{K}\rangle$ relative to $|\eta\pi\rangle$, $\underline{A}$ may be taken to be real symmetric. In the usual notation,

$$\underline{A} = \underline{K}^{-1} \quad (3.4)$$

so that

$$\underline{T} = \underline{K} + i \underline{T} \underline{\rho} \underline{K}. \quad (3.5)$$

Our normalization convention is such that, for the S-wave,

$$\frac{d\sigma}{dt}(i \rightarrow j) = \frac{16\pi}{s^2 \rho_i^2} |T_{ij}|^2. \quad (3.5)$$

On inverting (3.3), we find

$$\underline{T} = \frac{\begin{bmatrix} a_2 - i\rho_2 & b \\ b & a_1 - i\rho_1 \end{bmatrix}}{(a_1 - i\rho_1)(a_2 - i\rho_2) - b^2} \quad (3.7)$$

Here a_1 and a_2 are the diagonal elements of the matrix $\underline{A}$, and b is its off-diagonal element. Each is a real analytic function of s , which in particular is analytic at both $s = (m_\pi + m_\eta)^2$ and $s = 4m_K^2$. For $s < 4m_K^2$, $-i\rho_2$ is real and positive. It is easy to verify that for $(m_\pi + m_\eta)^2 < s < 4m_K^2$, T_{11} satisfies single-channel unitarity.

We construct models for T by making simple choices for a_1 , a_2 and b . In doing so, we have to be careful not to give the denominator in (3.6) zeros at unwanted places: the resonance poles of $\underline{T}$ must all be on the unphysical Riemann sheet¹⁶⁾.

When we construct models, it is convenient to use ρ_2 as the variable rather than s . Because we are interested in only a small range of s -values near the $\bar{K}\bar{K}$ threshold, ρ_1 may be treated as effectively constant, $\rho_1 \approx 0.66$. The statement that $\underline{T}$ shall have no poles on the physical sheet corresponds to there being no

poles in the quadrant A of the complex ρ_2 plane (Fig. 2a), that is in the region $0 < \phi < 90^\circ$, where

$$\rho_2 = r e^{i\phi}. \quad (3.8)$$

The curve in the Figure is a portion of the curve $r^2 = \cos 2\phi$, across which $\text{Re}(s - 4m_K^2)$ changes sign. The region B is where the poles corresponding to resonances above the $K\bar{K}$ threshold are normally found, while those below the threshold appear in region C. However, if they are very close to the threshold they may be in D or E, respectively. The region F may also contain poles; usually these have no physical significance, but if they are very close to threshold they can be important.

In Figs. 2b and c we illustrate how one reaches the various regions by analytic continuation in the complex s plane. In Fig. 2b we start from the physical amplitude above the $K\bar{K}$ threshold, and in Fig. 2c we start from the physical amplitude below the threshold.

Having constructed models for the two-body $\rightarrow$ two-body scattering amplitude $\underline{T}$, we then use them to describe the final-state interaction in a three-body resonance decay. If we write the two decay amplitudes $\iota \rightarrow \eta\pi\pi$ and $\iota \rightarrow K\bar{K}\pi$ as a single-column matrix $\underline{M}$, we can impose unitarity in the $\eta\pi/K\bar{K}$ final-state subenergy by writing

$$\underline{M} = \underline{Q} + i \underline{T} \underline{Q}, \quad (3.9)$$

where $\underline{Q}$ then is free of the $\eta\pi$ and $K\bar{K}$ thresholds in that subenergy. This equation is the analogue of (3.5) and is easy to derive. If we use (3.5), we can rewrite (3.8) in the form

$$\underline{M} = \underline{T} \underline{K}^{-1} \underline{Q}. \quad (3.10)$$

In writing these equations, we are assuming that the η is not simultaneously resonating with both pions. In the approximation where the only thresholds that matter are those in the resonating subenergy, $\underline{Q}$ is real because it does not have those thresholds. Then, when that subenergy is below the $K\bar{K}$ threshold (3.9) tells us that the phase of $\underline{M}$ is just $e^{i\delta}$, where δ is the elastic-scattering $\eta\pi$ phase shift. This is the usual final-state interaction theorem¹⁷⁾. In order to

use this to calculate the energy dependence of M , it is necessary to make some assumption about the energy dependence of $K^{-1}Q$. In the absence of any further information, we shall assume that it is a valid approximation to replace the elements of the matrix $\tilde{K}^{-1}Q$ by constants in some range of the subenergy to either side of the $K\bar{K}$ threshold: this assumption is consistent with the requirement that they be real analytic functions that do not have this threshold. With this assumption, the energy dependence of $\tilde{M}$ will be the same as that of $\tilde{T}$: near the $K\bar{K}$ threshold

$$\tilde{M} = \tilde{T} \tilde{C}, \quad (3.11)$$

where $\tilde{C}$ is a constant single-column matrix.

A fuller treatment of final-state interactions requires¹⁷⁾ isolating the $\tilde{N}$ and $\tilde{D}$ matrices in the representation $\tilde{T} = \tilde{D}^{-1}\tilde{N}$ with some prescription, and replacing $\tilde{T}$ in (3.11) with $\tilde{D}^{-1}$. However, in our problem there is not enough dynamical understanding to do this.

The above analysis applies to the decay of the spinless ι . In the case of D decay, the spin of the D has an important effect. Taking account of the quantum numbers of the D , and taking the resonating particles to be S-wave, the amplitude must be of the form

$$\epsilon \cdot p_\pi \tilde{M} \quad (3.12)$$

where ϵ is the polarization vector of the D and p_π is the four-momentum of the non-resonating pion. The presence of the p_π in (3.12) is particularly important in the case of the $K\bar{K}\pi$ decay mode: it depresses the upper end of the $K\bar{K}$ mass spectrum. With this reinterpretation of $\tilde{M}$, we use the form (3.11) near the $K\bar{K}$ threshold for both ι and D decay.

We remark that the extreme upper end of the $K\bar{K}$ spectrum should be suppressed also in ι decay, because of an Adler zero in the amplitude¹⁸⁾. This means that it is not correct to use S-wave phase space together with a constant matrix element to estimate from the data the magnitude of the non-resonating background to the $K\bar{K}$ peak.

In order to give ourselves the best chance of explaining the difficulty that arises from (2.1) and (2.3), we shall assume that for ι decay, $C_1 = 0$,

while for D decay $C_2 = 0$. That is, we assume that the τ has some initial decay (probably depending on the short-distance structure of its wave function) which does not involve the δ and which for some reason prefers $K\bar{K}\pi$ to $\eta\pi\pi$. The δ is then generated solely by the final-state interaction. Likewise, the D has an initial decay that instead prefers $\eta\pi\pi$.

4. - MODELS FOR $\delta(980)$

We now review four possible models for the observed peak structure and compare them with experiment. In each case, we choose a simple form for the inverse K matrix $\underline{A}$ of (3.3): because we are interested in the region near the $K\bar{K}$ threshold, the resulting structure for $\underline{T}$ is complicated.

(i) Flatté's model

Flatté^{19),20)} uses a model for $\underline{T}$ which is a special case of our form (3.7). The matrix $\underline{A}$ is given the form

$$\underline{A} = \lambda \underline{G} + \underline{H} , \quad (4.1)$$

where $\underline{G}$ is a constant matrix such that $\det \underline{G} = 0$:

$$\underline{G} = \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix} . \quad (4.2)$$

Then λ is made to go to infinity, with the result

$$\underline{T} = \frac{\begin{bmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{bmatrix}}{f(s) - i\rho_1 g_1^2 - i\rho_2 g_2^2} . \quad (4.3)$$

Here $f(s)$ is a linear combination of the elements of the matrix $\underline{H}$ and can be any real analytic function. It may easily be verified directly that (4.3) indeed satisfies unitarity both above and below the $K\bar{K}$ threshold. Flatté chooses

$$f(s) = M^2 - s, \quad (4.4)$$

so that (4.3) is Breit-Wigner-like, but with an s -dependent "width".

The standard treatment is to take $g_1^2/g_2^2 = 2/3$, the SU(3) value. With²⁰⁾, $M = 974$ MeV and $g_1^2 \rho_1 = M\Gamma$ at $s = M^2$, where $\Gamma = 72$ MeV, we obtain the curves shown in Figs. 3a and 3b. It is clear that, while the shapes of these curves may be good, there is relatively far too much $\delta \rightarrow \eta\pi$ to agree with the branching ratios given in (2.1).

Changing the value of the ratio g_1^2/g_2^2 still cannot give agreement with experiment. This is because the form (4.3) factorizes for all s , in the sense that

$$\frac{T_{11}}{T_{12}} = \frac{T_{21}}{T_{22}}. \quad (4.5)$$

This implies that the branching ratio for the decay of the resonance is independent of the initial channel. As we have already seen, if we combine the information from D and τ decay, this seems to be incompatible with experiment: the "delta" in D decay prefers $\eta\pi$, while the "delta" in τ decay prefers $K\bar{K}$.

Within the Flatté model, we can improve the situation a little by supposing, as we have described in Section 3, that the τ has some initial decay that does not involve the δ and which for some reason prefers $K\bar{K}\pi$ to $\eta\pi\pi$. The effect of this, at least crudely, is to remove the portion of the $\eta\pi$ peak below the $K\bar{K}$ threshold and hence partially suppress the $\eta\pi$ mode. However, this only gives about 50% suppression and is not sufficient to give agreement with experiment. While we do not believe that the $\eta\pi$ peak will really be cut off sharply at threshold, if the peak is eventually found in τ decay it will be interesting to see whether it has been shifted to higher mass.

In order to compare with our other models, let us analyze the pole structure of the Flatté model. Because we are interested in s very near the $K\bar{K}$ threshold we may approximate $\rho_1 \approx 0.66$, as we have said before, and $\rho_2^2 \approx (s - 4m_K^2)/4m_K^2$. So the poles correspond to the solution of the quadratic equation

$$4m_K^2 \rho_2^2 + i g_2^2 \rho_2 + i g_1^2 \rho_1 = 0$$

and are at $\rho_2 = 0.14 - 0.33i$ and $\rho_2 = -0.14 + 0.16i$. In Fig. 2a, these are respectively in region D and in E close to the boundary with C. At each value of s , both contribute to the elements of the matrix $\underline{T}$ in the same proportions. Below threshold, the region-E pole dominates, but above threshold both are numerically important.

(ii) Two-resonance model

Another special case of (3.7) is to choose $b = 0$. Then $T_{12} = T_{21} = 0$ and

$$\begin{aligned} T_{11} &= \frac{1}{a_1 - i \rho_1} , \\ T_{22} &= \frac{1}{a_2 - i \rho_2} . \end{aligned} \quad (4.6)$$

We choose a_1 and a_2 such as to make each of these amplitudes Breit-Wigner-like:

$$a_i = \lambda_i - \mu_i \rho_i^2 \quad (i = 1, 2) . \quad (4.7)$$

The curves in Figs. 4a and 4b correspond to the choices

$$\begin{aligned} \lambda_1 &= -0.64 & \mu_1 &= 16 \\ \lambda_2 &= 0.18 & \mu_2 &= 2.3 \end{aligned} . \quad (4.8)$$

With $b = 0$ we have two independent resonances. Of course we do not need strictly to choose $b = 0$: we can introduce a very small interchannel coupling without changing the essential features. As long as the ρ is coupled primarily to the resonance in channel 2 and the D is coupled primarily to the resonance in channel 1, we can achieve the observed branching ratios. The physical explanation for the difference between the strengths of the couplings to the two resonances could be the same as we discussed in the context of the Flatté model. Nevertheless, because of possible mixing either at the level of the primary

decay or in the final-state interaction ($b \neq 0$), there is the possibility that in the $\eta\pi$ channel two distinct resonances will be seen, one just below threshold and the other above and overlapping the first resonance. This two-resonance model may not seem very economical, but it is more plausible if one of the resonances is some sort of exotic, such as a four-quark state²¹⁾.

With the values of the parameters given in (4.8), and with $b = 0$, the amplitude T_{11} has two poles in the ρ_2 plane, at $\rho_2 = \pm(-0.08+0.18i)$. In Fig. 2a these are in regions C and D respectively. In the s -plane these are at the same point, but, if we introduce a very small coupling b so that the $K\bar{K}$ threshold appears in T_{11} , on different sheets of the $K\bar{K}$ branch point. The two poles of the amplitude T_{22} are at $\rho_2 = -0.22i \pm 0.18$, that is in regions D and F respectively. The important poles numerically are those in regions C and D.

(iii) One-resonance models

In the two-resonance model above, either $b = 0$ or it is small enough for it to be neglected in the equation that gives the pole locations, namely the condition that the denominator of (3.7) vanishes. This equation then factorizes: one factor is associated with channel 1 and the other with channel 2.

In contrast with this, we now examine two models where the interchannel coupling b cannot be neglected in this equation.

In the first of the two models, we choose μ_1 in (4.7) to be 0. In the absence of interchannel coupling, this would make T_{11} essentially constant. However, because we take $b \neq 0$ the resonance that we put into channel 2 also affects channel 1. We show in Figs. 5a and 5b the curves corresponding to the choices

$$\begin{aligned} \lambda_1 &= 2.5 & \mu_1 &= 0 \\ \lambda_2 &= 0.17 & \mu_2 &= 3.5 \\ b^2 &= 1.0 & & (4.9) \end{aligned}$$

In this model it is not possible at the same time to obtain a sufficiently wide $\pi\eta$ peak and a sufficiently narrow $K\bar{K}$ peak, but the model does overcome the factorization problem of the Flatté model.

The poles of the model are at $\rho_2 = -0.05+0.14i$ and $\rho_2 = 0.05-0.42i$, in regions C and D respectively of Fig. 2a. In fact the pole of region C is numerically more significant. It is straightforward to see from (3.7) that, even if the complete amplitude $\tilde{T}$ does not factorize in the way that it does in the Flatté model, its residue at each pole always does factorize. If each pole has a factorizing residue, there are two ways in which the complete amplitude may fail to factorize. The first way, which is realized in the above one-resonance model, is that, although only one pole is numerically significant, there is also an important non-pole contribution. Here, both the numerator and the denominator of T_{11} are quadratic in ρ_2 , so that if we decompose T_{11} into partial fractions we obtain a constant term in addition to the poles. (If we were to switch off the interchannel coupling b , this constant would be the only part of T_{11} to survive.) It turns out that this constant gives an important contribution to T_{11} above the $K\bar{K}$ threshold, and is enough to break the factorization.

The other way in which $\tilde{T}$ may fail to factorize is for more than one pole to be important. Even though the residue of each pole factorizes, the sum need not factorize. This is the case in our second one-resonance model. In this model, the interchannel coupling b again is non-negligible, but the inputs to the two channels are essentially interchanged. It is a sufficient illustration of this model to introduce only two non-zero parameters:

$$\begin{aligned} \lambda_1 &= 0 & \mu_1 &= 17 \\ \lambda_2 &= 0 & \mu_2 &= 0 \\ \mathcal{L}^2 &= 0.07 \end{aligned} \tag{4.10}$$

which leads to the curves shown in Figs. 6a and 6b. The amplitude T_{11} vanishes at the $K\bar{K}$ threshold because we have taken $a_2 = 0$; if instead a_2 is non-zero but small this amplitude still displays a structure at threshold. Present data do not have sufficiently good resolution to detect whether there is such a structure in, say, the $\eta\pi$ channel in D-decay.

The model has three poles, at $\rho_2 = 0.18-0.15i$, $\rho_2 = -0.09+0.17i$ and $\rho_2 = 0.09-0.02i$. In Fig. 2a, these are in regions B, C and F. Although the regions B and C are, respectively, the regions in which resonances above and below the $K\bar{K}$ threshold are found, in a sense the first two poles do not correspond to separate resonances. This is because if we imagined reducing b to zero they would occur at the same value of s : it is the non-zero coupling which causes them to separate. The poles in region F are usually irrelevant and not directly associated with physical resonances. This is why we have described the

model as a one-resonance model. Nevertheless, because the pole in region F is so very near to the $K\bar{K}$ threshold, it does have an important influence on the behaviour of T_{22} and $T_{12} = T_{21}$.

A careful examination of the amplitudes shows that above the threshold the pole in region F dominates T_{22} and T_{12} . Below the threshold the poles in regions F and C both contribute significantly to these amplitudes. The behaviour of T_{11} is quite different. Above the threshold the effects of the poles in C and F tend to cancel, leaving the pole in region B to dominate. Below the threshold, the pole in C is the most important. All three poles are thus essential for the description. Note that this intricate structure arises from a very simple K-matrix, with only two non-zero parameters.

5. - CONCLUSIONS

The existing data for D and ι decay cannot be described by an amplitude that factorizes. The presence of the $K\bar{K}$ threshold very close to the $\delta(980)$ considerably complicates the structure of the amplitude. Although the model of Flatté¹⁹⁾ contains the correct $K\bar{K}$ threshold structure in a particularly simple way, it cannot agree with the data because it factorizes.

One way to take account of the required $K\bar{K}$ threshold structure is to construct the amplitude from a K matrix. With very simple forms for the K matrix, it is possible to construct more than one model that is consistent with the data. Although the residue at each pole has to factorize, the complete amplitude need not, either because more than one pole is important or because there is a non-negligible non-pole background.

One of our models simply has two distinct resonances, which are rather little affected by the presence of the threshold. In this model, the $\eta\pi$ channel of the δ formed in D decay should contain both these resonances, though they would overlap. Although two separate peaks may not be obviously visible, it is important to have high-resolution data to look for them.

In our other two models, there is a more complicated interplay between the poles and the threshold structure. In a sense, a single resonance is at work and corresponds to more than one important pole, thus the usual simple correspondence between poles and resonances is obscured. In the model that seems more

compatible with present data, the $\eta\pi$ channel of the δ from D decay has a minimum exactly at the $K\bar{K}$ threshold. In the other model, the $\eta\pi$ peak in D decay cuts off rather sharply at threshold and does not rise again. In both models, our assumption that the ι decays initially to $K\bar{K}$, with the resonance generated only by the final-state interaction, moves the $\eta\pi$ peak in ι decay to just above the $K\bar{K}$ threshold and makes it very narrow.

We consider these models to be representative of a large class in which there is complicated structure near the $K\bar{K}$ threshold. In order to pick out the model that is most appropriate, and to determine the nature of the δ , it is necessary to have much more high-resolution data for δ decay, particularly in the $\eta\pi$ channel, with the δ being produced in different reactions. One should study D decay, ι decay and any other process that can produce the δ copiously, such as photoproduction. Because of the complexity of the two-channel problem, the decay of the δ may depend on how it was produced. It is important to understand in detail both the decays $\delta \rightarrow K\bar{K}$ and $\delta \rightarrow \eta\pi$ in more than one production reaction. Such studies of the δ are important because it plays a central rôle in discussion of three outstanding issues in meson spectroscopy: the scalar nonet, four-quark states and glueballs.

ACKNOWLEDGMENTS

We are grateful to several of our experimentalist colleagues for discussions of their data, especially J. Richman, A. Donnachie and J.-P. Stroot. The research of R.N.C. was supported in part by the Director, Office of Energy Research, Office of High Energy Physics, Division of High Energy Physics of the U.S. Department of Energy under contract DE-AC03-76SF00098.

REFERENCES

- 1) Review of Particle Properties, Particle Data Group, C.G. Wohl et al., Rev. Mod.Phys. No 2, part II (1984).
- 2) C. Edwards et al. - Phys.Rev.Lett. 51 (1983) 859.
- 3) W. Toki - Proceedings of Summer Institute on Particle Physics, July 18-29, 1983, Dynamics and Spectroscopy at High Energy, Ed. P.M. McDonough, Stanford (1984), p. 471.
- 4) J. Richman - Ph.D. Thesis, California Institute of Technology (1984).
- 5) K. Ishikawa - Phys.Rev.Lett. 46 (1981) 978;
M. Chanowitz - Phys.Rev.Lett. 46 (1981) 981;
See also:
M. Chanowitz - Proceedings of Summer Institute on Particle Physics, July 27-August 7, 1981, ed. A. Mosher, Stanford (1982), p. 41;
M. Chanowitz - "J/ ψ Physics at BEPC", LBL-17930 (1984).
- 6) K.A. Milton, W.F. Palmer and S. Pinsky - in Proceedings of the XVIIth Rencontre de Moriond, Les Arcs, France (1982), Ed. J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, 1982).
- 7) M. Chanowitz - SLAC Summer Institute, Ref. 5), p. 77;
M. Chanowitz and S.R. Sharpe - Phys.Lett. 132B (1983) 413.
- 8) C. Edwards et al. - Phys.Rev.Lett. 49 (1982) 259.
- 9) C. Edwards - Ph.D. Thesis, California Institute of Technology (1984);
G. Szklary - XXII International Conference on High Energy Physics, Vol. 1, Leipzig, July 19-25, 1984, p. 180.
- 10) J.B. Gay et al. - Phys.Lett. 63B (1976) 220.
- 11) M. Atkinson et al., The Omega Photon Collaboration - Phys.Lett. 138B (1984) 459.
- 12) A. Astier et al. - Phys.Lett. 25B (1967) 294.
- 13) M. Chanowitz - Ref. 5).
- 14) H. Grässler et al. - Nucl.Phys. B121 (1977) 189.
- 15) See, for example, R.H. Dalitz and S.F. Tuan - Ann.Phys. 3 (1960) 307.
For a more recent discussion in the context of the $I = 0$, $\pi\pi$ and $K\bar{K}$ channels, see: F.J. Yndurain - Nucl.Phys. B88 (1975) 318.
See also: B.R. Martin, D. Morgan and G. Shaw - "Pion-Pion Interactions in Particle Physics", Academic Press, London, 1976, pp. 112-117.
- 16) Analyticity is discussed in: R.J. Eden, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne - "The Analytic S-Matrix", Cambridge University Press 1966, and R.G. Newton - "Scattering Theory of Particles and Waves", McGraw-Hill, New York (1966).
- 17) See: J. Gillespie - "Final State Interactions", Holden-Day Inc., San Francisco (1964).

- 18) See, for example: S. Adler and R. Dashen - "Current Algebras and Applications to Particle Physics", W.A. Benjamin, Inc., New York, 1968 pp. 109-147.
- 19) S.M. Flatté - Phys.Lett. 63B (1976) 224.
- 20) A.D. Martin, E.N. Ozmutlu and E.J. Squires - Nucl.Phys. B121 (1977) 514.
- 21) R. Jaffe and K. Johnson - Phys.Lett. 60B (1976) 201;
R. Jaffe - Phys.Rev. D15 (1977) 267; *ibid.*, p. 281;
R. Jaffe and F. Low - Phys.Rev. D19 (1979) 2105.

FIGURE CAPTIONS

- Fig. 1 The $K^0_S K^{\pm}$ invariant mass spectrum from $\psi \rightarrow \gamma K^0_S K^{\pm} \pi^{\mp}$. The $K^0_S K^{\pm} \pi^{\mp}$ mass is constrained to be in the ψ region: 1.30-1.58 GeV. The Figure is taken from Ref. 4). The distribution for $K^+ K^-$ in $\psi \rightarrow \gamma K^+ K^- \pi^0$ is similar. Note that the first bin extends below the kinematic threshold. Thus the rise of the amplitude at threshold is rather more abrupt than it appears.
- Fig. 2 (a) The complex ρ_2 plane. The origin corresponds to the $K\bar{K}$ threshold. Region A must not contain any singularities. Region B is the usual region for resonances above threshold. Region C contains resonances below threshold. Singularities in regions D, E and F can be important if they are near threshold.
 (b) Paths of analytic continuation in the complex s -plane to reach poles in regions A, B, D and F from the physical region above the $K\bar{K}$ threshold.
 (c) Paths of analytic continuation in the complex s -plane to reach poles in regions A, C, E and F from the physical region below the $K\bar{K}$ threshold.
- Fig. 3 Distributions for $K\bar{K}$ invariant mass (solid line) and $\eta\pi$ invariant mass (dashed line) in the Flatté model. See Eqs. (4.3)-(4.4).
 (a) For ψ decay, with a portion of the histogram for $\psi \rightarrow K^0_S K^{\pm} \pi^{\mp}$ from Ref. 4) (our Fig. 1). The abrupt lower edge for the $\eta\pi$ distribution results from our assumption that the initial primary decay of the ψ is to $K\bar{K}\pi$. In this model, $\Gamma(\psi \rightarrow K\bar{K}\pi)/\Gamma(\psi \rightarrow \eta\pi\pi) = \frac{1}{2}$.
 (b) For D decay. In this model, $\Gamma(D \rightarrow K\bar{K}\pi)/\Gamma(D \rightarrow \eta\pi\pi) = 1/7$.
- Fig. 4 Distributions for $K\bar{K}$ invariant mass (solid line) and $\eta\pi$ invariant mass (dashed line) in a model with two unmixed resonances. See Eqs. (3.7), (4.6)-(4.8).
 (a) For ψ decay, with a portion of the histogram for $\psi \rightarrow K^0_S K^{\pm} \pi^{\mp}$ from Ref. 4) (our Fig. 1).
 (b) For D decay.
- Fig. 5 Distributions for $K\bar{K}$ invariant mass (solid line) and $\eta\pi$ invariant mass (dashed line) in the one-resonance model of Eq. (4.9).
 (a) For ψ decay, with a portion of the histogram for $\psi \rightarrow K^0_S K^{\pm} \pi^{\mp}$ from Ref. 4) (our Fig. 1). The abrupt lower edge for the $\eta\pi$ distribution results from our assumption that the initial primary decay of the ψ is to $K\bar{K}\pi$. In this model, $\Gamma(\psi \rightarrow K\bar{K}\pi)/\Gamma(\psi \rightarrow \eta\pi\pi) = 2$.
 (b) For D decay. In this model, $\Gamma(D \rightarrow K\bar{K}\pi)/\Gamma(D \rightarrow \eta\pi\pi) = 1/8$.
- Fig. 6 Distributions for $K\bar{K}$ invariant mass (solid line) and $\eta\pi$ invariant mass (dashed line) in the one-resonance model of Eq. (4.10).
 (a) For ψ decay, with a portion of the histogram for $\psi \rightarrow K^0_S K^{\pm} \pi^{\mp}$ from Ref. 4) (our Fig. 1). The abrupt lower edge for the $\eta\pi$ distribution results from our assumption that the initial primary decay of the ψ is to $K\bar{K}\pi$. In this model, $\Gamma(\psi \rightarrow K\bar{K}\pi)/\Gamma(\psi \rightarrow \eta\pi\pi) = 5$.
 (b) For D decay. In this model, $\Gamma(D \rightarrow K\bar{K}\pi)/\Gamma(D \rightarrow \eta\pi\pi) = 1/7$.

- Figure 1 -

- Figure 2 -

- Figure 3a -

- Figure 3b -

- Figure 4a -

- Figure 4b -

- Figure 6a -

- Figure 6b -

- Figure 5a -

- Figure 5b -