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COLOUR DIELECTRICS - A WAY TO UNDERSTAND QUARK BAGS

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A B S T R A C T

We discuss the colour dielectric model which introduces as new effective variable the colour dielectric field for quark confinement. The model may be constructed from the QCD action and leads to the very successful bag model of hadrons. It allows to calculate the change of nucleon properties in nuclei which the EMC effect indicates.

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1. CALCULATING THE BAG FROM LATTICE GAUGE THEORY

Everybody is familiar with dielectrics from its first course in electrodynamics. A macroscopic amount of matter has of the order of 10^{23} charges in it, therefore it is impractical to solve the Maxwell's equations from the charge and current distributions of all these charges. It is better to introduce a dielectric constant characterizing the medium. In QCD not only the quarks have colour charges but also the gluons carry colour. So even vacuum will have a colour dielectric field. From the macroscopic Hamiltonian

$$\mathcal{H} = \int \sum_{i=1}^8 \frac{1}{\epsilon} [\vec{D}_i^2 + \vec{H}_i^2] d^3 \vec{x}$$

it is easily seen that colour displacement fields $\vec{D}_i$ and magnetic fields $\vec{H}_i$ cannot subsist in a medium where $\epsilon = 0$, because the energy associated with them would be infinite. The vacuum therefore has $\epsilon = 0$. The dynamics of confinement is reduced to find the value of the dielectric field in every point in space around the colour charges. The region with $\epsilon = 0$ forms a bag around the $\epsilon \approx 1$ region inside of the hadron, where colour is present and not influenced by large scale vacuum fluctuations.

The colour dielectric model^{1,2} proposes a local order parameter $\chi(x)$ which is related to the dielectric constant ϵ as $\epsilon = \chi^4$ and can be calculated by averages over the gluon charges in a small volume L^4 :

$$\chi_L(x_0) = \frac{1}{N} \text{Tr} \frac{a}{L^4} P \exp(i \oint_{x_0} A_\mu dx^\mu) \quad (1)$$

On the lattice with a lattice constant a , one starts with all Wilson loops which fit into a box $L^4 = (2a)^4$ and pass through x_0 . From the moments

$$\langle \chi_L^n \rangle = \int d(U) e^{-S_{\text{QCD}}(U)} \chi^n(U) \quad (2)$$

given by the integral over all link variables $U_{ij} = \exp i \int_{x_i}^{x_j} A_\mu dx^\mu$ one reconstructs the effective action of the χ -field given by a kinetic term and a fourth order polynomial potential $U(\chi)$

$$\mathcal{L}_L(\chi) = \frac{1}{2} \sigma_V^2 (\partial_\mu \chi)^2 - U(\chi) \quad (3)$$

Note after the first averaging step $\langle \chi_L \rangle \neq 0$, so one has to restart the procedure by calculating the moments

$$\langle \chi_{2L}^n \rangle = \int d\chi d(U) e^{-S_{QCD}(\chi U) - \int \mathcal{L}_L(\chi) d^4x} \chi_{2L}^n$$

for a larger cell of size $(2L)^4 = (4a)^4$.

Hopefully, this procedure converges fast when $1/nL \rightarrow \Lambda_{QCD} \approx 0.2$ GeV. The effective potential $U(\chi)$ determines, e.g., whether confinement favours surface dominated (SLAC) or volume (MIT) bag solutions. In the effective Lagrangian also the quarks can be included using local gauge invariance. I do not know how to integrate out the quark fields. When the scale size becomes very large, probably chiral fields $\vec{\pi}(x)$ and $\sigma(x)$ should be considered. The success of quenched QCD calculations attributes a less important rôle to these fields. The effective Lagrangian of the colour-dielectric model is:

$$\mathcal{L} = \mathcal{L}(\chi) + i \chi(x) \bar{\psi} \gamma_\mu (\partial^\mu - iB^\mu/\chi) \psi - \bar{\psi} \psi m_q - \frac{1}{4g^2} \chi^4 F_{\mu\nu} F^{\mu\nu} \quad (4)$$

with

$$F_{\mu\nu} = (\partial_\mu - iB_\mu/\chi) B_\nu/\chi - (\partial_\nu - iB_\nu/\chi) B_\mu/\chi$$

and m_q as current mass. The effective potential $U(\chi) = 0$ at $\chi = 0$ and can be expanded as

$$U(\chi) = \frac{1}{2} \partial^2 U / \partial \chi^2 \Big|_{\chi=0} \chi^2 = \frac{1}{2} m_{GB}^2 \sigma_V^2 \chi^2.$$

The mass m_{GB} of lightest 0^+ bound state in pure gluon theory is given by the correlation length of the χ field in the effective Lagrangian.

Before discussing experimental consequences of this model, let me place it in relation to the others mentioned in this meeting. Of course, the colour dielectric model is closest to the other bag models³, like the MIT and cloudy

bag models and the non-topological soliton model. In contrast to the first, it introduces a dynamical field to describe confinement and can be used in scattering calculations. It is very similar to the Friedberg-Lee soliton model⁴, the main difference consists in the coupling of the soliton field. In the colour dielectric model the effective quark mass m_q/χ is identical to the current quark mass inside the bag ($\chi=1$) and infinite outside ($\chi=0$). In the Friedberg-Lee model the quark mass is large outside of the bag ~ 1 GeV and negative inside. This last feature does not go well with asymptotic freedom. In colour dielectric theory, one can calculate the string tension, once $U(\chi)$ is given. One also obtains the profile of the string, which in the strong coupling models is infinitely thin. In spectroscopy, the Roper resonance gives an indication for collective bag motion⁵. Fake six-quark states disappear when dynamical confinement is taken into account for NN-scattering⁶.

2. TESTING THE BAG INSIDE AND OUTSIDE OF THE NUCLEUS

There has been a lot of fine work⁷ with regard to low energy properties of hadrons in bag models. The important feature of the quark bag model is its relation to deep inelastic scattering⁸. Hopefully a translational invariant description helps to avoid the infinities⁹ of the static bag model, when one calculates sea quark distributions. The valence quark distributions in the colour-dielectric model are essentially given by the confinement radius, which separates the inside $\chi = 1$ region from $\chi = 0$ outside for a free nucleon. The colour dielectric field in nuclear matter, however, changes to $\chi_N \approx 0.02 \neq 0$ outside of the bag and lets the quarks tunnel outside of the nucleon bags. The change of the χ -vacuum costs energy, the quark tunnelling wins energy, both effects together make the nucleon bound in the nucleus. The effective potential depths W_N of a nucleon in the nucleus and the rms confinement radius of the inflated nucleon can be related. We find, using the formulas of Ref. 2, the results of the following Table.

A	He ³	Be ⁹	C ¹²	Fe ⁵⁶	Au ¹⁹⁷
$\sqrt{\langle r^2 \rangle}$ [fm]	0.655	0.658	0.695	0.746	0.805
χ_N	0.0073	0.0076	0.0116	0.016	0.020
W_N [MeV]	-15.5	-16.1	-24.6	-33.9	-42.4
$\bar{\epsilon}$ [MeV]		-17 (Li ⁶)	-25	-36* (Ni ⁵⁸)	-44* (Pb ²⁰⁸)
ρ_{th} ($x=0.62$)	1.05	1.05	1.02	1	0.97
ρ_{exp}	1.09	1.06	1.05	1	0.95

The shift $\bar{\epsilon}$ of the quasi-elastic peak in electron scattering¹⁰ should be compared to the theoretical potential depth W_N . A change of the quark distributions in the nucleus is directly shown in the EMC and SLAC results¹¹. At $x = 0.62$, the ratios $\rho = F_2(x, Q^2, A)/F_2(x, Q^2, Fe) \Big|_{x=0.62}$ are given in the last two rows of the Table. They are calculated with a scaling violation parameter $b_Y = -0.25$ and $\bar{\alpha}_s/\alpha_s(Q^2=10 \text{ GeV}^2) = 0.8$ extrapolated from $\bar{\alpha}_s/\alpha_s(Q^2=50 \text{ GeV}^2)$ ¹² at EMC. We get a somewhat weaker A-dependence than the experimental one. This is not surprising. Indeed we left out any possibility for nucleons to overlap. To investigate this clustering effect we¹³ have done Monte Carlo simulations of nucleons distributed randomly over the nuclear volume, such that the density distribution is correct. Nucleons with their centres closer than d_c form clusters and thereby the confinement size also increases. If we calculate for all the nuclei the modified confinement size as a function of d_c and then fit to the SLAC data, we find $d_c \approx 1.3 \text{ fm}$ as critical centre-centre distance. At this d_c the average probabilities for a n-quark cluster in Fe⁵⁶ are $p_3 = 45\%$, $p_6 = 20\%$, $p_9 = 11\%$ and $p_{12}+p_{15}+\dots = 24\%$. Certainly a calculation based on six-quark clusters alone can be quite misleading. In fact we interpret these high cluster probabilities as an indication that with increasing effective radius of the nucleon, the quarks percolate through the nucleus and the nucleus becomes colour-conducting as proposed in Ref. 12.

Experiments at large Q^2 make other models than the quark model an academic exercise. The Skyrme model would have to include an infinity of mesons to obtain scaling. At low energy, the bag models are less predictive. I expect, however, a qualitative description of the transition from low to high density nuclear matter. The leakage of colour given by $\chi_N = 0.02$ in the colour

dielectric model makes the quarks in the inter-nucleon medium strongly interacting ($O(g^2/\chi^4)$). It is only their small density which may allow a meaningful calculation of their interaction energy. This problem is currently investigated¹⁴.

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