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*Model Selection and
Multimodel Inference*

A Practical Information-Theoretic Approach
Second Edition

With 31 Illustrations

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