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TeV ELECTRON-POSITRON COLLISIONS WITH LUMINOSITIES OF $10^{34} \text{cm}^{-2} \text{s}^{-1}$

USING THE CERN TUNNELS FOR THE DAMPING RINGS

Ugo Amaldi

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ABSTRACT

The fundamental equations of linear electron-positron colliders are used to obtain a new expression of the beam invariant emittance which depends upon very few parameters and is valid for all acceleration methods. Since we look for upper limits, emittance growth during acceleration and chromatic effects in the low- β focusing system are neglected. Even with these optimistic assumptions, for luminosities of the order of $10^{34} \text{cm}^{-2} \text{s}^{-1}$, at few TeV of center-of-mass energy, the required emittances are very small and many kilometers of damping rings are needed to damp the positron (and electron) bunches. It is suggested that few rings having a large circumference are better suited than a very large number of small rings.

1. INTRODUCTION

It is by now generally recognized that parton-parton collisions at center of mass energies as large as 1-2 TeV are needed to collect new experimental data which could help in understanding the many mysteries of the standard model. Present techniques allow the acceleration of quarks and gluons to such energies and plans have already been made both in USA and in Europe to move along this line. As far as electron-positron collisions are concerned, linear colliders are the only possible scheme of acceleration in this energy range, and the SLC under construction in Stanford⁽¹⁾ will hopefully demonstrate within few years the possibilities of this new technique. Even before this much needed proof, it is worthwhile to look forward to colliders of much larger energies and prepare ourselves for their construction. This paper originates from the observation that soon we shall be faced not only with an energy barrier but also with an even more relevant luminosity barrier. One should not forget that, for all existing electron-positron colliders, the luminosity has been always smaller than $10^{32}\text{cm}^{-2}\text{s}^{-1}$.

Fig.1 shows the energy dependence of the total electron-positron cross-section. The by now very familiar landscape of few, isolated, very high peaks is unfortunately superimposed on the steeply falling slope which is dictated by the wave nature of the colliding particles. Of course we can hope to find in the TeV energy range another peak to climb, but for the moment we must prepare ourselves to study the pointlike nature of the produced particles with a cross section which is such that at $(1 + 1)$ TeV about 300 $\mu^+\mu^-$ pairs will be produced in one year (10^7 seconds) at a luminosity of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. W^+W^- pairs will be thirty times more abundant, but probably their production will not be, by then, very interesting and we cannot plan now an expensive accelerator for such a low $\mu^+\mu^-$ rate.

We conclude that by entering in the TeV energy range we have also to aim for luminosities of the order of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ or larger. In this paper we explore the consequences of such a request. After recalling in Section 2 the basic relations, in Section 3 we derive a new expression for the emittance of the electron and positron beams as a function of a few parameters (Eqs 13 and 20 and Fig.6). This expression allows us to conclude in Section 4 that, to get to $10^{34}\text{cm}^{-2}\text{s}^{-1}$ with relatively large beam powers, one needs emittances smaller than the SLC emittance by at least one order of magnitude. If one wants to work in the regime in which the beam-beam radiation is drastically reduced by quantum effects, the emittances have to be further reduced by at least a factor 10^3 .

Thus, the damping of the transverse emittance of the beams appears as a major problem in high-luminosity colliders, independently of the method used for acceleration. In Section 5 it is shown that to obtain such small emittances at the needed high bunch frequency the number of small damping rings to be built is very large. Few large rings seem much more interesting. In Section 6 the general case is discussed and, in particular it is shown that the ISR and the LEP tunnels are well suited to install cooling rings for a high-luminosity e^+e^- linear collider.

2. COLLIDER FORMULAE

This Section is devoted to a review of the relations between the various quantities used to describe linear electron-positron colliders. The knowledgeable reader can go directly to the next Section, after having noted eqs (8) and (9) which are extensions of previously known results and give the critical energy and the correction to the usual beamstrahlung treatment due to quantum effects.

Table 1 summarizes the symbols and the practical units used in this paper for all the relevant quantities.

Table 1. Symbols and practical units used in this paper

Quantity	Symbol	Practical unit
energy per beam	E	TeV
luminosity	L	$10^{33} \text{cm}^{-2} \text{s}^{-1}$
particles per bunch	N	10^{11}
peak bunch frequency	f	kHz
duty cycle	C	1
average bunch frequency	Cf	kHz
peak current	i	mA
average current = Ci	$\langle i \rangle$	mA
average power of a beam	P	MW
β -function in the crossing point (c.p.)	β	mm
x-transverse dimension in the c.p.(r.m.s.)	σ_x	μm
y-transverse dimension in the c.p.(r.m.s.)	σ_y	μm
aspect ratio = $\sigma_y/\sigma_x \geq 1$	R	1
invariant emittance = $\gamma\sigma^2/\beta$	ε	10^{-5}m
bunch length (r.m.s.)	σ_z	mm
disruption parameter	D	1
luminosity increase due to D	H(D)	1
beamstrahlung parameter	δ	1
effect of R on δ	$\Delta(R)$	1
effect of quantum correction on δ	Γ	1

For gaussian bunches in three dimensions which collide head-on the luminosity is

$$L = \frac{(Cf) N^2 H}{4\pi \sigma_x \sigma_y} \quad (1,1p)$$

(The "p" appearing in the number of the equation indicates, here and in the following, that the practical units of Table 1 have to be used.)

When an electron and a positron bunch meet the strong fields may have a focusing effect with a consequent increase of the luminosity. In eq.(1) H takes into account this pinch effect, computed at SLAC as a function of the "disruption parameter" D and shown in Fig.2^(2,3). For beams having the same transverse r.m.s. width in both dimensions Fig.2a shows that

$$R = 1: \quad D = 1 \quad H = 3.0 ; \quad D = 2 \quad H = 5.7 . \quad (2)$$

The disruption parameter, which is related to how many oscillations a particle will make as it passes through the opposite bunch, is defined as

$$D = \frac{r_e N \sigma_z}{\gamma \sigma_x \sigma_y} \frac{2 R}{1 + R} , \quad (3)$$

where r_e is the electron classical radius. Eq.(3) in practical units becomes

$$D = \frac{0.144 N \sigma_z}{E \sigma_x \sigma_y} \frac{2 R}{1 + R} . \quad (3p)$$

D cannot be too large otherwise instabilities will increase the beam size and reduce the luminosity.

When the bunches are not round so that the aspect ratio $R > 1$, the pinch factor H is smaller than for round beams. We shall use in the following the estimate

$$H(R) = H (1+R)/2R , \quad (4)$$

which represents reasonably well the results of numerical calculations made at Stanford⁽³⁾. Eq.(4) gives

$$R = 10: \quad D = 1 \quad H \approx 1.8 ; \quad D = 2 \quad H \approx 2.6 .$$

The average fractional energy lost by a particle due to synchrotron radiation in

the magnetic field of the opposite bunch is^(4,5)

$$\bar{\delta} = \frac{8}{21 \pi^{1/2}} \frac{r_e^3 N^2 \gamma}{\sigma_x \sigma_y \sigma_z} H \Delta(R) \quad (5)$$

where the factor H takes approximately into account the fact that the beamstrahlung is more intense when there is a pinch effect. Numerical calculations would be needed to take this effect properly into account. The function Δ (plotted in Fig.2b) describes the reduction of the beamstrahlung due to an aspect ratio R different from 1 and has been computed by Bassetti and Gygi-Hanney⁽⁴⁾:

$$\Delta = \frac{14 \arctan((3R^2 - 10 + 3/R^2)^{1/2} / (3R + 8 + 3/R))}{(3R^2 - 10 + 3/R^2)^{1/2}} \quad (6)$$

For $R \gg 1$ eq.(6) becomes $\Delta \approx 4.2/R$.

We are interested in the r.m.s. spread of the colliding particles due to the beamstrahlung phenomenon. As done in Ref.(1), we divide eq.(5) by $2\sqrt{2}$ to obtain the quantity generally called "beamstrahlung parameter":

$$\delta \approx \frac{4}{21 (2\pi)^{1/2}} \frac{r_e^3 N^2 \gamma}{\sigma_x \sigma_y \sigma_z} H \Delta, \quad (7)$$

which in practical units can be written as

$$\delta \approx 0.42 \frac{E L}{\sigma_z (Cf)} \Delta. \quad (7p)$$

This expression was derived with classical formulas for the radiation of a moving electron in the magnetic field produced by the opposite moving bunch. As recently discussed by Himel and Siegrist⁽⁵⁾ quantum correction may be important in some cases. We extended their calculation, which refers to round uniform beams, and obtained for the average critical energy of the radiated photons

$$E_c = \frac{\sqrt{3} r_e \hbar c \gamma^2 N H}{2 \sigma_z} \left(\frac{\Delta}{\sigma_x \sigma_y} \right)^{1/2}, \quad (8)$$

which becomes

$$E_c \approx \frac{0.185 E^2 N H}{\sigma_z} \left(\frac{\Delta}{\sigma_x \sigma_y} \right)^{1/2} \text{ TeV} \quad (8p)$$

when the natural units of Table 1 are used.

If $E_c \ll E$ the classical eq.(7) is valid. Otherwise a correction has to be applied. Generalizing the equations of Himel and Siegrist to the case of gaussian beams with $R \neq 1$, we find that to take into account "quantum effects" eq.(7) has to be multiplied by an approximate correction factor:

$$\Gamma = 1 / [1 + E_c / (3^{3/4} E)]^{4/3} \quad (9)$$

Finally, the average power (in MW) of each of the two beams is

$$P = (Cf) EN = 16 (Cf) EN \quad (10,10p)$$

which, combined with eqs (1p) and (3p), gives

$$P = \frac{4\pi mc^2 r_e \sigma_z}{D H} \frac{2 R}{1 + R} L = \frac{29\sigma_z}{D H} \frac{2 R}{1 + R} L \quad (11,11p)$$

This well known relation⁽⁶⁾ indicates that, to minimize the power for any given luminosity, one has to choose a large value of D and a small value of σ_z .

In designing a collider the energy E and the luminosity L are fixed a priori and the aspect ratio R may be chosen large to reduce, through Δ , the beamstrahlung parameter of eq.(7), even if this multiplies by a factor 2 the power, as shown in eq.(11).

3. THE EMITTANCE EQUATION

Eqs (1), (3), (7) and (9) (which multiplies eq.(7)) are the basis of the design of any e^+e^- collider which does not use the superposition of electron and positron bunches to reduce the disruption⁽⁷⁾. Such a possibility is disregarded in the rest of this paper, but may become important in future.

The choice of E , L , R , P , D , δ determines all the other parameters and, in particular, the invariant emittance in the x -plane $\epsilon_x = \gamma \sigma_x^2 / \beta_x$. In an ideal case the emittance does not vary during acceleration, but misalignments and other effects, which depend upon the type of linear accelerator, tend to increase it. Moreover, the final low- β focusing system is affected by chromatic aberrations, which are larger for smaller values of β_x : as a consequence a smaller initial emittance is required to compensate for these effects. Since we are interested in finding upper limits for the value of the invariant emittance before acceleration, from now on emittance blow-up and chromatic effects will be neglected, but it has to be kept in mind that the invariant emittances we shall obtain are upper limits for what will be really needed in any practical situation.

To have the largest possible luminosity, the β -value at the crossing point has to be as small as possible but still larger than the r.m.s. length of the beam σ_z by a factor of at least 5, so that we write

$$\beta_x = p \sigma_z \quad (p \geq 5) . \quad (12)$$

To avoid problems during acceleration we suppose to accelerate almost round beams of equal emittance ϵ in both planes and to obtain in the crossing point $\sigma_y / \sigma_x = R \geq 1$ by making $\beta_y = R^2 \beta_x = R^2 p \sigma_z$. In this case $\epsilon = \gamma \sigma_x \sigma_y / p R \sigma_z$ and by combining eqs (1), (3), (7) and (9) we obtain a new expression which clearly shows which are the important parameters and represents, as stressed above, an upper limit on the needed invariant emittance of the electron and positron bunches before acceleration:

$$\epsilon \approx \frac{21}{16(2\pi)^{3/2} r_e^3 E^2} \frac{H}{R\Delta} \frac{\delta}{p\Gamma} \left(\frac{P}{L}\right)^2 = \frac{0.145}{E^2} \frac{H}{R\Delta} \frac{\delta}{p\Gamma} \left(\frac{P}{L}\right)^2 . \quad (13, 13p)$$

This expression shows that the needed emittance is obviously proportional to

$p^{-1} \leq 0.2$ and varies as the square of the ratio between beam power and luminosity when quantum effects are negligible and $\Gamma \approx 1$. The scaling law of the emittance is completely different for $\Gamma \ll 1$, as we shall see at the end of this Section.

From the fundamental equations we can also derive all the other parameters:

$$\sigma_z = \frac{D}{4\pi mc^2 r_e} \frac{H(1+R)}{2R} \left(\frac{P}{L}\right) = \frac{D}{29} \frac{H(1+R)}{2R} \left(\frac{P}{L}\right) \quad (14, 14p)$$

$$Cf = \frac{8(2\pi)^{1/2} r_e^3 \gamma}{21 \sigma_z} \frac{\Delta \Gamma}{\delta} L = 0.42 \frac{E}{\sigma_z} \frac{\Delta \Gamma}{\delta} L \quad (15, 15p)$$

$$N = \frac{P}{(Cf) E} = \frac{P}{16 (Cf) E} \quad (16, 16p)$$

$$\sigma_x = (\epsilon \beta_x / \gamma)^{1/2} = 0.071 (\epsilon \beta_x / E)^{1/2} ; \sigma_y = R \sigma_x \quad (17, 17p)$$

It has to be noted that, for small beam powers and large luminosities, the bunch length σ_z of eq.(14) may be so small that it is not possible to design a low- β section which satisfies eq.(12) with $p = 5$. In this case the minimum value attainable for β will be chosen and in eq.(13) the value $p = \beta_{\min} / \sigma_z > 5$ has to be used. We shall come back to this point towards the end of the Section.

The strong R -dependence of the factors H/RA of eq.(13) and $H(1+R)/2R$ of eq.(14) are plotted in Fig.3 for some values of the disruption parameter. As a consequence the use of flat beams ($R > 1$) decreases both the required emittance and the bunch length. Since, as we shall see in the next Section, small emittances are not easy to get, we conclude that, for luminosities of the order of $10^{34} \text{cm}^{-2} \text{s}^{-1}$ and relatively small powers, round beams have to be preferred.

What happens when quantum effects are important? By combining eqs (8), (1), (14) and (15) we obtain

$$E_c / (3^{3/4} E) = q / \Gamma^{1/2} , \quad (18)$$

where the "quantum parameter" q measures the magnitude of quantum effects (α is the fine structure constant):

$$q = 3\sqrt{7} \left(\frac{\pi}{6}\right)^{3/4} \frac{r_e}{\alpha} \left(\frac{E\delta L}{DP}\right)^{1/2} = 2.39 \left(\frac{E\delta L}{DP}\right)^{1/2}. \quad (19, 19p)$$

By introducing eq.(18) in eq.(9) we obtain a nonlinear condition on Γ which has to be satisfied for the self-consistency of the equations: $\Gamma^{3/4} + q \Gamma^{1/4} = 1$. Its complicated exact solution can be approximated within $\leq 5\%$ by the simple expression:

$$\Gamma \approx 1 / \left[1 + \frac{4}{3} q \left(1 + \frac{4}{5} q \right) + q^4 \right] \quad (20)$$

which immediately indicates that there are two very different regimes. When the quantum parameter $q < 1$ (i.e. $P \gtrsim 10 E\delta L/D$) quantum effects are not important, the term q^4 is small, $\Gamma \gtrsim 1/3$ and the emittance of eq.(13) is approximately proportional to $(P/L)^2$, as anticipated. However, when $q \gg 1$ (i.e. $P \lesssim E\delta L/2D$) the first two terms in the denominator of eq.(20) can be neglected and, in this "quantum" regime,

$$1/\Gamma \approx q^4 = 3 \left(\frac{\pi}{2}\right)^3 \frac{r_e^4}{\alpha^4} \left(\frac{E\delta L}{DP}\right)^2, \quad (q \gg 1) \quad (21)$$

which, when introduced in (13), gives

$$\varepsilon \approx \left(\frac{7}{2}\right)^2 \left(\frac{\pi}{2}\right)^3 \frac{r_e}{\alpha^4} \frac{H}{R\Delta} \frac{\delta^3}{pD^2} \approx 4.72 \frac{H}{R\Delta} \frac{\delta^3}{pD^2} \quad (q \gg 1). \quad (22, 22p)$$

In this regime the required emittance (neglecting blow-up and chromaticity) does not depend on E , P , L !

Fig.4 and 5, obtained with eqs (13) - (20) for a typical set of parameters ($L = 10^{34} \text{cm}^{-2} \text{s}^{-1}$, $E = 1 \text{TeV}$, $D = 2$, $R = 1$, $\delta = 0.1$), show the flat behaviour of ε for low beam powers and prove that quantum effects are beneficial, because the bunch frequency is reduced. However, the limiting value of the emittance (which is obtained for $q \gg 1$ in the hypothesis that there is no limitation on the

smallness of the β -value at the crossing point) is very small: $\epsilon \approx 10^{-8} \text{m}$ for the parameters chosen (and it becomes ten times smaller for $R = 10$).

It is very important to remark once more that the emittance plotted in Fig.4 is required when the β -value at the crossing point is the one shown in Fig.5 and given by eq.(12) with $p = 5$. For instance, at $P = 1 \text{MW}$ one must have $\beta \approx 0.2 \text{mm}$; since such a small β -value cannot be obtained with negligible chromatic aberrations, the needed emittance has to be multiplied by the ratio $5/p$, which in some cases can be much smaller than 1. Fig.6 shows the power dependence of the emittance for various values of the minimum β -value. We conclude that for β -values of the order of one mm, which are already difficult to obtain even from a purely geometric point of view, the quantum effect does not really help to increase the required emittance with respect to the value computed without taking it into account.

In the quantum regime the expression for the frequency is also very different from eq.(15):

$$C_f = \frac{28}{3^{2/3} \pi} \left(\frac{2}{\pi}\right)^{1/2} \alpha^4 \frac{\Delta D}{H \delta^3} \frac{2R}{1+R} \frac{P}{E} = 0.373 \frac{\Delta D}{H \delta^3} \frac{2R}{1+R} \frac{P}{E} \quad (q > 1). \quad (23, 23p)$$

This formula and the curve of Fig.5 show that (i) frequencies as large as 10^2kHz are needed for $P \approx 2 \text{MW}$ and (ii) one has to choose $\Gamma \leq 10^{-2}$ to be helped by the quantum effect and have $C_f \leq 20 \text{kHz}$.

The different scaling laws which are applicable in the two regimes ($q \leq 1$ and $q > 1$) are clearly demonstrated by the changing slopes of the curves of Fig.4 and 5. It is necessary to underline that the effective energies with which the electrons and the positrons collide in the two regimes are different even if one keeps in the comparison the beamstrahlung parameter constant. Detailed calculations of the energy spectra after radiation are necessary before being sure that $\delta = 0.1$ is an acceptable value.

We conclude from Fig.6 that, having fixed L , δ and D , the needed emittance is a very rapidly increasing function of the beam power and that beam powers as large as 100MW are needed if one wants to reach $(1 + 1) \text{TeV}$ with $10^{34} \text{cm}^{-2} \text{s}^{-1}$ by using emittances similar to the one obtained in the SLC damping rings ($\epsilon \approx 2 \cdot 10^{-5} \text{m}$). With β -values of the order of few mm and beam powers $P \approx 1 \text{MW}$ quantum

effects are important in determining the bunch frequency and the upper limits on the emittance before acceleration are $\varepsilon \approx (10^{-9} \div 10^{-8}) \text{m}$, very small indeed.

4. GENERALITIES ON DAMPING RINGS

In the simplest linear colliders the accelerated beams are thrown away after one crossing, so that the minimum overall power needed is $P_t \approx 2P/\eta$, where η is the efficiency with which energy is transferred from the power plug to the two high energy beams. The upper scale in Figs.4 and 5 give the total power P_t computed with the optimistic assumption $\eta \approx 20\%$. We read from the graphs that to keep P_t smaller than $\sim 300 \text{ MW}$ (i.e. $P < 30 \text{ MW}$) and get $10^{34} \text{cm}^{-2} \text{s}^{-1}$ at $(1+1) \text{ TeV}$ with $D = 2$ and $R = 1$ one needs $\sigma_z < 1.2 \text{mm}$ and $\varepsilon < 2 \cdot 10^{-6} \text{m}$. The needed average repetition rate of the crossings is large: $Cf \approx 25 \text{ kHz}$. The emittance is thus ten times smaller than the SLC emittance and to work in the quantum regime one needs a further reduction of the order of 10^3 ($\varepsilon \approx 10^{-9} \text{m}$). In the rest of this Section we discuss the problem of how to obtain such small emittances by neglecting the possible blow-up of the emittance during acceleration.

Positron bunches produced by high energy electrons in a thick target have initial emittances $\varepsilon_0 \approx 10^{-2} \text{m}$ ⁽¹⁾. Similar emittances are obtained by using monochromatic photons of few tens of MeV produced in long undulators by high energy electron (and positron) bunches⁽⁸⁾. Thus in every scheme in which electron and positron bunches are used only once, the emittances of the low energy bunches have to be damped from $\varepsilon_0 \approx 10^{-2} \text{m}$ to $\varepsilon \approx 10^{-6} \text{m}$ (or 10^{-8}m) in suitable damping rings. Wiedemann and Steffen have studied two different types of damping rings^(9,10), which will be discussed in the next Section.

In general the waiting time t is defined as the time spent by each bunch of energy $E_D = \gamma_D mc^2$ in its damping ring before being extracted and accelerated. The ring is characterized by a damping time τ and an equilibrium emittance ε_D so that, after a time t , the emittance is

$$\varepsilon = \varepsilon_0 e^{-2t/\tau} + \varepsilon_D (1 - e^{-2t/\tau}) . \quad (24)$$

Table 2. Symbols and practical units used in the equations for the damping rings

Quantity	Symbol	Practical unit
number of e^+ (and e^-) damping rings	N_D	1
number of bunches per ring	N_B	1
circumference of each ring	$2\pi R_D$	m
energy of $e^\pm$ during damping	E_D	GeV
γ -factor at the energy E_D	γ_D	1
radius of curvature	ρ_D	m
damping time	τ	ms
waiting time of each bunch in its ring	t	ms
fraction of energy radiated per turn	$\Delta E_D/E_D$	1
total power radiated by the e^+ 's (and e^- 's)	P_D	MW
momentum dispersion	α_p	m
injection invariant emittance	ε_0	10^{-5}m
final invariant emittance	ε	10^{-5}m
equilibrium emittance for $t \rightarrow \infty$	ε_D	10^{-5}m
integral given in eq 21 along one ring	I_2	m^{-1}
integral given in eq 24 along one ring	I_5	m^{-1}
magnetic field in a bending magnet	B	tesla
length of each magnet	l_m	m
magnetic field in a wiggler	B_w	tesla
length of each magnet in the wiggler	l_w	m
total length of the wigglers in $2\pi R_D$	L_w	m

In a damping ring of length $2\pi R_D$ a particle radiates an average fraction of its energy per turn which is given by the expression (11):

$$\frac{\Delta E_D}{E_D} = \frac{2}{3} r_e \gamma_D^3 I_2 = 1.88 \cdot 10^{-15} \gamma_D^3 I_2, \quad (25, 25p)$$

where I_2 is the integral along the circumference of a function of the local radius of curvature ρ measured in m (see Table 2):

$$I_2 = \oint \frac{ds}{\rho^2}. \quad (26)$$

The damping time is inversely proportional to the fractional radiated energy

$$\tau = \frac{2/J_x}{(\Delta E_D/E_D)} \frac{2\pi R_D}{c}, \quad (27)$$

where J_x is the horizontal damping partition number (typically $J_x \approx 0.5-1$).

The coupled horizontal equilibrium emittance is (11)

$$\epsilon_D = \frac{55}{32\sqrt{3}} \frac{\hbar c}{1+K^2} \frac{\gamma_D^3}{mc^2} \frac{I_5}{J_x I_2} = \frac{3.83 \cdot 10^{-8}}{1+K^2} \gamma_D^3 \frac{I_5}{J_x I_2}, \quad (28, 28p)$$

where K is the coupling between horizontal and vertical oscillations (for full coupling $K \approx 1$) and

$$I_5 = \oint \frac{1}{|\rho|^3} (\alpha_p^2 + (\alpha_p' \beta_x - \beta' \alpha_p/2)) / \beta_x ds. \quad (29)$$

[As usual β' is the derivative with respect to s of the beta function β_x , and α_p is the momentum dispersion]. As shown in Table 2, I_5 is measured in m^{-1} .

Eqs (27) and (28) introduced in eq.(24) must give the desired emittance ϵ . Since $\epsilon_0 \approx 10^{-2} \text{ m}$, to obtain $\epsilon \approx 10^{-6} \text{ m}$ the limiting emittance ϵ_D has to be smaller than 10^{-6} m and the waiting time must be much longer than the damping time:

$$t/\tau \approx 5 \div 6. \quad (30)$$

On the other hand, eqs (15) and (20) fix the frequency f at which the bunches have to be extracted from the damping system and accelerated (and this frequency increases if one chooses a duty cycle C smaller than 1).

In general one will use N_D damping rings for electrons (and N_D for positrons), each containing at any moment N_B bunches in such a way that

$$N_B N_D = ft. \quad (31)$$

If in each of the $2N_D$ rings the average distance between the N_B bunches is ℓ_B , the total length of the damping system is

$$\ell_D = 2N_D N_B \ell_B = 2 (t/\tau) f \tau \ell_B . \quad (32)$$

To minimize ℓ_D it is necessary to chose ℓ_B as small as possible. At present to inject and extract a bunch one needs typically $\ell_B \geq 10\text{m}$. Less space would be needed if, as suggested by S.Tazzari, n bunches are stored at a small distance (say about 1m) and 10m are left free after the n bunches for their simultaneous injection and extraction. In the following we shall take conservatively $\ell_B \approx 10\text{m}$, but it is important to stress that schemes in which the average distance between the bunches is smaller require proportionally less damping rings.

The above equations allow also to compute the average power radiated in the damping rings. From eq.(27) we get for the waiting time the expression ($J_x = 1$)

$$t \approx \frac{2 (t/\tau)}{\Delta E_D / E_D} \frac{2\pi R_D}{c} , \quad (33)$$

where (t/τ) is given by eq.(30).

Since from eq.(31) the total number of electron (or positron) bunches, each made of N particles, at any time in all the rings is ft , the radiated power by the electrons (or the positrons) is

$$P_D = ft \left(\frac{\Delta E_D}{E_D} \right) E_D \frac{c}{2\pi R_D} N . \quad (34)$$

By using eq.(33) and introducing the practical units appearing in Tables 1 and 2, this power can be written as

$$P_D = 2 (t/\tau) f E_D N \approx 0.032 (t/\tau) f E_D N . \quad (35, 35p)$$

By comparing with eq.(10) we obtain the interesting result

$$\frac{P_D}{P} \approx \frac{2t/\tau}{C} \frac{E_D}{E} \quad (36)$$

which says that the fraction of the beam energy radiated in the damping rings is, in all cases, proportional only to the ratio E_D/E .

5. MANY SMALL DAMPING RINGS

In Ref 9, H.Wiedemann considered damping rings in which the focusing is obtained by simple FODO doublets in which the length of the quadrupoles is half of the length of the bending magnets. The bending radius ρ_D of the interspersed magnets of length l_m is such that they fill a fraction F_m of the circumference $2\pi R_D$. In the thin lens approximation he got:

$$\tau = \frac{\sin \mu}{0.0142g} \frac{1}{(Bl_m)^2}, \quad (37p)$$

$$\epsilon_D = 4.84 \frac{(Bl_m)^3}{F_m}, \quad (38p)$$

where $2\mu = 130^\circ$ is the optimum betatron phase advance per cell, l_m is in meters, g is the quadrupole strength in tesla/m and B is the magnetic field in tesla (see Table 2). By introducing these two expressions in eq.(24) with $\epsilon_0 = 10^{-2} \text{ m}$ and differentiating with respect to $X = Bl_m$ Wiedemann found the minimum emittance ϵ as a function of the waiting time t :

$$\epsilon = \epsilon_D \left(1 + \frac{3}{4} \frac{\tau}{t} \right). \quad (39)$$

Once the maximum quadrupole strength is chosen ($g = 100 \text{ tesla/m}$) the emittance can be reduced only by decreasing the quantity $X = Bl_m$ and this implies a lengthening of the damping time.

For $2\mu = 130^\circ$, $g = 100 \text{ T/m}$, and $F_m = 0.5$ Wiedemann procedure yields for the quantity X , which gives the minimum emittance of eq.39, an implicit relation of the form:

$$X^2 = 1.70 [1 + 0.187 \ln (t/X)]/t, \quad (40p)$$

where t is measured in ms and $X = Bl_m$ in tesla.m (see Table 2). Once the value of B is chosen, Fig.7 gives, as a function of ε , (i) the optimum length l_m of each bending magnet, (ii) the waiting time t , (iii) the energy E_D of the bunches in the damping ring and (iv) the length $2\pi R_D$ of each ring. Once the desired emittance is chosen ($\varepsilon \approx 2 \cdot 10^{-6} \text{m}$ in the high power example we are pursuing) all the parameters of the rings are fixed. From Fig.7 with $B = 1.0 \text{T}$ we get $l_m = 0.25 \text{m}$, $t = 50 \text{ms}$, $E_D = 0.55 \text{ GeV}$ and $2\pi R_D \approx 11 \text{m}$. In such a ring one can damp only one bunch ($N_B = 1$) and with $f \approx 25 \text{kHz}$ (which is obtained from Figs 4 and 5 for $\varepsilon \approx 2 \cdot 10^{-6} \text{m}$ and $C = 1$) the number of rings needed to damp all the electron bunches is from eq.(31)

$$N_D \approx 1250 . \quad (41)$$

Thus for accommodating both the electron and the positron bunches one has to construct bending magnets 25cm long and quadrupoles 12.5cm long to cover a total length $l_D = 4\pi N_D R_D \approx 28 \text{km}$! This number would be proportionally reduced if one could make $l_B < 10 \text{m}$. In the quantum regime, $\varepsilon \approx 10^{-9} \text{m}$. Already for 10^{-8}m Fig.7 gives $Bl_m \approx 0.05 \text{ Tm}$ and $t \approx 1 \text{ s}$. The magnets should be 10cm long at $B = 0.5 \text{T}$ and the energy $E_D \approx 50 \text{ MeV}$ with a circumference $2\pi R_D \approx 2 \text{m}$. (Clearly such a ring will have problems with the Touschek effect). These numbers illustrate the difficulty of obtaining the small emittances needed to work in the quantum regime.

In Ref.10, K.Steffen proposed an unconventional magnetic structure to obtain fast radiation damping and small transverse emittance. The "wiggler storage ring" consists of densely packed sector magnets of alternated polarity that generate a wiggled closed orbit. The total circumference $2\pi R_D$ contains n magnets which bend outwards the particles of energy E_D with a radius of curvature ρ_D . The length of each of this "short" magnet is

$$l_m = \frac{\pi}{n} (R_D - \rho_D) . \quad (42)$$

Alternate with these there are n magnets of length $l_m + 2\pi\rho_D/n$ which bend inwards. The field index is chosen such that the horizontal and vertical oscillations have equal damping. The horizontal damping time of such a ring is

$$\tau \approx \frac{5.23}{B^2 E_D} \approx 0.47 \frac{\rho_D^2}{E_D^3} , \quad (43p)$$

and the emittance is

$$\epsilon_D \approx 2.75 \frac{\rho_D}{l_m} (B l_m)^3 . \quad (44p)$$

In the example given by Steffen $B = 1.67 \text{ T}$, $E_D = 1 \text{ GeV}$, $\rho_D = 2\text{m}$, $n = 300$, $R_D = 49.75 \text{ m}$ and eqs (42), (43) and (44) give

$$l_m \approx 0.50\text{m}, \tau \approx 1.9\text{ms}, \epsilon \approx 6.4 \cdot 10^{-5} \text{m} . \quad (45)$$

To decrease this emittance by a factor ~ 30 we have to decrease l_m , B and ρ_D . With $l_m = 0.20\text{m}$, $B = 1 \text{ T}$, $E_D = 0.45 \text{ GeV}$, $\rho_D = 1.5\text{m}$ we obtain

$$\tau \approx 12\text{ms} , \epsilon_D \approx 0.17 \cdot 10^{-5} \text{m} \quad (46)$$

so that the waiting time has to be $t \approx 60\text{ms}$. These figures show that, in spite of the very different lattices, the strong focusing damping rings of Wiedemann and the weak focusing ones of Steffen have very similar requirements and properties: with magnets of $\sim 25\text{cm}$ length at 1 tesla filling almost the total circumference and energies $E_D \approx 0.5 \text{ GeV}$ they have emittances about 10 times smaller than SLC with damping times of the order of 10ms. The Steffen ring has the advantage that the circumference $2\pi R_D$ is not linked to the radius of curvature ρ_D .

6. FEW LARGE DAMPING RINGS : THE ISR AND THE LEP CASE

We now consider few large wiggler storage rings to obtain emittances of the order of $2 \cdot 10^{-6} \text{m}$ at frequencies $f \approx 25\text{kHz}$, which will give the $10^{34} \text{cm}^{-2} \text{s}^{-1}$ luminosity with beam powers $P \approx 30\text{MW}$ (Fig.5 and Fig.6). With the parameters of eq.(46) the total length of the positron and the electron rings is given by eq.(32): $l_D \geq 3000 l_B \approx 30\text{km}$. In a tunnel having a circumference $2\pi R = 1\text{km}$, as the ISR tunnel, one should then install 15 rings for damping the positrons and 15 rings for damping the electrons. For a beam power $P = 30\text{MW}$ and a duty cycle $C = 1$ eq.(36) gives a small total radiated power:

$$P_D = 0.15\text{MW} = 0.01\text{MW/ring} \quad (P = 30\text{MW}) . \quad (47)$$

from eqs (28) and (53) we also obtain

$$I_5 \leq \frac{3\sqrt{3}}{1.2} \frac{32}{55} \frac{\alpha}{r_e} \frac{1+K^2}{p_{CD}} \frac{2}{1+R} \frac{t/\tau}{N_D} \frac{\ell_B}{c} \frac{1}{\gamma_D^6} \frac{P}{E} \approx 2.39 \cdot 10^{-3} \frac{1+K^2}{p_{CD}} \frac{2}{1+R} \frac{t/\tau}{N_D} \frac{\ell_B}{E_D^6} \frac{P}{E} \quad (55, 55p)$$

The 1.2 in the denominator takes approximately into account the difference between the limiting emittance ε_D and the equilibrium emittance ε after a waiting time t (see eq.(39)).

I_2 and I_5 are plotted in Fig.8 for the usual parameters ($L = 10^{34} \text{cm}^{-2} \text{s}^{-1}$, $E = 1 \text{TeV}$, $\delta = 0.1$, $D = 2$, $R = 1$) and for $C = 1$, $t/\tau = 5$, $\ell_B = 10 \text{m}$, $N_D = 1$, $K = 1$, $p = 5$. Any collider design has to obtain these values of the integrals I_2 and I_5 for each damping ring. Since I_2 and I_5 are proportional to $1/N_D$, the values plotted in Fig.8 can be also interpreted as the integrals of eqs (26) and (29) over all the rings used for the positron (or electron) bunches.

7. CONCLUSIONS

The main results of this paper are summarized by eqs (13) and (20) and in Fig.6, which gives the invariant beam emittance as a function of the beam power P and shows that, for β -values at the interaction point larger than about a mm, the needed emittance increases monotonically and very rapidly with power. For luminosities of the order of $10^{34} \text{cm}^{-2} \text{s}^{-1}$ at few TeV in the center of mass, the emittances and bunch frequencies required to work in the "quantum regime" ($P < 1 \text{MW}$) are very difficult to achieve. For larger powers ($P > 10 \text{MW}$) we have shown that one needs many kilometers of damping rings and we concluded that few very large damping rings appear to be more convenient than many small ones for producing the needed emittances ($\varepsilon \approx 10^{-6} \text{m}$). In particular the LEP tunnel is well suited to accommodate one or more damping rings.

These arguments do not depend on the type of linear collider envisaged, but follow from a straightforward application of the fundamental equations (1), (3) (7) and (9), and in particular of the expression (13) we have obtained for the emittance, and the damping rings considered here certainly deserve further detailed optimization. Whatever optimization procedure will be used, eqs (53) and (55) fix the values of the integrals $N_D I_2$ and $N_D I_5$ which have to be obtained over the

N_D positron (electron) rings once the linear collider is defined, i.e. once the luminosity L , the energy E , the beamstrahlung parameter δ , the disruption parameter D , the aspect ratio R , the beam power P and the energy of the damping rings E_D are chosen.

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8. REFERENCES

1. "Slac Linear Collider Conceptual Design Report" SLAC-Report-229, June 1980
2. B.Richter, Proc.of 1984 ICFA Seminar on "Future perspectives in high energy physics", KEK, May 14-20, 1984, S.Ozaki, S.Kurokawa and Y.Unno Eds, KEK, 1984, p.226
3. A.Minten and R.Hollebeek, private communication
4. M.Bassetti and M.Gygi-Hanney, LEP note 221, CERN (1980)
5. T.Himel and J.Siegrist, "Quantum effects in linear collider scaling laws", SLAC-PUB-3572, Feb 1985.
6. U.Amaldi, Proc.1979 Int.Symposium on Leptons and Photon Interactions at High Energy, T.B.W.Kirk and M.D.I.Abarbanel Eds, FNAL, 1979, p.314
7. V.E.Balakin and A.N.Skrinsky, Proc.Second ICFA Workshop on Possibilities and Limitations of Accelerators and Detectors, Les Diablerets, 4-10 Oct, 1979, U.Amaldi Ed., CERN, 1980, p.31
8. U.Amaldi and C.Pellegrini, Proc.Second ICFA Workshop on Possibilities and Limitations of Accelerators and Detectors, Les Diablerets, 4-10 Oct, 1979, U.Amaldi Ed., CERN, 1980, p.21
9. H.Wiedemann, Proc.of the 11th Int.Conference in High-Energy Accelerators, Geneva, July 7-11, 1980, W.S.Newman Ed., Birkhauser Verlag, 1980, p.693
10. K.Steffen, "The wiggler storage ring", Internal Report DESY PET 79/05, Feb 1979
11. M.Sands, "The Physics of Electron Storage Rings", SLAC Report 121 (1970)

FIG.1 The e^+e^- cross-section as a function of the center-of-mass energy. In drawing the peaks it was assumed that the momentum spread of the colliding beams is the one of the storage rings which have been working around that energy.

Fig.2 (a) The factor H which describes the effect of the pinch effect on the luminosity is plotted versus the disruption parameter D ^(2,3).

(b) The factor Δ , by which the beamstrahlung parameter is reduced, decreases with the aspect ratio R ⁽⁴⁾.

Fig.3 The quantities which enter in the emittance ($H/R\Delta$) and in the bunch length ($H(1 + R)/2R$) decrease with the aspect ratio R .

Fig.4 The beam power P is the natural quantity towards which to plot the parameters of a linear collider: here the emittance ϵ , the quantum correction Γ and the bunch length σ_z are shown. In drawing the continuous curve it was assumed that it is always possible to make $\beta_x = 5 \sigma_z$ (see Fig.6 for different hypothesis). Note that the SLC damping rings emittance is $\epsilon \approx 3 \cdot 10^{-5} \text{ m}$.

Fig.5 The average bunch frequency C_f , the bunch length σ_z and the number of particles per bunch N are plotted versus the beam power. The continuous curve shows the value of β assumed in plotting ϵ in Fig.4. (See Fig.6 for different hypothesis).

Fig.6 Upper limit on the required emittance ϵ as a function of the beam power P for various minimum values of the β -function at the crossing point: $\beta_{\min} = 0, 0.1, 1, 10 \text{ mm}$.

Fig.7 Parameters of the optimized damping ring designed by Wiedeman⁽⁹⁾ plotted versus the required emittance ϵ .

Fig.8 I_2 and I_5 are the integrals which determine the damping time and the equilibrium emittance. The curves show the minimum and the maximum values, respectively, for these two quantities (integrated over the total length of the rings) as a function of beam power P in the hypothesis that the average distance between bunches in the rings is $\ell_B = 10\text{m}$ and that $p = \beta_{\min} / \sigma_z = 5$. I_2 and I_5 decrease proportionally to ℓ_B and $I_5 \propto p^{-1}$. E_D is the energy of the electrons and positrons in the damping rings measured in GeV.