KALUZA-KLEIN APPROACH TO THE HETEROTIC STRING

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A B S T R A C T

Both the gravitational and gauge field sectors of the $d = 10$ superstring, including the Chern-Simons terms, may be obtained from the purely gravitational bosonic string in $d = 10 + \dim G$ via a spontaneous compactification on the group manifold G with $\langle H_{mnp} \rangle$ the parallelizing torsion. The Kaluza-Klein relation between the coupling constants $\alpha' g^2 \sim \kappa^2$ is that of the heterotic string. Consistency of the Kaluza-Klein ansatz is essential and reduces the symmetry from $G_L \times G_R$ to G_L .

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The most appealing feature of Kaluza-Klein theories is that what we perceive to be internal symmetries are really space-time symmetries in a higher dimension. In particular, it is not necessary to postulate the separate existence of Yang-Mills fields; they are an automatic consequence of gravity [1].

It is ironic therefore that the recent spectacular successes of superstrings [2,3,4] seem to ignore this beautiful concept. For although formulated in 10 rather than 4 dimensions, the Yang-Mills fields of $E_8 \times E_8$ or $SO(32)$ are present as primary fields already in the $d=10$ Lagrangian. Moreover, the favoured compactification to $d=4$ [4] yields no Kaluza-Klein gauge fields since Calabi-Yau manifolds have no continuous symmetries.

In this paper we show, in fact, that Kaluza-Klein can provide an explanation for the gauge fields of the heterotic string [3], as might have been suspected from the Kaluza-Klein like relation between the Yang-Mills coupling constant g , the gravitational constant κ and the slope parameter α' , namely

$\alpha' g^2 \sim \kappa^2$. (This is to be contrasted with the corresponding relation $g^2 \sim \kappa \alpha'$ of the open string [2] which appears to admit no such K-K interpretation). The Yang-Mills gauge group will, as usual, be a subgroup of the higher dimensional general coordinate group. Although the theory compactifies on a group manifold G , however, our Kaluza-Klein ansatz will involve only the gauge bosons of G_L and not those of the full isometry group $G_L \times G_R$. As discussed elsewhere [1,5,6,7,8] this is essential for the consistency of the Kaluza-Klein ansatz. Surprisingly, consistency is achieved without the introduction of Kaluza-Klein scalars.

Our starting point is the off-shell effective action Γ of Fradkin and Tseytlin [9] for the infinite set of local fields corresponding to the modes of a free bosonic string spectrum.

Γ is the generating functional of all possible off-shell scattering amplitudes on arbitrary backgrounds. In the case of closed oriented strings Γ is defined (in Euclidean signature) by

$$\Gamma[\hat{\phi}(x), \hat{g}_{MN}(x), \hat{B}_{MN}(x), \dots] = \sum_{\alpha=2,0,-2,\dots} e^{\sigma\alpha} \int [d\gamma_{ab}] [dx^M] e^{-S} \quad (1)$$

where σ is a dimensionless parameter and where

$$S = \int_{M_2} \frac{d^2z}{4\pi\alpha'} \left\{ \sqrt{\gamma} \gamma^{ab} \partial_a x^M \partial_b x^N \hat{g}_{MN}(x) + \alpha' \sqrt{\gamma} R(\gamma) \hat{\phi}(x) + \varepsilon^{ab} \partial_a x^M \partial_b x^N \hat{B}_{MN}(x) + \dots \right\} \quad (2)$$

Here $x^M(z)$ defines the embedding of the two dimensional string worldsheet M_2 in a spacetime M_d ($M, N=1, \dots, d$); z^a ($a=1, 2$) are the coordinates on M_2 ; $R(\gamma)$ is the curvature scalar of the metric γ_{ab} . The graviton $\hat{g}_{MN}(x)$, the antisymmetric tensor $\hat{B}_{MN}(x)$ and the dilaton $\hat{\phi}(x)$ correspond to the massless modes of the bosonic string spectrum. The dots refer to terms describing the higher spin massive modes and the scalar 'tachyon'. In writing the action (2), and in the remainder of the paper, we revert to Minkowski signature for both the worldsheet and for spacetime.

For string consistency, the two dimensional theory must be conformally invariant and hence the two-dimensional worldsheet stress tensor must be traceless. The general structure of the trace is given by

$$\Theta_a^a = \beta^\phi \sqrt{\gamma} R(\gamma) + \beta_{MN}^g \sqrt{\gamma} \gamma^{ab} \partial_a x^M \partial_b x^N + \beta_{MN}^B \varepsilon^{ab} \partial_a x^M \partial_b x^N \quad (3)$$

where β^ϕ , β^g and β^B are local functionals of $\hat{\phi}(x)$, $\hat{g}_{MN}(x)$ and $\hat{B}_{MN}(x)$. As noted by Polyakov [10] the first term is just the two-dimensional gravitational trace anomaly [11] while the second and third terms are present even in flat space and arise from the self-interaction. The β functions in the tree approximation ($\alpha=2$) have been calculated by Callan et al. [12]. They find

$$16\pi^2 \beta^\phi = \frac{d-26}{3\alpha'} + \left[4(\partial\hat{\phi})^2 - 4\hat{\Box}\hat{\phi} - \hat{R} + \frac{1}{12}\hat{H}^2 \right] + O(\alpha') \quad (4)$$

$$\beta_{MN}^g = \hat{R}_{MN} - \frac{1}{4}\hat{H}_M{}^{\rho\alpha}\hat{H}_{N\rho\alpha} + 2\hat{\nabla}_M\partial_N\hat{\phi} + O(\alpha') \quad (5)$$

$$\beta_{MN}^B = \hat{\nabla}_\rho\hat{H}^\rho{}_{MN} - 2\hat{H}^\rho{}_{MN}\partial_\rho\hat{\phi} + O(\alpha') \quad (6)$$

where $\hat{H}_{MNP} = 3\partial_{[M}\hat{B}_{NP]}$ and satisfies

$$\partial_{[Q}\hat{H}_{MNP]} = 0 \quad (7)$$

The absence of a trace anomaly, i.e. the vanishing of β^ϕ , β^g and β^B , is therefore just equivalent to the Einstein-matter field equations obtained from the effective action

$$\Gamma \sim \int d^2x \sqrt{-\hat{g}} e^{-2\hat{\phi}} \left[\frac{d-26}{3\alpha'} - \hat{R} - 4(\partial\hat{\phi})^2 + \frac{1}{12}\hat{H}^2 + O(\alpha') \right] \quad (8)$$

One obvious solution of the field equations corresponds to

$\langle \hat{\phi} \rangle = \text{constant}$, $\langle \hat{g}_{MN} \rangle = \eta_{MN}$ and $\langle \hat{H}_{MNP} \rangle = 0$, but this is valid only for $d=26$. For $d > 26$, the cosmological term in (4) obliges us to look for solutions in which some of the dimensions are compactified and we can now follow the traditional Kaluza-Klein interpretation. Accordingly we split the indices $x^M = (x^\mu, y^m)$ where x^μ ($\mu=1, \dots, n$) refers to spacetime and y^m ($m=1, \dots, k$) refers to the extra dimensions. One solution which suggests itself corresponds to the case where the extra dimensions are a group manifold G with $k = \dim G$. In this case

$$\langle \hat{g}_{mn}(x, y) \rangle = g_{mn}(y) = \frac{1}{C_A} c_{mpq} c_n{}^{pq} \quad (9)$$

where c_{mnp} are the structure constants of the group, converted to world indices, and C_A is the second-order Casimir in the adjoint representation. This will indeed be a solution provided

$$\langle \hat{\phi}(x, y) \rangle = \text{constant} \quad (10)$$

and

$$\langle \hat{H}_{mnp}(x, y) \rangle = H_{mnp}(y) = m C_{mnp} \quad (11)$$

where m is a constant with dimensions of mass. The left-invariant Killing vector fields $K_m^i(y)$ are normalized by

$$g^{mn} K_m^i K_n^j = \delta^{ij} \quad (12)$$

and satisfy

$$[K_m^i, K_n^j] = m c^{ij}{}_R K^R \quad (13)$$

where $K^i = K^{im} \partial_m$. With these normalizations H_{mnp} corresponds to the parallelizing torsion⁽¹⁾. The curvature is given by

$$\langle R_{mnpq} \rangle = \frac{m^2}{4} C_{mn}{}^{\ell} C_{pq\ell} \quad (14)$$

$$\langle R_{mn} \rangle = \frac{m^2}{4} C_A \langle g_{mn} \rangle \quad (15)$$

$$\langle R_k \rangle = \frac{m^2}{4} C_A R \quad (16)$$

where R_k is the curvature scalar of the k-dimensional group manifold. Substituting (9)-(11) into the field equations (4)-(7) we see that (5)-(7) are identically satisfied while (4) yields

$$d - 26 = R \alpha' m^2 C_A / 2 + O(\alpha'^2) \quad (17)$$

Our aim is to obtain the bosonic sector of the heterotic string in ten dimensions. Neglecting the higher derivative terms, this corresponds to the bosonic sector of the Chapline-Manton [14,15] N=1 supergravity with fields $\phi(x)$, $g_{\mu\nu}(x)$, $H_{\mu\nu\ell}(x)$ coupled to Yang-Mills fields $A_{\mu}^i(x)$ with $G = E_8 \times E_8$ or $SO(32)$. This means that $d=506$ and $k=496$. We shall now write down a Kaluza-Klein ansatz which achieves this goal, but for the sake of generality we allow G and the dimensions n and d to be arbitrary. For the scalar, we write

$$\hat{\phi}(x,y) = \text{constant} + \phi(x) \quad (18)$$

For the metric ansatz, we employ the 1-forms $\hat{e}^A = \hat{e}^A_M dx^M$ for which

$$\hat{g}_{MN} = e_M^A e_N^B \eta_{AB} \quad (19)$$

and write

$$\hat{e}^A(x,y) = e^A_\mu(x) dx^\mu \quad (20)$$

$$\hat{e}^a(x,y) = e^a_m(y) dy^m - m^{-1} K^{ia}(y) A^i_\mu(x) dx^\mu \quad (21)$$

The quantities $\phi(x)$, $e^A_\mu(x)$ and $A^i_\mu(x)$ will be interpreted as the scalar, vielbein and Yang-Mills gauge potential in n-dimensional spacetime. We must also make an ansatz for $\hat{H}_{MNP}(x,y)$. Experience with antisymmetric tensors in d=11 supergravity [1,16] suggests that this must also involve the Yang-Mills fields as well as the d=10 three-form $H_{\mu\nu\gamma}(x)$. Accordingly, we write

$$\hat{H}_{\alpha\beta\gamma}(x,y) = H_{\alpha\beta\gamma}(x) \quad (22)$$

$$\hat{H}_{\alpha\beta c}(x,y) = -m^{-1} F^i_{\alpha\beta}(x) K^i_c(y) \quad (23)$$

$$\hat{H}_{abc}(x,y) = 0 \quad (24)$$

$$\hat{H}_{abc}(x,y) = m c_{abc} \quad (25)$$

where $F^i_{\alpha\beta}(x)$ is the Yang-Mills field strength

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + c^i_{jk} A^j_\mu A^k_\nu \quad (26)$$

It is important to note that (α, a) are tangent space indices and (μ, m) are world indices and that $c_{abc} = c_{ijk} K^i_a K^j_b K^k_c$.

Several comments are now in order. That equations (18) to (25) correspond to the correct Kaluza-Klein ansatz can only be justified a posteriori. First they must be mathematically consistent and, as emphasized elsewhere [1,5,6,7,8] this is a highly non-trivial requirement. A 'consistent' ansatz is one for which all solutions of the n-dimensional theory are solutions of the original d-dimensional theory. For a generic Kaluza-Klein theory with homogeneous extra dimensions, a consistent truncation is achieved by an ansatz which retains all those fields and only those fields invariant under a transitively acting subgroup of the isometry group [6]. In particular, for group manifolds the gauge bosons are only those of G_L and not those of the full isometry group $G_L \times G_R$ [5]. In certain cases, it may not be necessary to retain all the G_R singlets. Indeed, surprisingly, our ansatz (18)-(25) is consistent in spite of omitting the usual Kaluza-Klein scalars. Secondly, equations (18) to (25) should be physically consistent in that we recover the desired n-dimensional theory. In our case that means, for $n=10$, that we recover the low energy limit of the heterotic string [3], i.e. the Chapline-Manton $N=1$ supergravity coupled

to Yang-Mills with gauge group G including, of course, the correct Yang-Mills Chern-Simons terms [14,15] .

Note that we have not specified the ground-state values of the spacetime fields $\phi(x)$, $g_{\mu\nu}(x)$, $H_{\mu\nu\zeta}(x)$ or $A^i_\mu(x)$. Since, by definition, a 'consistent' ansatz is one for which all solutions of the n -dimensional field equations are solutions of the original d -dimensional field equations, it is not necessary at this stage to single out any preferred configuration such as $g_{\mu\nu} = \eta_{\mu\nu}$, $\phi = H_{\mu\nu\zeta} = A^i_\mu = 0$. (For example, there can be other solutions such as Calabi-Yau manifolds.)

We now substitute the ansatze (18) to (25) into the field equations (4) to (7). Most of the relevant calculations may be found in Appendix A of [1] . The $\beta^g_{\mu\nu}$ equation becomes

$$R_{\mu\nu} - \frac{1}{4} H_{\mu}{}^{\zeta\sigma} H_{\nu\zeta\sigma} + 2 \nabla_\mu \partial_\nu \phi - \tilde{m}^2 F^i{}_{\mu\zeta} F^i{}_{\nu}{}^\zeta = 0 \quad (27)$$

The $\beta^g_{\mu n}$ equation becomes

$$D_\zeta F^i{}_{\mu}{}^\zeta - \frac{\tilde{m}^2}{2} H_{\mu}{}^{\zeta\sigma} F^i{}_{\zeta\sigma} - 2 F^i{}_{\mu}{}^\zeta \partial_\zeta \phi = 0 \quad (28)$$

where D_ζ is the Yang-Mills covariant derivative. Note that since our ansatz is invariant under the transitive action of G_R , all the y dependence of the ansatz disappears from the n -dimensional equations. Note also that half the Yang-Mills contribution to the energy-momentum tensor comes from the standard metric ansatz and an equal contribution from the anti-symmetric tensor ansatz⁽²⁾. The β^g_{mn} equation is satisfied identically. (This is the equation in which one might have expected to encounter inconsistencies by omitting the Kaluza-

Klein scalars $S^{ij}(x)$ since in a generic theory one obtains an equation of the form $\square S^{ij} \sim F^i_{\mu\nu} F^{j\mu\nu}$. In our case, however, the inclusion of the Yang-Mills fields in the ansatz (23) for $\hat{H}_{abc}$ exactly cancels this right hand side and hence S^{ij} may consistently be set to zero).

The $\beta^B_{\mu\nu}$ equation becomes

$$\nabla_\lambda H^\lambda_{\mu\nu} - 2 H^\lambda_{\mu\nu} \partial_\lambda \phi = 0 \quad (29)$$

the $\beta^B_{\mu n}$ equation simply reproduces (28); while the β^B_{nn} equation is satisfied identically.

Next we consider the β^Φ equation, which becomes

$$4 \partial_\mu \phi \partial^\mu \phi - 4 \square \phi - R + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \frac{m^2}{2} F^i_{\mu\nu} F^{i\mu\nu} = 0 \quad (30)$$

after using (17). Finally, equation (7) becomes

$$\partial_\mu [H_{\nu\lambda\sigma}] + \frac{3m^2}{2} F^i_{[\mu\nu} F^i_{\lambda\sigma]} = 0 \quad (31)$$

or, in terms of differential forms,

$$dH + m^2 F^i \wedge F^i = 0 \quad (32)$$

In the case $n=10$, the reader will no doubt recognize equations (27)-(32) as the bosonic sector of the Chapline-Manton $N=1$ supergravity with coupling constant K coupled to $N=1$ Yang-Mills with

coupling constant g , where we make the identification

$$g^2 = \kappa^2 m^2 \quad (33)$$

It is perhaps remarkable that in (32) we correctly reproduce the Yang-Mills Chern-Simons term in n dimensions even though there was no such term in d dimensions. (The appearance of the $F^i \wedge F^i$ term on reducing from d to n dimensions is known in the mathematical literature as a transgression).

Equation (33) is of the standard Kaluza-Klein form since m^{-1} is just the radius of the extra dimensions. As in $d=11$ supergravity, however, the numerical coefficient will differ from that obtained in pure gravity Kaluza-Klein theories [17] owing to the Yang-Mills content of the antisymmetric tensor ansatz [16] .

If, on the other hand, we compare equations (27)-(32) with the low energy limit (≤ 2 derivatives) of the heterotic string [12] , we find

$$m^2 \alpha' = 1 \quad (34)$$

Hence we recover the well-known relation [3] between κ , g and α' of the heterotic string

$$\alpha' g^2 = \kappa^2 \quad (35)$$

One major difference from generic Kaluza-Klein compactifications is that we can obtain an n -dimensional theory with

zero cosmological constant. The particular $\hat{\phi}$ dependence of the action (8) ensures that the Einstein equation (5) contains no cosmological term, while the cosmological term in the d-dimensional $\hat{\phi}$ equation cancels out in the n-dimensional ϕ equation.

By ignoring the $O(\alpha')$ terms in the d-dimensional equations we have obtained in this paper the Chapline-Manton 10 dimensional equations including the Yang-Mills Chern-Simons term. In a subsequent paper we shall show that including the $\alpha' \hat{R}_{mnpq} \hat{R}^{mnpq} + \dots$ terms in d dimensions, we can obtain the modified field equations of Candelas et al. [4] in 10 dimensions with their $\alpha' (R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} - F_{\mu\nu}^i F^{i\mu\nu})$ terms. This Lagrangian will contain not only the Chapline-Manton Yang-Mills Chern-Simons term but also the Green-Schwarz [2] gravitational Chern-Simons term ensuring both Yang-Mills and gravitational anomaly cancellations in the case $G = E_8 \times E_8$ or $SO(32)$; i.e. equation (32) becomes

$$dH + \alpha' (F^i \wedge F^i - R^{ab} \wedge R^{ab}) = 0 \quad (36)$$

One is then in a position to seek a second compactification from $n=10$ to $n=4$ of the kind discussed by Candelas et al. [4]. If so desired, of course, one could descend directly from $d=506$ to $n=4$ in which case the embedding of the Yang-Mills potential in the spin connection [18] employed by Candelas et al. [4] could be reinterpreted as a purely gravitational phenomenon. Since, in our compactification from $d=506$ to $n=10$ the $\hat{H}_{mnp}$ background was non-trivial, moreover, there seems to be no compelling reason to insist that it be zero in going from $n=10$

to $n=4$. An interesting question is whether one can attach physical significance to compactifications which go directly from $d=506$ to $n=4$ and which do not admit any intermediate $n=10$ interpretation. The answer to this question would seem to require a better understanding of how the fermions are incorporated.

It is interesting to note that whereas in d dimensions the α' expansion corresponds to an expansion in numbers of derivatives (i.e. zero slope limit = low energy limit), in n dimensions this correspondence breaks down. For example, in the present paper we obtain $\alpha' F^i_{\mu\nu} F^i{}^{\mu\nu}$ terms in n dimensions even though we ignored $\alpha' \hat{R}^{\hat{MNPQ}}_{\hat{MNPQ}}$ terms in d dimensions. In fact we conjecture that the group manifold with parallelizing torsion solves the field equations to all orders in α' and that the Kaluza-Klein interpretation applies to the full string theory and not merely to its field theory limit.

All this, of course, is entirely consistent with previous work on strings in curved space and non-linear σ models [19-24]. The Lagrangian (2) with the fields set equal to their background values, i.e. $\hat{\phi} = \text{constant}$; $\hat{g}_{mn}$ the metric on the group manifold; and $\hat{H}_{mnp}$ the parallelizing torsion is nothing but the non-linear σ model with Wess-Zumino term. According to Witten [20], this model has vanishing β functions provided $\alpha' m^2 = 1$, which is just equation (34). Furthermore, the critical dimension of these theories is given by [23,24]

$$d - 26 = \frac{R C_A}{2 + C_A} \quad (37)$$

(e.g. with $G = SO(32)$, $k=496$, $C_A = 60$ and hence $d=506$ and $n=10$).

On the other hand, our equation (17) yields

$$\lambda - 26 = \frac{k C_A}{2} + \dots \quad (38)$$

on using (34). This suggests that solving the β^ϕ equation to all orders in α' would yield the exact equation (37).

Note, however, that the ordinary bosonic string formulated on the group manifold G would yield states belonging to representations of the full isometry group $G_L \times G_R$ in contrast to the heterotic string where the states belong only to representations of G_L . In our Kaluza-Klein formulation this is taken care of by the consistency of the truncation in the Kaluza-Klein ansatz which demands that we retain only G_R singlets.

In this Kaluza-Klein picture, the $E_8 \times E_8$ or $SO(32)$ symmetry of the heterotic string is a subgroup of the general coordinate group in $d=506$. This is to be contrasted with the Frenkel-Kac-Goddard-Olive [25] approach where the gauge symmetry emerges from compactification of the $d=26$ string on the maximal torus T^{16} [26,3,27]. No doubt the two approaches are ultimately equivalent and it will be interesting to establish the appropriate dictionary. In particular one can ask whether the $n=10$ fermions can be incorporated in any natural way into the $d=506$ bosonic theory. For example, one has the feeling that the gravitational, Yang-Mills and mixed anomaly cancellations of Green and Schwarz in $n=10$ [2] may have a simpler pure gravity interpretation in $d=506$.

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FOOTNOTES

- (1) Setting $H_{mnp} \sim C_{mnp}$ was first attempted (albeit unsuccessfully) in [13] .
- (2) Similarly in d=11 supergravity [5] one quarter of the Yang-Mills stress tensor comes from the standard metric ansatz and three quarters from the antisymmetric tensor ansatz.

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