CERN-TH.4198/85

THE SCHWINGER-FOCK GAUGE IN SUPERSPACE

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A B S T R A C T

The Schwinger-Fock gauge is generalized to supersymmetric Yang-Mills theories. In this gauge the gauge connection can be expressed in terms of the field strength. The gauge connection is represented in terms of a) the unconstrained field strength, b) the constrained field strength. In case b) the prepotentials are given as well.

CERN-TH.4198/85

June 1985

1. - INTRODUCTION

There is quite a large number of problems in quantum field theory where only some of all the fields involved are treated as genuine quantum fields, whereas the rest is kept classical. Let us mention only two examples: the calculation of anomalies and the background field method. Practically, classical gauge fields are of particular importance. From a technical point of view, the calculation of physical quantities in the background of classical gauge fields is often quite cumbersome and not very transparent. Whereas most quantities are initially defined in terms of the gauge field $A_\mu(x)$, physical quantities only depend on particular gauge invariant combinations of the gauge field, like the field strength and derivatives thereof. Due to the larger degree of gauge freedom which supersymmetric gauge theories have, compared with ordinary gauge theories, this problem is even more severe in these theories.

As long as one is dealing with gauge invariant quantities one can obviously choose a gauge which is adapted to the problem at hand. The experience, which has been gained in conventional, non-supersymmetric gauge theories, has shown that it is often extremely convenient to choose a gauge that allows to express the gauge field in terms of the physical field strength¹⁾⁻⁴⁾. This gauge is called the Schwinger-Fock gauge.

In the present paper we shall generalize the Schwinger-Fock gauge to supersymmetric gauge theories. Supersymmetric gauge theories are usually formulated in terms of the gauge connections. In contrast to ordinary gauge theories, however, the gauge connections are not independent objects. It is a common practice to express the gauge connections in terms of the so-called prepotentials which represent the independent fields. We shall show how both the gauge connections as well as the prepotentials can be expressed in terms of the supersymmetric field strength. Whereas the generalized Schwinger-Fock gauge is complete with respect to the gauge connection, i.e., the gauge connection is determined uniquely, the prepotentials are only determined up to a chiral, respectively antichiral, gauge transformation. We shall explicitly construct the gauge transformation which transforms an arbitrary gauge connection to the generalized Schwinger-Fock gauge. For simplicity we shall consider only supersymmetric Abelian gauge theories. Some remarks concerning the generalization to non-Abelian gauge groups can be found in Section 5. Our notation is in accordance with Ref. 5).

Before turning our attention to supersymmetric gauge theories let us briefly recapitulate some of the essential features of the Schwinger-Fock gauge in ordinary gauge theories. The Schwinger-Fock gauge⁶⁾⁻⁸⁾ is defined by

$$(x-x_0)^a A_a(x) = 0, \quad (1.1)$$

where x_0 is an arbitrary reference point, which plays the rôle of a gauge parameter. The condition (1.1) explicitly breaks translation invariance. This invariance is restored in gauge invariant quantities. The condition (1.1) fixes the gauge completely, i.e., (1.1) is only maintained under a gauge transformation with x -independent gauge parameter. It can be shown¹⁾ that in the gauge (1.1) $A_a(x)$ is expressible in terms of the field strength $F_{ab}(x)$. We have

$$A_a(x) = (x-x_0)^b \int_0^1 dt t \bar{F}'_{ba}(x_0 + t(x-x_0)), \quad (1.2)$$

where $\bar{F}'_{ab}(x)$ is the field strength $F_{ab}(x)$, gauge transformed into the Schwinger-Fock gauge. Expanding $F_{ab}(x)$ around $x = x_0$ we get

$$A_a(x) = (x-x_0)^b \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{(n+2)} (x-x_0)^{a_1} \dots (x-x_0)^{a_n} \times \partial_{a_1} \dots \partial_{a_n} \bar{F}_{ba}(x_0). \quad (1.3)$$

In many applications it is sufficient to keep only a finite number of terms in (1.3). Any arbitrary gauge field $A'_a(x)$ can be gauge transformed into the Schwinger-Fock gauge by performing a gauge transformation with the path ordered phase factor

$$U(x) = P \exp \left[- \int_C d\xi^a A'_a(\xi) \right], \quad (1.4)$$

where P denotes path ordering and the path C is a straight line from x_0 and x

$$C: \xi(t) = x_0 + t(x-x_0) \quad 0 \leq t \leq 1. \quad (1.5)$$

Instead of imposing (1.1) as the starting point, we could have begun with the transformation (1.4) which would have immediately given (1.2) and (1.1). In this paper we shall pursue the second path.

The paper is organized as follows. In Section 2 we shall construct a gauge transformation with the help of path dependent phase factors in superspace, which generalizes (1.4). Though the formalism which we use is quite standard in supersymmetric Yang-Mills theories, respectively in supergravity, we shall

include all the necessary notations in order to make the present paper self-contained. We shall give the gauge connections in terms of the unconstrained field strength. In Section 3 we shall impose the usual constraints on the field strength, which allows us to express the gauge connections in terms of the prepotentials. We shall demonstrate how the prepotentials can be calculated from the constraint field strength. In Section 4 we shall generalize the expansion (1.3). Finally in Section 5 we shall present some concluding remarks. The proof that the gauge connection constructed from the field strength in Section 2 gives indeed back the correct field strength is delegated to a separate appendix.

2. - RECONSTRUCTION OF THE GAUGE CONNECTION

Path dependent phase factors are well-known objects in ordinary Yang-Mills theories. As their properties depend mainly on the geometrical interpretation of the gauge field, they are readily generalized to superspace. Such a generalization has been performed in Refs. 9) to 11). As the path dependent phase factors form the key ingredient for the reconstruction of the gauge connections in terms of the field strength we shall recall some of their features briefly.

Let us first fix our superspace notation. A point in superspace is labelled by z^M :

$$z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}}) \quad , \quad (2.1)$$

where x^m is the Minkowski space co-ordinate and θ^μ and $\bar{\theta}_{\dot{\mu}}$ are anticommuting Grassmann parameters. Therefore

$$z^M z^N = (-)^{mn} z^N z^M \quad , \quad (2.2)$$

where m, n is 1(0) if M, N is fermionic (bosonic). We employ the following convention for the scalar product in superspace

$$U^M V_M = U^m V_m + U^\mu V_\mu + \bar{U}_{\dot{\mu}} \bar{V}^{\dot{\mu}} \quad . \quad (2.3)$$

The derivative ∂_M is defined by

$$\partial_M = (\partial_m, \partial/\partial\theta^\mu, \partial/\partial\bar{\theta}_{\dot{\mu}}) \quad (2.4)$$

Denoting the gauge connection by ϕ^M

$$\phi^M = (\phi^m, \phi^\mu, \bar{\phi}_{\dot{\mu}}) \quad , \quad (2.5)$$

we define the covariant derivative $\mathcal{D}_M(z)$ through

$$\mathcal{D}_M(z) = \partial_M + \phi_M(z) \quad . \quad (2.6)$$

Gauge transformations are parametrized by an arbitrary superfield $\Lambda(z)$. Under a gauge transformation ϕ_M changes like

$$\phi_M(z) \rightarrow \phi_M(z) + i \partial_M \Lambda(z) \quad . \quad (2.7)$$

Let a path $\mathcal{C}$ in superspace be given by $\zeta(t)$, $0 < t < 1$ with endpoints $\zeta(0) = z_2$ and $\zeta(1) = z_1$, then the path dependent phase factor $W(z, \mathcal{C}')$ is defined by

$$\dot{\zeta}^M \mathcal{D}_M(z) W(z, \mathcal{C}') \quad , \quad (2.8)$$

with

$$W(z_2, \mathcal{C}) = 1 \quad . \quad (2.9)$$

z is a point on $\mathcal{C}$, $\mathcal{C}'$ denotes the portion of $\mathcal{C}$ from z_2 to z and the dot in (2.8) stands for differentiation with respect to the parameter t . Equations (2.8) and (2.9) are equivalent to

$$W(z, \mathcal{C}') = 1 - \int_{z_2}^z d\xi^M \phi_M(\xi) W(\xi, \mathcal{C}'') \quad , \quad (2.10)$$

which in general is symbolically solved by

$$W(z, \mathcal{C}') = \mathcal{P} \exp \left(- \int_{z_2}^z d\xi^M \phi_M(\xi) \right) \quad . \quad (2.11)$$

A gauge transformation (2.7) acts on $W(z, C')$ like

$$W(z, e') \rightarrow e^{-i\Lambda(z)} W(z, e') e^{+i\Lambda(z_2)} . \quad (2.12)$$

The covariant derivative of $W(z_1, C)$ with respect to the endpoint z_1 of C can be expressed as an integral involving the field strength $R_{MN}(z)$

$$R_{MN}(z) = \partial_M \phi_N(z) - (-)^{mn} \partial_N \phi_M(z) . \quad (2.13)$$

In an Abelian gauge theory we have

$$\partial_M(z_1)W(z_1, e) = W(z_1, e) \int_0^1 dt (\partial'_M f^N) \frac{df^P}{dt} R_{PN}(f) . \quad (2.14)$$

Multiplying (2.14) from the left by $W^{-1}(z_1, C)$, one discovers that

$$\phi_M(z_1) = \int_0^1 dt (\partial'_M f^N) (df^P/dt) R_{PN}(f) , \quad (2.15)$$

where $\phi_M(z_1)$ is obtained from the gauge connection, which has originally been employed to construct W , by a gauge transformation with parameter

$$\Lambda(z_1) = -i \log W(z_1, e) . \quad (2.16)$$

In order to reconstruct the gauge connection ϕ_M from the field strength R_{MN} , it is convenient to take the path C in (2.11) to be a straight line from z_2 to z_1

$$f(t) = z_2 + t(z_1 - z_2) . \quad (2.17)$$

This leads to

$$\phi_M(z_1) = \int_0^1 dt t (z_1 - z_2)^N R_{NM}(z_2 + t(z_1 - z_2)) , \quad (2.18)$$

which is the generalization of (1.2). Due to the relation

$$R_{MN}(z_1) = -(-)^{mn} R_{NM}(z_1) , \quad (2.19)$$

the gauge condition satisfied by $\phi_M(z_1)$ is

$$(z_1 - z_2)^M \phi_M = 0 . \quad (2.20)$$

This is the natural extension of the Schwinger-Fock gauge (1.1) to superspace. The relation (2.18) will be the starting point of our further investigations.

The derivatives ∂_M in the basis used so far do not have simple commutation relations with the generators Q of supersymmetry transformations. Therefore one usually goes over into a basis where the derivatives with respect to the Grassmann parameters are replaced by the covariant derivatives D_α and $\bar{D}^{\dot{\alpha}}$. Such a transformation is accomplished by the vielbein $E_A^M(z)$

$$E_A^M(z) = \begin{pmatrix} \delta_a^m & & \\ i(\sigma^m \bar{\theta})_\alpha & \delta_a^\mu & \\ i(\theta \sigma^m \epsilon)^{\dot{\alpha}} & & \delta^{\dot{\alpha}}_{\dot{\beta}} \end{pmatrix} . \quad (2.21)$$

We have

$$D_A = E_A^M \partial_M , \quad (2.22)$$

where

$$D_A = (\partial_a, D_\alpha, \bar{D}^{\dot{\alpha}}) , \quad (2.23)$$

with the usual covariant derivatives D_α and $\bar{D}^{\dot{\alpha}}$.

The vielbein $E_A^M(z)$ has a non-vanishing covariant derivative

$$D_A E_B^M = -\frac{1}{2} T_{AB}^M , \quad (2.24)$$

where

$$T_{AB}^C = \delta_A^\alpha \delta_{B,\dot{\beta}} \delta_C^{\dot{\gamma}} (-2i)(\sigma^c \epsilon)_\alpha{}^{\dot{\beta}} . \quad (2.25)$$

Therefore we get an extra contribution to the field strength in the flat space basis

$$R_{AB} = (-)^{abm(n-1)} E_A^M E_B^N R_{MN} \quad (2.26a)$$

$$= D_A \phi_B - (-)^{ab} D_B \phi_A - T_{AB}{}^d \phi_d . \quad (2.26b)$$

The Bianchi identity of R_{MN} which in the "curved space basis" reads

$$(-)^{ml} \partial_L R_{NM} + (-)^{ln} \partial_N R_{ML} + (-)^{mn} \partial_M R_{LN} = 0 \quad (2.27)$$

becomes in the flat space basis

$$\begin{aligned} (-)^{ac} D_A R_{dB} - (-)^{b(a+c)} D_B R_{dA} &= D_d R_{AB} \\ - T_{Ad}{}^p R_{pB} + (-)^a T_{Bd}{}^p R_{pA} - T_{AB}{}^p R_{pd} &= 0 . \end{aligned} \quad (2.28)$$

When we rewrite (2.18) in the new basis, we have to take into account that the vielbein depends on the position in superspace. In the flat space basis (2.18) reads

$$\phi_A(z_1) = \int_0^1 dt t \oint \mathcal{R}_{dD}(\mathcal{C}) M^D{}_A(\mathcal{C}, z_1) , \quad (2.29)$$

with

$$M^D{}_A(\mathcal{C}, z_1) = E^M{}_A(z_1) E_M{}^D(\mathcal{C}) , \quad (2.30)$$

and

$$\oint = (z_1 - z_2)^M E_M{}^d(\mathcal{C}) . \quad (2.31)$$

Here $E_M{}^A(z)$ is the inverse of the vielbein, defined by

$$E_N{}^A(z) E_A{}^M(z) = \delta_N^M . \quad (2.32)$$

Despite its appearance, the vector ξ^C does not depend on the parameter of the curve $\mathcal{C}, t$. It depends only on the endpoints of $\mathcal{C}$, z_1 and z_2 . An explicit calculation gives

$$\xi^A = \left((z_1 - z_2)^a - i \theta_1 \bar{\theta}_2^a + i \theta_2 \bar{\theta}_1^a , \Delta^a , \bar{\Delta}_a \right) , \quad (2.33)$$

where

$$\Delta^A = (\Theta_1 - \Theta_2)^A, \quad \bar{\Delta}_i = (\bar{\Theta}_1 - \bar{\Theta}_2)_i. \quad (2.34)$$

Note that the distance ξ^A is invariant under a supersymmetry transformation

$$Z^M = (x^m, \Theta^\mu, \bar{\Theta}_\mu) \rightarrow Z'^M = (x^m + i\Theta^\mu \bar{\epsilon} - i\epsilon \bar{\Theta}_\mu, \Theta + \epsilon, \bar{\Theta} + \bar{\epsilon}). \quad (2.35)$$

Contracting the two vielbeins in the definition (2.30) of M^D_A one gets

$$M^D_A = \delta^D_A + \tilde{N}^D_A, \quad (2.36)$$

where

$$\tilde{N}^D_A = (1-t) N^D_A, \quad (2.37)$$

$$N^D_A = \delta^D_\alpha [\delta^A_\mu (i\bar{\Theta}^\mu \Delta)_\alpha + \delta_{A,i} (i\Delta \bar{\Theta}^\mu)_i] . \quad (2.38)$$

The gauge condition (2.20) is readily rewritten in the flat space basis as

$$\oint^A \phi_A(z_1) = 0. \quad (2.39)$$

The verification that the gauge connection given by (2.29) indeed gives rise to the field strength R_{AB} is somewhat involved. Therefore we shall perform the proof in a separate Appendix.

3. - THE GAUGE CONNECTION IN TERMS OF THE CONSTRAINT FIELD STRENGTH

The component fields represented by R_{AB} are far too numerous for physical applications. It is therefore natural to reduce the number of independent components by imposing constraints on R_{AB} (5), (12), (13). In this section we shall derive the gauge connection which corresponds to the constraint field strength.

In order to obtain those degrees of freedom which are required to construct a supersymmetric Yang-Mills theory the following constraints have to be imposed on the components of R_{AB}

$$R_{\alpha\beta} = D_\alpha \phi_\beta + D_\beta \phi_\alpha = 0 \quad , \quad (3.1a)$$

$$R_{\dot{\alpha}\dot{\beta}} = \bar{D}_{\dot{\alpha}} \bar{\phi}_{\dot{\beta}} + \bar{D}_{\dot{\beta}} \bar{\phi}_{\dot{\alpha}} = 0 \quad , \quad (3.1b)$$

$$R_{\alpha\dot{\beta}} = D_\alpha \bar{\phi}_{\dot{\beta}} + \bar{D}_{\dot{\beta}} \phi_\alpha + 2i \sigma_{\alpha\dot{\beta}}^a \phi_a = 0 \quad . \quad (3.1c)$$

So far we have considered all the components of the gauge connection ϕ_A as independent degrees of freedom. The constraints (3.1a-c) can be solved by introducing two new independent fields U and V , the so-called prepotentials, and letting the components of ϕ_A become dependent on them. Equations (3.1a) and (3.1b) are satisfied by

$$\phi_\alpha = D_\alpha V \quad , \quad (3.2a)$$

$$\bar{\phi}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} U \quad . \quad (3.2b)$$

The constraint (3.1c) allows us to express the gauge connection ϕ_a in terms of ϕ_α and $\bar{\phi}_{\dot{\alpha}}$

$$\phi_a = -\frac{i}{4} \{ \bar{D}_{\dot{\alpha}} \bar{\phi}_{\dot{\beta}} + D_\alpha \phi_\beta \} \quad . \quad (3.3)$$

Note that the prepotentials U and V are linearly shifted under the gauge transformation (2.7). Due to (3.2a,b) we can introduce an additional gauge freedom, which acts only on the prepotentials and does not affect the gauge connection. The new gauge transformations are parametrized by the (anti-) chiral superfield $(T)S$

$$\bar{D}_{\dot{\alpha}} S = D_\alpha T = 0 \quad . \quad (3.4)$$

Therefore a general gauge transformation acts on U and V like

$$U \rightarrow U + \Lambda + S \quad , \quad (3.5a)$$

$$V \rightarrow V + \Lambda + T \quad . \quad (3.5b)$$

In a similar way as ϕ_A is expressed in terms of the prepotentials U and V , we can express the components of the field strength tensor R_{AB} in terms of two new field strength W_α and $\bar{W}_{\dot{\alpha}}$. The inspection of the $(\dot{\alpha}\beta\gamma)$ and $(\dot{\alpha}\dot{\beta}\gamma)$ components of the Bianchi identity (2.28) reveals that the constraints (3.1a,b) force the spin 3/2 components of $R_{a\beta}$ and $R_{a\dot{\beta}}$ to vanish. Therefore we can introduce two new field strengths W_α and $\bar{W}_{\dot{\alpha}}$ through

$$R_{a\beta} = -\frac{i}{2} (G_a \bar{W})_\beta, \quad (3.6a)$$

$$R_a^{\dot{\beta}} = \frac{i}{2} (\bar{G}_a W)^{\dot{\beta}}. \quad (3.6b)$$

Inserting (3.2) and (3.3) into the definition (2.26b) of $R_{a\beta}$ and $R_{a\dot{\beta}}$ we obtain

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha (\bar{V} \cdot \bar{U}), \quad (3.7a)$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4} D^2 \bar{D}_{\dot{\alpha}} (V \cdot U). \quad (3.7b)$$

Imposing furthermore the requirement

$$W_\alpha = (\bar{W}^{\dot{\alpha}})^+, \quad (3.8)$$

we get

$$(U \cdot V) = (\bar{U} \cdot \bar{V})^+, \quad (3.9)$$

and

$$S = -T^+ \quad (3.10)$$

Note that (3.7a,b) imply the common relations

$$\bar{D}_{\dot{\alpha}} W_\alpha = D_\alpha \bar{W}_{\dot{\alpha}} = 0 \quad (3.11)$$

$$\bar{D} \bar{W} = D W \quad (3.12)$$

Finally we can express the remaining non-vanishing components of R_{AB} , R_{ab} in terms of W_α and $\bar{W}_{\dot{\alpha}}$

$$R_{ab} = \frac{1}{8} [\bar{D} \bar{G}_a G_b \bar{W} - D G_a \bar{G}_b W] . \quad (3.13)$$

Now we are ready to express the prepotentials U and V in terms of the field strength W and $\bar{W}$. To this end, let us first express the components of ϕ_A in terms of W_a and $\bar{W}_{\dot{a}}$. Inserting the relations (3.6a,b) and (3.13) into (2.29) we obtain

$$\begin{aligned} \phi_a(z_1) = \int_0^1 dt t \left\{ -\frac{1}{2} i (\not{G} \bar{W}(\not{z}))_a + (1-t) [\Delta_a (\bar{\Delta} \bar{W}(\not{z})) \right. \\ \left. + \bar{\Delta}^2 W_a(\not{z})] - \frac{1}{4} i (1-t) [(\bar{D}(\not{z}) \not{G})_a (\bar{\Delta} \bar{W}(\not{z})) \right. \\ \left. - (D(\not{z}) \not{G} \bar{\Delta}) W_a(\not{z})] \right\} , \end{aligned} \quad (3.14)$$

$$\begin{aligned} \bar{\phi}^{\dot{a}}(z_1) = \int_0^1 dt t \left\{ \frac{1}{2} i (W(\not{z}) \not{G} \epsilon)^{\dot{a}} - (1-t) [\bar{\Delta}^{\dot{a}} (\Delta W(\not{z})) \right. \\ \left. + \Delta^2 \bar{W}^{\dot{a}}(\not{z})] + \frac{1}{4} i (1-t) [(\bar{D}(\not{z}) \not{G} \Delta) \bar{W}^{\dot{a}}(\not{z}) \right. \\ \left. - (D(\not{z}) \not{G} \epsilon)^{\dot{a}} (\Delta W(\not{z}))] \right\} , \end{aligned} \quad (3.15)$$

and

$$\begin{aligned} \phi_a(z_1) = \int_0^1 dt t \left\{ \frac{1}{8} [\bar{D}(\not{z}) \not{G}_a \bar{W}(\not{z}) - D(\not{z}) \not{G}_a W(\not{z})] \right. \\ \left. + i \frac{1}{2} [\Delta G_a \bar{W}(\not{z}) - \bar{\Delta} \bar{G}_a W(\not{z})] \right\} . \end{aligned} \quad (3.16)$$

In (3.14)-(3.16) we have employed the convention

$$\not{G} = \not{G}_a G^a , \quad \bar{\not{G}} = \not{G}_a \bar{G}^a , \quad (3.17)$$

and the covariant derivatives $D_\alpha(\zeta)$ and $\bar{D}^{\dot{\alpha}}(\zeta)$ act always only on W and $\bar{W}$.

We can now calculate the prepotentials U and V by expressing both of them in terms of ϕ_α , $\bar{\phi}_{\dot{\alpha}}$ and covariant derivatives thereof. As

$$D_\alpha \phi_\beta = \frac{1}{2} \epsilon_{\alpha\beta} (D\phi) \quad , \quad (3.18a)$$

$$\bar{D}^{\dot{\alpha}} \bar{\phi}_{\dot{\beta}} = \frac{1}{2} \epsilon^{\dot{\alpha}\dot{\beta}} (\bar{D}\bar{\phi}) \quad , \quad (3.18b)$$

and

$$D^2 \phi_\alpha = \bar{D}^2 \bar{\phi}_{\dot{\alpha}} = 0 \quad , \quad (3.19)$$

we have

$$V = \Delta\phi + \frac{1}{4} \Delta^2 (D\phi) \quad , \quad (3.20a)$$

$$U = \bar{\Delta}\bar{\phi} + \frac{1}{4} \bar{\Delta}^2 (\bar{D}\bar{\phi}) \quad . \quad (3.20b)$$

The equations (3.20a,b) determine U and V only modulo S and T gauge transformations. $D\phi$ and $\bar{D}\bar{\phi}$ are readily calculated from (3.14) and (3.15), making use of

$$(D\mathcal{F})_i = 4i \bar{\Delta}_i \quad , \quad (3.21a)$$

$$(\bar{D}\bar{\mathcal{F}})^{\dot{\alpha}} = 4i \Delta^{\dot{\alpha}} \quad . \quad (3.21b)$$

We obtain

$$D_\alpha(z_1) \phi_\beta(z_1) = \epsilon_{\alpha\beta} \left\{ \frac{1}{2} \oint \int_0^1 dt [t(1-t)]^2 \partial^\alpha R_{\dot{\alpha}}(\mathcal{F}) \bar{\Delta}^2 \right. \\ \left. + \frac{1}{2} \int_0^1 dt t^2 (1-t) \bar{\Delta} \bar{\mathcal{F}}_{\dot{\beta}} \bar{\mathcal{F}}^{\dot{\beta}} \bar{W}(\mathcal{F}) - 2 \int_0^1 dt t(1-t) \bar{\Delta} \bar{W}(\mathcal{F}) \right\} , \quad (3.22)$$

and

$$\begin{aligned} \bar{D}^{\dot{\alpha}}(z_1) \bar{\phi}^{\dot{\beta}}(z_1) = & -\epsilon^{\dot{\alpha}\dot{\beta}} \left\{ \frac{1}{2} \oint \int_0^1 dt [t(1-t)]^2 \partial^d R_{dd}(\xi) \Delta^2 \right. \\ & \left. + \frac{1}{2} \int_0^1 dt t^2 (1-t) \Delta \not{\partial} \oint W(\xi) + 2 \int_0^1 dt t(1-t) (\Delta W(\xi)) \right\}. \end{aligned} \quad (3.23)$$

Note that the results (3.22) and (3.23) verify that ϕ_{α} and $\bar{\phi}_{\dot{\alpha}}$ as given by (3.14) and (3.15) indeed satisfy (3.18a,b). Taking a further covariant derivative one may check that the relations (3.19) are satisfied as well. This calculation can be performed using the same tricks as in the Appendix.

Let us finally rewrite the gauge condition (2.39) in terms of the gauge connections ϕ_{α} and $\bar{\phi}_{\dot{\alpha}}$. Inserting (3.3) into (2.39) and making use of (3.21a,b) one immediately derives

$$\left(\bar{D}(z_1) \oint \right)^{\dot{\alpha}} \phi_{\alpha}(z_1) + \left(D(z_1) \oint \right)_{\dot{\alpha}} \bar{\phi}^{\dot{\alpha}}(z_1) = 0, \quad (3.24)$$

where it is understood that $D(z_1)$ and $\bar{D}(z_1)$ act on both $\bar{\xi}$ and ϕ (ξ and $\bar{\phi}$).

Having derived the prepotentials in the gauge (3.24) we can take advantage of the fact that on the level of prepotentials gauge transformations act as linear shifts and bring U and V into any other gauge. For example in practical calculations the gauge in which $\bar{\phi}_{\dot{\alpha}} = 0$ is often preferable, because in this gauge the gauge covariant derivative $\bar{D}_{\dot{\alpha}}$ reduces to the supercovariant derivative $\bar{D}_{\dot{\alpha}} = D_{\dot{\alpha}}$. This gauge condition is easily obtained by choosing $\Lambda = -U$ in (3.5a,b).

4. - EXPANSION OF THE GAUGE CONNECTION

The expressions given so far for the gauge connection are non-local functions of the field strength. It is often desirable to have a formula which is local in the field strength and its derivatives. For the case of a non-supersymmetric Yang-Mills theory such an expression has been given in the Introduction, (1.3). In this section we shall generalize (1.3) to supersymmetric Yang-Mills theories.

In order to obtain an expansion of the gauge connection in terms of the field strength and its derivatives we have first to expand the field strength itself. It is quite natural first to expand the field strength in terms of the component fields, and then to perform an ordinary Taylor expansion of the components. Unfortunately in such an expansion manifest supersymmetry is lost. A manifest supersymmetric expansion can be obtained by expanding in terms of superfields. For the field $W^\alpha(z_1)$ such an expression is given by

$$W^\alpha(z_1) = \sum_n \frac{1}{n!} [(\gamma_1 - \gamma_2)^b \partial'_b + \Delta^\beta \partial'_\beta]^n W^\alpha(z_2) \quad (4.1)$$

$$= \sum_n \frac{1}{n!} [\xi_L^b \partial'_b + \Delta^\beta D'_\beta]^n W^\alpha(z_2), \quad (4.2)$$

where

$$\gamma_j = x_j + i \theta_j \sigma \bar{\theta}_j \quad j = 1, 2, \quad (4.3)$$

and

$$\xi_L^a = (\gamma_1 - \gamma_2)^a - 2i \Delta \sigma^a \bar{\theta}_2 = \xi + i \Delta \sigma^a \bar{\Delta}. \quad (4.4)$$

It is understood that the primed derivatives act exclusively on $W^\alpha(z_2)$. Note that the transition from (4.1) to (4.2) can be performed immediately, because the primed derivatives inside the bracket commute among each other. The analogue expression for $\bar{W}_{\dot{\alpha}}$ is given by

$$\bar{W}_{\dot{\alpha}}(z_1) = \sum_n \frac{1}{n!} [\xi_R^{\dot{b}} \partial'_{\dot{b}} + \bar{\Delta}_{\dot{\beta}} \bar{D}'^{\dot{\beta}}]^n \bar{W}_{\dot{\alpha}}(z_2), \quad (4.5)$$

with

$$\xi_R^{\dot{a}} = \xi + i \bar{\Delta} \bar{\sigma}^{\dot{a}} \Delta. \quad (4.6)$$

As we are ultimately interested in setting up an expansion in powers of ξ , Δ and $\bar{\Delta}$ it is useful to reexpand (4.2) and (4.5) such that

$$W^\alpha(z_1) = \exp(i \Delta \partial \bar{\Delta}) \sum_n \frac{1}{n!} [\xi \partial'_b + \Delta^\beta D'_\beta]^n W^\alpha(z_2), \quad (4.7)$$

and

$$\bar{W}_i(z_1) = \exp(i \bar{\Delta} \bar{\partial} \Delta) \sum_n \frac{1}{n!} [\delta^b \partial_b + \bar{\Delta}_{\dot{\beta}} \bar{\partial}^{\dot{\beta}}] \bar{W}_i(z_2). \quad (4.8)$$

Before we can use (4.7) and (4.8) in (3.14)-(3.16) we have to take into account that $W^\alpha(z_1)$ and $\bar{W}_\alpha(z_1)$ are not independent superfields. They are related by the condition (3.12). This condition relates some of the derivatives of W^α and $\bar{W}_\alpha$ at z_2 . In particular we have

$$D^i W^n(z_2) = \bar{D}^i \bar{W}^n(z_2), \quad (4.9)$$

$$4i(\bar{\partial}^i W^n)^{\dot{i}} = \bar{D}^{2\dot{i}} \bar{W}^{\dot{i},n}, \quad 4i(\partial^i \bar{W}^n)_\alpha = D^{i2} W_\alpha^n, \quad (4.10)$$

$$D'_\alpha (\bar{\partial}^i W^n)^{\dot{i}} = -(\partial^i \bar{D}')_\alpha \bar{W}^{\dot{i},n}, \quad \bar{D}'_{\dot{\alpha}} (\partial^i \bar{W}^n)_\alpha = (D^i \partial')_{\dot{\alpha}} W_\alpha^n, \quad (4.11)$$

$$i D'^2 (\bar{\partial}^i W^n)^{\dot{i}} = 4 \square' \bar{W}^{\dot{i},n}, \quad i \bar{D}'^2 (\partial^i \bar{W}^n)_\alpha = 4 \square' W_\alpha^n, \quad (4.12)$$

where

$$W_\alpha^n = \partial_{a_1}^i \dots \partial_{a_n}^i W_\alpha(z_2) \quad (4.13)$$

and $\bar{W}_\alpha^n$ analogously. Conversely, imposing (4.9)-(4.12) at z_2 guarantees that (3.12) is satisfied at z_1 .

Now we are ready to set up the expansion of the gauge connection. Rather than writing down the general expansion, which is given by a very unhandy expression, we shall content ourselves with keeping only the first few terms. With

$$W_\alpha(z_1) = W_\alpha + \Delta^\beta E_{\beta\alpha} + \Delta_\alpha \bar{F}, \quad (4.14)$$

and

$$\bar{W}^{\dot{i}}(z_1) = \bar{W}^{\dot{i}} + \bar{\Delta}_{\dot{\beta}} \bar{E}^{\dot{\beta}\dot{i}} + \bar{\Delta}^{\dot{i}} \bar{F}, \quad (4.15)$$

where

$$\bar{F} = -\frac{1}{2} D' W = -\frac{1}{2} \bar{D}' \bar{W}, \quad (4.16)$$

$$E_{\beta\lambda} = \frac{1}{2} D'_{c\beta} W_{\lambda}, \quad \bar{E}^{\beta\lambda} = \frac{1}{2} \bar{D}'^{c\beta} \bar{W}^{\lambda}, \quad (4.17)$$

we obtain

$$\begin{aligned} \phi_a(z_1) = & \frac{1}{6} \bar{\Delta}^2 W_a + \frac{i}{24} (E \not{\Delta})_a - \frac{1}{12} \bar{\Delta}^2 (E \Delta)_a - \frac{i}{6} (\not{\Delta} \bar{\Delta})_a \bar{F} \\ & + \frac{1}{6} \Delta_a \bar{\Delta}^2 \bar{F} - \frac{i}{4} (\not{\Delta} \bar{W})_a + \frac{1}{6} \Delta_a (\bar{\Delta} \bar{W}) + \frac{5i}{24} (\not{\Delta} \bar{E} \bar{\Delta})_a, \end{aligned} \quad (4.18)$$

$$\begin{aligned} \bar{\phi}^{\dot{a}}(z_1) = & -\frac{1}{6} \Delta^2 \bar{W}^{\dot{a}} - \frac{i}{24} (\bar{E} \not{\Delta})^{\dot{a}} + \frac{1}{12} \Delta^2 (\bar{E} \bar{\Delta})^{\dot{a}} + \frac{i}{6} (\Delta \not{\Delta})^{\dot{a}} \bar{F} \\ & - \frac{1}{6} \bar{\Delta}^{\dot{a}} \Delta^2 \bar{F} + \frac{i}{4} (\not{\Delta} W)^{\dot{a}} - \frac{1}{6} \bar{\Delta}^{\dot{a}} (\Delta W) - \frac{5i}{24} (\not{\Delta} E \Delta)^{\dot{a}}, \end{aligned} \quad (4.19)$$

and

$$\begin{aligned} \phi_a(z_1) = & -\frac{i}{4} \bar{\Delta} \not{\Delta}_a W + \frac{i}{4} \Delta \not{\Delta}_a \bar{W} + \frac{1}{16} \text{Tr}(\not{\Delta} \not{\Delta}_a \bar{E}) \\ & - \frac{i}{6} \Delta \not{\Delta}_a \bar{E} \bar{\Delta} - \frac{1}{16} \text{Tr}(\not{\Delta} \not{\Delta}_a E) + \frac{i}{6} (\bar{\Delta} \not{\Delta}_a E \Delta) + \frac{i}{3} \Delta \not{\Delta}_a \bar{\Delta} \bar{F}. \end{aligned} \quad (4.20)$$

We can use (3.20) to calculate U

$$\begin{aligned} \bar{U} = & \frac{i}{4} \bar{\Delta} \not{\Delta} W - \frac{1}{3} \bar{\Delta}^2 (\Delta W) - \frac{1}{6} \Delta^2 (\bar{\Delta} \bar{W}) \\ & - \frac{5i}{24} \bar{\Delta} \not{\Delta} E \Delta - \frac{i}{24} \bar{\Delta} \bar{E} \not{\Delta} \\ & - \frac{i}{6} \Delta \not{\Delta} \bar{\Delta} \bar{F} - \frac{1}{4} \Delta^2 \bar{\Delta}^2 \bar{F}. \end{aligned} \quad (4.21)$$

and V

$$\begin{aligned} \bar{V} = & -\frac{i}{4} \Delta \not{F} \bar{W} + \frac{1}{3} \Delta^2 (\bar{\Delta} \bar{W}) + \frac{1}{6} \bar{\Delta}^2 (\Delta W) \\ & + \frac{5i}{24} \Delta \not{E} \bar{\Delta} + \frac{i}{24} \Delta E \not{\Delta} \\ & - \frac{i}{6} \Delta \not{\Delta} \bar{F} + \frac{1}{4} \Delta^2 \bar{\Delta}^2 \bar{F} \end{aligned}$$

(4.22)

5. - FINAL REMARKS

In the present paper we have generalized the Schwinger-Fock gauge to supersymmetric Yang-Mills theories. This gauge has the remarkable property that it allows us to expand the gauge connection in terms of the field strength and its covariant derivatives. Therefore this gauge is useful for calculations in the presence of classical gauge fields.

As the gauge connection can explicitly be calculated from the field strength this gauge is unique. Besides the gauge connections, the prepotentials play an important rôle in supersymmetric Yang-Mills theories. We have shown that the prepotentials can be calculated as well. However, in contrast to the gauge connection, the prepotentials are not determined uniquely. The fact that the prepotentials are linearly shifted under a gauge transformation allows us to set up an expansion of the prepotentials in a gauge where one of them vanishes. Such a gauge is often chosen for convenience.

In this paper we have restricted our considerations to Abelian gauge theories. Let us briefly indicate how our work can be generalized to a non-Abelian gauge group. In principle such a generalization does not present any problem, though the generalization of the various steps performed in this paper may be technically involved due to the non-linearity of the gauge transformation. If we are working with a non-Abelian gauge group, we have to specify the gauge in which the field strength is to be evaluated. In particular $R_{PN}(\xi)$ in (2.14) and the relations following has to be evaluated in the Schwinger-Fock gauge. We can obviously get rid of this requirement by writing everything in a

covariant fashion, i.e., explicitly displaying the gauge transformation which takes us into the Schwinger-Fock gauge. Moreover, apart from the obvious need to use gauge covariant derivatives to define the field strength (2.13), etc., we have to expand the field strength in terms of its gauge covariant derivatives. Now

$$\phi_M(z_2) = 0, \quad (5.1)$$

and

$$(z_1 - z_2)^{M_1} \dots (z_1 - z_2)^{M_n} \partial_{M_1} \dots \partial_{M_n} \phi_M \Big|_{z_2} = 0, \quad (5.2)$$

which is obtained by expanding ϕ_M in the gauge condition (2.39) around the point z_2 . Therefore we can indeed replace the ordinary derivatives in the expansion of the field strength by gauge covariant derivatives.

ACKNOWLEDGEMENTS

The author would like to thank D. Amati, Y. Meurice, Y. Nicolai and G. Veneziano for useful discussions and for reading the manuscript.

APPENDIX

In the present appendix we shall verify that the gauge connection ϕ_A as given by (2.29) indeed leads to the correct field strength R_{AB} .

Let us begin by noting that

$$D_A(z_1) M^D_B = -\frac{1}{2} (1-t) T_{AB}^D \quad (A.1)$$

Making use of the product rule for D_A and of (A.1) we can write

$$D_A(z_1) \phi_B(z_1) - (-)^{ab} D_B(z_1) \phi_A(z_1) = \int_0^1 dt \Sigma_{AB}(t) = \int_0^1 dt t(1-t) \oint \xi^d R_{dD} T_{AB}^D, \quad (A.2)$$

where

$$\Sigma_{AB}(t) = \Sigma_{AB}^1(t) + \Sigma_{AB}^2(t), \quad (A.3)$$

$$\begin{aligned} \Sigma_{AB}^1(t) &= t \left\{ (-)^{ac} \oint \xi^d (D_A(z_1) R_{dD}(\xi)) M^D_B \right\} \\ &= (-)^{ab} (A \leftrightarrow B), \end{aligned} \quad (A.4)$$

and

$$\begin{aligned} \Sigma_{AB}^2(t) &= t \left\{ (D_A(z_1) \oint \xi^d) R_{dD}(\xi) M^D_B \right\} \\ &= (-)^{ab} (A \leftrightarrow B). \end{aligned} \quad (A.5)$$

Taking into account that

$$D_A(z_1) f(\xi) = [t D_A(\xi) + t \tilde{N}_A^D D_D(\xi)] f(\xi) \quad (A.6)$$

for an arbitrary function $f(\xi)$, where $\tilde{N}_A^D$ has been defined in (2.35), it is possible to express $\Sigma_{AB}^1(t)$ in terms of covariant derivatives of $R_{AB}(\xi)$. These covariant derivatives always appear in pairs, such that the Bianchi identity (2.28) can be applied. Thereby we can cast $\Sigma_{AB}^1(t)$ into the form

$$\begin{aligned} \Sigma_{AB}^1(t) = & t^2 \oint^G \left\{ D_d(f) R_{AB}(f) + \tilde{N}_A^D \tilde{N}_B^E D_d(f) R_{DE}(f) \right. \\ & \left. + (-)^c [N_A^D D_d(f) R_{DB}(f) - (-)^{ab} (A \leftrightarrow B)] \right\} - t^2 \oint^G T_{AB}^D R_{Dd} \\ & - 2t^2 \left\{ N_A^D R_{DB}(f) - \tilde{N}_A^D N_B^E R_{ED}(f) - (-)^{ab} (A \leftrightarrow B) \right\} \quad (A.7) \end{aligned}$$

The expression (A.7) can be simplified by noticing that

$$\oint^G T_{Ad}^D = 2N_A^D, \quad (A.8)$$

and

$$\oint^G D_d(f) f(f) = \frac{d}{dt} f(f(t)). \quad (A.9)$$

Applying (A.8) and (A.9) to (A.7) we get

$$\begin{aligned} \Sigma_{AB}^1(t) = & t^2 \frac{d}{dt} \left\{ R_{AB}(f) + \tilde{N}_A^D \tilde{N}_B^E R_{DE}(f) \right. \\ & \left. + [\tilde{N}_A^D R_{DB}(f) - (-)^{ab} (A \leftrightarrow B)] \right\} - t^2 \oint^G T_{AB}^D R_{Dd}(f) \\ & - t^2 \left\{ 2\tilde{N}_A^D N_B^E R_{DE}(f) + [N_A^D R_{DB}(f) - (-)^{ab} (A \leftrightarrow B)] \right\}. \quad (A.10) \end{aligned}$$

Let us now turn to $\Sigma_{AB}^2(t)$. Because

$$D_A(z_1) \oint^G = \oint_A^G + N_A^G \quad (A.11)$$

we can evaluate $\Sigma_{AB}^2(t)$ to be equal to

$$\begin{aligned} \Sigma_{AB}^2(t) = & 2t [R_{AB}(f) + N_A^G \tilde{N}_B^D R_{GD}(f)] \\ & + t(2-t) [N_A^G R_{GB}(f) - (-)^{ab} (A \leftrightarrow B)] \quad (A.12) \end{aligned}$$

Adding $\Sigma_{AB}^1(t)$ and $\Sigma_{AB}^2(t)$ results into

$$\begin{aligned} \Sigma_{AB}^1(t) + \Sigma_{AB}^2(t) = & \frac{d}{dt} \left\{ t^2 [R_{AB}(f) + \tilde{N}_A^c \tilde{N}_B^c R_{cd}(f) \right. \\ & \left. + (\tilde{N}_A^D R_{DB}(f) - (-)^{ab} (A \leftrightarrow B))] \right\} - t^2 \tilde{f}^c T_{AB}^D R_{cd}(f). \end{aligned} \quad (A.13)$$

As $\tilde{N}_A^B$ is proportional to $(1-t)$, almost all the terms inside the curly bracket vanish upon integration. Inserting the non-vanishing terms of (A.13) back into (A.2) and remembering that T_{AB}^D is only non-vanishing if D is a bosonic index, we get

$$D_A \phi_B - (-)^{ab} D_B \phi_A = R_{AB} - T_{AB}^D \phi_D, \quad (A.14)$$

which was to be shown.

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