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MASSIVE SQCD AND THE CONSISTENCY OF INSTANTON CALCULATIONS

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A B S T R A C T

Computing SQCD correlations in the large m (decoupling) regime and using their known m -dependence we check the internal consistency of our approach to supersymmetric condensates in which non-zero fermionic modes are systematically kept in instanton calculations even at small m .

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1. - INTRODUCTION

Instanton calculations¹⁾⁻⁶⁾ have been employed in the last couple of years in order to unravel the vacuum properties of supersymmetric gauge theories.

In vectorlike theories, such as SQCD, the results¹⁾⁻⁴⁾ have come out in essential agreement with those previously obtained (or conjectured) by more indirect methods, such as Witten-index calculations⁷⁾, effective Lagrangian techniques^{8),9)} and Ward identity constraints^{10),11)}. In a class of theories with chiral fermions, the new phenomenon of dynamical supersymmetry breaking has been found to occur^{5),6)}.

Two calculation techniques have actually been followed: in one of them^{4),6)}, one deals with a Higgs picture, using the fact that, in the massless situation, perturbative Higgs vacua exist with arbitrarily large scalar expectation values. Mass effects are then included by adding an extra term in the superpotential. In the other approach, initiated in Ref. 1) and developed by us^{2),3),5)}, one adopts a confining picture with vanishing expectation values for the scalar fields. Only gauge invariant condensates are allowed non-zero expectation values. This approach looks the only suitable one for direct computations in the massive theory which, perturbatively, has no Higgs vacuum.

As noticed in Ref. 3) - hereafter referred to as I - the effects generated by masses are highly non-trivial. While for massless theories, instanton effects may be reconduced to fermionic zero modes^{*)} (ZM), non-zero modes (NZM) play a crucial rôle for massive theories even for arbitrary small masses. This was signalled by singularities of the coefficients of an expansion in powers of m . It was conjectured that NZM would be responsible for the puzzling behaviour of correlation functions for $m \rightarrow 0$.

This conjecture has been recently challenged by Novikov et al.¹²⁾. These authors have argued that the above-mentioned problems with the ZM approximation should instead be cured by going, at small, finite mass, to a Higgs picture [like essentially the one of Refs. 4) and 6)]. Their argument was based on the fact that large scalar bilinear (gauge invariant) condensates occur at small m ⁸⁾ and on the belief that these imply a large elementary scalar expectation value.

*) Throughout this paper we shall denote by ZM (NZM) those fermionic modes which have zero (non-zero) eigenvalue of the Dirac operator in the instanton background in the limit $m \rightarrow 0$.

The main purpose of this paper is to show that the conjecture of I can indeed be checked and that the confining picture method is internally consistent at all $m \neq 0$.

The technical breakthrough which allows us to go beyond the conjecture stage of I is the consideration of a "large mass limit". On the one hand, we will be able to compute in such a limit the relevant correlation functions finding results that agree perfectly with decoupling considerations¹³⁾. On the other hand, Ward identities^{10),3)} allow us to extrapolate our large m results to arbitrarily small (finite) masses where they are completely consistent with Konishi's supersymmetric anomaly constraints¹¹⁾ and with the expected vacuum structure.

Since our confining picture calculation is supersymmetric we deduce that it is consistent at all m . The results exhibit a discontinuous behaviour when going from the small mass limit of the massive theory to the strictly massless one. This is precisely the behaviour conjectured in I on the basis of mass singularities of the NZM contributions.

In Section 2 we recall the constraints imposed by supersymmetric and chiral Ward identities on Green's functions. We shall show consistency with decoupling requirements at large m and recall why NZM are required in our confining approach. In Section 3 we show that Green's functions can be computed in a large m limit and check, in a few illustrative examples, the consistency of the results. In Section 4 we give some conclusions and compare our approach to the one put forward in Ref. 12). Details of calculations not reported in the text are presented in an Appendix.

2. - SUPERSYMMETRIC AND CHIRAL WARD IDENTITY CONSTRAINTS

We shall follow the notations of I and consider the following chiral superfields of SQCD:

$$W_\alpha : (\lambda_{\alpha a}^b, F_a^{\mu\nu b}) \quad \alpha = 1, 2, \quad a, b = 1 \dots N, \quad i = 1 \dots M$$

$$\Phi_{\alpha i} : (\phi_{\alpha i}, \psi_{\alpha i}) \quad , \quad \hat{\Phi}^{ai} : (\hat{\phi}^{ai}, \hat{\psi}^{ai})$$

belonging respectively to the adjoint, N and $\bar{N}$ representations of the gauge $SU(N)$ group. The mass term can be chosen to read:

$$-\mathcal{L}_{mass} = \sum_{i=1}^M m_i \int d^2\theta \hat{\Phi}_{ai}^{ai} \Phi_{ai} + \sum_{i=1}^M m_i^* \int d^2\bar{\theta} \bar{\Phi}^{ai} \bar{\Phi}_{ai} \quad (2.1)$$

The gauge invariant Green functions we shall be interested in are:

$$G_{N,M}^{p,q}(x_1, \dots, x_p, y_1, \dots, y_q) \equiv \langle \lambda \lambda(x_1) \dots \lambda \lambda(x_p) \hat{\phi}^{i_1}(y_1) \dots \hat{\phi}^{i_q}(y_q) \rangle \quad (2.2)$$

where $\lambda \lambda(x)$ and $\hat{\phi}^i(y)$ are shorthand notation for $g^2 \epsilon^{\alpha\beta} \lambda_{\alpha a}^b(x) \lambda_{b\beta}^a(x)$ and $\hat{\phi}^{i,a}(y) \phi_{j,a}(y)$, respectively.

Supersymmetry implies, for massive SQCD, the following properties for $G_{N,M}^{p,q}$:

- (i) G is independent of all co-ordinates x_i, y_i ^{1), 2)};
- (ii) G has to depend on masses as ^{3), 10)}:

$$G_{N,M}^{p,q} = \text{const.} \prod_{k=1}^M (m_k)^{\frac{p+q}{N}} \prod_{\ell=1}^q (m_{i_\ell} m_{j_\ell})^{-1/2} \quad (2.3)$$

where the constants are proportional to the appropriate power of Λ_{SQCD} in a mass independent renormalization scheme (see below for more discussion);

- (iii) using the supersymmetric anomaly constraint ¹¹⁾

$$\frac{1}{16\pi^2} \langle \lambda \lambda \rangle = m_i \langle \hat{\phi}^i \phi_i \rangle \quad i=1, \dots, M \quad (2.4a)$$

and clustering for large separations, $G_{NM}^{p,q}$ are shown to satisfy

$$G_{N,M}^{P,Q} = \delta_{i_1 j_1} \dots \delta_{i_q j_q} \frac{G_{N,M}^{P+Q,0}}{(16\pi^2)^q \prod_{i=1}^q m_i} \quad (2.4b)$$

The fact that our explicit calculation of $G_{NM}^{P,Q}$ will satisfy such constraints will represent a check of our method and of the assumption of a supersymmetric and gauge invariant vacuum.

In order to recall why NZM are necessary for the massive theory, let us remind that for the strictly massless case, $G_{NM(m=0)}$ is determined by fermionic zero modes only and is given by

$$G_{N,M}^{P,Q} = \text{const.} \delta_{Q,M} \delta_{P,N-M} \epsilon_{i_1 \dots i_M} \epsilon_{j_1 \dots j_M} \bigwedge^{3N-M} m_i \equiv 0 \quad N \geq M \quad (2.5)$$

As discussed in I, Eq. (2.5) shows a degenerate vacuum under $SU(M) \times SU(M)$ flavour symmetry signalling a spontaneous symmetry breaking. The anomaly constraint (2.4a) - which in this massless case reduces to $\langle \lambda \lambda \rangle = 0$ - forces, through clustering, the vanishing of G for $M < N$. This implies the lack of a well-defined (i.e., finite) vacuum expectation value $\langle \tilde{\phi} \phi \rangle$.

The point that interests us here is that (2.5) differs drastically from the expected result (2.4) at $m_1 \neq 0$. As noticed in I, this implies that NZM must enter at $m_1 \neq 0$. Indeed, zero modes alone would give the result (2.5) even at small finite mass due to the smoothness of their contribution around $m_1 = 0$. Besides (a fact not noticed in I) even when the mass and flavour dependences of (2.5) and (2.4) agree, the numerical factors do not¹⁴⁾. For the $N = 2, M = 1$ case, for instance, the zero modes give, at small mass:

$$\langle \lambda \lambda \tilde{\phi} \phi \rangle_{2M} = \frac{c \Lambda^5 g^4}{32 \pi^4} = \frac{5}{8} \frac{\langle \lambda \lambda \lambda \lambda \rangle_{2M}}{16 \pi^2 m} \quad (2.6)$$

and for $N = M = 2$,

$$\langle \tilde{\phi}^i \phi_j \tilde{\phi}^k \phi_l \rangle_{2M} = \epsilon^{ik} \epsilon_{jl} \frac{c \Lambda^4 g^4}{768 \pi^6} = \frac{5}{12} \frac{\langle \lambda \lambda \lambda \lambda \rangle_{4M} \epsilon^{ik} \epsilon_{jl}}{(16 \pi^2)^2 m_1 m_2} \quad (2.7)$$

where

$$c = g^{-8} \pi^6 2^{10} \quad (2.8)$$

In the Appendix we give the explicit evaluations (2.6) and (2.7) and show that all the g -dependent factors can be absorbed into renormalization group invariant quantities (Λ_{inv} and m_{inv}). The factors $5/8$ and $5/12$ in the right-hand side of (2.6) and (2.7) are at variance with the relations (2.4). Therefore something must be added to the ZM contribution at $m = 0$. We shall show that adding NZM contributions yields Green's functions whose flavour and mass dependence will be those of (2.4) with the correct proportionality constant. This will in turn justify the procedure adopted in I for obtaining all condensates, i.e., compute only those correlation functions which are free from NZM contributions (hence continuous at $m = 0$) and then imposing clustering and anomaly constraints to obtain all the others.

As already mentioned in Section 1 we shall be able to check consistency directly at large m and then indirectly, using supersymmetry, at all m . It is important for us to check that the m -dependence of correlation functions and condensates, given in Eq. (2.3), makes sense even at $m \gg \Lambda$. We shall now argue that this is indeed the case: even more, if we had no chiral argument to obtain the dependence (2.3), then decoupling at large m would prescribe exactly the same behaviour. Consider indeed the particular case of (2.3)

$$g_{N,1}^{1,0} \equiv \langle \lambda \lambda \rangle_{N,M=1} = \text{const. } m^{1/N} \Lambda^{3-1/N} \quad (2.9)$$

At first sight it seems hard to reconcile that result with the decoupling requirement¹³⁾ which implies:

$$\langle \lambda \lambda \rangle_{M=1} \xrightarrow{m \gg \Lambda} \langle \lambda \lambda \rangle_{M=0} \propto \Lambda_{\text{SYM}}^3 \quad (2.10)$$

As we shall see now, the contradiction is only apparent and, on the contrary, if care is taken with the definition of scales, the mass dependence in (2.9) is just what is required in order to fulfil decoupling for $(m/\Lambda) \rightarrow \infty$.

Indeed, Λ in (2.9) is the mass independent renormalization group invariant scale which controls the ultraviolet behaviour ($q \gg \Lambda, m$) of the $M = 1$ effective coupling $\bar{g}(q)$. If $m < \Lambda$, Λ represents also the scale of q for which $\bar{g}(q) \sim O(1)$. But if $m \gg \Lambda$, the value of q for which $\bar{g}$ is $O(1)$, call it Λ_{eff} , will be in the region where the heavy particles are decoupled:

$$\Lambda \ll \Lambda_{\text{eff}} \ll m \quad (2.11)$$

In order to determine Λ_{eff} , consider a SYM theory whose coupling coincides with that of one flavour SQCD at $q = m$. Because of decoupling, both couplings will evolve identically below m and will thus have the same effective scale, i.e., $\Lambda_{\text{eff}} = \Lambda_{\text{SYM}}$. We must thus require

$$\bar{g}_{\text{SQCD}}^{-2}(m) = \beta_{\text{SQCD}} \log \frac{m}{\Lambda} = \bar{g}_{\text{SYM}}^{-2}(m) = \beta_{\text{SYM}} \log \frac{m}{\Lambda_{\text{eff}}}. \quad (2.12)$$

But, recalling that for the single flavour SQCD we are discussing

$$\beta_{\text{SQCD}} = \frac{3N-1}{3N} \beta_{\text{SYM}}$$

Eq. (2.12) implies

$$\left(\frac{m}{\Lambda}\right)^{3N-1} = \left(\frac{m}{\Lambda_{\text{eff}}}\right)^{3N}$$

or

$$\Lambda_{\text{eff}}^3 = \Lambda^{3-\frac{1}{N}} m^{\frac{1}{N}} \quad (2.13)$$

We thus see that (2.9) is exactly equivalent to the decoupling condition (2.10) for the correctly defined scale as given by (2.13)*). It is easy to see that this property is of general validity.

3. - MASSIVE SQCD CORRELATION THROUGH A LARGE m LIMIT

We have been able to achieve control over all fermionic modes in the limit $m \gg \Lambda$. The fact that this limit sheds light on the small mass regime is yet another by-product of supersymmetry not very dissimilar from the one that allows to obtain vacuum properties (i.e., long-range correlations) from

*) As far as the g -dependence is concerned, this relation was already noticed in the second of Refs. 8). The argument given above about the mass dependence is our version of a statement made to us by M.A. Shifman.

short-distance calculations. Indeed, supersymmetry is able to provide information on space-time and mass dependence of some Green functions, allowing their computation in particular limiting conditions where approximations are under control.

We shall illustrate our method for the simple case of SU(2) gauge theory with one or two flavours (i.e., $N = 2$, $M = 1, 2$). The generalization to general N and $M \leq N$ will then be straightforward.

(A) $N = 2, M = 1$

The Green functions we will discuss are

$$G(x, y) = \langle \lambda \lambda(x) \lambda \lambda(y) \rangle \quad (3.1)$$

$$F(x, y) = \langle \lambda \lambda(x) \tilde{\psi} \psi(y) \rangle \quad (3.2)$$

which, due to supersymmetry, should be x, y independent.

In all calculations, we will consistently keep only the leading order quantum effect around the classical instanton. This amounts to neglecting terms $O(g^2)$ with respect to those considered. Supersymmetry is respected order by order in g and is therefore fulfilled by our approximation.

The calculation of $G(x, y)$ is straightforward because, as shown in I, non-zero modes cancel out exactly. The calculation, recalled in Appendix A, gives

$$G = m \Lambda^5 \frac{4c g^4}{5 \pi^2} \quad (3.3)$$

c being the constant given in Eq. (2.8).

As discussed in I and again in the Appendix, the Green function $F(x, y)$ of (3.2) is given, to lowest order in g , by

$$F(x, y) = 2 g^4 c \Lambda^5 \int d^4 x_0 d\vartheta \vartheta^3 \sum_{perm.} \text{Tr} \lambda_0(x) \lambda_0(x) \cdot \\ \cdot |m|^2 \int d^4 z_1 d^4 z_2 \text{Tr} \left\{ \left(\frac{1}{D^2 - |m|^2} \right)_{y z_1} \lambda_0(z_1) \left(\frac{1}{D \bar{D} - |m|^2} \right)_{z_1 z_2} \lambda_0(z_2) \left(\frac{1}{D^2 - |m|^2} \right)_{z_2 y} \right\} \quad (3.4)$$

As explained in Section 2, SUSY implies that $F(x,y)$ cannot depend on either m or $x-y$ with the consequence that the integral in (3.4) has to be a dimensionless constant. Being independent of the dimensionless combination $|m|^2(x-y)^2$ we may evaluate it for

$$|m|^2(x-y)^2 \gg 1 \quad (3.5)$$

Instanton dominance requires

$$(x-y)^2 \ll \frac{1}{\Lambda^2} \quad (3.6)$$

thus showing that we must work in a short distance, large mass ($m \gg \Lambda$) limit.

We shall identify now the integration regions of the variables z_1, z_2, ρ, x_0 which will contribute to (3.4) under the condition (3.5). The large mass m in the three propagators will exponentially suppress - for all values of the instanton collective co-ordinates - values of z_1 and z_2 that do not satisfy

$$(z_1-y)^2, (z_2-y)^2, (z_1-z_2)^2 \lesssim \frac{1}{m^2} \quad (3.7)$$

Then, from (3.5),

$$(x-z_1)^2 \sim (x-z_2)^2 \sim (x-y)^2 \gg \frac{1}{m^2} \quad (3.8)$$

The arguments x_1, x_2, z_1, z_2 of the zero mode eigenfunctions in (3.4) are thus pairwise very separated so that, in order to be all covered by an instanton, this must have

$$g \sim |x-y| \gg \frac{1}{m} \quad (3.9)$$

In the Appendix the intuitively obvious fact that values of ρ not satisfying (3.9) will not contribute to (3.4) under the condition (3.5) is actually shown. An instanton background field satisfying (3.9) gives a negligible contribution to the three propagators in (3.4) which therefore take their free expression. The constant value of F we are looking for will thus be given by

$$F = \lim_{|m|^2(x-y)^2 \rightarrow \infty} 2 g^4 c \Lambda^5 |m|^2 \int d^4 z_1 d^4 z_2 \Delta(y-z_1) \Delta(z_1-z_2) \Delta(z_2-y) \cdot \int d^4 x_0 d^4 y \sum_{perm.} Tr(\lambda_0(x) \lambda_0(x)) Tr(\lambda_0(z_1) \lambda_0(z_2)) \quad (3.10)$$

where

$$\Delta = \frac{1}{\square^2 - |m|^2} \quad (3.11)$$

is the free massive propagator.

By making use of the explicit expressions for λ_0 , (3.9) is evaluated in the Appendix. It is shown that the integral over the collective co-ordinates gives

$$const. + O\left[\frac{(z_1 - z_2)^2}{(x - z_1) \cdot (x - z_2)}\right]$$

contributing to the subsequent $z_1 z_2$ integral with

$$\frac{const.}{|m|^2} + O\left(\frac{1}{m^4(x-y)^2}\right)$$

respectively. Therefore in the limit implied in (3.10) only the constant contribution survives giving explicitly

$$F(x, y) = \frac{c g^4 \Lambda^5}{20 \pi^4} \quad (3.12)$$

Applying clustering to (3.3) and (3.12) for $x-y \rightarrow \infty$ we obtain

$$\langle \lambda \lambda \rangle = \left(\frac{4 m c g^4 \Lambda^5}{5 \pi^2} \right)^{1/2}, \quad \langle \hat{\phi} \phi \rangle = \frac{1}{40 \pi^2} \left(\frac{5 c g^4 \Lambda^5}{m} \right)^{1/2} \quad (3.13)$$

values that satisfy the expected mass dependence (2.3) and the anomaly condition (2.4).

We notice that the value (3.12) we obtained for F , independent of m as

expected, differs by a factor 8/5 from the value (2.6) obtained for the massless theory. This peculiar behaviour was understood in I on the basis of mass singularities (of the form m^2/m^2) of NZM. In the actual evaluation we have done here, the zero modes that were responsible for the massless result appear as less and less important as m increases, and give a vanishing result in the large mass limit (3.5) [see Eq. (A.14) in the Appendix].

(B) N = M = 2

Denote by m_1 and m_2 the two flavour masses. The three relevant Green's function we consider are

$$\begin{aligned} g(x, y) &= \langle \lambda \lambda(x) \lambda \lambda(y) \rangle \\ f^{ij}(x, y) &= \langle \lambda \lambda(x) \tilde{\phi}^i \phi_j \rangle \\ h^{ij,kl}(x, y) &= \langle \tilde{\phi}^i \phi_j(x) \tilde{\phi}^k \phi_l(y) \rangle \end{aligned} \quad (3.14)$$

$g(x, y)$ is the only one which, as before and as demonstrated in I, is saturated by fermionic ZM. One finds

$$g(x, y) = m_1 m_2 \epsilon g^4 \Lambda^4 \frac{4}{5 \pi^2} \quad (3.15)$$

In order to calculate f^{ij} , we follow for the pair of matter fields $\tilde{\phi}_i, \phi_j$ the large mass method utilized before. This will enforce the condition $i = j$. The integration over the remaining flavour fermions (i.e., $k \neq i = j$) is similar to the one developed for $\langle \lambda \lambda \lambda \lambda \rangle_{M=1}$ and gives rise to a contribution proportional to m_k . The ratio between f^{ij} and g will then be the same as the one between F and G of (3.11) and (3.2).

For $h^{ij,kl}$ we follow the large mass method for both flavours. For $h^{11,22}$ and $h^{12,21}$, for instance, we would find expressions having two integrals like the second one in (3.4), one for each flavour. These expressions would be represented by Fig. 1a for $h^{11,22}$ and by Fig. 1b for $h^{12,21}$. Again, the result having to be independent of m_1, m_2 and $(x-y)$ will be evaluated for

$$m_1^2(x-y)^2 \gg 1, \quad m_2^2(x-y)^2 \gg 1$$

It is then possible to realize that the second diagram will not contribute due to the exponential damping caused by the masses on a long distance propagation. For the first diagram, a situation similar to that encountered before will be met for each flavour triangle. This means that z_1 and z_2 will be near y by $O(1/m_1)$ and u_1 and u_2 near x by $O(1/m_2)$. Again, only instantons with $\rho \sim |x-y| \gg 1/m_1, 1/m_2$ will be relevant implying free massive propagators in the two triangles of diagram 1a. As before, the integrations may be done explicitly and give

$$h^{11,22} = \frac{c g^4 \Lambda^4}{320 \pi^6}, \quad h^{12,21} = 0 \quad (3.16)$$

verifying explicitly (2.4).

We notice, once more, that the v.e.v. are diagonal in flavour for all values of m_1 including $m_1 \rightarrow 0$. This shows spontaneous chiral symmetry breaking as an alignment of the vacuum with respect to our arbitrary small explicit symmetry breaking.

As discussed before, this differs from the strictly massless case both in the flavour structure and in the explicit numerical values. For the $N = M = 2$ case, for instance, the massless case result (2.7) implies

$$h_{m_i \equiv 0}^{11,22} = - h_{m_i \equiv 0}^{12,21} = \frac{c g^4 \Lambda^4}{768 \pi^6} \quad (3.17)$$

which differs from the massive result (3.16).

The generalization for all N and $M < N$ is formally simple. The only qualitatively new result worth mentioning is, for the $M = N$ case

$$\langle \det \tilde{\Phi}(x) \det \Phi(y) \rangle = 0 \quad (3.18)$$

where $\det \phi$ is the determinant of ϕ considered as a square matrix in its colour and flavour indices. The reason for the vanishing in (3.18) is analogous to the

vanishing of $h^{12,21}$ for $M = N = 2$ represented by diagram 1b. Equation (3.18) and clustering imply $\langle \det \phi \rangle = 0$ and thus the absence of spontaneous breaking of $U(1)_V$ in the massive case, the opposite being the case - as explained in I - for the strictly massless theory. The absence of the condensate $\langle \det \phi \rangle$ for the massive case is in agreement with the requirement implied by the non-anomalous commutator

$$\varepsilon^{\alpha\beta} \left\{ \bar{\phi}_\alpha, \varepsilon_{i_1 \dots i_N} \varepsilon_{j_1 \dots j_N} \phi_{j_1}^{i_1} \dots \phi_{j_{N-1}}^{i_{N-1}} \tilde{\psi}_{\beta j_N}^{i_N} \right\} = m \det \phi \quad (3.19)$$

4. - DISCUSSION AND COMPARISON WITH OTHER APPROACHES

The results of Section 3 have shown that the confining picture method, using the classical instanton solutions together with $\langle \phi \rangle = \langle \tilde{\phi} \rangle = 0$, is a self-consistent approach to the vacuum properties of massive SQCD in spite of recent claims to the contrary. As first suggested in I, non-zero mode contributions are crucial for the consistency of this method.

In massless SQCD, the situation can be different: classically the theory has an infinity of degenerate vacua (flat directions) but, for $M < N$, the theory has an ill-defined ground state⁸⁾. Both our method and that based on the Higgs picture explicitly confirm this pathology, absent only for the $N = M$ case.

For this massless $N = M$ case, the instanton calculations in the Higgs picture are consistent with the persistence of SUSY vacua with arbitrary large expectation values for ϕ and $\tilde{\phi}$. The confining picture instanton calculations of I gave instead a fixed value for the combinations

$$\det_{ij} \langle \tilde{\phi}^i \phi_j \rangle = \text{const. } \Lambda^{2N} \quad (4.1)$$

It is possible^{*)} that, by imposing vanishing boundary conditions on $\langle \phi \rangle$ and $\langle \tilde{\phi} \rangle$, our method selects one out of the many inequivalent vacua and is therefore "blind" to the Higgs vacua which the other approach is dealing with.

*) This was suggested to us by N. Seiberg

A recent effective Lagrangian analysis¹⁵⁾ for the case $N = M$ seems to confirm this possibility and to suggest a sort of complementarity between the Higgs and the confining picture. Indeed, at $m_1 = 0$, some of the conditions for a zero energy vacuum become trivially satisfied and, consequently, only one combination of $\langle \det \phi \tilde{\phi} \rangle$ and $\langle \det \phi \rangle \langle \det \tilde{\phi} \rangle$ gets fixed. This leaves, in the manifold of supersymmetric vacua, one extra flat direction, besides those due to spontaneous symmetry breaking. The extra flat direction is required, in the confining picture by 't Hooft anomaly matching conditions¹⁷⁾. At large $\langle \phi \tilde{\phi} \rangle$, it may be seen to correspond to a flat direction of the classical Higgs potential related to the anomalous $U(1)_A$ symmetry:

$$\phi_{ai} \rightarrow e^{i\beta} \phi_{ai} \qquad \tilde{\phi}^{ai} \rightarrow e^{i\beta} \tilde{\phi}^{ai}$$

and to its complexification.

In the effective Lagrangian approach, one can also find a framework where the peculiar difference between massive and massless theory vacua can be visualized. Indeed, in the massive case, the energy minimization (vanishing) conditions provide extra constraints (trivially satisfied in the massless theory) which fix both $\langle \det \phi \tilde{\phi} \rangle$ and $\langle \det \phi \rangle \langle \det \tilde{\phi} \rangle$ and force the flavour diagonal structure of the scalar bilinear condensates.

We want to comment finally on the alternative proposal of Ref. 12), aiming at achieving consistency in the massive case through a Higgs picture calculation at $m \ll \Lambda$. In that paper, the proposed solution of the inconsistency, say in the $N = 2, M = 1$ example, is as follows: one computes the correlation function $F = \langle \lambda \lambda \phi \tilde{\phi} \rangle$ using the Higgs picture with $v = \langle \phi \rangle \gg \Lambda$ (at $m \ll \Lambda$). The calculation, claimed to be fully under control, gives a v -independent result equal to twice the ZM contribution. If v is set to zero from the start, however, the ZM result is recovered (see Appendix). There is obviously an inconsistency between the above result and the one of our Section 3. $v \neq 0$ effects appear to provide a factor of two at $m \ll \Lambda$, while NZM have provided a factor 8/5 at $m \gg \Lambda$. However, F is m -independent (by Ward identities), hence the two estimates cannot be both correct.

On the other hand while our evaluation of F together with that of G - which as shown in I is dominated by zero modes - agree with the SUSY anomaly,

this is not the case for the evaluation of Ref. 12). The "8/5 discrepancy" that this gives rise to is assigned in Ref. 12) to some other contribution to G . It was suggested that this could be provided by a two-instanton configuration but it is easy to see that this cannot be the case. In Ref. 12), it is also argued that it may be impossible to set a reliable calculation of G , short of using the calculation of F and the anomaly constraint. Of course, by doing so, one does assume, rather than check, self-consistency.

Our feeling is rather that, if a sort of complementary description^{*)} for the massive case does exist this has to be looked for in some further modification of the Higgs picture calculation itself. It is not obvious, for instance, that NZM are completely absent in such a picture although the cut-off in ρ coming from $v = 0$ may reduce their importance at small m . In any case the possible existence of two complementary descriptions for the massive case is a very interesting issue, which certainly deserves further investigation.

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^{*)} We talk of a complementary description and not of complementarity "tout court" since, in the massive case, there is only one theory (i.e., no vacuum degeneracy except the N -fold degeneracy of Z_{2N} to Z_2 breaking).

APPENDIX

(A) We briefly recall some technical details of the instanton formalism as discussed in I, limiting ourselves for simplicity to an SU(2) gauge group. The zero-modes are normalized to 1 and read

$$\begin{aligned}\psi^{\beta\alpha} &= \frac{1}{\pi} \varepsilon^{\beta\alpha} \rho f^{3/2}, \quad f = \frac{1}{(x-x_0)^2 + \rho^2} \\ \lambda_{(\alpha)a}^{\beta b} &= \frac{1}{\pi} \left(\delta_a^\beta \delta_\alpha^b - \varepsilon^{\beta b} \varepsilon_{a\alpha} \right) \rho^2 f^2 \\ \lambda_a^{\beta(\dot{\alpha})b} &= \frac{1}{\sqrt{2} \rho} \sigma_\mu^{\dot{\alpha}\alpha} (x-x_0)_\mu \lambda_{(\alpha)a}^{\beta b}\end{aligned}\tag{A.1}$$

where $\alpha, \dot{\alpha}$ are mode labels, β is a spinor index, a and b are SU(2) indices, all with values 1 and 2. In the massless case the equation

$$(D^2 \phi)^\alpha = i\sqrt{2} g \psi^{\beta b} \lambda_b^{\alpha a} \varepsilon_{\beta a}\tag{A.2}$$

may be explicitly inverted for each zero mode for λ yielding

$$\begin{aligned}\phi_{(\alpha)}^a &= \frac{i\sqrt{2}}{4\pi^2} g \delta_\alpha^a \rho f^{3/2} \\ \phi^{\alpha(\dot{\alpha})} &= -\frac{i}{4\pi^2} \sigma_\mu^{\dot{\alpha}\alpha} (x-x_0)_\mu f^{3/2}\end{aligned}\tag{A.3}$$

The fermionic modes are integrated using Berezin's method. The bosonic measure is then¹⁶⁾

$$c \Lambda^{6-M} \int d^4 x_0 d\rho f^3, \quad c = g^{-8} 2^{10} \pi^6\tag{A.4}$$

A useful formula is

$$\int d^4 x_0 d\rho \rho^{2(m+n)-2} f(x)^n f(y)^m = \frac{\pi^2}{2(n-1)(m-1)(n+m-3)(x-y)^2}\tag{A.5}$$

In the massless case we obtain (all the calculations are in leading order in g)

$$\begin{aligned}
 \langle \lambda \lambda(x) \tilde{\phi} \phi \rangle_{M=1} &= c \Lambda^5 \frac{3g^4}{2\pi^6} \int d^4x_0 d\rho \rho^3 f(x) f(y) (x-y)^2 = \frac{c \Lambda^5 g^4}{32\pi^4} \\
 \langle \tilde{\phi}^i \phi_j(x) \tilde{\phi}^k \phi_l(y) \rangle_{M=2, m_i=0} &= \varepsilon^{ik} \varepsilon_{jl} \frac{c \Lambda^4 g^4}{32\pi^8} \int d^4x_0 d\rho \rho^5 (x-y)^2 f(x) f(y) = \varepsilon^{ik} \varepsilon_{jl} \frac{c \Lambda^4 g^4}{768\pi^6} \\
 \langle \phi_k \phi_l(x) \tilde{\phi}^i \tilde{\phi}^j(y) \rangle_{M=2, m_i=0} &= \varepsilon^{ij} \varepsilon_{kl} \frac{c \Lambda^4 g^4}{768\pi^6}
 \end{aligned} \tag{A.6}$$

In the massive case, we can also compute

$$\begin{aligned}
 \langle \lambda \lambda(x) \lambda \lambda(y) \rangle_M &= \left(\prod_{i=1}^M m_i \right) c \Lambda^{6-M} \frac{2g^4}{\pi^4} \int d^4x_0 d\rho \rho^9 f(x) f(y) (x-y)^2 = \\
 &= c \Lambda^{6-M} \frac{4g^4}{5\pi^2} \prod_{i=1}^M m_i
 \end{aligned} \tag{A.7}$$

because, as discussed in I, non-zero modes cancel out exactly in (A.7). However, when $m \neq 0$, Green's functions with matter field require more care. In the case $M = 1$, we use a path integral representation in terms of the eigenfunctions $\phi_p(x)$, $\psi_q(x)$ satisfying

$$\begin{aligned}
 D^2 \phi_p &= -h_p^2 \phi_p \\
 i \not{D} \psi_q &= h_q \bar{\psi}_q, \quad i \not{D} \bar{\psi}_q = \bar{h}_q \psi_q
 \end{aligned} \tag{A.9}$$

Finally, we obtain

$$\begin{aligned}
 \langle \lambda \lambda(x) \tilde{\phi} \phi(y) \rangle &= 2g^2 c \Lambda^5 \int d^4x_0 d\rho \rho^3 \sum_{\text{Perm}} \lambda_0 \lambda_0(x) \cdot \\
 &\sum_{p,n} \frac{\tilde{\phi}_p(y) \phi_n(y)}{(h_p^2 + m^2)(h_n^2 + m^2)} \left[\int d^4z_1 d^4z_2 (\tilde{\psi}_0(z_1) \lambda_0(z_1) \tilde{\phi}_p^*(z_1)) (\phi_n^*(z_2) \lambda_0(z_2) \psi_0(z_2)) + \right. \\
 &\left. + m^2 \sum_{s \neq 0} \frac{1}{h_s^2 + m^2} \int d^4z_1 d^4z_2 (\tilde{\psi}_s(z_1) \lambda_0(z_1) \tilde{\phi}_p^*(z_1)) (\phi_n^*(z_2) \lambda_0(z_2) \psi_s(z_2)) \right]
 \end{aligned} \tag{A.10}$$

which can be recast in the compact form (3.4).

Unfortunately, there exists no closed expression for the massive propagator in the instanton background and this is why we had to appeal to the large mass limit in order to actually compute (A.10). For the massless case,

$$\left(\frac{1}{D^2}\right)_{xy} = \frac{i}{4\pi^2} \frac{(x-x_0)_\mu (y-x_0)_\nu \sigma_{\mu\nu} \bar{\sigma}_\nu + \rho^2}{\sqrt{(x-x_0)^2 + \rho^2} \sqrt{(y-x_0)^2 + \rho^2}} \frac{1}{(x-y)^2} \quad (\text{A.11})$$

whose deviation from the free propagator is smooth in ρ and vanishes for $\rho \rightarrow \infty$. A detailed analysis of the limiting cases shows that this is also true for the massive case. Consequently, we can estimate the order of magnitude of the contributions coming from the instantons with size

$$\rho \leq \rho_0 \ll |x-y| \quad (\text{A.12})$$

without knowing the detailed form of the propagator in this region. Having in mind Eq. (3.7) we can see that in the leading order in $1/m|x-y|$, instantons of size (A.12) give contributions to the two terms of (A.10) which are respectively:

$$\begin{aligned} & \int d^4 x_0 \int_0^{\rho_0} d\rho \, \rho^2 f^4(x) f^4(y) (x-y)^2 \int d^4 \tilde{z}_1 d^4 \tilde{z}_2 \left(\frac{1}{D^2-m^2}\right)_{y\tilde{z}_1} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_2 y} \\ & \sim \frac{\rho_0^2}{(x-y)^6} \int d^4 \tilde{z}_1 d^4 \tilde{z}_2 \left(\frac{1}{D^2-m^2}\right)_{y\tilde{z}_1} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_2 y} \end{aligned} \quad (\text{A.13a})$$

and

$$\begin{aligned} & m^2 \int d^4 x_0 \int_0^{\rho_0} d\rho \, \rho^9 f^4(x) f^4(y) (x-y)^2 \int d^4 \tilde{z}_1 d^4 \tilde{z}_2 \left(\frac{1}{D^2-m^2}\right)_{y\tilde{z}_1} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_1 \tilde{z}_2} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_2 y} \\ & \sim \frac{m^2 \rho_0^6}{(x-y)^6} \int d^4 \tilde{z}_1 d^4 \tilde{z}_2 \left(\frac{1}{D^2-m^2}\right)_{y\tilde{z}_1} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_1 \tilde{z}_2} \left(\frac{1}{D^2-m^2}\right)_{\tilde{z}_2 y} \end{aligned} \quad (\text{A.13b})$$

where in the final integrals an average value of collective co-ordinates is understood. Then both expressions vanish in the limit implied in Eqs. (3.5) and (A.12).

On the other hand, for $\rho \gg 1/m$, the full propagator differs from the free one by corrections of order $1/m^2 \rho^2$. Using this result and Eq. (A.13) we can conclude that in the limit (3.5) the replacement of the free propagator in Eq. (A.10) induces corrections which are at most of order $1/m^2 |x-y|^2$.

The first term of (A.10) (which was the only one appearing in the massless case) becomes

$$\begin{aligned} & \int d^4 x_0 \int_0^\infty d\vartheta \vartheta'' f_1^4 \int d^4 z_1 d^4 z_2 f^{7/2}(z_1) f^{7/2}(z_2) \Delta(y-z_1) \Delta(z_2-y) \sim \\ & \sim \int d^4 z_1 d^4 z_2 \Delta(y-z_1) \Delta(z_2-y) \int \frac{d\alpha d\beta d\delta \delta(1-\alpha-\beta-\delta) (x-z_1) \cdot (x-z_2) \alpha^3 \beta^{5/2} \delta^{5/2}}{[\alpha\beta(x-z_1)^2 + \alpha\delta(x-z_2)^2 + \beta\delta(z_1-z_2)^2]^3} \sim \\ & \sim \frac{1}{m^4 (x-y)^4} \end{aligned} \quad (A.14)$$

which, again, vanishes in the limit (3.5). The dominant contribution comes thus from the second term of (A.10) which was given in Eq. (3.10) and reads

$$\begin{aligned} & m^2 \frac{144}{\pi^4} c g^4 \Lambda^5 \int d^4 x_0 \int_0^\infty d\vartheta \vartheta^9 f_1(x) \int d^4 z_1 d^4 z_2 f^2(z_1) f^2(z_2) (x-z_1) \cdot (x-z_2) \Delta(y-z_1) \Delta(z_1-z_2) \Delta(z_2-y) \\ & = m^2 \frac{288}{\pi^2} c g^4 \Lambda^5 \int d^4 z_1 d^4 z_2 \int \frac{d\alpha d\beta d\delta \delta(1-\alpha-\beta-\delta) \alpha^3 \beta \delta (x-z_1) \cdot (x-z_2)}{\alpha\beta(x-z_1)^2 + \alpha\delta(x-z_2)^2 + \beta\delta(z_1-z_2)^2} \Delta(y-z_1) \Delta(z_1-z_2) \Delta(z_2-y) \\ & = m^2 \frac{288}{\pi^2} c g^4 \Lambda^5 \left(\frac{1}{180} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 + m^2)^3} + O\left(\frac{1}{m^4 (x-y)^2}\right) \right) \end{aligned} \quad (A.15)$$

We can then conclude that

$$\lim_{m^2(x-y)^2 \rightarrow \infty} \langle \lambda \lambda(x) \tilde{\Phi} \Phi(y) \rangle_{N=1} = \frac{c \Lambda^5 g^4}{20 \pi^4} \quad (A.16)$$

which is the result reported in (3.12).

(B) We wish to comment now on the renormalization group properties of our Green's function. Let us note that the constant c , given in (2.8) for $N = 2$, depends on g as g^{-4N} so that

$$\langle \lambda \lambda(x_1) \dots \lambda \lambda(x_N) \rangle = (\text{numerical constant}) \cdot g^{-2N} m^N \Lambda^{3N-4} \Big|_{1\text{-loop}} \quad (A.17)$$

In this quantity, the factor g^{-2N} can be reabsorbed in higher order renormalization group-invariant quantities. Indeed, writing for the β function and for γ_m - anomalous dimension of m - the usual perturbative expansion

$$\gamma_m = \gamma_m^{(1)} + \gamma_m^{(2)} + \dots, \quad \beta = \beta^{(1)} + \beta^{(2)} + \dots$$

(A.18)

we see that the needed relation

$$\begin{aligned} g^{-2N} m^N \left(\mu e^{-\int \frac{dg}{\beta(g)}} \right)^{3N-M} &= \\ &= \left(\mu e^{-\int \frac{\gamma_m^{(1)} dg}{\beta^{(1)} + \beta^{(2)}}} \right)^N \left(\mu e^{-\int \frac{dg}{\beta^{(1)} + \beta^{(2)}}} \right)^{3N-M} (1 + O(g^2)) = m_{iuv}^M \Lambda_{iuv}^{3N-M} \end{aligned} \quad (A.19)$$

is satisfied provided we use the known results¹⁸⁾

$$\begin{aligned} \beta^{(1)} &= -\frac{g^3}{16\pi^2} (3N-M), \quad \beta^{(2)} = \frac{g^5}{(16\pi^2)^2} (-6N^2 + 2MN + 2M(N - \frac{1}{N})) \\ \gamma_m^{(1)} &= -\frac{1}{8\pi^2} \frac{N^2-1}{N} g^2 \end{aligned} \quad (A.20)$$

Equation (A.20) coincides with the expression of Ref. 1) which was obtained by similar considerations. Indeed the g -dependence of correlation functions such as $G_{MN}^{p,q}$ of Eq. (2.2) is determined by the anomalous dimensions of $\phi\phi$ which is $-\gamma_m$ (since $m\phi\phi$ is renormalization group invariant). One can easily check that this implies the g -dependence in Eqs. (A.6).

(C) Let us finally discuss the relation between our massless result and the $v \neq 0$ approach of Ref. 12) for the specific case $N = 2, M = 1$. The condensate $F(x,y)$ [defined in (3.2)] is given for $v \neq 0$ in Eq. (29) of Ref. 12) and, as discussed by the authors, it has a constant value independent both on v and $(x-y)$ and proportional to Λ^5 . But if $v = 0$ is set in their expression, a value which is a half of the constant discussed above is obtained¹²⁾. We thus see that their expression has in v the peculiar behaviour we encountered in m , i.e., that the $v \rightarrow 0$ limit of

a v independent result differs from the $v = 0$ result^{*)}. On the other hand, setting $v = 0$ in their Eq. (29) and integrating over the Grassmannian collective co-ordinates $\theta_0, \bar{\theta}_0$ and $\bar{\beta}$, one obtains

$$K \Lambda^5 \int d^4 x_0 d\rho \rho^7 f^4(x) f^3(y) (x-y)^2$$

which coincides with our massless result even before x_0 and ρ integration.

^{*)} This is due to the fact that $e^{-4\pi^2 \rho^2 v^2}$ is an infra-red regulator and terms in v^2/v^2 and v^4/v^4 subsist in the limit $v \rightarrow 0$.

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FIG. 1