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CONSISTENT SUPERSTRINGS AS SOLUTIONS OF THE $D = 26$ BOSONIC STRING THEORY

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ABSTRACT

Consistent closed ten-dimensional superstrings, i.e., the two $N = 1$ heterotic strings and the two $N = 2$ superstrings, are contained in the 26-dimensional bosonic closed string theory. The latter thus appears as the fundamental string theory.

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Consistent ten-dimensional closed superstrings^{1),2)} arise as particular solutions of the 26-dimensional closed bosonic string theory. Namely, we shall show that the states and the interactions defining such a superstring form a subset of states and interactions of the bosonic string; this subset is fully determined by choosing for the vacuum an appropriate soliton state. The emergence of space-time fermions and of supersymmetry, anticipated by Freund³⁾, is an impressive property of the bosonic string, whose apparent unphysical nature merely reflected the inadequacy of the tachyonic vacuum. The 26-dimensional bosonic string theory, which can also be viewed as conformal general relativity in two dimensions, now appears at a fundamental level as the only relevant string theory.

Before presenting the derivation of our result, we shall explain in qualitative terms why ten-dimensional supersymmetry is expected to hide in the simpler bosonic theory^{*)}. To generate space-time fermions from the bosonic string, one must meet at least two requirements. First, some dimensions $r = 24-d$ have to be compactified, leaving d uncompactified transverse dimensions, in such a way that the internal symmetry group G resulting from the compactification contains an internal group $\widetilde{SO}(d)$, the covering group of $SO(d)$. This can be achieved by the torus compactification for a suitably chosen simply-laced group G of rank r . Second, the transverse group $SO(d)$ of the non-compactified dimensions must be mapped onto $\widetilde{SO}(d)$ so that the diagonal subgroup $SO(d)_{\text{diag}}$ of $SO(d) \times \widetilde{SO}(d)$ becomes identified with a new transverse group. In this way, spinor representations of $\widetilde{SO}(d)$ describe fermionic states because a rotation in space induces a half-angle rotation on these states. The consistency of the above procedure critically depends on the possibility of extending the algebra $so(d)_{\text{diag}}$ to the full Lorentz algebra $so(d+1,1)_{\text{diag}}$. Supersymmetry necessitates a third requirement which will turn out to be crucial also for closure of the Lorentz algebra $so(d+1,1)_{\text{diag}}$ and for the removal of the tachyonic state: a consistent truncation must be performed on the spectrum of the bosonic string. This is to be expected because the consistency of supersymmetry in $D = d+2$ dimensions is guaranteed by a super Virasoro algebra rooted in a local supersymmetry of the world sheet; hence some bosonic degrees of freedom will be used in building a super Faddeev-Popov ghost, and will decouple from the physical transverse states. More precisely, in a supersymmetric sector of the closed string, the states involving p bosonic operators pertaining to the r compactified dimensions must decouple, except possibly for some zero modes. We thus determine p by the cancellation of two-dimensional conformal anomalies. In units such that the conformal anomaly is -1

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*) The results of this paper can also be extended to open strings if one allows for suitable Chan-Paton factors.

for a scalar and $-\frac{1}{2}$ for a Majorana fermion, the supersymmetric ghost has an anomaly -11 ⁴⁾. Matching the anomaly of the superghost and of the unphysical longitudinal and time-like Majorana fermions to the anomaly of the decoupled bosonic states yields the prediction $p \cdot (-1) = -11 + 2 \cdot (-\frac{1}{2})$, hence $p = 12$. To ensure a consistent decoupling, we require $\widetilde{SO}(d)$ to belong to a regular embedding of G so that the root lattice of $\widetilde{SO}(d)$ is contained in the root lattice of G . Hence d is even and the $d/2$ bosonic operators generating $SO(d)$ belong to a subset of the r operators generating G . As $p + (d/2) \leq r = 24-d$, we get when $p = 12$,

$$d \leq 8 \tag{1}$$

The highest available value of d corresponds to a ten-dimensional supersymmetry. In this case, the truncation has the effect of breaking G to $\widetilde{SO}(8) \times g$ where g is some group which can act only on the vacuum state; all states created by oscillators in the compactified dimensions which do not belong to the Cartan subgroup of $\widetilde{SO}(8)$ have indeed decoupled. This breaking, which is necessary to generate supersymmetry, has two additional but related implications. First, the original Lorentz group is broken, leaving room for a new Lorentz group whose transverse subgroup is $SO(8)_{\text{diag}}$. Second, if g contains an invariant subgroup which is contained in the Cartan subgroup of G , we expect, as in monopole theory⁵⁾, that a topologically stable soliton state can be created from a winding number quantized on the root lattice of G . This will lead to a new vacuum where the tachyon is annihilated by the soliton. In this way, a Poincaré supersymmetric, tachyon-free solution of the compactified bosonic string will emerge.

We now turn to the detailed derivation of these results. We first examine the consistency of the torus compactification for the closed bosonic string. The main point of torus compactification for open and closed strings is that momenta in the compact dimensions are quantized on the root lattice Λ_R of a simply-laced group G (or possibly on an even lattice containing the root lattice as a sublattice). It then follows that the full Lie algebra of G can be constructed out of the vertex operators in the compact dimensions⁶⁾ and the theory has local G invariance⁷⁾. The mass formulae for the closed strings are^{1), 8), 9), *)}

$$m^2 = \frac{1}{2} (m_R^2 + m_L^2) \tag{2}$$

where

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*) Whenever unspecified in the text, our notations and conventions are those of Ref. 8).

$$\frac{1}{8} m_L^2 = \frac{1}{8} (\vec{p} - 2\vec{L})^2 + N_L - 1 \quad (3)$$

$$\frac{1}{8} m_R^2 = \frac{1}{8} (\vec{p} + 2\vec{L})^2 + N_R - 1$$

Here $\vec{p}$ is a quantized momentum and $\vec{L}$ a "winding vector" which is constrained by the condition $m_R^2 = m_L^2$ to satisfy

$$\vec{p} \cdot \vec{L} = N_R - N_L \quad (4)$$

In the following, we shall take $\vec{p} \in \Lambda_R$ (the extension to the more general case is obvious). In Ref. 7), it was shown that the quantities

$$\vec{w}_L \equiv -\frac{1}{2} \vec{p} + \vec{L}, \quad \vec{w}_R \equiv \frac{1}{2} \vec{p} + \vec{L} \quad (5)$$

must belong to the weight lattice $\Lambda_W = \Lambda_R^*$ of G. Hence (3) and (4) now read

$$\frac{1}{8} m_L^2 = \frac{1}{2} \vec{w}_L^2 + N_L - 1; \quad \frac{1}{8} m_R^2 = \frac{1}{2} \vec{w}_R^2 + N_R - 1 \quad (6)$$

$$\frac{1}{2} (\vec{w}_R^2 - \vec{w}_L^2) = N_R - N_L \quad (7)$$

Equation (5) implies the constraint

$$\vec{w}_R - \vec{w}_L \in \Lambda_R = \Lambda_W^* \quad (8)$$

which further restricts (7). As the group constraint (8) was obtained by considering closed strings as bound states of open strings in Ref. 7), we shall now examine its necessity for a theory of closed strings only. To this effect, we consider one-loop amplitudes of closed strings which must be invariant under the modular group⁸⁾. Since the uncompactified D = 26 closed string loop amplitudes are modular invariant, it is sufficient to verify the invariance for the "correction factor"⁹⁾ which arises from the replacement of the integral over internal momenta by a discrete sum. This factor reads

$$R(\tau) \equiv \left(\frac{\text{Im} \tau}{2} \right)^{r/2} \sum_{\substack{\vec{w}_R, \vec{w}_L \in \Lambda_W \\ \vec{w}_R - \vec{w}_L \in \Lambda_R}} e^{i\pi\tau \vec{w}_R^2} e^{-i\pi\tau^* \vec{w}_L^2} \quad (9)$$

where the group constraint (8) has been taken into account. The modular group is generated by the discrete translations $\tau \rightarrow \tau + n$, $n \in \mathbb{Z}$, which leave $R(\tau)$ invariant because of (7), and by the inversion $\tau \rightarrow -1/\tau$. To establish the invariance of $R(\tau)$ with respect to the latter, one uses the Jacobi identity⁷⁾

$$F_{\Lambda_R}(\vec{v}|\tau) = \frac{1}{\sqrt{Z}} \frac{1}{(-i\tau)^{r/2}} e^{-\frac{i\pi \vec{v}^2}{\tau}} F_{\Lambda_W}\left(-\frac{\vec{v}}{\tau} \middle| -\frac{1}{\tau}\right) \quad (10)$$

where

$$F_{\Lambda}(\vec{v}|\tau) \equiv \sum_{\vec{q} \in \Lambda} e^{i\pi\tau \vec{q}^2} e^{2\pi i \vec{v} \cdot \vec{q}} \quad (11)$$

and Z is the order of the centre of the covering group of G . Substituting (10) into (9) but ignoring the group constraint (8), we would obtain

$$R(\tau) = Z \left(\frac{\text{Im}(-\frac{1}{\tau})}{2} \right)^{r/2} \sum_{\vec{P}_R, \vec{P}_L \in \Lambda_R} e^{-\frac{i\pi}{\tau} \vec{P}_R^2} e^{\frac{i\pi}{\tau^*} \vec{P}_L^2} \quad (12)$$

which is equal to $R(-1/\tau)$ only if $Z = 1$ and $\Lambda_R = \Lambda_W$, that is, if G is self-dual. Note that the group constraint (8) is then empty, provided that the same self-dual group acts on the left and right sectors, but we can also use two different self-dual groups in the two sectors. In fact, compactification on self-dual groups results in a complete factorization of the closed string into two open strings, except for the mass constraint (7); in this case and in this case only, the mass spectrum (6) coincides with the spectrum of an open string compactified on an even lattice.

If G is not self-dual, the group constraint (8) must be imposed and is indeed sufficient to guarantee modular invariance¹⁰⁾. To see this, we rewrite (9) as

$$R(\tau) = \left(\frac{\text{Im}\tau}{2} \right)^{r/2} \sum'_{\vec{w}_0} \sum_{\vec{P}_R, \vec{P}_L \in \Lambda_R} e^{i\pi\tau(\vec{w}_0 + \vec{P}_R)^2} e^{-i\pi\tau^*(\vec{w}_0 + \vec{P}_L)^2} \quad (13)$$

where the finite sum Σ' runs over the set of minimal weights $\vec{w}_0$ which label all distinct irreducible representations of the centre. On substituting (10) into (13), one gets an unrestricted sum over weights $\vec{w}_R$ and $\vec{w}_L$, which in addition

contains a factor

$$S(\vec{w}_R, \vec{w}_L) \equiv \frac{1}{Z} \sum'_{\vec{w}_0} e^{-2i\pi \vec{w}_0 (\vec{w}_R - \vec{w}_L)} \quad (14)$$

This factor reinstates the constraint in $R(-1/\tau)$ and hence proves modular invariance. Indeed, (14) is the sum over all irreducible representations of the centre of the element represented by $\vec{w}_R - \vec{w}_L \pmod{\vec{p}}$. Hence, from the standard orthogonality theorem for characters of finite groups, $S = 0$ except when $\vec{w}_R - \vec{w}_L \in \Lambda_R$, in which case $S = 1$.

As we require consistency at the level of the untruncated spectrum, we see from the above analysis of closed string compactifications that a supersymmetric solution of the heterotic type²⁾ can only arise from compactification of the closed bosonic string on self-dual groups: a group constraint between the left and right sectors of the string is not allowed in this case. Such compactifications are also a natural starting point for constructing $N = 2$ superstrings; the latter must factorize (except for the mass constraint) into two open strings, at least after the truncation, and this factorization is guaranteed if the original compactification is performed on self-dual groups. We shall therefore consider only such compactifications.

We next discuss the Lorentz algebra. The new transverse Lorentz algebra $so_{diag}(d)$ is generated by

$$J^{ij} \equiv L^{ij} + K_0^{ij} \quad (15)$$

where L^{ij} are the usual generators of the space-time $SO(d)$ (indices i, j run from 1 to d) and the operators K_0^{ij} generate the internal $\widetilde{SO}(d)$. They are the momenta and the constant mode in the expansion of the bosonic vertex operators in the internal dimensions⁶⁾. More precisely, the internal $\widetilde{SO}(d)$ is that subgroup of the internal group which is associated with the string co-ordinates X^A , $A = d+1, \dots, 3d/2$. The full Kac-Moody algebra of the vertex operators reads^{6), 11)}

$$\begin{aligned} [K_m^{ij}, K_n^{kl}] = & -i (K_{m+n}^{ie} \delta^{jk} + K_{m+n}^{jk} \delta^{ie} - \\ & - K_{m+n}^{ik} \delta^{je} - K_{m+n}^{je} \delta^{ik}) + 2m \delta_{kl}^{ij} \delta_{m+n,0} \end{aligned} \quad (16)$$

The crucial consistency check now consists in verifying whether the new transverse Lorentz algebra can be extended to the full Lorentz algebra. Thus, one has to demonstrate the existence of boost generators

$$J^{i-} \equiv L^{i-} + K^{i-} \quad (17)$$

such that the algebra closes. For L^{i-} , we take the usual expression^{1),8)}

$$L^{i-} \equiv x^i p^- - x^- p^i + \frac{i}{p_+} \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^- \alpha_n^i - \alpha_{-n}^i \alpha_n^-) \quad (18)$$

where

$$\begin{aligned} \alpha_m^- &\equiv \alpha_m^{-(d)} + \alpha_m^{-(d/2)} \equiv \\ &\equiv \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(: \alpha_{m-n}^i \alpha_n^i : + : \alpha_{m-n}^A \alpha_n^A : \right) \end{aligned} \quad (19)$$

includes the contributions from the internal directions $A = d+1, \dots, 3/2d$. The mass shell condition is

$$p^+ p^- = \alpha_0^- - C \quad (20)$$

where the intercept C is left unspecified for the moment. It is now obvious that, as a result of the truncation, the new Lorentz algebra will fail to close except for a very specific choice of K^{i-} in (17). If K^{i-} is to be constructed out of the available operators, one possibility is¹¹⁾

$$K^{i-} \equiv \frac{1}{p_+} \sum_{n=-\infty}^{\infty} K_{-n}^{ij} \alpha_n^j \quad (21)$$

(we have already adjusted a free coefficient). After a somewhat lengthy calculation, which only requires the use of (16) and of the bosonic commutator $[K_m^{ij}, \alpha_n^j]$, Ref. 11, besides the usual manipulations for the d -dimensional bosonic string theory, one obtains

$$[J^{i-}, J^{j-}] =$$

$$\begin{aligned}
 &= -\frac{1}{P_+^2} \sum_{n=1}^{\infty} \left(n \left(1 - \frac{d}{8} \right) - \frac{1}{n} \left(2C - \frac{d}{8} \right) \right) (\alpha_{-n}^i \alpha_n^j - \alpha_{-n}^j \alpha_n^i) + \\
 &+ \frac{1}{P_+^2} \left(2iC K_0^{ij} + \sum_{n=-\infty}^{\infty} \left(n K_{-n}^{ik} K_n^{jk} - 2i K_{-n}^{ij} \alpha_n^{-(d/2)} \right) \right) \quad (22)
 \end{aligned}$$

The first term vanishes for $d = 8$ and $C = \frac{1}{2}$. The fact that $C = \frac{1}{2}$ and not $= 1$ is essential to eliminate the tachyon and hence for the stability of the truncation; we will return to this point below. From now on, we will confine our attention to the case $d = 8$.

To show that the other term in (22) also vanishes for this choice, we express the bosonic vertices in terms of fermionic operators¹¹⁾. Since for each bosonic direction we get one complex fermion or, equivalently, two real ones, there are altogether eight real fermionic operators q^i . These can be assigned to any of the three eight-dimensional representations of $\widehat{SO}(8)$, but for the moment we take them to transform as vectors (this corresponds to the "old" superstring formalism). An explicit representation of the Kac-Moody generators K_m^{ij} is then furnished by

$$K_m^{ij} = -i \sum_{n=-\infty}^{\infty} q_{m-n}^{[i} q_n^{j]} \quad (23)$$

where the basic anticommutator relations are

$$\{q_m^i, q_n^j\} = \delta^{ij} \delta_{m+n,0} \quad (24)$$

A somewhat subtle point is that the bosonic normal ordered expression for $\alpha_m^{-(4)}$ and the corresponding fermionized normal ordered expression differ by a finite constant which may be determined by ζ -function regularization methods. The relevant result is

$$\begin{aligned}
 \frac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{m-n}^A \alpha_n^A : &= \\
 &= \frac{1}{2} \sum_{n=-\infty}^{\infty} n : q_{m-n}^i q_n^i : - \frac{d}{16} \delta_{m,0}
 \end{aligned} \quad (25)$$

Substituting this into (22) and using $C = \frac{1}{2}$, $d = 8$, the second term in (22) can be shown to vanish by (24).

One can alternatively assign the spinor q to one of the spinorial representations 8_s or 8_c of $\widetilde{SO}(8)$ [this corresponds to the "new" superstring formalism of Ref. 1)]. In this case, (23) is replaced by a new linear combination of bosonic vertices

$$K_m^{\dot{ij}} = - \frac{i}{4} \sum_{n=-\infty}^{\infty} q_{m-n}^a \delta_{ab}^{\dot{ij}} q_n^b$$

$$\text{or} = - \frac{i}{4} \sum_{n=-\infty}^{\infty} q_{m-n}^{\dot{a}} \delta_{\dot{a}\dot{b}}^{\dot{ij}} q_n^{\dot{b}} \quad (26)$$

All of the foregoing results remain valid with (26) instead of (23), as can be demonstrated by use of the triality properties of $\widetilde{SO}(8)$ [see, for example, Appendices C and D of Ref. 13)]. Observe that the proof of closure of the Lorentz algebra in the "old" and "new" formalism is reduced to virtually the same calculation.

This computation confirms the analysis in the introduction which led to (1), and furthermore shows that only the maximal value $d = 8$ can be realized in this way. Thus, we have to truncate $p = 12$ degrees of freedom. The internal symmetry group is reduced to $\widetilde{SO}(8) \times g$ where g acts only on the vacuum.

Let us now describe the emergence of space-time fermions out of the bosonic theory. In the first step of the compactification to $d = 10$, one obtains two self-dual internal groups G_L and G_R . In the next step, one analyzes the embedding of $\widetilde{SO}(8)$ into G . It is crucial at this point that $G \supset \widetilde{SO}(8) \times g$, so that the states obtained in the compactification contain both spinorial and vectorial representations of $\widetilde{SO}(8)$. Remarkably, the only simply-laced groups which contain the $\widetilde{SO}(8)$ subgroup in a regular embedding are the exceptional groups E_6 , E_7 and E_8 . Thus, we must compactify on $G = E_8 \times E_8$ if we are to get space-time fermions. In accordance with the group decomposition $E_8 \times E_8 \supset \widetilde{SO}(8) \times g$, we write

$$\vec{W}_{E_8 \times E_8} = \vec{W}_{SO(8)} + \vec{W}_g \quad (27)$$

Since the $E_8 \times E_8$ lattice is even, we have $|\vec{W}_{E_8 \times E_8}|^2 = 2$, which is what we need in order to get a massless state by (6). On the other hand, the 8_v , 8_s and 8_c representations of $SO(8)$ have $|\vec{W}_{SO(8)}|^2 = 1$, which by itself would not be enough

to have a massless state. Thus, we see that the ground state receives half of its mass from the "hypercharge" directions orthogonal to $\widetilde{SO}(8)$ on the $E_8 \times E_8$ lattice. Moreover, this state cannot be annihilated or transformed into a tachyonic state by the vertex operators of the truncated theory. This guarantees the stability of the truncation. We now also understand the significance of the value $C = \frac{1}{2}$ instead of $C = 1$ in (20). Namely, the "hypercharge" momentum contribution to α_0^- , which has been omitted in (19), removes $\frac{1}{2}$ from the original intercept value $C = 1$ in 24 dimensions. Equation (20) with $C = \frac{1}{2}$ then coincides with (6).

We now identify the hypercharge. The regular embedding of $\widetilde{SO}(8)$ in E_8 is contained in the 78-dimensional adjoint representation of E_6 . The decomposition of 78 with respect to the $\widetilde{SO}(8) \times U(1) \times U(1)$ subgroup of E_6 is¹⁴⁾

$$\begin{aligned} 78 = & 2 \times (1, 0, 0) \oplus (8_c, 2, 0) \oplus (8_c, -2, 0) \\ & \oplus (28, 0, 0) \oplus (8_s, 1, -3) \oplus (8_v, -1, -3) \\ & \oplus (8_s, -1, 3) \oplus (8_v, 1, 3) \end{aligned} \quad (28)$$

Identifying the ground state by its $\widetilde{SO}(8)$ content 8_s or 8_v , we see that the hypercharge group is $g = U(1) \times U(1)$. The full spectrum in the supersymmetric sector is then obtained by acting on these ground states with the operators q_{-n}^a , $n > 0$. Comparing (26) to the corresponding expression in superstring theory^{1),8)}, one realizes that the operator q_0^a , which was constructed out of the bosonic string, is nothing but the constant mode of the supersymmetry generator. Note that to cope with the hypercharge assignments of the (ket) ground states $(8_s, 1, -3)$ and $(8_v, -1, -3)$, we must identify the q 's in (26) with the operators $(8_c, \pm 2, 0)$ in (28), where the hypercharge component is restricted to its zero mode, in such a fashion that only states with hypercharge $(\pm 1, -3)$ are produced. The identification leaves K_m^{ij} unaffected but corrects q_0^a in such a way that the hypercharge shift between 8_v and 8_s is correctly accounted for.

It should now be clear how to get the spectrum of all consistent $D = 10$ closed superstrings in this way. Taking $G_L = E_8 \times E_8$ and $G_R = E_8 \times E_8$ or $Spin(32)/Z_2$ and performing the truncation in the left moving sector, we get the two $N = 1$ heterotic strings²⁾. On the other hand, taking $G_R = G_L = E_8 \times E_8$, we get both the chiral and the vectorlike $N = 2$ superstring theories of Ref. 1), as the

identification of 8_c and 8_s in the right and left moving sectors may be done independently. Note that in this case, the new transverse Lorentz group is the diagonal subgroup of $SO(8) \times \widetilde{SO}(8)_L \times \widetilde{SO}(8)_R$.

We still have to substantiate our claim that the truncation is not only consistent for the free states but also at the level of interactions. All scattering amplitudes of the $D = 10$ superstring theory are actually contained in those of the $D = 26$ theory. Namely, taking the purely bosonic vector emission vertex of the $D = 26$ theory (the tachyon emission vertex is irrelevant for the truncated theory), one first obtains an $SO(8)$ -invariant bosonic vertex operator V_B in 12 dimensions, cf. for example Eq. (3.23) of Ref. 8). This operator contains the operators K_m^{ij} , for which one may use the explicit representation in terms of q_n^a, q_0^a . Substituting this expression into an amplitude $\langle B_1 | \dots V_B | B_2 \rangle$, where $|B_1\rangle$ designates a bosonic ground state, and introducing a fermionic state $|F_2\rangle \equiv q_0^a |B_2\rangle$, one transfers the zero-mode q_0^a operator from the bosonic vertex to the state $|B_2\rangle$, thereby defining a fermion emission vertex V_F where q_0^a has been "peeled off" from V_B . In this fashion, the part of the bosonic amplitude $\langle B_1 | \dots V_B | B_2 \rangle$ containing q_0^a is reinterpreted as a fermion emission amplitude $\langle B_1 | \dots V_F | F_2 \rangle$, which coincides with that of superstring theory, cf. Eq. (3.34) of Ref. 8). Note that the fermionic vertex V_F contains the compensating hypercharge which renders the full amplitude hypercharge-independent. In this way, all amplitudes of the truncated theory are hypercharge-independent.

In this paper we have shown that the states, symmetries and interactions of the consistent $D = 10$ closed superstring theories are contained in the purely bosonic $D = 26$ theory. An interesting question is whether other consistent string theories may emerge in this way. For instance, are there solutions for $D < 10$ ($D = 4$?) as suggested by (1) which would in some sense correspond to torus compactification of superstrings embedded in the full $D = 26$ theory? However, the outstanding question, which remains unanswered, is by what dynamical mechanism the $D = 26$ theory selects one of the available ground states and how the decoupling we found is dynamically realized. The answer to this question requires a formulation of string theory which goes beyond the perturbative S-matrix theory approach and is ultimately connected to the very existence of a complete theory of the 26-dimensional bosonic string.

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