COMMENTS ON MULTIPLICITY IN PROTON NUCLEUS COLLISIONS

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We model particle creation in the central rapidity region in ultra-relativistic nuclear collisions by assuming an intermediate state of color flux. Our starting point is that nuclear-nuclear collisions are dominated by single gluon exchange.\(^1\) This causes the color to flip from singlet to octet so that the two receding nucleons become linked by a flux tube with the diameter of the nucleon. This flux tube then materializes to produce the observed phenomena. In the case of a proton-proton collision there will be several gluon exchanges for each quark in the projectile, which will change the details of the flux tube's formation and materialization. In what follows, we make some comments on this problem.

Suppose that the projectile nucleon suffers \(v\) interactions while traversing the target nucleus. In each collision, the projectile exchanges one soft gluon with a target nucleon. This process can be viewed as a random walk in the intrinsic color space. Therefore, the strength of the color charge built up after the collision will be proportional to the square root of \(v\):

\[
Q = \sqrt{v}
\]  

(1)

(Here, we consider a simple Abelian version of the color charge. The essential feature of the random walk process does not change for groups of higher rank.\(^2\)) Gauss' law demands that a color electric flux tube form between the projectile and the target. Since the color charge at the ends of this tube is greater than that produced in pp collisions, the field strength in the tube is initially stronger.

The particle production process can be modelled as the quantum creation of qq pairs in the strong color field \(^3\) (the Schwinger mechanism\(^4\) in QED and the subsequent combination of quarks into hadrons. The energy originally stored in the color field is gradually converted into the kinetic energy of the quarks. This process continues until the field energy is exhausted.

According to Schwinger, the pair creation rate (the probability of producing a pair per unit four-volume) is determined by the strength of the external color electric field \(E\) as

\[
P = aE^2,
\]

(2)

\(a\) is a dimensionless numerical constant. (We have taken the massless quark limit.) The total number of pairs produced out of the color field is given by the space-time integral

\[
\frac{dN}{d^4x} = \frac{d^4x}{2}\int F^2(x) = c_4AE_0^2.
\]

(3)

To evaluate this integral, we first assume that the transverse cross section of the tube is fixed at \(A\). We define the longitudinal coordinate \(z\) and time \(t\) in the center-of-mass frame and define the light cone variables

\[
\tau = \sqrt{t_0^2 - z^2}
\]

(4)

\[
y = \frac{1}{2} \ln \frac{\tau + z}{\tau - z}
\]

(5)

On average, the longitudinal velocity \(v_z\) of a secondary hadron would be related to the position \((\tau, z)\) where the particle is created by \(v_z = z/\tau\). The position of hadron creation is approximately equal to the position where qq pair creation occurs. Thus, \(y\) defined by (5) can be identified with the rapidity of hadrons.

Since the four-volume element is given by \(d^4x = dy d\tau A\), Equations (2) and (3) lead to

\[
\frac{dN}{dy} = -aA \int_0^{\infty} d\tau E_0^2.
\]

(6)

In the central rapidity region we assume that the physics is approximately invariant under Lorentz boosts in the \(z\) direction. Then the field strength \(E\) becomes a function only of the proper time \(\tau\) and hence takes the form

\[
E(\tau) = E_0 f(\tau/\tau_0),
\]

(7)

where \(E_0\), the initial field strength at \(\tau = 0\) and \(f(x)\) is a dimensionless function which satisfies \(f(0) = 1\). The constant \(\tau_0\) sets the time scale for attenuation of the color field. Since the attenuation of the field due to pair creation is controlled by the local strength of the field, and since \(\tau_0\) is the only parameter which has a dimension (of energy\(^2\) or length\(^2\)), \(\tau_0\) must be inversely proportional to \(v_E\):

\[
\tau_0 = \frac{b}{\sqrt{v_E}}.
\]

(8)

Substituting (7) into (6) and using (8) we find

\[
\frac{dN}{dy} = c_4AE_0^2 \int_0^\infty \frac{dx}{x^2} f^2(x) = c_4AE_0^2
\]

(9)

where \(c_4\) is a dimensionless constant. The initial field strength \(E_0\) is related by Gauss' law to the color charge \(Q\) built up in the collision as

\[
AE_0 = Q.
\]

(10)
Then we see that the particle density in the central rapidity region increases in proportion to $Q$,

$$\frac{dN}{dy} \propto Q.$$  \hspace{1cm} \text{(11)}

In the above discussion, we fixed the cross section of the flux tube. The tube will expand, however, because the field pressure ($\sim \frac{E^2}{2}$) is greater than the equilibrium pressure. The above derivation is right only if the field attenuation due to the pair creation is much faster than the expansion of the tube. Now let's consider the other extreme case, $\omega \ll 1$, that the pair creation is a very slow process and that the tube first expands and attains its equilibrium shape. The equilibrium cross section of the tube is given by

$$A_{eq} = \frac{Q}{E_{eq}}.$$  \hspace{1cm} \text{(12)}

where the equilibrium field $E_{eq}$ is defined to balance the external bag pressure via

$$\frac{1}{2} \frac{\pi}{\tau_{eq}} = E.$$  \hspace{1cm} \text{(13)}

In this case the number of pairs produced after the expansion can be estimated as

$$\frac{dN}{dy} = \frac{A_{eq}}{E_{eq}} \frac{2}{\tau_{eq}} E \, \mathcal{E} \, (t)$$

$$= c_{eq} A_{eq} E_{eq} = c_q Q.$$  \hspace{1cm} \text{(14)}

Thus, in both cases one sees that the particle density increases in proportion to $\sqrt{\omega}$,

$$\frac{dN}{dy} \propto \sqrt{\omega}$$  \hspace{1cm} \text{(15)}

where $c$ is a universal constant. Assuming that the number of $\bar{q}q$ pairs is proportional to the number of hadrons, this leads to

$$\frac{dN}{dy}_{AA} / \frac{dN}{dy}_{pp} = \sqrt{\omega}.$$  \hspace{1cm} \text{(16)}

This result seems to fit the data at the central rapidity region reasonably well if we use the phenomenological relation

$$v = \frac{A_{AA}}{A_{pp}} / \frac{dN}{dy}_{AA} = \sqrt{\omega}.$$  \hspace{1cm} \text{(17)}

The use of Equation (17) is equivalent, in effect, to the assumption that the number of proton interactions $v$ in the target nucleus with atomic number $A$ scales as $A^{1/3}$. This observation, combined with the foregoing discussion, allows us to make an interesting prediction for particle production and energy deposition in ultrarelativistic nucleus-nucleus collisions. The central collision of two identical heavy nuclei may be considered as the incoherent creation of many flux tubes whose cross section is given by

that of a proton. In this case, the number of interactions (color exchange processes) which take place in each tube creation would be proportional to $A^{1/3} \times A^{1/3} = A^{2/3}$. Thus, average local color charge density per unit transverse area built up after the nucleus-nucleus collision grows as $A^{2/3} = A^{1/3}$. This implies that one can expect an energy density in the central rapidity region $\sim A^{2/3}$ times that in a pp collision. This initial condition also leads to faster quark pair production characterized by $t_0 = A^{-1/3}$. Under such circumstances, we may suppose that the matter produced by the decay of color flux takes the form of a plasma of unconfined quarks and gluons, which later materializes into hadrons.

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REFERENCES

6. This fact was pointed out by M. Faessler.