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THE P-MATRIX AND A MATCHING RADIUS MODEL FOR PION HADRON SCATTERING

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Abstract

A matching radius model of pion hadron scattering is developed by assuming the continuity of the axial current. This model is applied to the simplified situation of a pion scattering off a static empty bag to study the relationship between P-matrix poles and the bag model primitives. It is found that only under extremely simplified assumptions can the P-matrix poles be identified with the bag model primitives.

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- 2 -

9

Quark bag models have enjoyed considerable popularity especially in the realm of spectroscopy [1,2]. For scattering problems generalizations of the MIT bag model are usually used, for example, the Cloudy Bag Model [3] (CBM) or Quark Compound Bag Model [4] (QCB). A more direct use of the MIT bag model in scattering is the P-matrix approach of Jaffe and Low [5]. The P-matrix approach has implications for both spectroscopy and scattering since it identifies bag-model states (primitives) with the poles of the P-matrix and not the poles of the S-matrix. Hence the primitives can not all be associated with resonances.

The purpose of the present paper is two fold; first to develop a matching radius model for pion-hadron scattering and second to apply this model to pion scattering off an empty static bag to illustrate that the bag model primitives can not be simply identified with the P-matrix poles. The idea of the model is to divide space into the two regions. In one region quark (and in principle gluon) degrees of freedom are used and in the other meson degrees of freedom. The two regions are connected by making an appropriate current continuous across the surface. For pion scattering the most convenient current is the axial current since it is linear in the pion field. To have the vector current also continuous it would be necessary to include vector mesons which couple directly to the vector current. We are only assuming the axial current is continuous not that it is conserved. The two are quite different.

For simplicity we consider the model problem of pion scattering off an empty static bag. This is a lowest order approximation to πN scattering where the only interaction between the pion and nucleon is due to the modification of the vacuum because of the quarks in the nucleon.

The effect of the perturbative gluon exchange interaction can then be added at a later stage.

Consider a spherical cavity of radius b . Inside the cavity ($r < b$) we use quark degrees of freedom and outside ($r > b$) meson degrees of freedom. In both regions consider a state of the form $(c_1 + c_2 \hat{a}_N^\dagger) |0\rangle$ where $|0\rangle$ is the vacuum state and $\hat{a}_N^\dagger$ creates a pion with quantum numbers N . The constants c_1 and c_2 , which for simplicity we take to be real, satisfy the condition $c_1^2 + c_2^2 = 1$ and determine the probability that a pion is present. For the matching we consider the expectation value of the axial current operator $\hat{A}^\mu(r)$ for this state. Thus we have

$$\hat{A}^\mu(r) = \langle 0 | (c_1 + c_2 \hat{a}_N^\dagger) \hat{A}^\mu(r) (c_1 + c_2 \hat{a}_N^\dagger) | 0 \rangle \quad (1)$$

The assumption now is that for $r > b$ it is sufficient to write

$$\hat{A}^\mu(r) = f \partial^\mu \phi(r) \quad r > b \quad (2)$$

where $\phi(r)$ is the pion wave-operator and f the pion decay constant, while for $r < b$ we must write the axial current operator in terms of the quarks degree of freedom, thus

$$\hat{A}^\mu(r) = \frac{\hat{A}}{\psi(r)} \gamma^\mu \gamma_5 \tau/2 \psi(r) \quad r < b \quad (3)$$

Considering first $r > b$, we have by combining eq. (1) and eq. (2)

$$\hat{A}^\mu(r) = 2fc_1c_2 \text{Re} [\partial^\mu \langle 0 | \phi(r) \hat{a}_N^\dagger | 0 \rangle] \quad (4)$$

Note that the term quadratic in c_2 vanishes. Defining $\phi_N(r) = \langle 0 | \phi(r) \hat{a}_N^\dagger | 0 \rangle$ as the pion wave-function with quantum numbers N , we have

$$\phi_{\omega_N, 0}(r) = c_N \frac{\sin(k_\pi r + \delta)}{k_\pi r} e^{-i\omega_\pi t} \quad (5)$$

for angular momentum $L=0$ and pion energy $\omega_\pi = \sqrt{k_\pi^2 + m_\pi^2}$ (for simplicity we consider only $L=0$ and suppress isospin indices). The actual value of the normalisation constant, c_N , will not be needed. Eq. (5) is the most

general S-wave wave-function since $\phi(r)$ must satisfy the Klein-Gordon equation. If eq. (7) was used in all space then δ would have to be zero in order to have $\phi(r)$ regular at the origin. In the present case δ will be determined from the matching condition at the radius b . The axial current as r approaches b from outside is given by

$$A^0(r) = -2fc_1c_2c_N\omega_\pi \sin(\omega_\pi t) \frac{\sin(k_\pi b + \delta)}{k_\pi b} \quad (6)$$

and

$$A^+(r) = 2fc_1c_2c_Nk_\pi \frac{\cos(k_\pi b + \delta)}{k_\pi b} - \frac{\sin(k_\pi b + \delta)}{(k_\pi b)^2} \quad (7)$$

Inside $r=b$ the situation is more complicated. Here we must express $\hat{a}_N^\dagger$ in terms of quark fields. The simplest assumption is that both quark and antiquark are in s-states with the same energy. In this case, using MIT bag states, the axial current is given by (using eq. (1) and eq. (3))

$$A^0(r) = -c_1c_2c_N \sin(2\omega_q t) \left[\frac{1}{2} \langle 0 | \psi_q(r) \gamma_0^z \psi_q(r) | 0 \rangle \right] \quad (8)$$

and

$$\hat{A}^+(r) = c_1c_2c_N \cos(2\omega_q t) r 2J_0(k_q r) J_1(k_q r) \quad (9)$$

where $\omega_q = \sqrt{k_q^2 + m_q^2}$ and $\Omega = \sqrt{(\omega_q - m_q)/(\omega_q + m_q)}$ and c_N' is another normalisation factor. Note that again the term quadratic in c_2 vanishes. For a one channel problem the P-matrix is just [5] $P = k_\pi \cot(k_\pi b + \delta)$, so from matching the ratio $\hat{A}/A^0$ we have

$$P(k_\pi, b) = k_\pi \cot(k_\pi b + \delta) = \frac{1}{b} + \frac{2\omega_\pi J_0(k_q b) J_1(k_q b)}{J_0^2(k_q b) - \Omega^2 J_1^2(k_q b)} \quad (10)$$

The quark energy and hence momentum are determined by requiring the same time dependences on both sides of $r=b$, thus $\omega_q = \omega_\pi/2$. The phase shift for low energy scattering is shown in the figure for $b=1$ fm, $m=140$ MeV and $m_q=0$. The size of the phase-shift is actually comparable in size

to the experimental results. The first pole in P is at $k_q=2.04/b$ corresponding very nicely to the first bag model primitive as suggested by Jaffe and Low. The second pole is at $k_q=3.81/b$, also a bag primitive, but corresponding to a quark in the $J=1/2^-$ state ($j_0(k_q b)=-j_1(k_q b)$) whereas the state we have used for the quark is $J=1/2^+$. Thus there are twice as many poles as naively expected. In the limit of large energy the phase shift goes to $\pi/2$ which is equivalent to scattering off a repulsive $1/r^2$ potential. It might be noted that although the poles in the P-matrix are all shifted up in energy suggesting repulsion the low energy phase-shift is positive suggesting attraction. Hence nothing can be learned about low energy scattering by merely looking at the poles of P-matrix. This may not be surprising considering that the first pole is already 800 MeV above threshold. Of course if we knew both poles and residues then the P-matrix would be completely determined.

Our choice for the quark wave-function is by no means unique. We have put both the quark and antiquark into $J=1/2^+$ states (the antiquark has negative intrinsic parity so it is responsible for the negative parity of pion). We could equally well have used the $J=1/2^-$ states. Despite the fact that the bag model primitives for the two sets of quantum numbers are different the P-matrices are the same. We can continue: Why not put the quark and antiquark into states of higher J? We can still couple the quark and antiquark pair to total angular momentum zero and make the axial current continuous. Each value of J would entail a whole new set of primitives which a priori is just as good as the one corresponding to $J=1/2^+$. Another option is to say that the S-wave P-matrix has a pole when there is a primitive for any value of J. This rather clearly leads to too many poles and a very rapidly increasing

phase shift. A similar criticism applies to the QCB model [4]. The origin of the problem will become clearer after the next example.

Let us consider another perhaps more reasonable choice for the quark wave-function. The pion is not composed of an uncorrelated quark-antiquark pair. So let us consider another extreme, namely the quark antiquark pair is strongly correlated in momentum space. Thus we take for the pion wave-function

$$\hat{a}_N^+ = \int d\Omega_k \left(\begin{matrix} b^+ & d^+ \\ k & k \end{matrix} \right)_{s=0} \quad (11)$$

where $\hat{b}^+$ and $\hat{d}^+$ are the creation operators for the quark and antiquark respectively. The superscript $s=0$ indicates that the quark-antiquark pair are coupled to spin zero. In terms of the quark (antiquark) creation operators $\hat{b}_{j\mu p}^+$ ($\hat{d}_{j\mu p}^+$) of good angular momentum J and parity P this new state can be written as

$$a_N^+ = - \sum_{j\mu p} (-)^J S \hat{b}_{j\mu p}^+ \hat{d}_{j\mu p}^+ \quad (12)$$

where S is 1 for even parity and -1 for odd parity. This new state is just a particular linear combination of the states discussed in the last paragraph. It can no longer be written as a simple product state. In this case the P-matrix is

$$P = k_q \cot(k_q b + \delta) = 2k_q \cot(2k_q b) \quad (13)$$

Again we have $\omega_q = \omega_\pi/2$. Eq. (13) is the same as the result for scattering off a square well, where the well depth is $2m_q - m_\pi$. Thus when $m_\pi = 2m_q$, δ is zero. In the chiral limit ($m_\pi = m_q = 0$) a pion scattering off an empty bag will give zero shift, a satisfying result.

The result for $m_\pi = 140$ MeV, $m_q = 0$ and $b=1$ are given by the dashed line in the figure. In this case the poles are given by $k_q = N\pi/2$, where

N is an integer. These poles have no simple relation to the bag model primitives. The lowest pole differs from the lowest primitive by about 20%. As the energy increases the phase shift and hence the cross-section goes to zero. This contrasts with the previous example where the phase-shift went to $\pi/2$ and the cross-section became as large as possible for an S-wave scattering, namely $\sigma=4\pi/k_n^2$. Although our second choice of wave-function is not perfect we believe it is much better than the original completely uncorrelated wave-function.

We would agree with Jaffe and Low [5] that all bag model primitive do not correspond to resonances since color singlet pairs are not confined. However, we do not believe they correspond to poles of the P-matrix either. The connection is broken once we allow correlated pairs since then the properties of individual terms (in eq. (12) for example) add coherently. Thus instead of the higher J states adding more poles they merely shift the P-matrix poles away from the MIT bag model primitives. We note that for the lowest pole the effect is of the order of 20% and we can retain the agreement with experimental data found by Jaffe and Low by, for example, adjusting relation between the bag model radius and the matching radius.

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FIGURE CAPTION

The phase shift versus pion energy for a pion scattering off an empty bag. The solid line corresponds to an uncorrelated quark-antiquark pair. The dashed line corresponds to having the quark-antiquark pair strongly correlated in momentum space.