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MAXIMAL SPONTANEOUS CP-VIOLATION AND LEFT-RIGHT SYMMETRY

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ABSTRACT

We discuss an interesting form of quark mass matrices that arises in left-right symmetric models of weak interactions. It leads to a maximal CP-phase in the left-handed charged-current quark mixing matrix and to predictions for the mixing matrix elements U_{us} and U_{cb} in agreement with experiment, leaving U_{ub} as a free parameter. We describe a way in which the model may be realized in a technically natural manner.

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Despite many successes, a major shortcoming of the standard model of electroweak interactions is its inability to explain quantitatively the phenomenon of quark mixing as well as CP-violation. It however provides a parametrization of this mixing in terms of a unitary 3×3 matrix with three real mixing angles and one phase. One of these angles is the Cabibbo angle, which is known rather precisely. Recent experimental progress in the study of the properties of b-quarks has led to strong restrictions on the allowed values of the other two angles¹⁾. This, in turn, has led to a situation where understanding of the observed magnitude of CP-violation in $K_L \rightarrow 2\pi$ decay, in terms of left-handed currents, requires the violation to be as large as it possibly could be.

It is very convenient to describe the quark mixing matrix in terms of the magnitudes $|U_{us}|$, $|U_{cb}|$, $|U_{ub}|$ and the corresponding convention-free²⁾ phase Φ . The latter is given, in general, by $\arg(U_{us} U_{cs}^* U_{cb} U_{ub}^*)$ and may be judiciously chosen to be the phase of U_{ub} in a convention in which the phases of the other elements vanish³⁾. All CP-violating amplitudes, being proportional to $|U_{us} U_{cb} U_{ub}| \sin \Phi$, become maximal for $|\Phi| = \pi/2$, when the three magnitudes $|U_{ij}|$, $i < j$ are fixed. We refer to this possibility, indicated by present observations, as maximal CP-violation²⁾.

In this letter we seek an understanding of this maximality of CP-violation in charged weak currents, as well as the magnitude of the weak mixing matrix elements U_{us} , U_{ub} and U_{cb} , in the framework of unified gauge models. As already mentioned, to achieve this goal we have to go beyond the standard model. A few proposals in this direction were recently made by means of certain ansatz about the form of the quark mass matrix⁴⁾. Although motivated by theoretical arguments, these forms with maximal phase structure cannot be easily derived in specific models⁵⁾. It is the purpose of the present paper to show that left-right symmetric models of weak interactions lead to a very interesting form for the quark mass matrices with a maximal phase parameter, which not only predicts correct values for the mixing angles but also to maximal value for the phase Φ (i.e., $|\Phi| \approx \pi/2$). This form for the mass matrix, however with an unknown phase, was first proposed by Ecker, Grimus and Neufeld⁶⁾, who considered its implications for left-handed mixing angles and for CP-violation arising specifically from the right-handed current sector. Our approach is different. We show that under plausible assumptions the model can lead to $|\Phi| \approx \pi/2$, and thereby its left-handed currents alone may be responsible for the observed CP violation.

DERIVATION OF THE MASS MATRIX

Let us consider the left-right symmetric models of weak interactions⁷⁾, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which reproduces the standard model at low energies and may lead to new physics at the TeV scale. The quarks $Q_a = (U_a, D_a)$ and leptons (ν_a, e_a) are assigned to left and right-handed doublets of the gauge group in a left-right symmetric manner⁷⁾. The symmetry breaking and fermion masses result from the introduction of a minimal set of Higgs multiplets⁸⁾ $\phi(2,2,0)$ and $\Delta_L(3,1,2) + \Delta_R(1,3,2)$.

It is well known that constraints of left-right symmetry reduce⁹⁾ the number of arbitrary parameters of the standard model that enter the fermion mass matrices. To see this, we write the most general Yukawa couplings of the quarks Q_a to the Higgs fields ϕ :

$$\mathcal{L}_Y = \sum_{a,b} \left[h_{ab} \bar{Q}_{aL} \phi Q_{bR} + \tilde{h}_{ab} \bar{Q}_{aL} \tilde{\phi} Q_{bR} + h.c. \right] \quad (1)$$

where a, b are generation indices that go over 1, 2 and 3 and $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$. Under parity, the fields in Eq. (1) and the gauge bosons transform as:

$$\begin{aligned} P: \quad Q_{aL} &\longleftrightarrow \gamma_0 Q_{aR} \\ \phi &\longleftrightarrow \phi^\dagger \\ W_{\mu L} &\longleftrightarrow (-1)^{1+\delta_{\mu 0}} W_{\mu R} \\ \Delta_L &\longleftrightarrow \Delta_R \end{aligned} \quad (2)$$

Parity symmetry leads to the following relations between the Yukawa couplings:

$$h_{ab} = h_{ba}^* \quad \text{and} \quad \tilde{h}_{ab} = \tilde{h}_{ba}^* \quad (3)$$

In addition, we require the theory to conserve CP prior to spontaneous symmetry breaking. This will impose additional constraints on the Yukawa couplings h and $\tilde{h}$. We will use the following CP transformation⁶⁾, in which the i -factor introduced in some of the charge-conjugation transformations plays a crucial role in the forthcoming study.

$$\begin{aligned}
 \text{CP : } \quad Q_L &\rightarrow C Q_L^* \\
 Q_R &\rightarrow i C Q_R^* \\
 \phi &\rightarrow i \phi^* \\
 \tilde{\phi} &\rightarrow -i \tilde{\phi}^* \\
 \Delta_L &\rightarrow \eta_L \Delta_L^* \\
 \Delta_R &\rightarrow \eta_R \Delta_R^* \\
 W_{\mu L R}^{\pm} &\longleftrightarrow W_{\mu L R}^{\mp} (-1)^{\delta_{\mu 0}}
 \end{aligned} \tag{4}$$

We have dropped the obvious transformation of space-time co-ordinates $(x_0, \vec{x}) \rightarrow (x_0, -\vec{x})$, and $|\eta_L| = |\eta_R| = 1$. It is easy to check that under these transformations one obtains:

$$h_{ab} = h_{ab}^* \quad \text{and} \quad \tilde{h}_{ab} = -\tilde{h}_{ab}^* \tag{5}$$

These, together with Eq. (3), imply that h is a real symmetric matrix and $\tilde{h}$ is an imaginary antisymmetric matrix. One can go to a new basis by an orthogonal transformation where h can be made diagonal and $\tilde{h}$ remains imaginary and antisymmetric. We will work in this basis while analyzing the mass matrices.

To obtain the up and down quark mass matrices from Eq. (1) we need the vacuum expectation values of the fields ϕ and $\tilde{\phi}$ at the minimum of the Higgs potential. We analyze this question in a subsequent section. For the moment, we assume that

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix} \quad (6)$$

where κ and κ' are real and $\alpha \ll 1$. We will argue subsequently that, under certain plausible assumptions, the Higgs potential can lead to such a minimum. Since $\alpha \ll 1$, we will ignore its effect in the first approximation when analyzing the mass matrices. We observe that the vacuum expectation value in Eq. (6) breaks CP-symmetry spontaneously.

The mass matrix for the up and down quarks can be written using Eqs. (1) and (6) after a phase redefinition of the quarks, as follows:

$$M_u = \kappa \begin{pmatrix} g_1 & h_1 \omega & i h_2 \omega \\ h_1 \omega & g_2 & h_3 \omega \\ -i h_2 \omega & h_3 \omega & g_3 \end{pmatrix} \quad (7)$$

$$M_d = \kappa \begin{pmatrix} g_1 \omega & h_1 & i h_2 \\ h_1 & g_2 \omega & h_3 \\ -i h_2 & h_3 & g_3 \omega \end{pmatrix} \quad (8)$$

where $\omega = (\kappa'/\kappa)$, $h_{aa} \equiv g_a$, $\tilde{h}_{12} \equiv i h_1$, $\tilde{h}_{13} = i h_2$, $\tilde{h}_{23} = -i h_3$, g_a and h_a are real. This is the form of the mass matrices that we analyze in this paper.

QUARK MIXING ANGLES AND CP-VIOLATION

In order to derive the consequences of the above matrices and obtain the correct pattern of quark masses, we impose the following hierarchy on the values of the coupling parameters g_i and h_i :

$$g_3 \gg g_2 > h_3 \gg h_1, h_2 \gg g_1 \quad (9)$$

Similar mass matrices have been studied earlier for the case¹⁰⁾ $h_2 = 0$ and we use the same procedure for extracting the weak left-handed charged-current mixing matrix, U . We will show that under the assumption Eq. (9), needed to guarantee the correct pattern of quark masses, the maximal phase in the mass matrix translates to a maximal invariant phase Φ in U , i.e., $|\Phi| \approx \pi/2$.

In the first approximation, we ignore h_1, h_2 and g_1 and study the remaining 2×2 mass matrices that connect the second and third generation quarks. Inequality (9) then implies that $\omega \approx (m_b/m_t)$ and the mixing angles in the up and down quark subsectors θ_u and θ_d are related as follows:

$$\tan 2\theta_u \approx \left(m_b/m_t\right)^2 \tan 2\theta_d \quad (10)$$

The mixing element U_{cb} is predicted, to lowest order in quark mass ratios, to be:

$$U_{cb} \approx \theta_d - \theta_u \approx \sqrt{\left(\frac{m_s}{m_b} - \frac{m_c}{m_t}\right)} \quad (11)$$

For¹¹⁾ $m_s \approx 175$ MeV, $m_b \approx 5.3$ GeV and $m_c \approx 1350$ MeV, the model requires $m_t > 41$ GeV. When m_t is close to this value, higher order corrections must be included. Furthermore, Eq. (11) requires the t-quark mass to lie in the range of 43 to 46 GeV for $U_{cb} = 0.05 \pm 0.01$ ¹⁾. We expect a further 30% uncertainty in m_t due to the theoretical uncertainty in the input values of m_s , m_c and m_b ¹¹⁾.

The next stage of the diagonalization of the full 3×3 matrix is performed keeping all small terms to lowest order and performing an approximate diagonalization to this order. We have checked that, to this order, it does not introduce off-diagonal terms in the already diagonal 2×2 subsector. The parameter h_1 is now determined to be $h_1 \approx \sqrt{m_d m_s}$, leading to a prediction for the Cabibbo angle: $U_{us} \approx \sqrt{m_d/m_s}$. Also, smallness of ω and the mixing angles implies that the final approximate form of the mixing matrix U is given by:

$$U \approx \begin{pmatrix} 1 & \sqrt{\frac{m_d}{m_s}} & i \frac{h_2}{m_b} \\ -\sqrt{\frac{m_d}{m_s}} & \cos(\theta_d - \theta_u) & \sin(\theta_d - \theta_u) \\ \sqrt{\frac{m_d}{m_s}} \sin \theta_d + i \frac{h_2}{m_b} & -\sin(\theta_d - \theta_u) & \cos(\theta_d - \theta_u) \end{pmatrix} \quad (12)$$

Note that $U_{ub} \approx i h_2 / m_b$ implies that $|\Phi| \approx \pi/2$, giving rise to maximality of CP-violation. Furthermore, since h_2 is a free parameter, $|U_{ub}|$ is not determined in this model. If $h_2 \approx h_1 \approx \sqrt{m_d m_s}$, we predict $|U_{ub}| \approx 0.007$, which is not far from its present experimental upper limit. Our model, therefore, appears to reflect the present status of the weak mixing matrix rather nicely.

MINIMUM OF THE HIGGS POTENTIAL

A crucial input into our study of the mass matrices is the vacuum expectation value of $\langle\phi\rangle$ in Eq. (6) with $\alpha \ll 1$. In this section we study the conditions under which a minimum of the above form can arise. For this purpose we need the Higgs potential, which can be written in the following form consistent with P and CP-conservation.

$$V = V_0 + V_1 + V_2 \quad (13)$$

where V_0 is a function of gauge-invariant monomials of form $\text{Tr } \phi^\dagger \phi$, $\text{Tr}(\Delta_L^\dagger \Delta_L)$, $\text{Tr}(\Delta_R^\dagger \Delta_R)$. V_1 is a function of quartic terms of form $\text{Tr } \phi_i \phi_j^\dagger \phi_k \phi_l^\dagger$ (where $i, j, k, l = 1, 2$ and where $\phi_1 \equiv \phi$, $\phi_2 \equiv \tilde{\phi}$). It is easily seen that all coupling and mass parameters in V_0 and V_1 are real due to the constraints of CP and P invariance.

$$\begin{aligned} V_2 = & i \lambda_1 \text{Tr}(\Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R) \text{Tr}(\phi \tilde{\phi}^\dagger - \tilde{\phi} \phi^\dagger) \\ & + \lambda_2 \left\{ \text{Tr}(\Delta_L^\dagger \phi \phi^\dagger \Delta_L + \Delta_R^\dagger \phi \phi^\dagger \Delta_R) \right\} \\ & + \tilde{\lambda}_2 \left\{ \text{Tr}(\Delta_L^\dagger \tilde{\phi} \tilde{\phi}^\dagger \Delta_L + \Delta_R^\dagger \tilde{\phi} \tilde{\phi}^\dagger \Delta_R) \right\} \\ & + [i \lambda_3 \text{Tr} \left\{ \Delta_L^\dagger \phi \tilde{\phi}^\dagger \Delta_L + \Delta_R^\dagger \phi \tilde{\phi}^\dagger \Delta_R \right\} \\ & + \text{h.c.}] \end{aligned} \quad (14)$$

where $\eta_{L,R}$ have been appropriately chosen to eliminate any other terms. To study this potential, we first use the fact that, in this case, we can choose $v_L = 0$. Denoting $\beta = \tan^{-1} \omega$ and using the fact that⁸⁾ $\langle \Delta_R \rangle = v_R \gg \rho = \sqrt{\kappa^2 + \kappa'^2}$, we find

for the leading terms in the potential at the classical level:

$$\begin{aligned}
 V \approx V_0 (V_R^2, \rho^2, \dots) \\
 + a V_R^2 \rho^2 \sin 2\beta \sin \alpha \\
 + b V_R^2 \rho^2 \cos 2\beta
 \end{aligned} \tag{15}$$

It is clear that the minimum of this potential corresponds to $\alpha = \pi/2$ and $\tan 2\beta \approx a/b$. This would correspond to a purely CP-conserving case and is not physical.

To obtain the desired pattern in Eq. (6) we add a real scalar field, σ , which is even under parity but odd under CP. In the presence of this, V_2 is modified to the following form, V_2' :

$$V_2' = V_2 + \mu \sigma (\text{Tr } \phi \tilde{\phi}^\dagger + \text{Tr } \tilde{\phi} \phi^\dagger) \tag{16}$$

Choosing the mass term for σ such that $\langle \sigma \rangle \equiv \sigma \neq 0$, we obtain a modified form for the classical potential V' as follows:

$$\begin{aligned}
 V' = V_0' (V_R^2, \rho^2, \sigma^2) \\
 + \mu \sigma \rho^2 \sin 2\beta \cos \alpha \\
 + a V_R^2 \rho^2 \sin 2\beta \sin \alpha \\
 + b V_R^2 \rho^2 \cos 2\beta
 \end{aligned} \tag{17}$$

At the minimum of this potential we obtain

$$\tan \alpha = a \left(v_R^2 / \mu \sigma \right)$$

$$\tan 2\beta = \frac{\mu \sigma \cos \alpha + a v_R^2 \sin \alpha}{b v_R^2} \quad (18)$$

It is now clear that, if we choose $a \ll b$ and $\mu \sigma = v_R^2$, we obtain $\alpha \ll 1$ and $\beta \approx b^{-1}$. This shows that, for small values of the parameter a in Eq. (17), the desired maximality of CP violation emerges.

The next question we consider is: is it plausible that a is small? To answer this, we note that a is proportional to λ_1 and λ_3 in Eq. (14). We find that, if at the tree level $\lambda_1^{\text{tree}} = \lambda_3^{\text{tree}} = 0$, then the antisymmetry of $\tilde{h}$ and symmetry of h implies that no divergent correction to λ_1 and λ_3 arises up to the three-loop level. At the four-loop level non-zero divergent contributions may arise. If we adopt the attitude that such divergences are cut off by some underlying grand unified framework which may include gravity (perhaps supergravity or strings?), the cut-off will be replaced by M_{Planck} . Then $\lambda_1, \lambda_3 \lesssim g^4 h^3 \tilde{h}^3 \ln(M_{\text{Pl}}/\mu)$ and is therefore a very small number. In contrast, parameters such as λ_2 and $\tilde{\lambda}_2$, which contribute to b , receive one-loop divergent contributions and are therefore expected to be bigger. We thus conclude that there exist plausible arguments in support of our minimum where $\alpha \ll 1$.

In conclusion, we have analyzed a new class of quark mass matrices that arise in left-right symmetric theories, which can lead to maximal CP-violation in the left-handed charged weak current as required by observation, as well as reasonable predictions for two mixing angles, U_{us} and U_{cb} . The model can easily be extended to include the leptons, making it a completely realistic theory of quark and lepton interactions.

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