EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
CERN - SPS DIVISION

CERN SPS/85-38 (ARF)

RF FOR THE CERN PROTON-ANTIPROTON COLLIDER

D. Boussard

Abstract

The succession of RF manipulations or special features which are implemented in the CERN proton-antiproton complex will be reviewed. This description more or less follows the particle's path, starting from the handling of high intensity beams for antiproton production, up to the end of the chain where an almost noise free RF system keeps the proton and antiproton beams bunched for many hours.

To look at things from a somewhat more general and perhaps more academic point of view, two selected topics, which received considerable attention at CERN, will be considered separately in Appendices I and II. One is the problem of beam loading on RF cavities with high beam currents, the other is the RF noise problem which, during the first collider tests, was the major cause for bad lifetime.

Lecture given at the 1984 US Summer School on High Energy
Particle Accelerators, Fermilab, August 13-24, 1984.

Geneva, August, 1985.

I. ANTIPROTON PRODUCTION

1. Damping of longitudinal instabilities by active feedback
2. Second harmonic cavity in the PSB
3. Cavity compensation in the PS and PSB
4. Controlled beam-blow-up in the PS
5. Beam merging at 26 GeV/c in the PS
6. Future technique for ACOL

II. ANTIPROTON ACCUMULATION

1. The Antiproton Accumulator (AA) RF system
2. The Antiproton collector (ACOL) RF system

III. PROTON AND ANTIPROTON TRANSFERS

1. Acceleration and deceleration in the PS
2. Bunch compression at 26 GeV/c

IV. BEAM STORAGE IN THE SPS

1. The SPS RF cavities
2. The low level RF system

REFERENCES

APPENDIX I - BEAM LOADING ON RF CAVITIES

1. Equivalent circuit
2. Peak generator power
3. The RF drive generation

APPENDIX II - THE RF NOISE PROBLEM

1. Synchrotron oscillation with noise excitation
2. RF noise seen by the beam
3. Equilibrium distribution and life time

RF FOR THE CERN PROTON-ANTIPROTON COLLIDER

D. Boussard

European Organization For Nuclear Research (CERN)

1211 Geneva (Switzerland)

The succession of RF manipulations or special features which are implemented in the CERN proton-antiproton complex will be reviewed. This description more or less follows the particle's path, starting from the handling of high intensity beams for antiproton production, up to the end of the chain where an almost noise free RF system keeps the proton and antiproton beams bunched for many hours.

To look at things from a somewhat more general and perhaps more academic point of view, two selected topics, which received considerable attention at CERN, will be considered separately in Appendices I and II. One is the problem of beam loading on RF cavities with high beam currents, the other is the RF noise problem which, during the first collider tests, was the major cause for bad lifetime.

I ANTIPROTON PRODUCTION

Antiprotons are produced by a primary beam of high energy protons which impinge on a copper target (Fig. 1). The old CERN synchrotron (PS) and its booster (PSB) are used for this purpose, delivering up to $1.6 \cdot 10^{13}$ protons at 26 GeV/c squeezed into only one quarter of the PS ring. This is because the length of the injection orbit in the antiproton accumulator (AA) corresponds to one quarter of the PS circumference. Table I summarizes the most relevant parameters of the PS and PSB, from the RF point of view.

Fig. 1 The network of machines and transfer lines at CERN

Table I

	<u>PSB</u>	<u>PS</u>
Injection	$T = 50 \text{ MeV}$	$T = 800 \text{ MeV}$
Ejection	$T = 800 \text{ MeV}$	$p \pm 26 \text{ GeV}$
	Four rings	
Radius	$R = 25 \text{ m}$	$R = 100 \text{ m}$
Beam intensity	10^{13} p/ring	$2 \cdot 10^{13} \text{ p}$
f_{RF}	3-8MHz	2.7-9.5 MHz
Harmonic number	$h = 5$	$h = 20$
Cavities	4 x 1 (12kV each)	11 (20 kV each)
Harmonic system		
f_{RF}	6-16 MHz	200 MHz (fixed)
	$h = 10$	$h \text{ variable } (\approx 420)$
Cavities	4 x 1 (8 kV each)	8 (40 kV each)

Antiproton production requires the handling of very high intensity beams in the PSB rings. In the PS, in addition to the problem of high density beams (only one half of the ring is filled during acceleration), special manipulations for merging two half beams at 26 GeV must be performed just before targeting.

1 Damping of longitudinal instabilities by active feedback

In the PSB, with five bunches, all coupled bunch modes appear ($n = 1, 2$) and are extremely harmful to the beam. Longitudinal dipole ($m = 1$), quadrupole ($m = 2$) and even sextupole ($m = 3$) oscillations are present and must be damped. In the PS, mode $n = 1, m = 1$ develops just after the 800 MeV injection and must be cured by feedback; later on in the cycle the bunches can be blown up in longitudinal phase space and coupled bunch instabilities are avoided.

For one particular mode (m, n) the beam spectrum contains the revolution frequency lines and the associated synchrotron sidebands at the frequencies $pf_{RF} \pm n f_{rev} \pm m f_s$, where f_{RF} , f_{rev} and f_s are the RF, the revolution, the synchrotron frequencies respectively (Fig. 2). Damping of this particular mode of oscillation can be achieved if a real impedance, which changes sign at the frequency $pf_{RF} \pm n f_{rev}$ is seen by the beam ^{1,2}. This impedance is synthesized by a feedback path from a longitudinal pick-up electrode to the main RF amplifier - cavity combination (Fig. 3). The latter has a sufficient bandwidth to accept feedback signals at frequencies $f_{RF} \pm n f_{rev} \pm m f_s$. (This is the case for the PS and PSB RF systems).

Fig. 2 Frequency spectrum of longitudinal coupled bunch instability

Fig. 3 Equivalent impedance of the longitudinal feedback

Fig. 4 Synthesis of filter transfer function (two path active filter)

The proper shape of the transfer function is obtained by two high pass filters $G(s)$ (Fig. 4) which are translated by coherent mixing (real and imaginary components) at the frequency $f_{RF} + n f_{rev}$. With this arrangement the transfer function changes sign through a zero at $f_{RF} + n f_{rev}$ as required. The non oscillatory component of the beam current at that frequency (resulting from unequal bunch populations, for instance) is not transmitted through the filter, which automatically avoids saturation of the amplifiers.

Moreover, when the RF frequency changes (from 3 to 8 MHz for instance in the PSB), the position of the zero of the filter follows. The overall delay of the system can be corrected by a phase advance circuit which adjusts the relative phase of the two mixers (input carrier - output carrier) as a function of frequency.

2 Second harmonic cavity in the PSB ³

Transverse space charge detuning at the 50 MeV injection energy limits the maximum current which can be accelerated in the PSB. With an additional second harmonic RF system ($h=10$, 6-16 MHz frequency range), the bucket trajectories are distorted and a new particle distribution in longitudinal phase space is established (Fig. 5). Longer flat topped bunches, having a reduced peak density, are produced, and more particles per bunch can be accommodated for the same space charge detuning (which depends essentially on the peak line density).

Each PSB ring is equipped with a second harmonic cavity (Fig. 6) of a very classical design (push-pull configuration and ferrite rings tuning). The final amplifier tube, mounted very close to the cavity, is driven by a fast solid state preamplifier. There is a direct feedback path from the gap to the preamplifier input, via a capacitive gap voltage divider. This arrangement provides a broad band reduction of the cavity impedance. Reduction

factors from 14 to 22 dB are achieved over the 5-17 MHz frequency range by the use of a solid state amplifier with a very short propagation delay (5ns).

Fig. 5 Accelerating wave shape, bucket shape and line density

Fig. 6 Simplified schematic of second harmonic cavity and amplifier in the PSB.

To make flat topped bunches the second harmonic cavity runs at half the voltage of the fundamental RF system, and with a relative phasing such that the synchrotron frequency, zero in the bunch center, increases linearly with amplitude (Fig. 7). One would expect therefore a strong Landau damping effect for all the longitudinal instabilities, because of the large synchrotron frequency spread present in the beam. Nevertheless experiments with beam have shown that this is not the case, and in particular a sextupole ($m = 3$, $n = 0$) mode appears, which had to be cured by a special feedback system. Even with a large overall spread in synchrotron frequencies, there are regions in the phase plane, at large amplitudes (Fig. 7) where there is no dispersion at all, and this consideration might explain why Landau damping is not effective, and why the use of all feedback loops is absolutely necessary to stabilize long, intense bunches.

Fig. 7 Synchrotron frequency versus amplitude.

3 Cavity compensation in the PS and PSB

At injection, when the RF voltage should be low (adiabatic capture in the PSB, matching long incoming bunches in the PS), the beam induced voltage in the cavities is considerably larger than the voltage produced by the RF power generator. Under these conditions, there is complete loss of control of the RF voltage just after injection. This effect can also be described as an unstable behaviour of the beam cavity feedback loops system⁴.

Similar situations happen also during other RF manipulations when the RF voltage has to be lowered, i.e. synchronisation at 800 MeV of the four PSB rings, controlled beam blow up, beam merging at 26 GeV in the PS.

An obvious solution to this problem is to lower the cavity impedance during the transient period. This technique has been tested in the PSB where an additional RF tube can be pulsed in order to bring a low impedance in parallel to the cavity gap. Another solution, already mentioned for the second harmonic cavity, is to build a broad band feedback system through a fast RF preamplifier. In the PS case, where all the RF preamplifiers are located in the center of the ring, long delays are unavoidable and this technique cannot easily be implemented. The problem has been solved differently by a feedforward technique (cavity compensation).

The beam cavity feedback loop combination becomes unstable when beam-loading introduces too large coupling terms between the cavity tuning and amplitude loops and the main phase loop ⁴. Although such couplings can, in principle, be removed by a proper design of compensating feedback paths ⁴, a much simpler solution consists in injecting into the cavity drive a signal proportional to $-I_b$ (I_b : RF component of beam current) such that the voltage induced by the beam on the cavity will be cancelled. It can be shown ^{5, 6} that cross couplings between loops disappear in this case, which results in a smooth, non oscillatory behaviour of the loops.

It is relatively easy, although may be cumbersome, to implement such a feedforward technique when the RF frequency is practically constant (injection transient, intermediate and high energy flat-tops). This is done on all 9.5 MHz PS cavities and is absolutely essential for high intensity operation; it has also been tested successfully on the PSB at 50 and 800 MeV ⁷.

4 Controlled beam blow-up in the PS

In the PS it is extremely attractive to make use of the fact that the longitudinal acceptance of the machine rapidly increases with energy. By blowing up the bunch area on several occasions during acceleration as much as the acceptance permits, one considerably increases the threshold for any bunch instability. This is why only one feedback system against coupled-bunch instabilities is needed, just after injection⁶. Soon afterwards the bunch area is blown up from the injected 8 mrad to about 20 mrad which is sufficient to keep the beam stable. Moreover, to cross transition in the PS at very high intensity ($>10^{12}$ per bunch), even with the help of the γ transition jump technique⁸, a minimum longitudinal emittance of about 25 mrad is needed. Controlled blow up of the bunch area is done in two steps, the first on an intermediate flat top at 806 MeV (700 Gauss) lasting 50 ms, the second on another intermediate flat top at 3.5 GeV/c where the bunch area is further blown up from 20 to 25 mrad. The overall magnetic cycle of the PS in the antiproton production (and antiproton transfer) mode is illustrated in Fig. 8.

Fig. 8 Antiproton production cycle in the PS

Blow-up of the bunch area is the long term result of filamentation in phase space⁹ which occurs whenever the bunch boundary does not match the phase space trajectories. However, this is naturally a very slow process which relies on the dispersion of synchrotron frequencies within the bunch. Experience has shown that in cases where Landau damping is lost (above the instability threshold), filamentation does not take place as expected and controlled beam blow-up is almost impossible. Fortunately, the PS is equipped with a second RF system, working at 200 MHz. This system was initially intended to prebunch the PS beam, just before ejection towards the SPS (operating in the fixed target mode). By running the two systems in parallel (Fig. 8) very complicated phase space trajectories appear inside the bucket the result being a filamentation (or mixing between particles) which is extremely fast (~ 10 ms) compared to the case of a single frequency RF waveform. The phase of the 200 MHz sinewave with respect to the low frequency RF is unimportant: only the "degree of complexity" of the trajectories matters. This is why it is possible to synchronize the 200 MHz frequency on a multiple of the PS revolution frequency, not necessarily on a multiple of the RF frequency. In this first case the relative phase of the two RF systems is different from bunch to bunch, but many more harmonic numbers (e.g. $h=433$, $h=496$), not multiples of 20 are possible.

The excitation mechanism for the controlled blow-up is a phase modulation of the 200 MHz waveform with respect to the normal RF. The phase modulation frequency is typically 2 to $3 f_s$ and the amplitude of the order of $\pm 8^\circ$ at the normal RF frequency.

The bunches, initially parabolic in shape develop tails during the process and show typical $\cos^2$ shapes after blow-up; however there is no evidence of long tails which would show up as additional beam losses when the machine acceptance shrinks.

Because the 200 MHz cavities are fixed-tuned, the controlled blow-up operations take place on dedicated flat-tops, but there is no fundamental reason which would prevent achieving the same results without lengthening the magnetic cycles, if fast tunable cavities were available.

5 Beam merging at 26 GeV/c in the PS

In contrast to the previously mentioned features of the PSB and PS RF systems which are of interest for any high intensity operation of these machines, the beam merging operation is specific to the antiproton production process in the CERN complex. The aim is to obtain at 26 GeV/c a proton beam with the highest possible number of particles localized in one quarter of the PS circumference.

Two strings of five bunches (string A and string B in Fig. 9) are accelerated with the $h=20$ RF system of the PS. Each set is either the full beam (5 bunches) of one PSB ring ($h=5$) or the result of the vertical recombination in the PS, at 800 MeV, of two beams from two PSB rings ¹⁰.

Fig. 9 Two strings of five bunches accelerated before beam merging.

On the 26 GeV extraction flat top we want the two strings to move azimuthally with respect to each other and eventually coincide in azimuth. At that time the extraction kicker is fired and the two sets A and B are sent simultaneously on the target.

Imagine that the RF wave is of the form:

$$V_{RF}(t) = V_0 (\sin \omega_1 t + \sin \omega_2 t) \quad (1)$$

ω_1, ω_2 , constant, no acceleration.

It can be shown by theoretical arguments¹¹ and computer simulations^{12, 13} (Fig. 10), that this waveform generates two sets of quasi-independent buckets if the approximate condition:

$$|\omega_1 - \omega_2| > 4 f_s \quad (2)$$

is satisfied.

Fig. 10 Bunch distortion during drifting (concentric lines correspond to various bunch sizes), for $\omega_1 - \omega_2 = 4f_s$

In other words, at each frequency ω_1 , and ω_2 , one can associate a bucket with its inside trajectories, as if each frequency were alone. If the two buckets related to ω_1 and ω_2 do not touch, the inside trajectories are relatively little perturbed by the other frequency, and consequently bunches sitting in the center of these buckets can drift azimuthally while being kept bunched.

Although it is not difficult to synthesize the RF waveform with two sets of RF cavities (with separate fine tuning circuits),

the problem of accelerating bunches A to the energy corresponding to ω_1 and decelerating bunches B to ω_2 is a little more tricky¹³.

To control independently bunches A and B at the start of the process, before the condition (2) is met, we make use of the time separation between the two sets: this is the reason for the empty space (5 buckets) left between bunches A and bunches B. Imagine now that the RF waveform is fully amplitude modulated (from 0 to 100%) at the revolution frequency f_{rev} . Bunches sitting on the crest of the modulation will see the full RF voltage, while those which are diametrically opposite will see no RF voltage at all. The full amplitude modulation is in fact synthesized (approximately) with two RF cavities driven at the carrier frequency ($h = 20$), plus one at the lower side band ($h = 19$) and one at the upper side band ($h = 21$). Proper phasing of these cavities ensure that, bunches A, say, are sitting on the crest of the modulation and can be controlled (in energy) by this set of cavities which, on the other hand has very little influence on bunches B. Similarly, a second set of cavities controls bunches B which can be decelerated while bunches A are accelerated. In fact, two independent phase loops and cavity compensation circuits control the frequency splitting. During the drift periods the frequency difference is accurately servoed in order to ensure that merging of the two sets of bunches occurs exactly when the extraction kicker has to be fired. Figure 11 shows the overall result (three dimensional picture) of the process.

Fig. 11 Three-dimensional picture of beam merging at 26 GeV in the CPS.

6 Future technique for ACOL

For the future antiproton collector ring (ACOL) a bunch rotation scheme (see Chapter II.2) is envisaged. This implies that the parent proton bunches in the PS have a small bunching factor (bunch length over bunch separation), typically 0.2. The longitudinal merging described above, which is inherently a non adiabatic process, even for $|\omega_1 - \omega_2| > 4 f_s$, leads to a large blow-up of the bunch area, especially at the highest intensities. Experience has shown that with the present PS parameters ($V_{RF} \text{ max} = 200 \text{ kV}$ at $h = 20$) the bunching factor requirements cannot be met. With the new proposal ¹⁴, longitudinal merging of the two sets of bunches is done in a quasi adiabatic way, and the expected blow-up factors and bunch distortions are much smaller.

10 bunches $h = 20$

5 bunches $h = 10$

Fig. 12 Distribution of bunches in the PS for the future ACOL technique.

Initially the two sets of five bunches (each coming from one booster ring) are arranged in one string (Fig. 12) held by several $h = 20$ cavities. Another set of RF cavities is tuned on $h = 10$ (4.75 MHz) and smoothly powered while the $h = 20$ holding voltage is adiabatically decreased. The bunches coalesce by pairs, and at the end of this operation five bunches, each having hopefully twice the initial bunch area, are held by the $h = 10$ RF system (Fig. 12). However, even if the process is made slow enough

(adiabatic) beam blow-up may occur due to the microwave instability when only a thin filament joins the two initial bunches. Preliminary experiments, although at low intensity, have demonstrated the feasibility of the method (Fig.13).

Fig. 13 Harmonic number change, $h = 20 \rightarrow h = 10$ at low intensity

Fig. 14 Buckets (top) and RF waveforms (bottom) during the harmonic number change $h = 10$ to $h = 12$.

To further compress the beam from one half to one quarter of the PS circumference, step by step changes of the harmonic number will be used. While the beam is held by several $h = 10$ cavities, some other RF cavities will be smoothly powered on $h = 12$. Adiabatic distortion of the phase space indicated in Fig. 14 takes place and brings the five bunches a little closer together. The same process can be repeated from $h = 12$ to $h = 14$ and so on until one reaches the original $h = 20$, at which point the five bunches occupy one quarter of the circumference (Fig. 15). With the relatively small expected bunch area, further compression can take place just before ejection if the RF voltage is abruptly raised to its maximum value.

This scheme, although somewhat cumbersome from the hardware point of view (separate coarse tuning circuits are needed for each set of RF cavities), is almost adiabatic and should provide a better final bunching factor. Reduction of the beam-loading effects by cavity compensation as described above would be extremely complicated because of the large number of operating RF frequencies. It is therefore envisaged to move the RF preamplifiers close to the cavities and build fast feedback systems to reduce the impedance of the RF cavities over a large frequency range.

Fig. 15 Overall evolution of the azimuthal charge density during beam merging.

1 The Antiproton Accumulator (AA) RF System

The diagram illustrates the SSB transmitter section of the SSB-SSB system. It is organized into three main vertical sections: INTERFACE, COMPUTER, and a central transmitter section.

- INTERFACE Section:** Contains a FUNCTION GENERATOR with MEM, PROC, and DIA sections. It outputs a 10.81V signal to the LOGARITHMIC DETECTOR. A VOLTAGE input is also shown.
- COMPUTER Section:** Contains a FUNCTION GENERATOR with VECTOR, MEMORY, VECTOR, PROCESS, 12 BIT DIA, and CONV sections. It outputs a 12.121V signal to the QUAD. VCO.
- Central Transmitter Section:**
 - The QUAD. VCO (10-30 kHz) outputs to a 558 MIXER.
 - The 558 MIXER outputs to a LOG MODULATOR.
 - The LOG MODULATOR outputs to a DRIVER.
 - The DRIVER outputs to a POWER AMPLIFIER.
 - The POWER AMPLIFIER outputs to a 5V-14kV output.
 - The 5V-14kV output is connected to a 185 MHz CAVITY.
 - The 185 MHz CAVITY outputs to a BEAM.
 - The LOGARITHMIC DETECTOR outputs to an AVC LOOP AMPLIFIER.
 - The AVC LOOP AMPLIFIER outputs to a PHASE LOOP AMPLIFIER.
 - The PHASE LOOP AMPLIFIER outputs to a PHASE LOOP CONTROL.
 - The PHASE LOOP CONTROL outputs to a HIGH PASS filter.
 - The HIGH PASS filter outputs to a PHASE DISCRIM. block.
 - The PHASE DISCRIM. block outputs to two AVC blocks.
 - The two AVC blocks output to a PU (Pulse Unit).
 - The PU outputs to a BEAM.

Additional components and labels include: QUADRATURE SYNTHESIZER 1830 kHz, 10.81V, 10.81V, LOGARITHMIC DETECTOR, 1830 kHz, 10-30 kHz, 558 kHz, LOG MODULATOR, DRIVER, POWER AMPLIFIER, 5V-14kV, 185 MHz CAVITY, PHASE LOOP AMPLIFIER, PHASE LOOP CONTROL, HIGH PASS, PHASE DISCRIM., and BEAM.

A special feature of the AA RF System is its very wide dynamic range (gap voltage varying from 5V to 14 kV) achieved by the use of logarithmic detectors and modulators in the amplitude control circuitry. Small effective impedance of the cavity is essential to operate the RF system at very low voltages.

The various operations to be performed by the AA RF System are under the control of the RF computer. From a model describing the beam behaviour in longitudinal phase space, with the AA parameters, elementary operations are derived, which can be combined into complex sequences. As a result the RF frequency and RF amplitude functions of time are loaded into the corresponding function generators.

Many different sequences are used during AA operation; for instance

- A bunch of protons, injected from the PS is captured (bunch into bucket transfer) and accelerated across the vacuum chamber to explore the machine aperture.
- An unbunched stack of antiprotons can be accelerated (or decelerated) by phase displacement¹⁶ (successive traversals of the stack with an empty bucket).
- More specific to the accumulator operation is the stacking procedure, where the precooled antiproton beam is captured, moved across the vacuum chamber and deposited alongside the existing stack.
- The inverse operation, or unstacking, is the adiabatic capture of a pilot or a dense bunch of antiprotons inside the dense part of the stack (with a very low voltage) and its transfer, through the stack onto the ejection orbit.
- With the low noise RF system of the AA (phase loop operation) this process is quasi adiabatic and restacking is even possible, in case the bunch waiting on the ejection orbit cannot be taken immediately, (the PS or SPS not being ready for instance).

In the case of adiabatic trapping the relative change of bucket area dA/A in a time interval dt is made proportional to the actual synchrotron period T_s :

$$\frac{dA}{A} = \alpha \frac{dt}{T_s} \quad (3)$$

α is the adiabaticity coefficient which should be kept small for little bunch distortion and hence little beam blow-up. Typical values are between 0.25 and 0.5. Usually the adiabatic trapping function is calculated at constant α . With, in addition, the condition $\phi_s = 0$, one obtains the time evolution:

$$A(t) = \frac{A_i}{1 - \frac{t}{\Delta t} \left(\frac{A_f - A_i}{A_f} \right)} \quad (4)$$

where A_i and A_f are the initial and final bucket areas and Δt the time of the process. The voltage function $V(t)$ is finally derived from equation (4).

Moving the beam across the stack is done at low (and usually constant) $\sin \phi_s$ in order to minimize phase space distortions. However to unstack a low intensity beam, but with the same transverse emittance (i.e. coming from the stack core) a faster change of f_{RF} (high $\sin \phi_s$) is used. A fraction of the captured beam is lost outside the bucket and is finally restacked with the help of the momentum cooling system. This technique used for LEAR filling gives an unstacked bunch with a smaller intensity.

Another way to achieve the same result is to unstack the beam with a higher harmonic RF system which provides, for the same RF voltage a much smaller bucket (the bucket area per bunch is proportional to $h^{-3/2}$). This technique was proposed to unstack the antiproton beam for LEAR, using the stack tail cooling kickers as wide band RF cavities working on $h = 128$ ¹⁷.

Every 2.4 seconds, a new burst of antiprotons is injected into the AA. After the initial precooling period, these antiprotons are captured by the RF system and moved towards the accumulated stack. If the stack and bunch densities were constant and equal it would be sufficient to deposit the full bucket just along the existing stack. In reality the stack density increases from the edge to the core, and the bunch density is also higher in the bucket center. A better match of the distributions can be obtained by combining adiabatic debunching (shrinking of bucket area) and bucket displacement, such that the low density part of the bunch is deposited in the tail of the distribution and the dense core is pushed furthest inside the stack.

The limiting case is when the phase space reduction dA/dt equals the area traversed, i.e:

$$\frac{dA}{dt} = \frac{dE}{dt} T_{RF} \quad (5)$$

where T is the RF period and A is expressed in units of eV.s (energy x time).

With $dE/dt = V \sin \phi_s f_{rev}$, one obtains:

$$\frac{dA}{dt} = \frac{V \sin \phi_s}{h} \quad (6)$$

which, for a constant value of $\sin \phi_s$ normally used in this operation shows that the quantity dA/dt should be proportional to A^2 . This is the same as equation (3) which describes capture at constant adiabaticity coefficient. Similarly the time functions $A(t)$, $V(t)$ and $f(t)$ can be computed from the solution (4) of this equation, and used to control the RF system for the stacking process.

2 The Antiproton Collector (ACOL) RF Systems

The new ACOL ring which is being built surrounds the existing AA machine^{18,19}. Its purpose is to collect a much larger number of antiprotons, using, in particular the bunch rotation technique described later, and to separate the cooling and collection functions in two independent, better optimized machines.

After precooling in ACOL the antiproton burst must be transferred into the AA machine first, and then pushed into the already existing stack. Bunching and deceleration on the ACOL ejection orbit is achieved by a dedicated RF system.

The required voltage (3.5 kV at $h = 1$) is generated in a single ferrite loaded cavity driven by a 10 kW amplifier. The iso adiabatic law (constant adiabaticity coefficient α) is used to capture the antiproton batch (with $\alpha = 0.5$ the time needed is $\Delta t = 167\text{ms}$).

As the orbits of the two machines have different lengths (the ratio, at transfer is 1.16), the two RF frequencies, in AA and ACOL cannot be made equal. A suitable rational approximation of the frequency ratio is:

$$\frac{f_{AA}}{f_{AC}} = \frac{36}{31}$$

The buckets of AA and ACOL are therefore in phase every 19.5 μs (36 times the AA revolution period or 31 times the ACOL revolution period). Ejection and injection kickers are triggered from this subharmonic frequency.

The time structure of the antiproton burst leaving the target is the same as that of the incoming proton beam. On the other hand, the momentum acceptance of the transfer channel from the target into ACOL and of ACOL itself limits the momentum width of

the injected antiproton beam. The incoming beam is therefore represented in the phase space diagram by a rectangle (Fig. 17) with its $\Delta p/p$ limited by the acceptance and its length by the proton bunching factor. It is obvious that, for the same number of protons impinging on the target, the shorter the bunch, the higher the final antiproton density in longitudinal phase space. This is why a new merging procedure (Chapter I.6) giving a smaller bunching factor, is envisaged to fill ACOL.

Fig. 17 Longitudinal emittance of incoming antiproton burst

Fig. 18 Antiproton rotation in ACOL

With bunch rotation the tightly bunched antiproton beam can be converted into a quasi continuous beam with a much smaller momentum spread. This is illustrated figure 18 which also shows the limits of the method. The non linearity of the synchrotron motion distorts the final shape if the $\Delta p/p$ of the bucket is not large compared to that of the injected beam.

A dedicated RF system for ACOL has the specifications given in table II. It must provide a large RF voltage (1.32 MV) at the frequency of the incoming bunches (i.e. the PS frequency at ejection: 9.537 MHz). Moreover, as soon as the bunch has rotated a quarter of a turn in phase space, the RF voltage must be abruptly reduced to a value which approximately matches the

rotated bunch, ($V_{RF}=300\text{kV}$), and then adiabatically brought to zero (Fig.19). The cavity is severely limited in length due to the lack of space in ACOL and heavy capacitive loading is necessary. Note that, with the high voltage involved the whole cavity is under vacuum, without ferrites ¹⁸.

Table II

Voltage, rotation	1.32 MV _p
Bucket/bunch height	1.41
Harmonic number	6
Frequency	9.537 MHz
Rotation time	60.5 μs
Revolutions during rotation	96
Matching voltage (for initial bunch length of 25ns)	297 kV _p
Debunching time	10 ms

Fig. 19 Typical voltage program

III PROTON AND ANTIPROTON TRANSFERS

In the CERN proton-antiproton complex, the CERN PS is certainly the heart of the transfer technique. Its various functions are summarized below:

- Antiprotons transferred from the AA at 3.5 GeV are accelerated to 26 GeV/c and extracted towards the SPS.
- Protons from the PSB are accelerated to the 3.5 GeV intermediate flat top where, after RF manipulations, they are either accelerated to 26 GeV/c for filling the SPS collider, or injected into the AA at 3.5 GeV/c for test purposes or even decelerated down to 600 MeV/c for injection into LEAR.
- Antiprotons from the AA at 3.5 GeV/c are decelerated to 600 MeV/c for LEAR filling.

Up to spring 1984, the PS also delivered antiproton beams to the ISR, using the same technique as for the SPS, except for the final RF manipulation at 26 GeV/c which was not needed in the case of ISR filling.

For all these operations only two magnetic cycles are used. From 800 MeV to the 3.5 GeV/c intermediate flat top the two cycles are the same, with identical magnetic fields, low energy corrections and RF functions. After the 3.5 GeV/c flat top the beam is either accelerated to 26 GeV/c or decelerated for LEAR filling.

1 Acceleration and deceleration in the PS

For colliding beam operations in the CERN complex, the PS must run with the highest longitudinal acceptance per bucket (only one bunch is accelerated at a time). The largest acceptance is needed to:

- capture long antiproton bunches from the AA (70 ns),
- compress the bunches in a large bucket (small non linearities) before extraction to the SPS,
- decelerate the bunches to low energy (shrinkage of the bucket).

Fortunately the PS RF system was designed to cope with the old 50 MeV injection and accepts a large frequency swing, from 2.7 to 9.5 MHz. It is therefore possible to accelerate from 3.5 GeV/c to 26 GeV/c on a lower harmonic number than the usual $h=20$. With $h=6$, for which the RF frequency is near the lower edge of the band, one gains a factor $(20/6)^{3/2} = 6.08$ on the bucket area as compared to $h=20$. This is the technique used to accelerate to 26 GeV/c for SPS, or ISR filling. For LEAR where deceleration to 600 MeV/c is required, $h=10$ is the lowest possible harmonic number.

The 11 PS cavities are spaced by multiples of 1/20th of the ring circumference. When in use with $h=20$ they are all driven in phase, and the beam sees the sum voltage of all cavities. With different harmonic numbers ($h=6$, $h=10$, or $h=19.21$ for antiproton production) each cavity must be driven with a different phase to ensure that the beam sees the full voltage available. Even on $h=6$ the phases are different for proton or antiproton acceleration, because of the opposite particle velocities. Programmable phase shifters are installed on each PS cavity to cope with these various requirements.

The intensities of the pilot antiproton pulses can be as small as a few 10^8 per bunch. With these tiny currents, it is very difficult to accurately measure the beam radial position, which is normally used to control the RF frequency. Instead a precise frequency programme is derived from a magnetic field measurement in the reference bending magnet. The normal phase loop, using a

more sensitive sum pick up electrode, is kept running to suppress coherent phase oscillations. In the vicinity of transition one expects large radial excursions for small frequency errors, due to the relation:

$$\frac{\Delta R}{R} = \frac{\gamma^2}{\gamma^2 - \gamma_{tr}^2} \frac{\Delta f}{f} \quad (7)$$

However, fast transition crossing with the γ_{tr} jump scheme⁸ considerably reduces the maximum ΔR ; moreover, with modern digital techniques Δf can be made very small. The overall result is that no difficulties are experienced at transition, neither with protons, nor with antiprotons.

To fill the SPS with one bunch of protons per PS cycle (before antiproton filling) a complicated procedure takes place during acceleration. First one accelerates only one booster ring (5 bunches) from 800 MeV to 3.5 GeV/c on $h = 20$. On the 3.5 GeV/c intermediate flat-top four out of five bunches are ejected, and the remaining one is blown up with the 200 MHz RF system as required. The harmonic number is then adiabatically changed from $h = 20$ to $h = 6$ using one group of cavities at $h = 20$ to hold the bunch while another group, tuned on $h = 6$ is smoothly powered; acceleration on $h = 6$ at full voltage and transition crossing follow.

2 Bunch compression at 26 GeV/c

On the 26 GeV/c ejection flat top the bunches are typically 15 to 16 ns long with the full 200 kV RF voltage, on $h = 6$. This is not compatible with the 200 MHz SPS bucket which can only accept bunches shorter than 5 ns. The bunch compression process, which takes place in the PS just before ejection reduces the bunch length to 3-4 ns, with a corresponding increase in the momentum spread of the beam.

A mismatch followed by bunch rotation is a well known technique to achieve bunch compression. In the PS case, mismatch is obtained by jumping the RF phase by 180° : particles follow hyperbolic trajectories in phase space, and the bunch gets elliptical. The degree of mismatch depends on the duration of the stay on the instable fixed point.

The RF phase is brought back to its original setting and the elliptical mismatched bunch rotates until it reaches the "upright" position when its length is minimum and when it is ejected towards the SPS. If the bunch size is not very small compared to the bucket area, the non linearity of the synchrotron movement produces an S shaped bunch at the end of the rotation (Fig. 20a). This is the reason why the lowest possible harmonic number has been chosen ($h = 6$) to maximize the bucket size.

Fig. 20 Left: bunch rotation in a sinusoidal, constant bucket
Right: Optimized bunch rotation with second harmonic
($h=12$) cavity

With a harmonic cavity (for instance $h=12$) it is possible to linearize the RF waveform and minimize the bunch shape distortions. The linearized part of the RF waveform does not need to be longer than the bunch, during the rotation (Fig. 21). By changing the relative amplitudes and phases (0° or 180°) of the $h = 6$ and $h = 12$ cavities, one makes the best use of the available voltage (200 kV for all cavities). This results in an almost elliptical bunch shape at the end of the process (Fig. 20b).

Fig. 21 Top: bunch rotation around stable phase point
 Bottom: linearized part of RF waveform during rotation
 ($\frac{1}{12}, \frac{1}{6}, \frac{1}{4}$ turn)

The phase jitter of the compressed bunch is about ± 0.5 ns, which is just acceptable for the SPS bucket (5 ns long). To achieve this figure, the PS revolution frequency is first synchronized with the SPS reference, long before the compression. This is done in two steps: coarse phase comparison at the revolution frequency, first, and then at the RF frequency for optimum accuracy. The phase loop is switched off during bunch compression to avoid phase errors coming from different bunch populations or different bunch shapes. Care must be taken also to ramp slowly enough the magnetic bumps for extraction: the stable phase must be kept as close to zero as possible.

IV BEAM STORAGE IN THE SPS

1 The SPS RF cavities ²⁰

The RF cavities of the SPS are travelling wave structures, working at 200 MHz, which have been optimized for operation under heavy beam loading conditions (typically 3.10^{13} protons per pulse in fixed target operation). Travelling wave structures are interesting, because of their inherent wide bandwidth, (no tuner is necessary) and also because they present a matched load to the RF power amplifier, even in the case of heavy beam loading. They are directional devices where the group and phase velocity are in opposite directions (backward wave structures). With a group velocity of 0.094 C, the filling time of the 20 m long structures is about 0.7 μ s (~ 1 μ s if one includes the RF power amplifier rise time).

This time being short compared with the distance in time between two bunches (7.6 μ s in the three bunch mode), one can change the RF phase or amplitude between two bunches and hence control each bunch separately.

Contrary to the standing wave cavities, which can accelerate simultaneously both particles, protons and antiprotons, the travelling wave structures can only accelerate those particles for which phase and particle velocity are equal (or almost equal). Amongst the four SPS cavities (each giving about 2MV at the nominal power) two are fed in reverse direction (through a high power coaxial switch) for antiproton acceleration. Only 4 MV per beam is available in this configuration, but independent control of protons and antiprotons is possible; in particular the exact location of the interaction point can be adjusted by phasing the two groups of cavities.

For the acceleration of leptons in the SPS (LEP filling) a new set of 200 MHz standing wave cavities is being constructed. A number of those cavities could be used in the future for proton and antiproton acceleration and storage. In this situation the position of the intersection point would be fixed.

To accept bunches with a bigger bunch area (1 eV.s instead of 0.5 eVs) with the new ACOL ring, it is very attractive to make a longer bucket at injection in the SPS. This would considerably ease the bunch compression scheme at 26 GeV/c in the PS and reduce the transverse space charge problems on the SPS injection flat-top. A new 100 MHz RF system is being built for the SPS collider. Five standing wave, high Q cavities will deliver a total of 2 MV to each beam. By combining the two RF systems, travelling wave structures and 100 MHz cavities, it will be possible to control the bunch length in order to satisfy the different machine requirements: long bunches at injection against transverse space charge effects and microwave instabilities, optimum bunch length during storage (compromise between intrabeam scattering and average luminosity).

2 The low level RF system ²¹

As it is customary in proton machines a phase loop circuitry is the heart of the low level RF System. The master oscillator (VCO) is locked onto the first injected bunch (always protons), and any dipole oscillation of that bunch is very strongly damped. When the second bunch is injected, 2.4 s later, its phase error which can be as large as $\pm 40^\circ$ must be damped independently. This is achieved by correcting the RF phase when the bunch passes through the cavities. The phase error signal, from the sampled phase detector, is used to drive a fast phase shifter on the RF drive line, through a multiplexer working at the revolution frequency (Fig.22). The damping rate is much smaller than that provided by the phase lock configuration, but is sufficient to

suppress any phase oscillation in a few synchrotron periods. If no damping is provided on the individual bunches, severe losses occur during acceleration.

Fig. 22 Simplified layout of the low level RF system in proton-antiproton mode

The antiproton bunches are damped in exactly the same way, the correcting signals being applied on the antiproton cavities.

During acceleration and storage quadrupole oscillations usually develop on the intense proton bunches. These must also be damped to avoid a large blow up of the bunch area and hence a poor life time. A gating technique is also used here to provide independent damping of quadrupole oscillations on each proton bunch. The signal from a wide band pick-up is analysed in a sampled peak detector to extract the quadrupole oscillations of each bunch. Feedback is provided on the RF amplitude via a multiplexer at the revolution frequency.

The slow control of RF frequency is, as in the PS, via a frequency loop, which compares the actual RF frequency with a precise frequency programme derived from the bending magnetic field. As compared with the PS, the situation is more favourable here as there is no transition crossing. On the other hand, deceleration must be possible for the "pulsed collider" experiments where the magnetic field is cycled during storage, between 100 GeV/c and 450 GeV/c. Proper rejection of the RF noise generated in the frequency and phase programs is of great concern for this type of operation.

The most striking feature of the collider low level RF system is its extremely low noise necessary to achieve long life time during storage. In the first bunched beam storage experiments, where the fixed target low level RF system was used, the beam life time was less than one hour²². With sampled phase detectors, much better suited to the long intervals between bunches, considerable improvement has been achieved (several 10 hours). A new VCO (master oscillator) design further reduced the high frequency noise and brought the life time in the 100 hours range. For the antiprotons the low frequency noise of the antiproton cavities regulation circuits working at 10.7 MHz (Fig. 22) is also an important parameter. With the new design, now in operation, the antiproton life time is comparable to that of the protons, whereas it was previously limited to about 40 hours. Our understanding of the RF noise problem is given in Appendix II, together with the experimental result on the SPS, which agree well with the theoretical estimates.

REFERENCES

- [1] F. Pedersen, F. Sacherer, Theory and Performance of the longitudinal active damping system for the CERN PS booster, IEEE Trans. on Nucl. Science, Vol. NS 24, No. 3, June 1977, p. 1396.
- [2] B. Kriegbaum, F. Pedersen, Electronics for the longitudinal active damping system for the CERN PS booster, IEEE Trans. on Nucl. Science, Vol. NS 24, No. 3, June 1977, p. 1695.
- [3] J. M. Baillod, L. Magnani, G. Nassibian, F. Pedersen, W. Weissflog, A second harmonic RF System with feedback reduced gap impedance for accelerating flat-topped bunches in the CERN PS booster, IEEE Trans. on Nucl. Science, Vol. NS 30, No. 4, August 1983, p. 3499.
- [4] F. Pedersen, Beam loading effects in the CERN PS booster, IEEE Trans. on Nucl. Science, Vol. NS 22 No. 3, June 1975, p. 1906.
- [5] D. Boussard, Cavity compensation and beam loading instabilities, CERN SPS/ARF/Note/DB/78-16.
- [6] D. Boussard, E. Brouzet, R. Cappi, J. Gareyte, Collective effects at very high intensity in the CERN PS, IEEE Trans. on Nucl. Science, Vol. NS 26, No. 3, June 1979, p. 3568.
- [7] N. Rasmussen, Counteracting beam loading effects in the PS Booster, CERN PS/BR Note NR/84-3.
- [8] W. Hardt, Gamma-Transition jump scheme of the CPS, Proc IXth Int. Conf. on High Energy Accelerators, Stanford, USA, p. 434.
- [9] D. Boussard, RF Techniques for $p\bar{p}$, CERN Accelerator School, September 1983 and CERN/SPS/84-2 (ARF).
- [10] J.P. Delahaye, P. Lefèvre, J.P. Riunaud, Improved recombination of the 20 PSB bunches and merging into 5 dense bunches in the CPS, IEEE Trans. on Nucl. Science, Vol. NS 26, No. 3, June 1979, p. 3565.
- [11] F. Mills, Stability of phase oscillation under two applied frequencies, BNL Internal Report AADD 176 (1971).
- [12] D. Boussard, Y. Mizumachi, Numerical computation of the behaviour of bunches in the azimuthal combination process, CERN SPS/ARF Internal Note 79-12.
- [13] D. Boussard, Y. Mizumachi, Production of beams with high line density by azimuthal combination of bunches in a synchrotron, IEEE Trans. on Nucl. Science, Vol. NS 26, No. 3, June 1979, p. 3623.

- [14] R. Garoby, Proposal for a new process realizing longitudinal merging of bunches in the CPS, CERN PS/RF/Note 83-15.
- [15] R. Johnson, S. van der Meer, F. Pedersen, G. Shering, Computer control of RF manipulations in the CERN antiproton accumulator, IEEE Trans. on Nucl. Science, Vol. NS 30, No. 4, August 1983, p. 2290.
- [16] G. Dôme, Theory of RF acceleration and RF noise, CERN Accelerator School Sept. 1983 and CERN/SPS 84-14.
- [17] R. Johnson, S. van der Meer, F. Pedersen, Measuring and manipulating an accumulated stack of antiprotons in the CERN antiproton accumulator, IEEE Trans. on Nucl. Science, Vol. NS 30, No. 4, August 1983, p. 2123.
- [18] E.J.N. Wilson, Editor, Design study of an antiproton collector for the antiproton accumulator (ACOL) CERN 83-10.
- [19] B. Autin, The CERN Antiproton Collector, CERN Accelerator School, Sept. 1983 and CERN/PS/85-1 (AA).
- [20] G. Dôme, The SPS acceleration system, Proc. 1976 Proton Linac Conf. Chalk River, Canada.
- [21] D. Boussard, T.P.R. Linnecar, RF noise performance of the CERN proton antiproton collider, Proc. of the 12th Internal Conf. on High Energy Accelerators Fermilab, August 1983, p. 226.
- [22] D. Boussard, G. Dôme, C. Graziani, The influence of RF noise on the lifetime of bunched proton beams, Proc. XIth Conf. on High Energy Accelerators, CERN, Geneva, July 1980, p. 620.
- [23] D. Boussard, RF power estimates for a hadron collider, CERN SPS/ARF note 84-9.
- [24] E. Haebel, Superconducting cavities and minimum RF power schemes for LHC, CERN EF/RF 84-4.
- [25] D. Boussard, G. Lambert, Reduction of the apparent impedance of wide band accelerating cavities by RF feedback, IEEE Trans. on Nucl. Science Vol. NS 30 No. 4, August 1983, p. 2239.
- [26] H. G. Hereward, The elementary theory of Landau damping, CERN 65-20.
- [27] S. Hansen, A. Hofmann, E. Peschardt, F. Sacherer, W. Schnell, Longitudinal bunch dilution due to RF noise, IEEE Trans. on Nucl. Science Vol. NS 24, No. 3, June 1977, p. 1452.
- [28] U. Wehrle, The low noise oscillator for pp, SPS/ARF/UW Note 83-10.

- [29] Y. Mizumachi, Partial linearization of the RF voltage of a storage ring to reduce bunch diffusion, Proc XIth Conf on High Energy Accelerators, CERN, Geneva, July 1980, p. 615.
- [30] D. Boussard, Quantization noise in digital beam feedback systems, CERN/SPS/ARF/Note/DB/84-13.
- [31] D. Möhl, G. Petrucci, L. Thorndahl, S. van der Meer, Physics and Technique of stochastic cooling, CERN/PS/AA 79-23.

APPENDIX I

BEAM LOADING ON RF CAVITIES

1 Equivalent circuit

In the following, we shall only consider the case of hadron machines, where the bunches are not very short compared to the RF wavelength. It is therefore legitimate to neglect, as a first approximation, the interaction of a bunch with the high order modes of the cavity and only consider its fundamental resonance.

The usual RLC equivalent circuit (Fig. I.1) represents the RF cavity in the vicinity of the main resonance, seen from the accelerating gap. By a proper transformation from the RF generator to the gap, the power tube can be described by the current source i_g , its impedance being embedded into the RLC circuit elements. If there is a long transmission line between power generator and cavity, this model is still valid if the generator presents a matched load, seen from the cavity (for instance, with a circulator, Fig. I.4). We also assume here that the cavity is a standing wave structure, in which the phase of the electric field is the same everywhere, and allows the two current sources i_b (beam current) and i_g to be added directly.

Fig. I.1 Equivalent circuit of accelerating cavity.

At the RF frequency, f_{RF} , the phasor diagram of Fig. I.2 describes the circuit behaviour. Note that i_b is the component of the beam current at f_{RF} , and that V (gap voltage) is taken as reference. For a single cavity, or if all cavity voltages are in

phase, V and i_b are in quadrature for no acceleration: no active power is delivered to the beam ($\phi_s = 0$). In the general case the phase difference between V and i_b is $\pi/2 + \phi_s$ (Fig. I.2).

Fig. I.2 Phasor diagram related to the equivalent circuit of Fig. I.1.

The amplitude of the vector i_b , obviously proportional to the DC beam current, also depends upon V , which for given machine parameters and beam emittance determines the bunch form factor. However this dependence is weak (bunch length varies as the inverse fourth power of the RF voltage), except for adiabatic capture, and is usually neglected in beam loading stability studies. The vector sum $i_b + i_g = i_t$ is the total current which flows into the resonator. It is related to V by the cavity admittance Y :

$$i_t = VY = \left(\frac{1}{R} + jB\right)V \quad (\text{I.1})$$

the real part of which $1/R$ is constant, whereas its imaginary part jB can be varied by adjusting the tuning of the resonator. As a consequence i_t follows the dotted line on Fig. I.2 according to the cavity tuning. The generator current i_g is then fully determined by the relation

$$i_g = i_t - i_b \quad (\text{I.2})$$

Obviously the minimum value of $|i_g|$ is obtained when i_g and V are in phase. This is the best operating point for the power generator, and usually there is a servo tuning system designed to maintain this condition: the tune of the RLC circuit (with respect

to f_{RF}) is controlled with a feedback system which keeps V (gap voltage) and i_g (generator current) in phase. Under these conditions, the generator works into a resistive load and the generator power is entirely converted into cavity losses and beam acceleration.

All this is perfectly satisfactory as long as we stay at equilibrium, namely as long as i_b is constant. During transients, for instance, when a prebunched beam is injected into the machine, the limited speed of the servo tuning system will result in a variation of V which, if not very short compared to a synchrotron period may affect the longitudinal motion of the beam. This is also the case during adiabatic trapping where the RF component of the beam grows initially much faster than the RF voltage.

2 Peak generator power

Consider again the case of a prebunched beam (i_b) injected into an empty machine. Before injection, the servo tuning system keeps $i_t = i_g$ and V in phase (Fig. I.3).

Fig. I.3 Phasor diagram at injection.

Immediately after injection, the new vector i_b destroys the equilibrium, and V changes by a large amount, until the loop retunes the cavity to a different value. The only way to maintain V constant during the transient phase of the tuner is to act via the RF power generator which provides a fast control of V . The obvious solution is to change i_g into i_g' (Fig. I.3) when i_b is injected. If we make:

$$i_g' = i_g - i_b \quad (I.3)$$

the total current in the cavity does not change and, at constant tuning, V stays constant.

In the simple case of no acceleration, the amplitude of the peak current i_g' which must be delivered by the tube to suppress beam loading is given by:

$$i_g'^2 = i_g^2 + i_b^2 \quad (I.4)$$

A similar analysis can be done for an RF generator connected to the cavity via a circulator (Fig. I.4).

Fig. I.4 Use of a circulator to match the RF generator to the cavity.

Assume the cavity matched without beam and $\phi_s = 0$, for simplicity. All the generator power P_o is delivered to the cavity and there is no reflected power flowing into the load. To keep V constant with beam during the servo tuner transient, the required peak power can be calculated^{23,24}:

$$P' = P_o \left[1 + \left(\frac{V i_b}{4 P_o} \right)^2 \right] \quad (I.5)$$

As the voltage across the gap does not change and $\phi_s = 0$, the excess power $P' - P_o$ is simply wasted in the matching load to keep V constant.

One can optimize P' by selecting the cavity impedance R . One obtains:

$$R_{opt} = \frac{2V}{i_b} \quad (I.6)$$

which gives the minimum peak power required:

$$P'_{opt} = 2P_0 = \frac{V i_b}{2} \quad (I.7)$$

This requirement can nevertheless be somewhat relaxed if the cavity is detuned before injection.

The transient RF drive i'_g must be synthesized to achieve condition (I.3) and keep V constant in amplitude and phase. The response of the circuitry which generates i'_g must be fast compared with the synchrotron period, assuming that the cavity voltage is not modulated at f_{rev} ($Q \gg h$). If, on the contrary $Q \ll h$, unequal filling of the machine will result in a large modulation of V at f_{rev} and its multiples. The same analysis applies in this case: at each "batch" passage transient beam loading must be corrected by an RF drive i'_g generated by a circuitry fast compared to the revolution period.

3 The RF drive generation

a) Amplitude and phase servo loops (Fig. I.5)

The problem of the synthesis of i_g in such a way as to maintain V constant in amplitude and phase can be solved by two separate servo loops. The first keeps the amplitude of V constant by acting on the amplitude of i_g (amplitude loop) and the second maintains the relative phase of V and i_b through the control of the phase of i_g (phase loop). Neglecting the fluctuations of V at multiples of f_{rev} ($Q \gg h$), the phase and amplitude loops must be fast compared to the synchrotron period ($f_c \gg f_s$) f_c being the cut off frequency of the loop. The loops are

nevertheless band limited by the cavity itself which is obviously included into the feedback paths (bandwidth: $\propto f_{\text{rev}}/2Q$). Fortunately, in hadron machines $Q_s = f_s/f_{\text{rev}}$ is very small ($Q_s \ll 1$) and f_c can be conveniently chosen between f_s and f_{rev} .

Fig. I.5 Amplitude and phase servo loops.

Dotted line: feedforward correction.

Independent amplitude and phase control of V is a well known technique for proton machines; it works satisfactorily for relatively small beam currents, i.e. when the gap voltage is mostly determined by the generator current (typically $|i_b| < |i_g|$). For higher beam currents, a variation of the amplitude of i_g for instance, not only results in a variation of the amplitude of V , but also of its phase and vice versa. In other words, the two loops which were independent at low beam currents, become coupled together, and one understands that an unstable behaviour of the system occurs above some beam current threshold. A detailed analysis⁴ and machine experiments confirm this intuitive result. Although it is, in principle possible to compensate loop couplings by additional cross coupling circuits⁴, feedforward correction offers a much simpler solution to the problem.

b) Feedforward correction

If i_b (RF component of the beam current) is measured separately, with a pick-up electrode followed by a filter centred at f_{RF} , for instance, the RF drive with beam can be generated from the RF drive without beam, by making use of the relation

$$i_g' = i_g - i_b \quad (I.8)$$

Applied to the amplitude and phase servo loops described above, the method consists in reinjecting directly on the input of the RF amplifier the signal $-i_b$ (Fig. I.5). It can be shown analytically⁵ that, with i_g being now the parameter controlled by the loops (i_g corresponds to the case $i_b = 0$) instead of i_g' , the cross couplings between amplitude and phase loops disappear and the system becomes stable even at very high beam currents.

The signal corresponding to $-i_b$ does not need to be synthesized with high precision, as it only removes the loop couplings and restores stability. For a varying RF frequency, the pick-up to cavity delay must be properly compensated, as well as the variations in gain and phase of the RF power amplifier. In the case of the PS a coarse, feedforward correction (or cavity compensation) covers the whole RF frequency swing during acceleration, but finer settings are achieved at a few fixed frequency points.

Note that, in the case of a low gain, slow, amplitude loop cross coupling between loops is not critical, but the RF gap voltage is not held constant during the transient phase of the amplitude loop. Here the feedforward correction restores the constant value of V , but more precision on the synthesis of $-i_b$ may be required. This technique was used successfully on the Brookhaven AGS.

In the case of a wide band cavity ($Q < h$), feedforward correction can also be used to suppress the transient beam loading effects, at multiples of f_{rev} . There is nevertheless, an additional constraint, namely that the overall delay between pick-up and cavity be exactly equal to the beam transit time plus an integer number of turns.

c) RF feedback around the power amplifier (Fig. I.6)

To generate the compensating current $-i_b$ delivered by the RF power amplifier, we can take the information from the cavity itself, which is by its very nature a pick-up element tuned at f_{RF} . This leads to the scheme of Fig.I.6, in which one obviously recognizes a feedback loop built around the RF amplifier.

Fig. I.6 RF feedback around the RF amplifier.

With the loop equations one can easily calculate the new value of i_g when the beam is injected (i_g'):

$$i_g' = i_g - \frac{GZ}{1 + GZ} i_b \quad (I.9)$$

which, for $GZ \gg 1$ reduces to equation (I.3), i_g being here the generator current for no beam:

$$i_g' = i_g - i_b \quad (I.10)$$

The feedback loop automatically generates the correct compensating signal, which is another way of saying that it keeps the controlled parameter V constant.

Stability of the loop is determined by the cavity bandwidth (pole at $f_{RF}/2Q$) and the total delay of the feedback path. The amplifier and preamplifiers (Figs. 6 and 16) are selected for the shortest propagation delays and are located as close as possible to the cavity itself. For a varying RF frequency, one could, in principle, adjust the delay of the return path to keep the 180° phase condition at the RF frequency. However, in many designs (2nd harmonic RF in PSB, future PS RF system for instance), the total delay is kept short enough to ensure stability over the entire RF frequency swing, without programming the phase.

RF feedback reduces the effective impedance of the cavity over a bandwidth which is intrinsically limited by the overall open loop delay; if technically feasible (i.e. short delay) it offers the best solution to the beam loading problem, with all the advantages of feedback systems in terms of adjustment tolerances.

Unfortunately, it is not always applicable in the simplest form described above, because of unavoidable delays (in large RF systems for instance) which would restrict the overall bandwidth to only a small fraction of the cavity pass-band and leave, for instance, transient beam loading at multiples of the revolution frequency uncorrected. To correct beam loading in the vicinity of nf_{rev} the overall delay must be either very small (phase = 0) or equal to one turn (phase = $2\pi n$). Although, it is always possible to increase the overall delay to one turn in order to make the phase correct, this is not sufficient because the system would be completely unstable at frequencies $(n+\frac{1}{2}) f_{rev}$ where the phase would be completely wrong. The solution is to program the gain to be maximum at $n f_{rev}$ and minimum at $(n + \frac{1}{2})$

f_{rev} , by a so called "comb filter" inserted into the feedback path²⁵. Overall gain limitations result from the particular transfer function shape, which are not too severe for small Q_s . This solution is in use on the SPS system in its high intensity mode.

APPENDIX II

THE RF NOISE PROBLEM

1 Synchrotron oscillation with noise excitation

We shall present a very simple approach of the problem, although certainly not perfectly rigorous. Consider first a single particle oscillating in an elliptical bucket (a linear RF waveform is assumed), for which the synchrotron frequency f_s is constant. Like a lossless resonator, the transfer function of the system has a pole at $\omega = \omega_s = 2\pi f_s$.

If the excitation contains all frequencies (white noise spectrum, for example), the steady state response will have an infinite amplitude at $\omega = \omega_s$, but reached only after an infinitely long time. We must therefore look at the transient response of the lossless resonator and consider a sinusoid starting at $t = 0$ as the exciting waveform²³:

$$f(t) = H(t) \exp j\omega t \quad (\text{II.1})$$

where $H(t)$ is the unity step function: $H(t) = 0$ for $t < 0$; $H(t) = 1$ for $t > 0$.

The linearized synchrotron oscillation writes:

$$\ddot{\phi} + \omega_s^2 \phi = \omega_s^2 f \quad (\text{II.2})$$

where $\omega_s^2 f$ is the excitation term, $f(t)$ being the phase fluctuation of the RF wave.

With the initial conditions:

$$\phi = \dot{\phi} = 0 \quad \text{for} \quad t \leq 0$$

one can easily verify that:

$$\phi(t) = \frac{\omega_s^2}{\omega_s^2 - \omega^2} (\exp j\omega t - \exp j\omega_s t) H(t) \quad (\text{II.3})$$

is a solution of II.2, if $\omega_s \neq \omega$.

Imagine now that $f(t)$ is one particular sample of an exciting phase noise. For a limited band of frequencies Δf , centred at f_s , the noise signal $f(t)$ can be represented by a sinusoid at a frequency f with $|f - f_s| < \Delta f$, having a random complex amplitude A from sample to sample such that:

$$\langle |A|^2 \rangle = 2 S_\phi(\omega_s) \Delta f ; \quad \langle A \rangle = 0 \quad (\text{II.4})$$

where $S_\phi(\omega_s)$ is the spectral density of the phase noise excitation at ω_s and the brackets represent an average over many samples.

This is valid for a time τ of the order of $1/\Delta f$:

$$\tau = k/\Delta f \quad (\text{II.5})$$

k being of the order of unity.

The effect of the band of frequencies of width Δf on the harmonic oscillator is obtained with (II.3). For $t < \tau$, and with the conditions (II.5) and $|f - f_s| < \Delta f$, we can expand the quantity in brackets and obtain, to first order:

$$\phi(t) = -jt \frac{\omega_s}{2} \exp j\omega t H(t) A \quad (\text{II.6})$$

The amplitude of the phase response, at time $t = \tau$ writes:

$$\hat{\phi}(\tau) = \frac{\omega_s}{2} \tau |A|$$

which, combined with II.4 and II.5 finally gives:

$$\langle \frac{\hat{\phi}(\tau)^2}{\tau} \rangle = k \omega_s^2 S_{\phi}(\omega_s) \quad (\text{II.7})$$

The quantity $\hat{\phi}(\tau)^2$, averaged over many noise samples increases linearly with time, the proportionality coefficient being the noise spectral density $S_{\phi}(\omega)$ of the exciting waveform.

The rigorous approach ¹⁶ gives the following results, valid for phase and amplitude noise:

$$\langle \frac{\Delta x}{\Delta t} \rangle = \frac{\omega_s^2}{4} [S_{\phi}(\omega_s) + 2x S_a(2\omega_s)] \quad (\text{II.8})$$

$$\langle \frac{(\Delta x)^2}{\Delta t} \rangle = \frac{\omega_s^2}{2} [x S_{\phi}(\omega_s) + x^2 S_a(2\omega_s)] \quad (\text{II.9})$$

where $x = \hat{\phi}^2/4$ and S_a is the spectral density of the RF amplitude noise.

For a stationary, sinusoidal bucket, one should take $x = \sin^2(\hat{\phi}/2)$ ($x=1$ on the separatrix) and add higher order terms at multiples of ω_s ¹⁶.

For many particles, submitted to the same sample of RF phase noise, Liouville theorem predicts conservation of phase space area. If the bucket is rigorously linear, any elliptical bunch will remain elliptical and there will be no macroscopic blow-up, but only a random displacement of the center of gravity of the bunch according to II.8.

However, over a sufficiently long time scale, the assumption that all particles have exactly the same frequency, which ensure that the excitation is coherent for all of them is not realistic. In other words, the smallest synchrotron frequency spread destroys the coherence between particles, and the average over noise samples in II.8 can be replaced by an average over particles in

the bunch provided the time scale Δt is chosen long enough (many synchrotron periods).

The quantity $\langle x \rangle$ is therefore proportional to the bunch emittance. Its evolution, over a sufficiently long time scale is again governed by equation II.8: the emittance increases linearly with time, proportionally to the noise spectral densities $S_\phi(\omega_s)$ and $S_a(2\omega_s)$.

If a harmonic cavity is used, phase noise between the two RF systems (fundamental and harmonic) may be present. In this case not all particles see the same noise samples (if the bunch is long compared to the period of the harmonic RF system) and the coherence between particles is lost immediately. Smooth blow-up of the beam emittance can be achieved in only a few synchrotron periods in this case, contrary to the very long time scales considered for a sinusoidal bucket. Application of this idea is in the controlled blow-up in the PS with the 200 MHz RF system.

Figure II.1 shows the results of ISR experiments where the linear increase of emittance versus time is clearly demonstrated²⁴.

Fig. II.1 Increase of bunch emittance in the ISR

2 RF noise seen by the beam

For RF amplitude noise, it is relatively easy to measure S_a ($2\omega_s$) with a simple detector on the cavity gap. On the contrary the phase noise of the RF generator is very strongly modified by the presence of the phase loop, which is mandatory on machines where natural damping is absent (hadron colliders).

Take the typical example of Fig. II.2 with the following noise sources:

- u_p : phase discriminator noise
- u_f : oscillator (VCO) noise, magnet noise,
noise from the frequency loop.

Fig. II.2 Typical phase loop system

From the loop equations, the frequency noise y seen by the beam can easily be calculated:

$$y = \frac{u_f + G u_p}{1+GB} \quad (\text{II.10})$$

where G is the gain of the loop amplifier and B is the beam transfer function (which relates the beam-cavity phase to the RF frequency deviation). In the case of a linear bucket B is given by:

$$B = \frac{j\omega}{\omega_s^2 - \omega^2} \quad (\text{II.11})$$

By making G large, the contribution of u_f can be made negligible, and one is left with the ultimate source of noise u_p coming from the phase detection. With a careful design of the phase detector (RF mixer followed by a low noise amplifier and sampler) typical values of u_p in the order of 10^{-6} rad/ $\sqrt{\text{Hz}}$ can be achieved ($1.6 \cdot 10^{-6}$ rad/ $\sqrt{\text{Hz}}$ in the SPS). The best technique to measure u_p is to feed the two arms of the phase detector with signals coming from the same bunch. In a machine with several cavities, the total RF voltage seen by the beam must be properly reconstructed from the individual cavity signals to avoid any additional sources of noise on u_p .²¹

In the multibunch case, like in the SPS, the damping loops for individual bunches have a much smaller gain G than the main phase loop. The contribution of u_f , at multiples of the revolution frequency, becomes dominant for the bunches not directly damped by the main phase loop. The VCO design should provide a low noise spectral density at multiples of f_{rev} (Fig. II.3)²⁵.

Fig. II.3 Noise spectral density of the SPS main oscillator

If many bunches are to be stored, e.g. in a proton-proton collider, damping of individual bunches would rely on digital feedback techniques. In this case quantization noise of the phase detector signal may also become important. The corresponding noise spectral density S_u is of the order of:

$$S_u \approx 2^{-2N} / 3f_{\text{rev}}$$

where N is the number of bits ²⁷.

The influence of the beam transfer function B is obvious from equation II.10. In the ideal case of a linear bucket $B(\omega)$ has a pole at $\omega = \omega_s$ and $y(\omega_s) = 0$: the phase loop completely suppresses any noise of the system at $\omega = \omega_s$. For a sinusoidal bucket, in the vicinity of ω_s , the quantity $|B(\omega_s)|$ can be approximated by $1/s$, where s is the synchrotron frequency spread in the bunch. It follows:

$$S_{\phi}(\omega_s) = S_u(\omega_s) \frac{s^2}{\omega_s^2} \quad (\text{II.12})$$

S_u is the noise spectral density of the electronic noise u , and the term ω_s^2 comes from the conversion of y (frequency noise) into phase noise. As far as RF noise is concerned, short bunches are preferable, because s is small. With a harmonic cavity, one could linearize the RF waveform to minimize s even further²⁶. Unfortunately a small value of s also means little Landau damping and a low intensity threshold against instabilities.

3 Equilibrium distribution and life time

The blow-up of the longitudinal emittance of the bunch, when submitted to RF noise is, of course, limited when particles reach the separatrix and leave the bucket. At that point the particle density ρ vanishes; in other words

$$\rho(x = 1) = 0 \quad (\text{II.13})$$

is the boundary condition of the RF noise diffusion process.

In the following, we shall introduce, again in a simple, but not rigorous way, the two dimensional Fokker-Planck equation describing the long term evolution of many particles submitted to RF noise. In the theory of momentum stochastic cooling²⁸, the one dimensional Fokker-Planck equation can be derived from the transport and continuity equations. A very similar approach will be used here.

For an "intermediate" time scale dt , long enough for having perfect mixing between particles, but very short compared with the diffusion process, the average particle radius $\hat{\phi}$ does not grow: $\langle d\hat{\phi}/dt \rangle = 0$, but the average fluctuation $\langle (d\hat{\phi})^2/dt \rangle$ is given by equation II.9:

$$\langle \frac{(d\hat{\phi})^2}{dt} \rangle = \frac{1}{2} \omega_s^2 S_\phi(\omega_s) \quad (\text{II.14})$$

valid for a linearized bucket ($x = \hat{\phi}^2/4$) and phase noise only ($S_a = 0$).

Now calculate the number of particles dN in a layer of width $d\hat{\phi}$ between $\hat{\phi}-d\hat{\phi}$ and $\hat{\phi}$. We limit the expansion to the second order in $d\hat{\phi}$ and obtain:

$$dN = 2\pi \hat{\phi} \left(\rho d\hat{\phi} - \frac{1}{2} \frac{\partial \rho}{\partial \hat{\phi}} d\hat{\phi}^2 \right) \quad (\text{II.15})$$

The outgoing flux of particles, through the circle of radius $\hat{\phi}$: $\Psi = dN/dt$ reduces to :

$$\Psi = \frac{dN}{dt} = - \frac{1}{2} \pi \hat{\phi} \omega_s^2 S_\phi(\omega_s) \frac{\partial \rho}{\partial \hat{\phi}} \quad (\text{II.16})$$

Combined with the continuity equation:

$$2\pi \hat{\phi} \frac{\partial \rho}{\partial t} + \frac{\partial \Psi}{\partial \hat{\phi}} = 0 \quad (\text{II.17})$$

and the relation: $x = \hat{\phi}^2/4$, one obtains:

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} \left(x \frac{\omega_s^2}{4} S_{\phi}(\omega_s) \frac{\partial \rho}{\partial x} \right) \quad (\text{II.18})$$

This classical diffusion equation describes the evolution of particle density inside the bucket. The rigorous approach¹⁶ gives the complete formula valid for phase and amplitude noise:

$$\boxed{\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} \left[\frac{\omega_s^2}{4} (x S_{\phi}(x) + x^2 S_a(x)) \frac{\partial \rho}{\partial x} \right]} \quad (\text{II.19})$$

Analytical solutions of the diffusion equation (II.18) can be found in a few interesting cases (G. Dôme)¹⁶.

- Amplitude noise only

A good approximation is to take $S_a(x) = \text{constant}$, as there is no strong feedback loop to perturb the electronic noise. The solutions are of the form :

$$\rho(x,t) = R(x) e^{-t/T} \quad (\text{II.20})$$

The longest time constant T_o , giving the long term equilibrium distribution and long term beam life time is given by:

$$T_o = \frac{16}{\omega_s^2 S_a(2\omega_s)} \quad (\text{II.21})$$

Experiments carried out on the SPS collider where white noise was injected in the amplitude regulation circuits of the RF cavities, have shown an excellent agreement with this formula.

- Phase noise only

Here we must model $S_{\phi}(x)$ according to equation II.9 and take, for G large:

$$S_{\phi}(x) = S_u \frac{1}{\omega^2} \frac{1}{|B|^2} \quad (\text{II.22})$$

Assume S_u constant and, as a first approximation for $B(x)$ take the case of no spread: $B = j\omega/(\omega_0^2 - \omega^2)$ where $\omega_0/2\pi$ is the synchrotron frequency in the center of the bunch:

$$S_{\phi}(x) \approx S_u \left(\frac{\omega_0^2 - \omega^2}{\omega^2} \right)^2 \quad (\text{II.23})$$

For a sinusoidal bucket ω is a function of x : $\omega = \omega(x)$, hence, we can calculate numerically:

$$S_{\phi}(x) \approx S_u g(x)$$

and approximate the function $g(x)$ by the quantity $0.04 (1-x)^{-2}$ which will allow an analytical solution to be found for the diffusion equation. The long term distribution has an equilibrium life time given by:

$$T_0 \approx \frac{20}{\omega_s^2 S_u(\omega_s)} \quad (\text{II.24})$$

Here again the SPS collider experiments, with white noise injected in the phase detector, were in very good agreement with this formula.