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GAUGE SYMMETRIES OF THE FREE BOSONIC STRING FIELD THEORY

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ABSTRACT

The gauge covariant local formulations of free bosonic string theories that contained a finite number of supplementary fields are extended to include an infinite number of supplementary fields. These new formulations allow the generators of the Virasoro algebra to appear on a more equal footing.

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Recently, a procedure to find a second quantized gauge covariant formulation of string theories was given¹⁾. This procedure involved the introduction of supplementary fields, and when applied to the open bosonic string it was used to find a covariant formulation for the first six levels of the free theory as well as the interaction at the first level. In a subsequent paper²⁾ a gauge covariant formulation of all known free string theories was given for all levels. These formulations involved only a finite set of supplementary fields. For the open bosonic string, one required seven fields in all, six supplementary string fields and the original string field ψ . In this paper we extend this formulation to include more supplementary fields and more gauge invariances. This has the advantage that the L_n 's appear on a more symmetrical footing.

We recall²⁾ that the action of the local gauge covariant formulation of the open string was of the form

$$\begin{aligned} & \frac{1}{2}(\psi, (L_0 - 1)\psi) + \sum_{n=1}^{\infty} \left\{ (L_n \psi, \phi^{(n)}) + n (\phi^{(n)}, \phi^{(n)}) \right\} \\ & + \sum_{n,m=1}^{\infty} \left\{ (L_n \phi^{(m)} + (2n+m)\phi^{(n+m)}, \zeta^{(n,m)}) \right. \\ & \left. - \frac{1}{2} (\zeta^{(n,m)}, (L_0 + n + m - 1)\zeta^{(m,n)}) \right\} \end{aligned} \quad (1)$$

It is invariant under the gauge transformations

$$\begin{aligned} \delta \psi &= \sum_{n=1}^{\infty} L_n \Lambda_n \\ \delta \phi^{(n)} &= - (L_0 + n - 1) \Lambda_n \\ \delta \zeta^{(n,m)} &= - L_m \Lambda_n - (2m+n) \Lambda_{n+m} \end{aligned} \quad (2)$$

We now introduce further supplementary fields leading to the set

$$\phi^{(n)}, \zeta^{(n,m)}; \quad n, m = 1, 2, 3, 4, \dots \quad (3)$$

Remarkably, the action of Eq. (1) when sums are extended over all positive integers n is invariant under the gauge transformations of Eq. (2), which now also hold for all positive integers n . The proof of this statement requires the following identity

$$-\sum_{m=1}^{n-1} (2n-m)(n+m) + \frac{D}{12} n(n^2-1) - 2n(n-1) = 0 \quad (4)$$

which is only valid provided D , the dimension of space-time, is 26.

One may verify, by the same technique as given in Ref. 2), that it is possible to choose a gauge such that

$$(L_0 - 1) \psi = 0, \quad L_n \psi = 0 \quad n \geq 1$$

while

$$\zeta(n, m) = \phi^n = 0 \quad (5)$$

This demonstrates that this new gauge invariant formulation is equivalent to the previous one and contains the correct spectrum.

It is straightforward to translate these results from the above operator formalism into functional form. We first recall some of the elementary definitions which relate these two formalisms. When the open string defined from 0 to π has its range extended from $-\pi$ to π , in the usual way, we find that

$$x^\mu(\sigma) = \sum_{n=-\infty}^{\infty} x_n^\mu e^{in\sigma} \quad (6)$$

where

$$x_n^\mu = x_{-n}^\mu.$$

The Virasoro generators are given by

$$L_n = \alpha' \frac{\pi}{2} \int_{-\pi}^{\pi} d\sigma e^{-in\sigma} P^2(\sigma) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_m^\mu \alpha_{n-m}^\nu \eta_{\mu\nu}$$

where

$$P^2(\sigma) = \left[-i \frac{\delta}{\delta x^\mu(\sigma)} + \frac{1}{2\pi\alpha'} \left(\frac{dx^\mu(\sigma)}{d\sigma} \right)^2 \right] \quad (7)$$

The fields $\phi^{(n)}$ are collected into an object which is not only a functional of $x^\mu(\sigma)$ but is also an explicit function of σ ; namely

$$\phi [x^\mu(\sigma), \sigma'] = \sum_{n=1}^{\infty} e^{in\sigma'} \phi_n [x^\mu(\sigma)]$$

Similarly, we collect $\zeta^{(n,m)}$ into one object as follows

$$\zeta [x^\mu(\sigma), \sigma', \sigma''] = \sum_{n,m=1}^{\infty} e^{in\sigma'} e^{im\sigma''} \zeta^{(n,m)} [x^\mu(\sigma)] \quad (8)$$

The transformation laws of Eq. (2) when the sums run over all positive integers become

$$\begin{aligned} \delta \psi &= \frac{\alpha' \pi}{2} \int_{-\pi}^{\pi} d\sigma \ P^2(\sigma) \Lambda(\sigma) \\ \delta \phi(\sigma) &= \left[-(\mathcal{L}_0 - 1) + i \frac{\partial}{\partial \sigma} \right] \Lambda(\sigma) \\ \delta \zeta(\sigma', \sigma'') &= \frac{\alpha' \pi}{2} \hat{P}^2(\sigma') \Lambda(\sigma'') \end{aligned} \quad (9)$$

where

$$\begin{aligned} \frac{\alpha' \pi}{2} \hat{P}^2(\sigma') \Lambda(\sigma'') &= \left[\frac{\alpha' \pi}{2} P^2(\sigma') \right. \\ &\quad \left. + \frac{i}{2\pi} \left\{ \frac{\partial}{\partial \sigma''} \delta(\sigma' - \sigma'') - \delta(\sigma'' - \sigma') \frac{\partial}{\partial \sigma''} \right\} \right] \Lambda(\sigma'') \end{aligned} \quad (10)$$

and we have suppressed the $x^\mu(\sigma)$ dependence. The action of Eq. (1) now takes the form

$$\begin{aligned} &\frac{1}{2} (\psi, (\mathcal{L}_0 - 1) \psi) + \int_{-\pi}^{\pi} d\sigma \ (\psi, \frac{\alpha' \pi}{2} P^2(\sigma) \phi(\sigma)) \\ &+ \frac{1}{2\pi} \int_{-\pi}^{\pi} d\sigma d\sigma' \left\{ \left(\frac{\alpha' \pi}{2} \hat{P}^2(\sigma') \phi(\sigma), \zeta(\sigma', \sigma) \right) \right. \\ &\quad \left. - \frac{1}{(2\pi)^2} \left(\zeta(\sigma, \sigma'), (\mathcal{L}_0 - 1 - i \frac{\partial}{\partial \sigma} - i \frac{\partial}{\partial \sigma'},) \zeta(\sigma', \sigma) \right) \right\} \\ &- \frac{i}{2\pi} \int_{-\pi}^{\pi} d\sigma \left(\phi(\sigma), \frac{\partial}{\partial \sigma} \phi(\sigma) \right) \end{aligned} \quad (11)$$

All the gauge covariant formulations of the free string theories given in Ref. 2) can be extended, as was done above, for the open bosonic string. As a final

example, we consider the closed bosonic string. The action of Eq. (5.3) of Ref. 2) can be extended to include new supplementary fields and becomes

$$\begin{aligned}
 & \frac{1}{2} (\Psi, (L_0 + \bar{L}_0 - 2) \Psi) + (\Psi, \sum_{n=1}^{\infty} (L_{-n} \phi^{(n)} + \bar{L}_{-n} \bar{\phi}^{(n)})) \\
 & + \sum_{n=1}^{\infty} n \{ (\phi^{(n)}, \phi^{(n)}) + (\bar{\phi}^{(n)}, \bar{\phi}^{(n)}) \} \\
 & + \sum_{n,m=1}^{\infty} \{ (L_n \bar{\phi}^{(m)} + (2n+m) \bar{\phi}^{(n+m)}, \bar{\mathcal{Z}}^{(n,m)}) \\
 & + (L_n \phi^{(m)} + (2n+m) \phi^{(n+m)}, \mathcal{Z}^{(n,m)}) + (\chi^{(m,n)}, L_m \phi^{(n)})^{(12)} \\
 & + (\bar{\chi}^{(m,n)}, L_m \bar{\phi}^{(n)}) - (\mathcal{Z}^{(n,m)}, (L_0 + n + m - 1) \mathcal{Z}^{(m,n)}) \\
 & - (\bar{\mathcal{Z}}^{(m,n)}, (L_0 + n + m - 1) \bar{\mathcal{Z}}^{(m,n)}) \\
 & - (\chi^{(m,n)}, (L_0 + \bar{L}_0 + m + n - 2) \bar{\chi}^{(n,m)}) \}
 \end{aligned}$$

This action is invariant under the transformations

$$\begin{aligned}
 \delta \Psi &= \sum_{n=1}^{\infty} L_{-n} \Lambda_n, \quad \delta \phi^{(n)} = -2(L_0 + n - 1) \Lambda_n \\
 \delta \mathcal{Z}^{(n,m)} &= -L_m \Lambda_n - (2m+n) \Lambda_{m+n} \\
 \delta \bar{\chi}^{(n,m)} &= -\bar{L}_m \Lambda_n \\
 \delta \bar{\phi}^{(n)} &= \delta \bar{\mathcal{Z}}^{(n,m)} = 0
 \end{aligned} \tag{13}$$

There is also a similar transformation which for ψ has the form

$$\delta \Psi = \sum_{n=1}^{\infty} \bar{L}_{-n} \bar{\Lambda}_n \tag{14}$$

The variations of the other fields are found by adding and removing bars in an obvious way. The field ψ is subject to the constraint

$$(L_0 - \bar{L}_0) \psi = 0$$

which implies the existence of related constraints for the other fields and gauge parameters, one example being

$$(L_0 - \bar{L}_0 - n + m) \bar{\chi}^{(m, n)} = 0 \quad (15)$$

These results for the closed bosonic string can easily be put in their functional form along the same lines as for the open bosonic string above.

The gauge covariant formulations of the free supersymmetric open and closed strings as well as the heterotic string given in Ref. 2) admit similar extensions which can easily be found by generalizing the results of this paper. The interactions in these theories, as well as the corresponding vertex algebras, will be reported elsewhere³⁾. Although the two formulations discussed in this paper are equivalent at the level of the free theory, it is possible that one of them may be preferred for the construction of the interacting theory.

REFERENCES

- 1) A. Neveu and P.C. West, CERN preprint TH.4200/85 (1985).
- 2) A. Neveu, H. Nicolai and P.C. West, CERN preprint TH.4233/85 (1985).
- 3) A. Neveu and P.C. West, in preparation.