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ON THE VECTOR MESON DOMINANCE IN
EFFECTIVE CHIRAL LAGRANGIANS

Y. Brihaye
Univ. of Mons
Mons, Belgium

N.K. Pak ^{*)}
CERN - Geneva

and

P. Rossi
Physics Department, Univ. of Pisa
and
INFN, Sezione di Pisa, Pisa, Italy

ABSTRACT

We generalize a recently proposed effective chiral vector meson model to the N flavour case. That the low energy predictions involving purely pseudoscalar and electromagnetic processes should not be altered without any constraints on the parameters of the model, due to the new vector meson contributions, is confirmed in the presence of the Wess-Zumino term. At the end, we critically compare our scheme with another one recently proposed which introduces the vector mesons as composites of pions.

^{*)} Permanent address: Dept. of Physics,
Middle East Technical University,
Ankara, Turkey.

Lately there have been attempts^{1),2),3)} to incorporate the vector mesons into the Skyrme model⁴⁾, based on two different motivations. The first one is the desire to have a complete effective theory of all the low-lying hadrons, baryons being generated as the solitonic excitations. The second one is the hope triggered by a recent claim⁵⁾ that the vector mesons may render the solitonic excitations of the pseudoscalars stable without any need for ad-hoc quartic terms.

Presently, there are two alternative approaches to the problem^{2),3)}. One of them²⁾, based on an early attempt by Kaymakçalan et al.¹⁾, identifies the vector mesons with the gauge bosons associated with the gauging of the chiral group, softly broken by explicit mass terms. The physical action is obtained by rigorously imposing the Vector Meson Dominance (VMD)⁶⁾ at the functional level. It was shown that this method is tantamount to adding suitable counterterms systematically which remove all the spurious local vertices introduced by the propagation of the vector mesons, thus reproducing all the purely pionic and electromagnetic low energy theorems⁷⁾ after an integration over the vector meson degrees of freedom are performed. The principal feature of this scheme is that the dimensionless parameter, $\alpha = (f_\pi g/m_\rho)^2$, which appears naturally in chiral vector meson theories, is unconstrained and stays a free parameter.

In the second approach³⁾, the vector mesons are identified with the dynamical gauge bosons (thus composites of pions) of a hidden local symmetry present in the non-linear chiral model. Deferring a critical comparison of the two approaches to the end, we would like to stress one important difference in the viewpoints. VMD is built-in in the first approach by construction, without any constraint on the value of α , whereas in the second one, it follows after a judicious choice of $\alpha = \frac{1}{2}$. However, as we shall demonstrate at the end, even this is not enough to get VMD for processes involving more than one photon, in that scheme.

Because of the special nature of the SU(2) group which enabled us to implement our functional formalism exactly in the closed form, our previous work was restricted to the two-flavour case. The main purpose of this work is to generalize it to the arbitrary flavour case, thus confirming our results in the presence of the Wess-Zumino term^{8),9)}.

Our starting point is the effective chiral action which describes all the low energy pseudoscalar interactions satisfactorily

$$\begin{aligned}
 \Gamma_0(U) &= \Gamma_0^N(U) + \Gamma_0^A(U) \\
 \Gamma_0^N(U) &= \frac{1}{4} f_\pi^2 \int d^4x \operatorname{tr}(\partial_\mu U^\dagger \partial^\mu U) + \dots \\
 \Gamma_0^A(U) &= \frac{N_c}{240\pi^2} \int d^5x \varepsilon_{ijklm} \operatorname{tr}(U^\dagger \partial^i U U^\dagger \partial^j U U^\dagger \partial^k U U^\dagger \partial^l U U^\dagger \partial^m U)
 \end{aligned} \tag{1}$$

where

$$\begin{aligned}
 U &= e^{i\phi} \in SU(N) \\
 \phi &= \frac{1}{f_\pi} \pi^a \lambda^a \quad ; \operatorname{tr}(\lambda^a \lambda^b) = 2 \delta^{ab} \\
 f_\pi &\simeq 93 \text{ MeV}.
 \end{aligned} \tag{2}$$

In Γ_0^N the dots are for the eventual Skyrme terms which is not the main issue here. Γ_0^A is the Wess-Zumino term; the integral is taken over a five-dimensional disk whose boundary is the space-time.

The global symmetry group of Γ_0 , $U(N)_L \times U(N)_R$, or a subgroup of it can be gauged in order to incorporate the vector mesons. This was motivated by the successful gauging of the $U(1)_{em}$ subgroup, thanks to the photon being massless. Here we will be concerned with the gauging of the $SU(N)_V$ subgroup of the full group (keeping an eye on the physically interesting $N = 3$ case)

$$\Gamma_0(U, V) = \frac{1}{4} f_\pi^2 \operatorname{tr}(D_\mu U^\dagger D^\mu U) + \frac{1}{2} \operatorname{tr}(V_{\mu\nu}^2) + \Gamma_0^A(U, V) \tag{3}$$

where

$$\begin{aligned}
 D_\mu U &= \partial_\mu U + g[V_\mu, U] \\
 V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu + g[V_\mu, V_\nu] \\
 V_\mu &\equiv V_\mu^a \lambda^a / 2i
 \end{aligned} \tag{4}$$

The full expression^{9),10)} for $\Gamma_0^A(U, V)$ is extremely cumbersome, therefore we refrain from presenting it in here. Instead we will use the gauge invariant expansion in powers of ϕ given in Ref. 11)

$$\begin{aligned}
 \Gamma_0^A(U, V) = & -\frac{N_c}{48\pi^2} \varepsilon^{\mu\nu\sigma\tau} \int d^4x \operatorname{tr} \left\{ \frac{1}{5} \phi D_\mu \phi D_\nu \phi D_\sigma \phi D_\tau \phi + \frac{3}{2} \phi V_{\mu\nu} V_{\sigma\tau} + \right. \\
 & + \frac{1}{4} V_{\mu\nu} \{ [\phi, D_\sigma \phi], D_\tau \phi \} + \frac{1}{5!} V_{\mu\nu} \left(\frac{5}{2} [D_\sigma \phi^4, D_\tau \phi] \right. \\
 & - [D_\sigma \phi^3, D_\tau \phi] - 2 \{ \phi^3, [D_\sigma \phi, D_\tau \phi] \} + 3 \{ \phi^2, [D_\sigma \phi^2, D_\tau \phi] \} \\
 & + \frac{3}{2} \{ \phi, [D_\sigma \phi^2, D_\tau \phi^2] \} - 4 \{ \phi, [D_\sigma \phi^3, D_\tau \phi] \} \\
 & \left. \left. + O(\phi^7) \right\} \right. \quad (5)
 \end{aligned}$$

We clearly recover the well-established effective electromagnetic action by restricting the gauged subgroup to the $U(1)_\gamma$ group

$$\Gamma_{em}(U, a_\mu) \equiv \Gamma_0(U, V_\mu) \Big|_{V_\mu = -i a_\mu Q^{em}} \quad (6)$$

where Q^{em} is the generator of the $U(1)$ subgroup.

The central idea in Ref. 2) consists in constructing an action, $\Gamma(U, V)$, such that VMD in the functional formalism reads as

$$\exp \{ i \Gamma_{em}(U, a) \} = \int \mathcal{D}V_\mu \exp \left\{ i \Gamma(U, V_\mu) + i \frac{m^2}{2} \int \operatorname{tr} (V_\mu + i a_\mu Q)^2 \right\} \quad (7)$$

Then, we propose the combination

$$\Gamma_{eff}(U, V, a) = \Gamma(U, V) + \frac{m^2}{2} \int \operatorname{tr} (V_\mu + i a_\mu Q)^2 \quad (8)$$

to be the effective action describing the pseudoscalar-vector meson-photon system. It consists of two parts. One is a purely hadronic part and the other is the part which directly couples the photon to the neutral vector meson, through which all the hadrons interact with the photon. To simplify the computation procedure employed in Ref. 2), we first formally integrate the integral equation in (7), and get

$$\exp (i \Gamma(U, V)) = \int \mathcal{D}Z_\mu \exp \left\{ i \Gamma_{em}(U, Z) - i \frac{m^2}{2} \int \operatorname{tr} (Z_\mu - V_\mu)^2 \right\} \quad (9)$$

For the $SU(2)$ the computation can be carried out exactly (to all orders in terms of the physical pion field) and successfully yields our previous result²⁾. For the arbitrary flavour case, there is no closed form which involves the physical pions directly, therefore we use the expanded action in powers of physical pseudoscalars in (9).

If we restrict our attention momentarily to the non-anomalous part of Γ , the integral can be performed without problems because of its quadratic nature in Z_μ . The result in the expanded form is the following

$$\begin{aligned} \Gamma^N = & \frac{1}{4} \int_{\pi}^2 d^4x \left[\text{tr} (D_\mu \phi D^\mu \phi) + \frac{1}{2} \left(\frac{1}{3} - \alpha \right) \text{tr} (\phi D_\mu \phi \phi D^\mu \phi - \phi^2 D_\mu \phi D^\mu \phi) \right. \\ & + \frac{1}{8} \left(\alpha^2 - \frac{2}{3} \alpha + \frac{32}{6!} \right) \text{tr} (\phi^4 D_\mu \phi D^\mu \phi \\ & - 4 \phi^3 D_\mu \phi \phi D^\mu \phi + 3 \phi^2 D_\mu \phi \phi^2 D^\mu \phi) \\ & \left. + O(\phi^8) \right] \end{aligned} \quad (10)$$

Notice that the terms independent of α are exactly those which appear in the expansion of Γ_0^N , whereas the α -dependent terms are those counterterms needed to implement VMD¹²⁾.

The anomalous part requires special care, because it is a quartic polynomial of V_μ . As a consequence the functional integration in (9) cannot be performed exactly [this is true even for the anomalous part of the SU(2) case, see Ref. 2)]. However, since we are dealing with a low energy effective action, to lowest order in momentum, only those diagrams containing one anomalous vertex will dominate. Therefore, we can restrict our attention to those parts of Γ which is linear in anomalous vertices. Furthermore, the non-anomalous part of the action is not affected by the presence of anomalous vertices. This enables us to use a special saddle-point method in the integration of (9). Namely, we solve the classical equation of motion for the non-anomalous part for the effective action appearing in the integrand of (9), finding

$$Z_\mu = V_\mu - \frac{\alpha}{4} [\phi, D_\mu \phi] \quad (11)$$

Next, we substitute this in the anomalous part. This is justified by the previously stated fact that we are only interested in the linear part in the anomalous vertices. Thus, we finally get the following subtracted anomalous action expanded up to sixth order in a (vector) gauge invariant fashion.

$$\begin{aligned}
\Gamma^A(U, V) = & -\frac{N_c}{48\pi^2} \int d^4x \, \varepsilon^{\mu\nu\sigma\tau} \text{tr} \left[\left(\frac{1}{5} - \alpha \right) \phi D_\mu \phi D_\nu \phi D_\sigma \phi D_\tau \phi \right. \\
& + \frac{3}{2} \phi V_{\mu\nu} V_{\sigma\tau} + \\
& + \frac{1}{4} (1-3\alpha) V_{\mu\nu} (\phi D_\sigma \phi \phi D_\tau \phi + D_\sigma \phi D_\tau \phi \phi - 2 D_\sigma \phi \phi D_\tau \phi) \\
& + \left(\frac{1}{5!} \left(-\frac{5}{2} \right) + \frac{\alpha}{16} (1-3\alpha) \right) V_{\mu\nu} (\phi^3 D_\sigma \phi D_\tau \phi + D_\sigma \phi D_\tau \phi \phi^3) \\
& + \left(\frac{3}{5!} - \frac{\alpha}{8} (1-3\alpha) - \frac{\alpha}{4} \right) V_{\mu\nu} D_\sigma \phi \phi^3 D_\tau \phi \\
& - \frac{4}{5!} V_{\mu\nu} \phi D_\sigma \phi \phi D_\tau \phi \phi \\
& + \left(\frac{1}{5!} \frac{13}{2} - \frac{3\alpha}{16} (1-3\alpha) - \frac{3}{8} \alpha \right) V_{\mu\nu} (\phi^2 D_\sigma \phi \phi D_\tau \phi + D_\sigma \phi \phi D_\tau \phi \phi^2) \\
& + \left(-\frac{1}{5!} \frac{9}{2} + \frac{3\alpha}{16} (1-3\alpha) + \frac{3}{8} \alpha \right) V_{\mu\nu} (D_\sigma \phi \phi^2 D_\tau \phi \phi + \phi D_\sigma \phi \phi^2 D_\tau \phi) \\
& + \frac{1}{5!} V_{\mu\nu} (\phi^2 D_\sigma \phi D_\tau \phi \phi + \phi D_\sigma \phi D_\tau \phi \phi^2) + O(\phi^7) \Big]
\end{aligned} \tag{12}$$

The combination of (10) and (12) is the main result of this work. Several comments on this manifestly gauge-invariant result are in order.

The α -independent terms in (12) are exactly the same as those in Γ_0^A given in (5). The α -dependence clearly reflects the presence of the counterterms.

The second term in $\Gamma^A(U, V)$ is not affected by the subtraction scheme as expected, since it is normalized in such a fashion as to reproduce the correct $\pi^0 \rightarrow 2\gamma$ amplitude, through use of VMD. The Wess-Zumino term is also affected by the subtraction scheme. That the successful predictions of the purely pseudo-scalar Wess-Zumino term on processes like $K^+K^- \rightarrow \pi^+\pi^-\pi^0$ are not altered by the introduction of the vector mesons is an interesting (and controversial) question which we discuss next.

Although the 5ϕ contact term is affected, there are two more terms in the expansion of Γ^A , the second and third terms, which through contraction with two and one $V\phi\phi$ vertices respectively, conspire with the contact terms:

$$\begin{aligned}
 & \frac{ig}{2} \text{tr}(V^\alpha[\phi, \partial_\alpha \phi]) \otimes \frac{3}{2} \varepsilon^{\mu\nu\sigma\tau} \text{tr}(\phi V_{\mu\nu} V_{\sigma\tau}) \otimes \frac{ig}{2} \text{tr}(V^\beta[\phi, \partial_\beta \phi]) \\
 & \rightarrow 3\alpha^2 \varepsilon^{\mu\nu\sigma\tau} \text{tr}(\phi \partial_\mu \phi \partial_\nu \phi \partial_\sigma \phi \partial_\tau \phi) \\
 & \frac{ig}{2} \text{tr}(V^\alpha[\phi, \partial_\alpha \phi]) \otimes \frac{1}{4} (1-3\alpha) \varepsilon^{\mu\nu\sigma\tau} \text{tr}(V_{\mu\nu} (\phi D_\sigma \phi D_\tau \phi + D_\sigma \phi D_\tau \phi - 2 D_\sigma \phi \phi D_\tau \phi)) \\
 & \rightarrow (1-3\alpha) \alpha \varepsilon^{\mu\nu\sigma\tau} \text{tr}(\phi \partial_\mu \phi \partial_\nu \phi \partial_\sigma \phi \partial_\tau \phi)
 \end{aligned} \tag{13}$$

Thus, adding the contributions of the three diagrams, we get

$$-\frac{N_c}{240\pi^2} \varepsilon^{\mu\nu\sigma\tau} \int d^4x \text{tr}(\phi \partial_\mu \phi \partial_\nu \phi \partial_\sigma \phi \partial_\tau \phi) \tag{14}$$

which is the purely pseudoscalar result. This confirms our previous claim for the SU(2) that α is a free parameter and is in no way constrained by the VMD mechanism, on a special process which is governed by the Wess-Zumino term exclusively. This is an even stronger result than the SU(2) one²⁾ which was based on reproducing $\pi^0 \rightarrow 2\gamma$, and $\gamma \rightarrow 3\pi$ through use of VMD, whereas the above result is obtained on a purely pseudoscalar process which did not necessitate a further use of VMD. This result should be compared with that of Ref. 13), whose authors were even claiming not only that α is constrained by the use VMD, but also that one could get different values of α , depending on the process in question.

As the VMD is built into our formalism by construction, we would like to briefly discuss the question of ρ -universality, namely, the fact that ρ couples to all hadrons with equal strength, to complete our picture. To compare the couplings of ρ to pions with the other hadrons, namely nucleons which are to be generated as the solitonic excitations, we resort to the usual prescription of introducing collective co-ordinates

$$U \rightarrow U_\pi U_0 U_\pi \tag{15}$$

with

$$\begin{aligned}
 U_\pi &= e^{i\phi/2} \\
 U_0 &= A(t) U_S(\vec{x}) A(t)^{-1}
 \end{aligned} \tag{16}$$

Here $U_s(\vec{x})$ is Skyrme's soliton solution⁴⁾. The vertex which describes the coupling of ρ to other hadrons can be deduced from (2) as

$$\frac{1}{2} g f_\pi^2 \text{tr} [V_\mu (R_\mu - L_\mu)] \quad (17)$$

$$R_\mu = U^\dagger \partial_\mu U, \quad L_\mu = \partial_\mu U U^\dagger$$

Using the substitution (15), we get

$$R_\mu - L_\mu = r_\mu^\pi - \ell_\mu^\pi + (U_\pi^\dagger r_\mu^0 U_\pi - U_\pi \ell_\mu^0 U_\pi^\dagger) \\ + (U_\pi^\dagger U_\pi^\dagger r_\mu^\pi U_0 U_\pi - U_\pi U_0 \ell_\mu^\pi U_0^\dagger U_\pi^\dagger) \quad (18)$$

Now, to compare the couplings of V to π and N , we have to sort out among the terms in (18) those which involve one type of hadron:

$$U_\pi^\dagger r_\mu^0 U_\pi - U_\pi \ell_\mu^0 U_\pi^\dagger \simeq (r_\mu^0 - \ell_\mu^0) - i[\phi, r_\mu^0 - \ell_\mu^0] \quad (19)$$

The first term is the only relevant one for the VNN coupling. By a similar reasoning one can show that the third term in (18) always contains πN mixing to lowest order, and thus is irrelevant. Therefore, finally

$$\frac{1}{2} g f_\pi^2 \text{tr} (V^\mu (R_\mu - L_\mu)) \simeq \frac{1}{2} g f_\pi^2 \text{tr} (V^\mu (r_\mu^\pi - \ell_\mu^\pi) + V^\mu (r_\mu^0 - \ell_\mu^0)) \quad (20)$$

which manifestly depicts the coupling of ρ to π and N with equal strength

$$g_{\rho\pi\pi} = g_{\rho NN} = g \quad (21)$$

We would like to finish with a brief comparison of our scheme with that of Ref. 3), apart from the already discussed difference on the nature of α .

Their non-anomalous Lagrangian, together with the electromagnetism, in our normalization, is given as

$$\begin{aligned} T_J &= -\frac{1}{4} f_\pi^2 \int d^4x \left\{ \text{tr} (\partial_{\bar{L}} \bar{\xi}_L^+ - \partial_{\bar{R}} \bar{\xi}_R^+)^2 + a \text{tr} (\partial_{\bar{L}} \bar{\xi}_L^+ + \partial_{\bar{R}} \bar{\xi}_R^+)^2 \right\} \\ \partial \bar{\xi} &= (\partial_\mu - i V_\mu) \bar{\xi} + i e_0 B_\mu \bar{\xi} \tau^{3/2} \end{aligned} \quad (22)$$

Before discussing the VMD in this scheme, let us point out an amusing formal resemblance between the two schemes. In the absence of electromagnetism, replacing in (22) by D defined in (4) and expanding in terms of the physical pion fields, one gets

$$\begin{aligned} T_J^{\text{modified}} &= \frac{1}{4} f_\pi^2 \int d^4x \left\{ \text{tr} (D_\mu \phi D^\mu \phi) + \frac{1}{4} \left(\frac{a}{2} - \frac{2}{3} \right) \text{tr} (D_\mu \phi D^\mu \phi^2 - \phi D_\mu \phi D^\mu \phi) \right. \\ &\quad + \frac{1}{32} \left(\frac{1}{9} + \frac{8}{5!} - \frac{a}{6} \right) \text{tr} (\phi^4 D_\mu \phi D^\mu \phi - 4 \phi^3 D_\mu \phi D^\mu \phi \\ &\quad \left. + 3 \phi^2 D_\mu \phi \phi^2 D^\mu \phi) \right. \\ &\quad \left. + O(\phi^6) \right\} \end{aligned} \quad (23)$$

The interesting point is that (23) and (10) coincide for the value of $a \equiv 1/\alpha = 2$ which is the KSFR value¹⁴⁾. Thus, the two schemes are formally (only formally, because of the different definitions of the covariant derivatives) the same for $a = 2$. Given the fact that the two schemes have been constructed using different principles, this is quite surprising though. Moreover the dependence on a of (25) is only linear, whereas (10) involves arbitrary powers of $\alpha = 1/a$.

Despite this formal resemblance, the two schemes differ significantly as far as VMD is concerned. It is claimed³⁾ that setting $a = 2$ yields VMD and ρ -universality. Since the ρ couplings to pions are the same in both schemes for $a = 2$, the claim on ρ -universality is all right. However the choice of $a = 2$, although sufficient to eliminate the coupling of the single photon to pions, fails to do so for vertices involving more than one photon. For instance, the $2\gamma-2\pi$ coupling is generated by the following effective piece in (22)

$$\begin{aligned} \frac{1}{16} e_0^2 f_\pi^2 B_\mu^2 \text{tr} [(\bar{\xi}_L \tau^3 \bar{\xi}_L^+ - \bar{\xi}_R \tau^3 \bar{\xi}_R^+)^2 \\ + a (\bar{\xi}_L \tau^3 \bar{\xi}_L^+ + \bar{\xi}_R \tau^3 \bar{\xi}_R^+)^2] \end{aligned} \quad (24)$$

which can be rewritten as (without any need for fixing the gauge as $\xi_L^+ = \xi_R = U^{-\frac{1}{2}}$)

$$\frac{1}{2} \left(\frac{e_0}{g} \right)^2 m_\rho^2 B^2 \left\{ \frac{a+1}{a} + \frac{a-1}{4a} \text{tr}(U \tau^3 U + \tau^3) \right\} \quad (25)$$

Neglecting the π^2 terms, one gets for $a = 2$

$$\frac{1}{2} \left(\frac{e_0}{g} \right)^2 m_\rho^2 B^2 \quad (26)$$

which indeed yields, together with the V^2 term after diagonalization, a massless photon. However, the presence of the $\pi^2 B^2$ term clearly means a breakdown of VMD for their scheme, whereas in our scheme it is built in and holds for all the processes.

Another difference between the two schemes is the lack of a $V^2 \pi^2$ contact term in theirs. This would yield a small difference in the $\rho\pi$ scattering amplitude. However, the agreement is complete on the π -Compton scattering process, because of the existence of the $B^2 \pi^2$ contact term which compensates the lack of the $V^2 \pi^2$ term, at the expense of losing the full VMD in their scheme. Recently, they have themselves also noticed¹⁵⁾ that VMD breaks down in anomalous processes like $\pi^0 \rightarrow 2\gamma$, $\gamma \rightarrow 3\pi$ in their scheme.

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FIGURE CAPTION

Contribution to the five pseudoscalar process. The blobs denote the anomalous vertices.

$$\frac{1}{5} - \alpha$$

+

$$3\alpha^2$$

+

$$\alpha (1 - 3\alpha)$$