1984 CERN SCHOOL OF PHYSICS

Lofthus, Hardanger, Norway
11–24 June 1984

PROCEEDINGS
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CERN—Service d'information scientifique—RD/676-4000—août 1985
ABSTRACT

The CERN School of Physics is intended to give young experimental physicists an introduction to the theoretical aspects of recent advances in elementary particle physics. These Proceedings contain reports of lecture series on the following topics: proton antiproton physics, experimental tests of gauge theories, QCD, phenomenology of Higgs particles, the electroweak model, unification and supersymmetry. In addition, there is a report of a special lecture on elementary supersymmetry.
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PREFACE

The 1984 CERN School of Physics was held at Lofthus, on the Hardanger Fjord, Norway, from 11 - 24 June. The School was organized jointly by CERN and by the Particle Physics Group of the University of Bergen.

The famous Norwegian composer Edward Grieg used to visit Lofthus for musical inspiration. The Organizers hope that the 77 students who attended the School were also inspired by the beautiful surroundings as well as by the lectures and the discussion sessions! We wish to thank the lecturers and discussion leaders for their hard work in teaching and in stimulating the students.

Our thanks are also due to the Utne family of the Ullensvang Hotel for their cooperative spirit and for their excellent food.

We are indebted to the University of Bergen for assistance with technical facilities and for hosting a lunch for the participants during a visit to Bergen.

Special thanks are due to the Norwegian Research Council (NAVF) for generous financial support.

Finally, I wish to express my personal gratitude to Alf Halsteinslid, Bergen, for doing more than his share of the local organizing and to Ann Caton, CERN, who did an excellent job in preparing and running the School.

Cecilia Jarlskog
Chairman of the
Organizing Committee
PROTON ANTIPROTON PHYSICS

J. D. Dowell

University of Birmingham, Birmingham, B15 2TT, England.

ABSTRACT

Results from the 1981-1983 operation of the collider at 540 GeV obtained in experiments UA1, UA2, UA3, UA4 and UA5 are presented. After a brief discussion of the accelerator performance and the details of the experimental apparatus the results on elastic and total cross sections are given, as well as those for particle production at low transverse momenta, largely as experimental facts. The main emphasis of the lectures concerns the study of jets and the consequent tests of QCD, the production and decay of the W and Z bosons in comparison with the standard electroweak theory and the observation of a variety of new and intriguing phenomena, some of which suggest physics beyond the standard model. The lectures conclude with a brief review of the future prospects. They rely on material presented at this school by C. Jarlakog, R. Petronzio and D. Haldt concerning Gauge Theories, QCD and their experimental tests, and by J. Iliopoulos on Supersymmetry.

CONTENTS

1. Accelerator performance and details of experiments.
2. Elastic scattering and total cross section.
4. Particle production (minimum bias physics).
5. Jet production and QCD.
7. New phenomena.
9. Acknowledgements.

1. ACCELERATOR PERFORMANCE AND DETAILS OF EXPERIMENTS

The CERN proton-antiproton collider\textsuperscript{(1)} first operated in July 1981 with a centre of mass energy $\sqrt{s} = 540$ GeV, almost ten times higher than the highest energy, 63 GeV, previously obtained at the CERN Intersecting Storage Rings, and is the first machine to provide sufficient energy to produce the W and Z particles, the carriers of the weak force\textsuperscript{(2)}. So far there have been three data taking periods, each with improved luminosity which reached $1.6 \times 10^{29}$ cm$^{-2}$s$^{-1}$ in 1983. The integrated luminosities for each of the periods are:

<table>
<thead>
<tr>
<th>Period</th>
<th>$\int L dt$ (nb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 1981</td>
<td>0.023</td>
</tr>
<tr>
<td>Oct. - Dec. 1982</td>
<td>28</td>
</tr>
<tr>
<td>Apr. - Jul. 1983</td>
<td>150</td>
</tr>
</tbody>
</table>
The collider operates with 3 proton and 3 antiproton bunches, each 30 cm long and 1 mm high at the collision points, which are stored for 15 - 20 hours at 270 GeV, the energy limit being determined by the power dissipation.

The time between successive bunch crossings, 7.6 usec, is used to receive data from the detectors and is sufficient to allow a trigger decision to be made whether or not to record the current event on magnetic tape (only a few of the 10^6 events per second can be written). The collider experiments are housed in two underground areas at long straight sections L8S4 and L8S5 and there is also an experiment using a gas jet target.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Purpose</th>
<th>Participating Institutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA1</td>
<td>General purpose detector (W, Z, jets etc.)</td>
<td>Aachen, Annecy (LAPP), Birmingham, CERN, Harvard, Helsinki, Kiel, Queen Mary College London, NIKHEF Amsterdam, Paris (College de France), Riverside, Roma, Rutherford Appleton Laboratory, Saclay (CERN), Vienna, Wisconsin.</td>
</tr>
<tr>
<td>UA2</td>
<td>- ditto -</td>
<td>Bern, CERN, Copenhagen (NEFI), Orsay, Pavía, Saclay (CERN).</td>
</tr>
<tr>
<td>UA3</td>
<td>Monopole search</td>
<td>Annecy, (LAPP), CERN.</td>
</tr>
<tr>
<td>UA4</td>
<td>Elastic scattering and total cross sections</td>
<td>Amsterdam (NIKHEF), CERN, Genova, Napoli, Pisa.</td>
</tr>
<tr>
<td>UA5</td>
<td>Streamer chamber (particle production characteristics)</td>
<td>Bonn, Bruxelles, Cambridge, CERN, Stockholm.</td>
</tr>
<tr>
<td>UA6</td>
<td>Gas jet target</td>
<td>CERN, Lausanne, Michigan, Rockefeller.</td>
</tr>
</tbody>
</table>

UA6 is a fixed target experiment designed to compare pp and \( \bar{p}p \) reactions\(^{(3)} \). The gas jet may be polarised. As the experiment has not yet run it will not be discussed further.

The highest luminosity is achieved with "low \( \theta \)" quadrupoles energized in order to focus the beam strongly to give the smallest possible size. This is not ideal for elastic scattering studies which require a low angular spread and therefore "high \( \theta \)". As the product of angular divergence times beam size is conserved (Liouville's theorem), in each plane, this implies low luminosity. Consequently there is an element of incompatibility between elastic scattering studies and W, Z physics. In addition to UA4, UA1 is equipped to study elastic scattering. Details of the various experiments are summarised in the following paragraphs, also in Figures 1 - 9 and Tables 1 - 3.

Throughout these lectures \( \theta \) is defined as the polar angle of a particle with respect to the beam direction and \( \phi \) the azimuthal angle around the beam. The rapidity of a particle is defined as

\[
y = \frac{1}{2} \ln \frac{E + p_\perp}{E - p_\perp}
\]

where \( E \) is the particle energy and \( p_\perp \) its longitudinal momentum. It can be approximated by the "pseudo-rapidity" \( \eta \) given by

\[
\eta = -\ln \tan \theta/2
\]

Intervals of rapidity or pseudo-rapidity are Lorentz invariant which is convenient in
Figure 1  A general view of the UA1 detector

Figure 2  A side view of the UA1 detector.
\[ \Delta \phi = 0.3 \cdot 10^{-3} \text{ rad} \]
\[ \Delta x = 6.3 \cdot 10^{-3} \text{ cm} \]
from pulse ht.

Figure 3 Arrangement of the large angle calorimeters in UA1 showing the 'gondolas' and 'Cs'.

Figure 4 Layout of the muon drift tubes in UA1.

Figure 5 The elastic scattering detectors in UA1. The Roman pots house the drift chambers shown in the lower half of the figure which contain a typical event.
Figure 6  The UA2 detector  a) a plan showing the side detector in place  b) the central calorimeter and the side detector. The wedge aperture was closed for the 1983 run.
### Table 1

<table>
<thead>
<tr>
<th>Type</th>
<th>Drift chamber with charge division readout of the second coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas mixture</td>
<td>Argon (40%) + ethane (60%)</td>
</tr>
<tr>
<td>Drift field and gap length</td>
<td>1.5 kV/cm, 18 cm</td>
</tr>
<tr>
<td>Drift velocity</td>
<td>5.3 cm/μs</td>
</tr>
<tr>
<td>Drift angle</td>
<td>23° at</td>
</tr>
<tr>
<td>Anode plane arrangement:</td>
<td></td>
</tr>
<tr>
<td>a) Distance between sense wires</td>
<td>10 mm</td>
</tr>
<tr>
<td>b) Wire length</td>
<td>80 cm min., 220 cm max.</td>
</tr>
<tr>
<td>c) Sense wire charac.</td>
<td>35 m Ni-Fe stretched at 80 g</td>
</tr>
<tr>
<td>d) Field wire charac.</td>
<td>100 μm gold-plated Cu-Be stretched at 200 g</td>
</tr>
<tr>
<td>Cathode plane structure:</td>
<td></td>
</tr>
<tr>
<td>a) Distance between wires</td>
<td>5 mm</td>
</tr>
<tr>
<td>b) Wire characteristics</td>
<td>150 μm gold-plated Cu-Be stretched at 200 g</td>
</tr>
<tr>
<td>Total number of wires</td>
<td>22800</td>
</tr>
<tr>
<td>Total number of sense wires</td>
<td>6110</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Calorimeter</th>
<th>Angular coverage</th>
<th>No. rad. lengths</th>
<th>No. abs. lengths</th>
<th>θ8 (°)</th>
<th>θ4 (°)</th>
<th>Sampling step</th>
<th>Segmentation in depth</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.m.: gondolas</td>
<td>26.6/sinθ/1.1/sinθ</td>
<td>5</td>
<td>180</td>
<td>1.2 mm Pb</td>
<td>1.5 mm scint.</td>
<td>3.3/1.6/1.0/1.6 X₀</td>
<td>0.15/οE</td>
<td></td>
</tr>
<tr>
<td>Barrel</td>
<td>25-155</td>
<td></td>
<td></td>
<td>4 mm Pb</td>
<td>50 mm Fe</td>
<td>10 mm scint.</td>
<td>2.5/2.5 Λ</td>
<td>0.8/οE</td>
</tr>
<tr>
<td>End-caps</td>
<td>27/cosθ/1.1/cosθ</td>
<td>11</td>
<td>10</td>
<td>4/7/9/7 X₀</td>
<td>0.12/οE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hadr.: C's</td>
<td></td>
<td>5.0/sinθ</td>
<td>15</td>
<td>6 mm scint.</td>
<td>50 mm Fe</td>
<td>10 mm scint.</td>
<td>3.5/3.3 Λ</td>
<td>0.8/οE</td>
</tr>
<tr>
<td>Calcom</td>
<td></td>
<td></td>
<td></td>
<td>3 mm Pb</td>
<td>4 x 7.5 X₀</td>
<td>0.15/οE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hadr.</td>
<td>0.7-5</td>
<td>30</td>
<td>1.2</td>
<td>45</td>
<td>40 mm Fe</td>
<td>8 mm scint.</td>
<td>6 x 1.7 Λ</td>
<td>0.8/οE</td>
</tr>
<tr>
<td>e.m.</td>
<td>175-179.3</td>
<td></td>
<td>10.2</td>
<td>45</td>
<td>8 mm scint.</td>
<td>0.15/οE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very forward</td>
<td>0.2-0.7</td>
<td>24.5</td>
<td>1.0</td>
<td>90</td>
<td>6 mm scint.</td>
<td>0.15/οE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hadr.</td>
<td>179.3-179.8</td>
<td></td>
<td>5.7</td>
<td>90</td>
<td>40 mm Fe</td>
<td>10 mm scint.</td>
<td>5 x 1.25 Λ</td>
<td>0.8/οE</td>
</tr>
</tbody>
</table>

UAI: angular coverage, segmentation and resolution of calorimeters.
Table 3
Main parameters of the UA2 detector

I. SOLID ANGLE COVERAGE
\[ \Delta \phi = 2, \Delta \theta = 100^\circ, \Delta \phi = 300^\circ \]
Forward–backward regions. Electron calorimetry and magnetic spectroscopy.
\[ \Delta \phi = 1.5, \Delta \theta = 35^\circ, \Delta \phi = 82\% \text{ of } 360^\circ \]
Wedge region. electron calorimetry and magnetic spectroscopy.
\[ \Delta \phi = 1.3, \Delta \theta = 68^\circ, \Delta \phi = 28^\circ \]

II. CENTRAL CALORIMETER
200 cells, each covering \[ \Delta \theta \times \Delta \phi = 10^\circ \times 15^\circ \].
Longitudinal segmentation
17 r.l. (lead–scintillator) + 2 x 2 abs. l. (iron–scintillator)
Electron calorimetry 26 x 3.5 mm lead plates
27 x 4 mm NE 104 scintillator plates
Hadron calorimetry (18 + 22) x 15 mm iron plates
(18 + 22) x 5 mm scintillator plates
(PMMA, 10% naphthalene, 1% PBD, 0.01% POPOP)
Light guides: 2 mm lucite, 80 mg/1 BBQ
Photobus: 7 per cell, XP2012 for electron calorimetry and XPD7008 for hadron calorimetry

III. FORWARD DETECTORS
24 identical sectors each covering \[ \Delta \theta \times \Delta \phi = 17.5^\circ \times 25^\circ \]
Field integral 0.38 Tm.
9 drift chambers per sector
- wire orientation with respect to field: -70°, 0°, +70°
- drift cell width 15 cm
- field shaping wires every 5 mm
- total number of signal wires 2304

Table 3 (continued)
preshower counter
- preconverter 1.4 r.l. lead + iron
- 4 tube planes (brass) 20 mm O.D., 0.3 mm thick
- tube orientation with respect to field: 0°, 0°, 77°, 77°
- anode 30 micron gold plated tungsten
forward calorimeters
- 10 cells per sector, each covering \[ \Delta \theta \times \Delta \phi = 4^\circ \times 15^\circ \]
- cell transverse sizes 27 x 33 to 27 x 60 cm²
- longitudinal segmentation: 33 x (4 mm lead + 4 mm Altustipe 10105) + 8 x (4 mm lead + 4 mm Altustipe 10105) = 24 + 6 r.l.
- light guides and phototubes as in central calorimeters

IV. VERTEX DETECTOR
- five proportional chambers with cathode strip read-out one of which is located behind a 1.5 r.l. tungsten converter.
- number of strips 480, 480, 528, 672 and 480.
- number of wires 288, 384, 576, 864 and 576.
- chamber radii 100, 124, 236, 315 and 355 mm.
- chamber lengths 104, 110, 150, 178 and 80 cm.
- wire pitch 2.2, 2.0, 2.6, 2.3 and 3.9 mm.
- strip angle \[ \tan \alpha = \pm 0.9, 1.3, 1.0, 1.0, 1.0 \]
- half gap 4 mm.
- strip pitch = 4 mm.
- two drift chambers of 24 azimuthal cells each, 6 sense wires/cell (charge division, multihit capability), sense wires lengths 1520 and 1785 mm.
- 24 scintillator plates.
comparisons with cosmic ray data where the energy of the primary parent particle is not known. Another quantity commonly used is transverse energy defined as

\[ E_T = E_1 \sin \theta_1 \]

where \( E_1 \) is the energy of a particular particle and \( \theta_1 \) its polar angle. It is relevant where calorimeters are used which measure energy. In the case of jets, local sums of calorimeter energy may be used

\[ E_T = \sum_i E_i \sin \theta_i \]

where the sum is over calorimeter elements contributing to an energy cluster. Furthermore, one can define the total transverse energy of an event as the sum of all transverse energy depositions.

The UA1 detector\(^4\) (figures 1 - 5) was designed as a general purpose instrument with an almost 4\(\pi\) solid angle coverage extending down to polar angles of a few mrad. Its central part is a 6 m long, 2.4 m diameter drift chamber system with 18 cm drift spaces. The image readout gives space points at centimetre intervals along the tracks and records ionization information. The central track detector is surrounded along its length by 48 semicylindrical electromagnetic calorimeters and at each end by 32 similar radial sectors. All of the above are inside the coil of a dipole magnet (7 m x 3.5 m x 3.5 m) which produces a highly uniform field of 0.7T. The laminated return yoke of the magnet, equipped with scintillation counters, also serves as a hadron calorimeter. The outer shell of the detector is a large-area muon detector consisting of 8 layers of drift tubes (2 separated chambers each with 4 layers). Calorimeterized compensator magnets, small angle calorimeters and further track detectors extend the detection to angles less than 50°. The measured rms accuracy of the central drift chambers is 290 \(\mu\)m giving a momentum resolution \( dp/P \approx 0.005 \) p for a 1 m track perpendicular to the field. The electromagnetic and hadron calorimeters have energy resolutions of \( dE/E \approx 0.15/\sqrt{E} \) and \( \approx 0.8/\sqrt{E} \) respectively. An independent means of measuring the luminosity is provided by small drift chambers at \( \pm 22 \) m to measure elastic scattering by detecting collinear particles.

The UA2 detector\(^5\) (figure 6), more specifically matched to the W\(^\pm\), Z\(^0\) search, is composed of finely segmented calorimeter cells, 240 in the central region and 240 in the forward and backward cones. They are arranged in tower structures pointing at the intersection region. An inner detector, using drift and proportional chambers, determines the vertex position. The forward and backward detectors include magnetic spectrometers (toroidal magnets and drift chambers) to measure the asymmetry of the electrons from W\(^\pm\) decays. For the 1981 and 1982 runs a 60° azimuthal wedge of the central calorimeter was replaced by a magnetic spectrometer covering \( \pm 0.7 \) units of rapidity which used drift chambers, time of flight counters and a lead-glass array, outside the main apparatus, to identify charged particles and neutral pions.

Experiment UA3\(^6\) shares the same intersection region as UA1 and is a search for magnetic monopoles by looking for their expected ionization in 125 \(\mu\)m thick kapton foils. These are placed both inside the vacuum chamber near the intersection point and around the
Figure 7  The UA4 elastic scattering and total cross section experiment a) general layout b) Roman pot c) details of drift chambers d) inelastic detector showing the UA2 calorimeter.
Figure 8  The UA3 magnetic monopole experiment.

Figure 9  The UA5 streamer chamber experiment.
outside of the UA1 central track detector. The foils have been scanned but no evidence for monopoles has yet been observed\(^{(6)}\). The specific ionization of a monopole is related to that of the specific ionization of a minimum ionizing particle by

\[
\frac{\langle dE/dx \rangle_{\text{mono}}}{\langle dE/dx \rangle_{\text{min,ion}}} = \beta^2 (g^2/e^2)
\]

where \(\beta\) is the monopole velocity and \(g\) the magnetic charge. For Dirac monopoles \((g^2/e^2) \approx 5 \times 10^3\). The experimental threshold is \(\sim 2 \times 10^3\) minimum ionization.

Experiment UA4\(^{(7)}\) coexists with UA2 (figure 7). It measures elastic and inelastic scattering and (using the optical theorem) the \(p^2\) total cross section. Small drift and proportional chambers can be placed very near to the beams inside "Roman pots" situated 20 m and 40 m on either side of the intersection point. Smaller angles of scattering can be detected than in UA1.

The main part of the UA5 detector\(^{(8)}\) consists of two 6 m long streamer chambers above and below the intersection region incorporating lead-glass plates to allow photon detection. There is no magnetic field and the experiment is an alternative to UA2. It is designed to study simple characteristics of particle production. (See figure 8). A 90° hadron calorimeter has been added for triggering on high \(p_T\) processes but not yet used.

As data acquisition rates are limited some form of triggering is needed to select events of interest. Up to now, three types of trigger have been used to study inelastic events: a) simple hodoscopes at each end of the detector (UA1, UA2, UA5) giving a 'minimum bias' trigger; b) transverse energy triggers (UA1, UA2) where \(E_T = \sum E_i \sin \theta_i\) is summed over electromagnetic and hadronic calorimeter cells and a threshold is imposed to select high \(p_T\) processes (for example a high \(p_T\) electron from \(W\) decay would give a localised electromagnetic deposition); c) muon triggers (UA1).

It is worth commenting on the relative merits of the UA1 and UA2 detectors. The strong features of UA1 are the magnetic field and central track detector which make it a good general purpose instrument. Furthermore its calorimeters extend to very forward angles \((0.2^\circ)\) giving excellent sensitivity to missing transverse energy (neutrinos). On the other hand, the electromagnetic calorimeters are not segmented in \(\phi\) in the barrel region (gondolas) or in \(\theta\) in the end-cap regions (bouchons) which can cause problems if more than one particle strikes a cell. UA2 has better calorimeter granularity and position detection for electromagnetic showers which compensate to some extent for the absence of a magnetic field in recognising electrons (see later) but has less complete angular coverage which is bad for missing \(E_T\). There is also only one depth segmentation in the UA2 electromagnetic calorimeters compared to four in UA1.

Calibration of the electromagnetic calorimeters is essential for \(W\) and \(Z\) mass determinations. UA1 uses an intense \((7 \text{ Ci})\) Co\(^{60}\) source to map the calorimeters which is normalised to test beam measurements on a single element. A pulsed laser is used to monitor the photomultiplier gains. UA2 can put all cells in a beam and uses a Co\(^{60}\) source and photodiodes for local monitoring.
Figure 10. The elastic scattering slope parameter $b$ at $\sqrt{s} = 540$ GeV and $\sqrt{s} = 53$ GeV (pp collisions), when $\text{d} \sigma / \text{d}t$ is parametrised as $e^{bt}$.

Figure 11. The $pp$ and $pp$ total cross section. The curves are fits up to ISR energies (12, 13, 14). The solid curve (12) has an $(\ln s)^2$ dependence.

Figure 12. $\rho$, the ratio of the real part to the imaginary part of the forward amplitude for pp and pp elastic scattering. The curves are dispersion relation fits by Amaldi et al (12) (solid curves) and Block et al (13).
2. ELASTIC SCATTERING AND TOTAL CROSS SECTION

W and Z production account for only about $10^{-7}$ of the total cross section. Even QCD processes producing high $p_T$ ($>20$ GeV/c) jets are only a fraction of a percent. The majority of what happens therefore comes under the heading of total cross section, particle production etc., discussed in the next three sections. If one is designing an experiment, say for a 20 TeV collider, one needs a way of extrapolating these things to higher energies in order to judge what detectors will be required to handle the events. Furthermore, one hopes that what is called "soft" physics may ultimately be described by QCD which would therefore have to explain the details. Finally, there is always the prospect of some anomalous behaviour pointing to new phenomena even at low $p_T$. However, in the absence of any precise theory of low $p_T$ process, I am presenting only the data and refer the reader to published work for theoretical attempts to explain the results.

$\bar{p}p$ elastic scattering has been measured by UA1\(^{(9)}\) and UA4\(^{(10)}\) in the 1981 and 1982 running periods. Good agreement for the slope $b$ of the differential cross section, parametrised as $\frac{d\sigma}{dt} (t) = \frac{d\sigma}{dt} (0) e^{bt}$, were obtained for $|t|$ values up to 0.8 (GeV/c)^2 showing a change of slope near $|t| = 0.15$ (GeV/c)^2 (Figure 10) similar to that observed at the ISR ($\sqrt{s} = 63$ GeV). Improved measurements by UA4 in 1983\(^{(11)}\), however, give a smaller value of the low $t$ slope (15.3 ± 0.3 (GeV/c)^2 compared to 17.3 ± 0.6 (GeV/c)^2). Using this smaller value gives a slope change $\Delta b$ of 1.6 ± 0.4 (GeV/c)^2 compared to 2.7 ± 0.4 (GeV/c)^2 at 63 GeV, where $\Delta b$ is the difference in slope between $-t = 0.1$ (GeV/c)^2 and $-t = 0.3$ (GeV/c)^2. This necessarily affects the value obtained for the total cross section which requires an extrapolation to $t = 0$.

The total cross section $\sigma_t$ is obtained using the optical theorem

$$(\text{Im } f(0))^2 = \frac{\sigma_t^2}{16\pi(\hbar c)^2}$$

where $f(0)$ is the forward elastic scattering amplitude. The differential elastic event rate can be written

$$\frac{dN_{el}}{dt} = L \frac{\sigma_t^2(1+\rho^2)}{16\pi(\hbar c)^2} e^{bt}$$

where $\rho$ is ratio of the real to imaginary parts of $f(0)$ and $L$ the luminosity. Furthermore

$$N_{el} + N_{inel} = L \sigma_t$$

where $N_{el}$ and $N_{inel}$ are the total elastic and inelastic event rates.

Combining the two expressions and extrapolating $\frac{dN_{el}}{dt}$ to $t = 0$ allows $\sigma_t$ to be calculated (the extrapolation can allow an arbitrary form for the shape of the elastic differential cross section at small $t$ which we have written as $e^{bt}$ for simplicity). This is the method
Figure 13 The forward slope parameter $b$ for $\bar{p}p$ and pp elastic scattering. The curves are fits up to ISR energies (13, 15, 16).

Figure 14 a) Differential cross section for $\bar{p}p$ elastic scattering measured by UA4. The curve is a fit with two interfering exponential amplitudes. b) A comparison between collider data and ISR data at $\sqrt{s} = 53$ GeV.
used by UA4 who measure \( \frac{dN_{el}}{dt} = N_{el}, N_{inel} \) yielding \((1 + p^2)^{1/2} q_t\). UA1 measure only
\( \frac{dN_{el}}{dt} = N_{el} \) and require a separate measurement of the luminosity (provided by wire scanners
in LSS2 diametrically opposite to LSS5, with an error of \( \pm 8\% \)) which leads to a
determination of \((1 + p^2)^{1/2} q_t\). The 1982 measurements of the two experiments with low
statistics were in excellent agreement (figure 11) giving an average value of 67 \( \pm 5 \) mb.
The 1983 UA4 result is 61.9 \( \pm 1.5 \) mb at 546 GeV, somewhat lower but statistically
compatible (figure 11). In all cases a value of 0.15 has been assumed for \( p \) as suggested by
a dispersion relation fit to lower energy measurements (figure 12) by Amaldi et al\(^{(12)}\).
Both the old and new results are compatible with a \((1ns)^2\) dependence of \( q_t \) and agree well
with a dispersion relation extrapolation\(^{(12)}\). The energy variation of the forward elastic
slope (and as a consequence of the total elastic cross section) is more controversial, the
old results favouring a \((1ns)^2\) dependence and the new UA4 result an \(1ns\) dependence (figure
13). This is important in assessing whether an asymptotic situation has been reached\(^{(17)}\).
The Froissart bound\(^{(18)}\) forbids the total cross section to rise faster than \((1ns)^2\). If
the bound is qualitatively saturated then asymptotically \( \frac{dN_{el}}{d\omega} \) should tend to a constant
which also means \( \frac{b}{q_t} \) \( \rightarrow \) constant. The new UA4 data indicate a significant change
in \( \frac{dN_{el}}{d\omega} \) from 0.185 \( \pm 0.005 \) at the ISR to 0.215 \( \pm 0.005 \) at the collider, a change of
\((16 \pm 4)\%\).

The tentative conclusion is that asymptopia has not yet been reached. In geometrical
terms the proton is becoming bigger and blacker (more opaque). A totally black
proton would have \( \frac{dN_{el}}{d\omega} = 1/2 \).

UA4 have also made elastic measurements at larger \(|t|\) which have revealed a shoulder
at \(|t| = 0.8 \text{ (GeV/c)}^2\) which is presumably related to the dip at 1.2 \(\text{ (GeV/c)}^2\) observed at
53 GeV (figure 14). An empirical fit of the expression
\[
\frac{d\sigma}{dt} = b_1 (t-t_0)^{-1/2} + b_2 (t-t_0)^2 + \delta^2
\]
gives \( b_1 = 10.9 \pm 1.1 \text{ (GeV/c)}^{-2} \)
\( b_2 = 4.6 \pm 1.1 \text{ (GeV/c)}^{-2} \)
\( t_0 = 0.81 \pm 0.01 \text{ (GeV/c)} \)
\( \delta = -0.39 \pm 0.16 \text{ rad.} \)
\( \delta = \pi \) would give a dip at \( t = t_0 \) (destructive interference).

Several authors\(^{(17,19)}\) have described models to explain the elastic scattering but
none is particularly successful in explaining all the features. They will not be
discussed here.
Figure 15. The invariant cross section for $pp + pX$ at the collider plotted versus $M^2/s$, where $M$ is the mass of the diffractive system $X$, for two different $t$-values. ISR data are shown and exhibit scaling for larger values of $M^2/s$.

Figure 16. Data on the non-invariant cross section $d^2\sigma/dtdM^2$ at various energies and fixed $t$, in the interval $0.01 < M^2/s < 0.04$, plotted versus $M^2$. 
3. **SINGLE DIFFRACTION DISSOCIATION** \((pp \rightarrow \bar{p}X)\)

UA4 have studied single diffraction dissociation\(^{(20)}\) in which one of the colliding particles fragments but the other remains intact. The technique is to detect the scattered \(\bar{p}\) (or \(p\)) and measure its momentum \(p\) and angle \(\theta\). This allows the four momentum transfer and the mass of the recoiling system to be calculated.

\[
-t = m_p^2(1-x)^2/x + 2xpQ(1-\cos\theta)
\]

\[M^2 = (1-x)s\]

where \(P_0\) is the incoming momentum and \(x = p/P_0\). \(P\) is measured from the bending in an SPS quadrupole to a precision of about 0.6%. Consequently \(\Delta(\frac{M^2}{s}) = x\ \frac{\Delta p}{p}\) is \(\approx 0.006\).

The main results are (figures 15 and 16)

1) Scaling of the invariant cross section \(\frac{s d^2\sigma}{dtdM^2}\) at fixed \(t\) with the results obtained at the ISR\(^{(21)}\) to about 20%, apart from a kinematic effect at low \(M^2/s\) which is due to the fact that the minimum mass that can be produced is independent of \(s\),

2) A \(1/M^2\) behaviour for \(\frac{d^2\sigma}{dt dM^2}\) at fixed \(t\).

Both of these results are expected in a Regge picture with Pomeron exchange.

\[\frac{d^2\sigma}{dt dM^2} = f(t) \left(\frac{s}{M^2}\right)^2 q_p(t-1) \frac{1}{M^2} q_p\ (N^2, t)\]

where \(f(t)\) is a function of \(t\) and \(q_p\) is the Pomeron-proton cross section which should be constant for large \(M^2\). \(q_p(t)\) is the Pomeron Regge trajectory.

Summarising the main results on total cross sections, elastic and single diffractive scattering at 540 GeV we have:–

1) \(q_t\) is compatible with an \((\ln s)^2\) increase with \(s\)

ii) the elastic slope parameter \(b\) (where \(\frac{d\sigma}{dt} = e^{bt}\)) increases as \((\ln s)^2\)

iii) the above results mean that \(\sigma_{el}/\sigma_t\) is varying with energy and is \((16 \pm 4)\%\) larger than at 53 GeV.

iv) therefore we are not yet in an asymptotic regime

v) single diffractive production of large masses (\(\sim 100\) GeV/c\(^2\)) is observed which scales with ISR results to \(\sim 20\%\).

4. **PARTICLE PRODUCTION (MINIMUM BIAS PHYSICS)**

4.1 **Pseudo-rapidity density distribution**

The study of particle production (multiplicity, \(p_T\) spectrum etc.) would ideally be unbiased, including all types of events. In practice it is necessary to apply a trigger to signify that an event has occurred. It is easy to trigger from inelastic events in which both colliding particles fragment because they will generally throw particles into
Figure 17  The pseudo-rapidity density for non-single diffractive inelastic events observed by UA1 and UA5. The dashed curve is the expectation based on cylindrical phase space for $<p_T> = 0.35$ GeV/c. The solid curve is for $<p_T> = 0.5$ GeV/c. The shape of the ISR data is shown for comparison.

Figure 18  The central rapidity plateau height showing the lns dependence. Note the difference between the plateau heights for inclusive and non single diffractive events.
the triggering hodoscopes. However, single diffraactive events are more difficult to
detect because one particle stays in the beam pipe. The term "minimum bias" therefore
usually refers to non single diffraactive events. UA1 and UA5 studied such events in
1981 (23, 24) and, in 1982 (25), UA5 added a single diffraction trigger. The UA1 data were
taken with field off to make acceptance calculations easier and UA5 in any case has no
field. Thus, the data on charged particle multiplicities have been obtained as a function
of pseudo-rapidity $n = \ln \tan \theta/2$.

In 1969 Feynman (26) conjectured that, if the transverse momentum was limited, there
should be a uniform distribution of particles as a function of pseudo-rapidity giving rise
to the rapidity plateau, and that the pseudo-rapidity density $\frac{dN}{dn} \frac{d^2}{d\eta d\phi}$ should be
independent of $s$, known as Feynman scaling. As the range of rapidity available increases
as in s, the average multiplicity would then increase as in s. It was already known from
the ISR that the average central rapidity density $\frac{dN}{dn} \frac{d^2}{d\eta d\phi}$ at $n = 0$ increases by 40% from
$\sqrt{s} = 23$ GeV to $\sqrt{s} = 63$ GeV violating Feynman scaling. A continued rise, as in s, is
observed in going to the collider energy, $\sqrt{s} = 540$ GeV (figure 18). As a consequence, the
average inclusive charged particle multiplicity increase to $\langle n_{ch} \rangle = 28.9 \pm 0.4$
(figure 19) requires an $(\ln s)^2$ term to account for its energy dependence. However, the
width of the pseudo rapidity distribution has grown by only 2 units from ISR to collider
energy compared to the 4.6 units available kinematically. Furthermore, a cylindrical
phase space distribution with $\langle p_t \rangle = 0.35$ GeV/c (see figure 17) that describes the ISR
data (dashed curve) fails to account for the collider results. One explanation could be
that the mean particle transverse momentum has increased, so limiting the rapidity.
Better agreement is obtained (solid curve) with $\langle p_t \rangle = 0.50$ GeV/c. However, UA1
measurements (27) (see later) give $\langle p_t \rangle = 0.42$ GeV/c so this may not be the whole reason.

4.2 Multiplicity distributions

If particles were produced randomly and independently their multiplicity might be
expected to obey a Poisson distribution $P_n = e^{-\langle n \rangle} \frac{\langle n \rangle^m}{m!}$ which would become relatively
narrower (as $\sim 1/\sqrt{n}$) as the energy and hence multiplicity increased. (The fact that some
particles result from the decay of resonances already modifies this expectation. Any
correlation between particles (29) will result in a broader distribution). Koba, Nielsen
and Olesen (28) showed, starting from Feynman scaling, that on the contrary the shape of
the distribution should tend to become constant as $s \rightarrow \infty$ (KNO scaling). Thus if
$\langle n \rangle$ is plotted against $n/\langle n \rangle$, where $P_n$ is the probability of observing a
multiplicity n, the distributions at different energies should coincide at sufficiently
high energies. Early collider results (1981) from UA1 (23) and UA5 (24), when compared with
ISR data were consistent with KNO scaling in spite of the violation of Feynman scaling.
However, the UA5 data taken in 1982 (25) show a deviation from KNO scaling particularly at
high multiplicities (figure 20). Nevertheless scaling is observed for a restricted range of $|\eta|$ both by UA1 (23) and UA5 (25) (figure 21). A natural explanation for KNO scaling
would be that a limited number of subprocesses is responsible for particle production, the
number varying little with energy. The collider results hence have prompted renewed
theoretical interest in the subject (30) but it will not be discussed further here.
Figure 19  The average charged particle multiplicity for fully inclusive and non-single diffractive inelastic events versus energy. The curve (parameters given) is a fit to the non-diffractive events.

Figure 20  The charged multiplicity distributions at collider and lower energies plotted in KNO variables (see text). A deviation from KNO scaling is observed by UA5.
Figure 21. The charged particle multiplicity distributions for a restricted range of $|\eta| \leq 1.3$. In this case KNO scaling is observed as seen also by UA1 (23) (points not shown).

Figure 22. The invariant differential cross section as a function of $p_T$ for different regions of rapidity density showing the multiplicity dependence.

Figure 23. The growth of average $p_T$ with rapidity density.
4.3 Single particle $p_t$-spectrum

The transverse momentum spectrum for unidentified charged hadrons observed by UA1\(^{(27)}\) is given in figure 22 for three different bands of multiplicity averaged over the interval $|y| < 2.5$. The three spectra have been normalised to the full inclusive cross section at $p_t = 0$ [n.b. the invariant cross section $E \frac{d^3 \sigma}{dp_t^2 dy} = \frac{d^2 \sigma}{dp_t dy}$ since $\frac{dp_t}{dy} = E$]. Even at low $p_t$ values the spectrum becomes flatter with increasing multiplicity, a result first noticed in cosmic ray experiments\(^{(31)}\). The average transverse momentum $\langle p_t \rangle$ increases with increasing multiplicity, an effect that begins to be apparent at the highest ISR energy $\sqrt{s} = 63$ GeV (figure 23), and becomes constant at high multiplicities. Averaged over all multiplicities $\langle p_t \rangle = 0.42$ GeV/c. According to Van Hove\(^{(32)}\) the observed behaviour could be an indication of a phase transition to a quark-gluon plasma. Indeed, an accompanying effect would be the observation of local density fluctuations in the rapidity plateau for which there is evidence from UA5\(^{(33)}\). Explanations of the $p_t$-variation exist also within conventional fragmentation models\(^{(34)}\) but the effect in this case would be confined to large $|y|$. Further study would clearly be interesting but tends to receive a lower priority than $W$, $Z$, top, etc.

4.4 Strange and neutral particle production

UA2 have studied $K$, $p$ and $\pi$ productions using their wedge spectrometer\(^{(35)}\). UA5 have measured $\gamma$-ray production from their conversion in the beam vacuum pipe and in lead-glass plates: also $K^0$ and $\Lambda^0$ and $\Xi$ decays\(^{(36)}\). Kaons have a higher average $p_t$ than pions. Roughly summarised there are about 12% $K_s$, 5% baryons or antibaryons and 38% neutral particles per event.

4.5 Summary on particle production

In summary the following features have been observed, some of which are intriguing:

i) The width of the rapidity plateau ($\frac{dn}{d\eta} \approx \eta$) grows from the ISR to the collider but less than would be expected for constant $\langle p_t \rangle$.

ii) However, $\langle p_t \rangle$ rises from 0.35 to 0.42 GeV/c over the same energy range.

iii) The height of the plateau ($\frac{dn}{d\eta} \approx 0$) rises as $n_s$ (continued violation of Feynman scaling).

iv) $\langle n_{ch} \rangle$ as a consequence requires an $(1ns)^2$ term.

v) There is a deviation from KNO scaling at large multiplicities for $|\eta| < 5$, but scaling is observed for $|\eta| < 1.5$.

vi) $\langle p_t \rangle$ rises with multiplicity - a new feature that begins to set in at the ISR.

vii) A typical collision contains about 43 particles of which 28.9 $\pm$ 0.4 are charged (62%) with 12% $K_s$ and 5% baryons/antibaryons.

viii) So far there has been no indication of bizarre processes such as the Centauro events observed in cosmic ray experiments\(^{(37,38)}\).

5. JET PRODUCTION AND QCD

5.1 Introduction

One of the striking features of collider results is the cleanliness of jets (see
Figure 24. A typical two-jet event in the UA1 experiment showing the central detector tracks.

Figure 25. "Lego" plots of two-jet events in the UA1 experiment. The axes are pseudo rapidity, $\eta$, and azimuth, $\phi$. The heights of the bins are proportional to transverse energy.
figures 24 and 25) resulting from parton-parton scattering and the excellent agreement with expectations from QCD. Unlike the situation at $e^+e^-$ machine where the basic jets are from quarks and antiquarks

$$e^+e^- \rightarrow q\bar{q}$$

the initial partons at the $pp$ collider may be quarks, antiquarks or gluons which may collide in any combination. Furthermore their relative contributions depend on the proton structure functions and, of course, there are spectator partons. As the gluon structure function is softer (concentrated at low values of Bjorken $x$) gluons tend to dominate scattering at low $x_T$, where $x_T = p_T/p_T^{\text{max}} = 2p_T/s$ is the fractional transverse momentum of the produced jet.

For larger values of $x_T(\gg 0.2)$, i.e. $p_T \gg 50$ GeV/c, valence quarks and antiquarks become increasingly important as only they can carry enough momentum to provide the required $p_T$. Both UA1 (39-42) and UA2(43-46) have obtained results on cross sections, angular distributions and fragmentation of jets, as well as three-jet events consistent with gluon radiation.

5.2 Data taking and trigger

Both experiments used a calorimeter trigger:

UA1

1. $\sum E_T(|n|<1.5) > 20, 30, 40, 50$ GeV
2. 'jet' trigger $E_T(8C + 2C) > 15$ GeV
   $E_T(\text{end cap quadrant}) > 15$ GeV

UA2

$\sum E_T(|n|<1.0) > 25, 40$ GeV

Cuts were applied to the analyzed data to remove beam-gas and halo events yielding, after cuts:

<table>
<thead>
<tr>
<th></th>
<th>UA1</th>
<th>UA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background</td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>Event losses</td>
<td>2%</td>
<td>5%</td>
</tr>
</tbody>
</table>

The resulting data samples are:

<table>
<thead>
<tr>
<th></th>
<th>UA1</th>
<th></th>
<th>UA2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1982</td>
<td>14 nb$^{-1}$</td>
<td>1983</td>
<td>118 nb$^{-1}$</td>
<td>1982</td>
</tr>
<tr>
<td>1983</td>
<td>112 nb$^{-1}$</td>
<td></td>
<td></td>
<td>1983</td>
</tr>
</tbody>
</table>

5.3 Jet Algorithms

Somewhat different algorithms are used in the two experiments to select jets. In UA1 the following procedure is used:
Figure 26 The UA1 jet algorithm selects calorimeter cells according to their proximity in $\eta, \phi$ space. A cut is made at $r = 1$, where $r = \sqrt{\Delta \eta^2 + \Delta \phi^2}$.

Figure 27 a) The fractional transverse energy carried by the leading jet and the leading two jets in UA2. b) The ratio of the transverse energies of the second jet to the first and the third to the second.
i) Construct an $E_T$ 'vector' for each of the 548 electromagnetic and hadronic cells with $|\eta| < 3$.

ii) Order cells with $E_T > 2.5$ GeV.

iii) Using these as initiators, associate cells with $\sqrt{\Delta \eta^2 + \Delta \phi^2} < \Delta_0 = 1$ to form clusters (see figure 26).

iv) Add the remaining cells to the nearest cluster if $p_T$ relative to the jet axis is $< 1.0$ GeV/c and $\Delta \theta_{\text{relative}} < 45^\circ$.

Note that, because of the background from soft processes, the jets should be uniform in $\eta - \phi$ space.

The UA2 algorithm has two stages:-

i) Group cells with a common side for cells with $E_T > 0.4$ GeV.

ii) Split the clusters if there is a valley deeper than 5 GeV between two local maxima.

UA2 then order the jets so found in decreasing $E_T$: $E_T^1 > E_T^2 > E_T^3$ etc.

They define $h_1 = \frac{1}{E_T} \sum E_T$

$h_2 = \frac{(E_T^1 + E_T^2)}{E_T} \sum E_T$

$r_{21} = \frac{E_T^2}{E_T^1}$

$r_{32} = \frac{E_T^3}{E_T^2}$ etc.

For a pure 2-jet event $h_1 = 0.5$ and $h_2 = 1$. Also $r_{21} = 1$ and $r_{32} = 0$.

Figure 27 shows the results from the 1983 data for these quantities which clearly demonstrate the 2-jet dominance for $\frac{1}{E_T} > 100$ GeV where the jet algorithm is correctly finding the jets.

Internal properties of jets

Four jet characteristics have been studied by the two experiments:-

i) Transverse energy flow

ii) Fragmentation function

iii) $p_T$ of charged particles with respect to the jet axis

iv) Charged particle multiplicity.

Figure 28 shows the transverse energy density in jets with $E_T > 35$ GeV for the UA1 experiment.

The curves are the ISAJET Monte Carlo programme(47) and cylindrical phase space(48).

The former is a closer approximation to the data. Note that the bin size of the data does not limit the resolution.

Figure 29 shows the charged particle density in UA1 versus the angle relative to the jet axis for various cuts on the $p_T$ of the particles with respect to the beam direction. The dashed curve is a Monte Carlo calculation with no jets. For $p_T$ (beam) $> 2.0$ GeV/c essentially all jet fragments are contained within a $35^\circ$ cone around the jet axis. Consequently this has been used as a somewhat arbitrary cut to determine the fragmentation function. Obviously the selection (angle, $p_T$) and background processes (from the remaining partons) affect the choice of particles belonging to a jet. This is clearly a
Figure 28 The transverse energy flow in jet events (UA1) jet events (UA1) as a function of $\Delta \eta$ with respect to the jet axis compared to Monte Carlo calculations.

Figure 29 The charged particle density versus the particle angle relative to the jet axis in UA1 for various cuts on the $p_T$ of the particle with respect to the beam direction. The dashed curve is a Monte Carlo calculation with no jets.
Figure 30  The charged particle fragmentation function for UA1 jets with $E_\text{T} > 30$ GeV compared to equivalent data from the TASSO $e^+e^-$ experiment at $\sqrt{s} = 34$ GeV.

Figure 31  Comparison between the UA1 fragmentation function and a QCD Monte Carlo calculation (B. Webber (39)) for quark and gluon jets.
Figure 32. The transverse momentum distribution of charged particles with respect to the jet axis for $z > 0.1$ and three bands of jet $E_T$ (UA1).

Figure 33. A comparison between the transverse momentum distribution of charged particles with respect to the jet axis and a QCD Monte Carlo calculation(58).
Figure 34. The charged particle density in UA2 jets as a function of $\Delta \phi$ with respect to the jet axis. The background level ($\Delta \phi = \pi/2$) is approximately twice the level for minimum bias events as observed also by UA1.

Figure 35. A lower bound on the mean charged multiplicity in TASSO and UA2 jets as a function of $s$ or $m_{jj}$ (i.e. after subtracting a constant background of $2 \times$ minimum bias).

Figure 36. A comparison between the mean charged multiplicity in UA2 and TASSO jets and a QCD Monte Carlo calculation with two different background subtractions.
more difficult problem than with $\mu^+\mu^-$ jets where there are no spectator particles. The fragmentation function is defined as

$$D(z) = \frac{dN}{dz}/N(\text{jet})$$

where $z = p^\cdot n(\text{jet})/E_\text{jet}$, i.e. the fractional component of momentum along the jet axis. $N$ is the number of charged tracks and $N(\text{jet})$ the number of jets in the sample.

$$\therefore N/N(\text{jet}) = \int_0^1 D(z)dz$$

As an example of the magnitude of the bias that may be introduced by the selection procedures it is estimated that the $35^\circ$ cut in UA1 loses 35% of the particles for $0.02 < z < 0.03$ and 5% for $z < 0.07$, for jets with $E_T > 30$ GeV.

Figure 30 shows the fragmentation function obtained compared to the $\mu^+\mu^-$ result from TASSO with total energy $W = 34$ GeV (49). No correction has been applied for the loss of particles at low $z$ or for the uncertainty on the jet energy ($\pm 15\%$) which smears $z$. The shapes of the distributions are remarkably similar. At first sight this is somewhat surprising as the $p\bar{p}$ data are expected to be dominated by gluon jets at these $E_T$ values whereas the $\mu^+\mu^-$ data are quark jets. Gluon jets naively should have a softer fragmentation function than quark jets since a gluon must first turn into a $q\bar{q}$ pair as part of the fragmentation process. However, a comparison with a QCD Monte Carlo calculation (50) in figure 31 indicates that the UA1 results (apart from a few high-$z$ points) are somewhere between the quark and gluon expectations, a not unreasonable result at the present stage of the data, showing that the higher energy of the collider data also affects the shape of the fragmentation function.

Figure 32 shows the $p_T$ of charged particles with respect to the jet axis for $z > 0.1$ and three bands of jet $E_T$. No significant differences between the spectra are noticeable. A comparison with the QCD Monte Carlo (50) in figure 33 (same cuts) shows perhaps a slight preference for the gluon prediction. The average value $p_T$ is 0.6 GeV/c.

Figure 34 shows the charged particle density for two bands of jet-jet invariant mass, obtained in the UA2 experiment (46), plotted against $\Delta \phi$, the difference in $\phi$ between the particle and the jet-axis. There is a constant background level which is approximately twice the density in minimum bias events. The behaviour is seen also by UA1 (40). Presumably this has to be interpreted as soft gluon emission from the jets. UA2 have obtained a lower bound on the mean charged multiplicity of their jets as a function of $N_{jj}$ (after subtracting a constant background of 2 x minimum bias) which is compared in figure 35 with results from TASSO. Figure 36 is a comparison with the QCD Monte Carlo (50) for two different background assumptions, the UA2 results appearing to favour the expectations for gluon jets.

In summary, the data are dominated by two-jet events which at the present level of study are certainly compatible with being mainly gluon jets as expected.

5.4 Multi-jet events

Events with three or more jets can arise from gluon bremsstrahlung from initial or final state partons.
Figure 37 The fraction of 1, 2 and 3-jets with $E_T > 15$ GeV, $|\eta| < 2.5$ observed in UA1. The 'trigger' jet has $E_T > 30$ GeV, $|\eta| < 1.5$ and is included in the count.

Figure 38 A plot of $p_{\text{out}}$ for multijet events in UA1. $p_{\text{out}}$ is the amount of momentum perpendicular to a plane containing the beam and the trigger jet. The data agree with a 3-jet QCD Monte Carlo calculation.\(^{31}\)
The ratio of 3-jet to 2-jet events should therefore be of the order $\alpha_s^{\sim 15\%}$. In principle this offers a way to measure $\alpha_s$ but is obviously sensitive to cuts and to the fact that there is a continuous transition between two and three jet events as the radiated gluon becomes harder. Up to now only the general features have been studied at the $p\bar{p}$ collider but the events appear to have the expected characteristics.

UA1 define a "trigger" jet as one with $E_T > 30$ GeV and $|\eta| < 1.5$ and count the fraction of one, two and three jet events where the additional jet has $E_T > 15$ GeV and $|\eta| < 2.5$. As seen in figure 37, the fraction of 3-jet events is indeed of the order of 15% but no allowance has been made for acceptance. A more quantitative comparison has been made noting that the multijet events do not in general lie in a plane containing the trigger jet and the beam. A comparison of the amount of momentum perpendicular to this plane ($p_{out}$) shows good agreement with a QCD Monte Carlo ($^{51}$) (see figure 38). More refined analyses are in progress.

5.5 Angular distribution for 2-jet events and structure functions

An extensive study of the formulae describing parton-parton scattering has been carried out by B. Combridge et al. ($^{52,53}$). For a 2-jet process one can write

$$\frac{d^2\sigma}{dx_1 dx_2 d(\cos \theta)} = \sum_{ij} \frac{F_i(x_1) F_j(x_2)}{x_1 x_2} \frac{d\sigma_{ij}}{d(\cos \theta)}$$

where $\theta$ is the centre of mass scattering angle and $F_i(x)$, $F_j(x)$ are the structure functions. The sum is over the different combinations of partons in the proton (1) and antiproton (j). $\frac{d\sigma_{ij}}{d(\cos \theta)}$ are the respective cross sections (e.g. $q\bar{q}$, $gg$, $q\bar{q}$, etc.) and are calculable in QCD. Elastic sub-processes (i.e. ones where the initial and final state partons are the same) are mediated by $t$-channel gluon exchange and are expected to dominate for small $t$, the sub-process four-momentum transfer squared. In fact for vector gluons they depend essentially on $\frac{1}{t^2} = \frac{1}{(1-\cos \theta)^2}$ which is the familiar angular dependence of Rutherford scattering
Processes involving s-channel gluon exchange or quark exchange are generally inelastic - the final state partons are different from the initial ones - and have a much weaker angular dependence\(^{(53)}\). Consequently, at sufficiently large values of \(\cos \theta\), the angular dependences of the dominant sub-processes \((gg + gg, gq + gq, gg + q\bar{q}, q\bar{q} + q\bar{q})\) are approximately the same. In the approximation that they are identical one may make a considerable simplification of equation (1) which factorises to give

\[
\frac{d^3 \sigma}{dx_1 dx_2 d(\cos \theta)} = \frac{F(x_1)}{x_1} \frac{F(x_2)}{x_2} \frac{d\sigma}{d(\cos \theta)}
\]

where \(\frac{d\sigma}{d(\cos \theta)}\) is a common differential cross section and \(F(x)\) is an effective structure function which incorporates "colour factors" to account the different relative couplings of \(qq, q\bar{q}\), etc. In terms of the gluon, quark and antiquark structure functions \(G(x), Q(x)\) and \(\bar{Q}(x)\) one finds

\[
F(x) = G(x) + \frac{4}{9} (Q(x) + \bar{Q}(x))
\]

which gives the appropriate relative rates for all the contributing subprocesses when \(F(x_1)\) and \(F(x_2)\) are multiplied. As gluon-gluon scattering is expected to be the most important for \(E_T^\text{jet} < 50\) GeV we write the gluon-gluon differential cross section in full

\[
\frac{d\sigma}{d(\cos \theta)} = \frac{\pi \alpha_s^2}{8} \frac{(3 + \cos^2 \theta)^3}{2x_1x_2 (1 - \cos \theta)^2}
\]

where \(x_1x_2 = \frac{s}{\hat{s}}\) is the parton-parton centre of mass energy squared. In summary, the approximation that the angular distributions have the same shape allows us to extract a combined structure function from the data. It remains only to determine the values of \(x_1\), \(x_2\) and \(\cos \theta\) on an event by event basis so that the rate can be determined in terms of \(x_1\), \(x_2\) and \(\cos \theta\). Integrating over \(\cos \theta\) then allows \(F(x_1)\) or \(F(x_2)\) to be found if factorization holds.

Let \(p_1\) and \(p_2\) be the three-momenta of the incoming partons, \(p_3\) and \(p_4\) the three-momenta of the outgoing partons. (n.b. the lowest \(E_T^\text{jet}\) jet in 3-jet events is ignored and the effect corrected by Monte Carlo). \(p_1 = x_1 \sqrt{s}/2; p_2 = x_2 \sqrt{s}/2; p_3\) and \(p_4\) are measured.
Figure 39 Angular distributions in the parton-parton centre of mass for various x-ranges of the colliding partons. The curves are for vector gluon exchange (normalised to the data).

Figure 40 The combined parton-parton angular distribution in UA1 corrected for acceptance. The upper curve is for gluon-gluon scattering allowing for the variation of $\alpha_s$ with $Q^2$. Scalar gluon exchange is clearly excluded. The theoretical curves are normalised at $\cos \theta = 0$. 
Now $\phi$ (measured) = $x_1 x_2 s$ and $x_F = x_1 - x_2$ where $x_F = \frac{1}{p_{beam}}$. Solving for $x_1, x_2$ one finds

$$x_1 = \frac{x_F + \sqrt{x_F^2 + 4t}}{2}$$

$$x_2 = \frac{-x_F + \sqrt{x_F^2 + 4t}}{2}$$

Finally \( \cos \theta = \frac{(p_3 - p_4) \cdot (p_1 - p_2)}{|p_3 - p_4| |p_1 - p_2|} \)

The limited y (rapidity) acceptance means that the accessible $\cos \theta$ range depends on $x_1$ and $x_2$. Figure 39 shows the angular distributions for regions of full acceptance obtained by UA1(42) for various $x_1, x_2$ ranges. The curves are for vector gluon exchange. Figure 40 shows the combined distribution obtained in UA1 with curves for the three main subprocesses, normalised at $\cos \theta = 0$. The expectations for scalar gluon exchange are also shown and are clearly excluded by the data. Interestingly, even better agreement is obtained if allowance is made for the $Q^2$ dependence of $\alpha_s$ in the differential cross section formula (2) where $Q^2$ is taken to be $-t$. Similar results have been obtained by UA2(46).

To extract the effective structure function $F(x)$ it is important to verify that factorization is working, i.e. that $F(x_2)$ is independent of the value of $x_1$ (and vice versa). The data determine a quantity $S(x_1, x_2)$ if $d\sigma/d(\cos \theta)$ is known

$$S(x_1, x_2) = \frac{x_1 x_2 (d^2\sigma/dx_1dx_2)}{\cos \theta_{\text{max}} \int_0^1 K \frac{d\sigma}{d(\cos \theta)} d(\cos \theta)}$$

where $\cos \theta_{\text{max}}$ is the maximum value of $\cos \theta$ permitted for given values of $x_1, x_2$. The theoretical form of $d\sigma/d(\cos \theta)$ was used by UA1(53) and $K$ is a factor to allow for higher order corrections (K-factor). In the theoretical expressions for $d\sigma/d(\cos \theta)$

$$\alpha_s = 12\pi/[23 \ln(Q^2/A^2)]$$

which assumes 5 effective flavours of quarks. $A$ was taken to be
Figure 42 A demonstration that the product of the structure functions obtained from the UA1 data factorises. i.e. $F(x_2)$ is independent of the value of $x_1$.

Figure 43 The structure function $G(x) + \frac{4}{9} (Q(x) + \bar{Q}(x))$ derived from the UA1 jet data [$Q^2 \sim 2000 \text{ GeV}^2$]. The curves are the results for the same quantity measured in the CDHS and CHARM experiments ($Q^2 = 20 \text{ GeV}^2$) extrapolated in $Q^2$. 
0.2 GeV and $Q^2 = -t$. As can be seen from figure 42, factorization appears to work

$$i.e. \, S(x_1,x_2) = F(x_1)F(x_2).$$

The resulting structure function, assuming $K = 2$, $F(x) = G(x) + \frac{4}{9} (Q(x) + \overline{Q}(x))$ is shown in figure 43. Also shown is the same function from the CDES and CHARM experiments\(^{(54)}\) obtained at $Q^2 = 20$ GeV\(^2\) but extrapolated to $Q^2 = 2000$ GeV\(^2\), the average $Q^2$ of the UA1 data. A similar analysis has been performed by UA2\(^{(46)}\) and is in good agreement with UA1. The conclusion is that the collider data are entirely compatible with QCD expectations, showing the expected angular distribution for vector gluon exchange and compatibility with low energy results evolved to collider energies for the structure functions.

5.6 Jet production cross sections

In determining jet cross sections it is necessary to allow for various systematic effects, the most important being the luminosity, the content (correctness of the assignment of calorimeter elements to jets by the algorithms) and the calibration of the energy scale. A Monte Carlo procedure is used to estimate a correction factor for the jet content. Jets are generated in the apparatus, with shower simulation in the calorimeters, and then reconstructed by the algorithm. A comparison of $\frac{dN}{dE_T}$ (found) with $\frac{dN}{dE_T}$ (generated) then allows a correction to the cross section to be obtained which is $\pm 1.25$ and only weakly dependent on $E_T$. There is, of course, a dependence on the reliability of the Monte Carlo simulation itself.

The calibration of the energy scale includes an allowance for the different response of the electromagnetic calorimeters to hadrons and photons which is a correction of $\pm 1.2$. For UA1 the uncertainties on the energy scale are

$$\begin{align*}
\text{absolute calibration e.m.} & \pm 3\% \\
\text{had.} & \pm 5\%
\end{align*}$$

$$\text{e.m./had. response correction} \pm 5\% \pm 7\%$$

The resulting cross section errors for the two experiments are:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Energy Scale</th>
<th>Luminosity</th>
<th>Jet Algorithm</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA1</td>
<td>$\pm 50%$</td>
<td>$\pm 20%$</td>
<td>$\pm 20%$</td>
<td>$\pm 45%$</td>
</tr>
<tr>
<td>UA2</td>
<td>$\pm 20%$</td>
<td>$\pm 20%$</td>
<td>$\pm 35%$</td>
<td>$\pm 65%$</td>
</tr>
</tbody>
</table>

Figure 44 shows the UA1 inclusive jet cross sections\(^{(40)}\). The shaded band is the QCD prediction\(^{(55)}\). Figure 45 shows a comparison between UA2\(^{(45)}\) and UA1 (1981 and 1982 data) cross sections and figure 47 a comparison between UA2 results for inclusive cross sections and 2-jet masses with QCD expectations. In all cases the agreement is excellent.
Figure 44 a) UA2 inclusive jet cross sections compared to 1981-2 data
b) jet-jet mass distribution from UA2.

Figure 45 a) inclusive jet cross sections compared to QCD (hatched band)
b) jet-jet mass distribution compared to QCD.
5.7 Summary of jet physics

i) 2-jet events dominate at high $E_T$ (85%)

ii) the rate of 3-jet events is compatible with QCD ($O(a_s)$)

iii) the fragmentation function for pp jets (mainly gluons) is similar to that for $e^+e^-$ jets (quarks) but agrees with QCD expectations when the higher collision energy is taken into account

iv) the $\langle p_T \rangle$ of charged particles with respect to the jet axis is ~0.6 GeV and agrees with QCD

v) the 2-jet centre of mass angular distribution, $(1-\cos\theta)^{-2}$, supports vector gluon exchange

vi) the effective structure function $F(x) = G(x) + \frac{4}{3} (Q(x)+\bar{Q}(x))$ shows the expected evolution from $Q^2 = 20 \text{ GeV}^2$ to $Q^2 = 2000 \text{ GeV}^2$

vii) the jet cross sections and 2-jet mass distributions agree with QCD up to $E_T$ of 150 GeV and $m_{jj}$ of 270 GeV respectively.

6. W AND Z PHYSICS

6.1 Introduction

The main features of the standard electroweak model have been described elsewhere in this school\(^{(56,57)}\). We shall concentrate on the properties of the W and Z particles and how they are determined experimentally. Then we shall review the extent to which the standard model is checked by the UA1 and UA2 results and the expected improvements in the future. An excellent account of the collider results is given in reference 58.

6.2 The W and Z masses

Irrespective of the fine details one may obtain an order of magnitude estimate of the W mass under the assumption that the electromagnetic and weak coupling strengths are the same (unification) but that the two interactions are mediated by a massless photon and a massive W respectively. The point-like four fermion interaction is characterised by a single constant $G_F$ (Fermi constant) which determines the rate of $\beta$-decay (strictly d $\rightarrow$ u$e^-$). Under the W boson hypothesis this is replaced by a coupling constant $g$ at each of the quark and lepton vertices, and the boson propagator.

\[ G_F \rightarrow \frac{g^2}{m_W^2}, \text{ where } q^2 \text{ is the four-momentum squared of the W. At low } q^2 (\ll m_W^2) \]

\[ G_F \sim \frac{g^2}{m_W^2} \]

If $g = e$ (equal electromagnetic and weak coupling)
\[ M_W \sim \frac{\alpha}{\sqrt{G}} \times 100 \text{ GeV/c}^2. \]

According to the standard model

\[ M_W = (\pi a/\sqrt{2} G_F)^{1/2}/\sin \theta_W \]

where \( e^2 = 4\pi a \) and \( a = 1/137.032 \) at low energy\(^{(59)}\). From neutrino neutral current experiments\(^{(60)}\) \( \sin^2 \theta_W = 0.233 \pm 0.009 \) giving \( M_W = 77.2 \pm 1.6 \text{ GeV/c}^2 \). However, at the \( W \)-mass both \( a \) and \( \sin^2 \theta_W \) are different (renormalised)\(^{(61)}\).

\[ a(m_W) = 1/127.7 \]

\[ \sin^2 \theta_W(m_W) = 0.217 \pm 0.014 \]

leading to the prediction

\[ m_W = 38.65/\sin \theta_W = 83.0 \pm 3.0 \text{ GeV/c}^2. \]

which is measurable, already providing a test of radiative corrections at the present level of experimental study.

In the GSW model, the parameter \( \rho \), which determines the relative strengths of the charged and neutral current couplings is given by

\[ \rho = \frac{m_W^2}{m_Z^2} \cos^2 \theta_W \]

If there are only Higgs doublets, \( \rho = 1 \), i.e. \(1/2I_1\), where \( I_1 \) is the Higgs isospin. Therefore \( \rho = 1 \) in the simplest theory. Using \( \sin^2 \theta_W(m_W) \) gives

\[ m_Z = 93.8 \pm 2.5 \text{ GeV/c}^2 \]

6.3 Decay of the \( W \) and \( Z \)

The decays of the \( W^- \) are

\[ W^- + \bar{e} \nu_e \]

\[ W^- + \bar{\mu} \bar{\nu}_\mu \]

\[ W^- + \bar{\tau} \bar{\nu}_\tau \]

and \[ qq \] where \( \bar{e} \nu_e \) is \( e^\nu_e, \mu^\nu_\mu, \tau^\nu_\tau \) and \( qq \) is \( d\bar{u}, s\bar{c}, b\bar{t} \). The \( W^- \) decays are the charge conjugates. Here we have ignored the Cabibbo type quark mixing but this is not important for calculating the decay width. For each of the three leptonic channels

\[ \Gamma(W^- + \bar{e} \nu_e) = \frac{G_F m_W^3}{6\pi^2} = 250 \text{ MeV} \]

The quark channels are the same except for three effects
1) a colour factor of 3

ii) a QCD correction of \((1 + \alpha_s/\pi)\)

iii) a phase space factor for \(e^+e^-\)

Assuming \(m_t = 30\) GeV/c\(^2\) the total hadronic width is

\[\Gamma(W \rightarrow \text{hadrons}) = 2.20\ \text{GeV}\]

and \(\Gamma(W \rightarrow \text{all}) = 2.95\ \text{GeV}\)

Hence \(\Gamma(W \rightarrow e^+e^-)/\Gamma(W \rightarrow \text{all})\) is 8.5\% or approximately 1/12. Furthermore the decay is purely \(V-A\) in other words the \(W\) decays purely into left-handed leptons or quarks. It is also produced by left-handed quarks. Of course, it couples to right handed antiquarks and antileptons.

The \(Z\) decay is more complicated because of mixing with the photon which modifies its vector couplings. Consequently its vector and axial vector couplings are not equal, except for the neutrino decays, and it couples to both left and right handed quarks and leptons.

\[
\Gamma_Z = \frac{G_F m_Z^3}{12\pi\sqrt{2}} \left[ 3\lambda(q^2 + v_q^2) + \frac{1}{2} (a_q^2 + v_q^2) \right]
\]

where \(v_q (a_q)\) is the vector coupling to quark (lepton)

\(a_q (a_q)\) is the axial coupling to quark (lepton)

and the factor 3 is for colour. According to the model (with the definition that \(a_\nu = v_\nu = 1\)),

\[
\begin{align*}
  v_q &= 2I_3^q - 4Q_q \sin^2 \theta_W; \quad a_q = 2I_3^q \\
  v_\ell &= 2I_3^\ell - 4Q_\ell \sin^2 \theta_W; \quad a_\ell = 2I_3^\ell
\end{align*}
\]

\(I_3\) is the 3rd component of weak isospin and \(Q_q, Q_\ell\) the lepton and quark charges.

Consequently

\[
\Gamma(Z \rightarrow \ell^+\ell^-) = \frac{G_F m_Z^3}{12\pi\sqrt{2}} = 181\ \text{MeV}
\]

\[
\Gamma(Z \rightarrow \ell^+\ell^-) = 92\ \text{MeV}
\]

and, allowing for \(m_t\) and the QCD factor

\[
\Gamma(Z \rightarrow \text{hadrons}) = 2.12\ \text{GeV}
\]

\[
\Gamma(Z \rightarrow \text{all}) = 2.94\ \text{GeV}
\]

assuming 3 families of neutrinos. The branching ratio

\[
\frac{\Gamma(Z \rightarrow \ell^+\ell^-)/\Gamma(Z \rightarrow \text{all})}{3} = 3.12
\]

6.4 Production of W and Z; cross sections

The production is analogous to the Drell-Yan mechanism, which has been well studied experimentally\(^{(63)}\).
The $Y$ is replaced by a $W$ or $Z$ and there are mass peaks for the final state particles

For $W$ decay into $e\bar{v}_e$, only the electron is seen and it has a transverse momentum peak at $p_T = m_W/2$, smeared by the finite width of the $W$, assuming no transverse motion of the $W$. This is then the characteristic signature of the $W$. The remaining spectator quarks will, of course, produce additional particles in the event.

The Drell-Yan differential cross section is given by (63)

$$\frac{d^2\sigma}{dx_1dx_2} = \left(\frac{4\pi\alpha^2}{3M^2}\right)^2 \frac{Q^2}{3} [q_1(x_1)\bar{q}_2(x_2) + \bar{q}_1(x_1)q_2(x_2)]$$

where $q_1(x_1)$ is the probability density of finding a quark of fractional momentum $x_1$ in hadron 1 etc., and $M$ is the mass of the produced lepton pair. The second factor 3 allows for the fact that the annihilating quark and antiquark have to have the same colour.

For $W$ production we have

$$\bar{u}d \rightarrow W^+; \quad \bar{u}s \rightarrow W^+$$

$$\bar{u}d \rightarrow W^-; \quad \bar{u}s \rightarrow W^-$$

where we now include Cabibbo suppressed channels. In this case the differential cross section is
Figure 46  The cross sections for $W^+$ (or $W^-$) production versus energy (Paige\textsuperscript{(64)}). The solid curves are for non-scaling quark distributions. The dashed curves are for scaling distributions.

Figure 47  The probability that a pion entering a UA1 electromagnetic calorimeter (gondola) will deposit less than $E_{c,\text{THR}}$ in the hadron calorimeter (C) behind it. The data are from test beam measurements. For $p_\pi = 40$ GeV/c and $E_{c,\text{THR}} = 0.2$ GeV the probability is $4 \times 10^{-3}$. 
\[
\frac{d^2\sigma}{dx_1dx_2} = \frac{\sqrt{2}G_F}{3} \left[ \left[ u_1(x_1)\bar{d}_2(x_2) + \bar{d}_1(x_1)u_2(x_2) \right] \cos^2 \theta_c \\
+ \left[ u_1(x_1)\bar{s}_2(x_2) + \bar{s}_1(x_1)u_2(x_2) \right] \sin^2 \theta_c \right]
\]

For the Z, only \( u\bar{u}, d\bar{d} \) and \( s\bar{s} \) contribute.

\[
\frac{d^2\sigma}{dx_1dx_2} = \frac{\sqrt{2}G_F}{3} \left[ \left[ u_1(x_1)\bar{u}_2(x_2) + \bar{u}_1(x_1)u_2(x_2) \right] F_1(\theta_W) + \\
\left[ d_1(x_1)\bar{d}_2(x_2) + \bar{d}_1(x_1)d_2(x_2) + s_1(x_1)\bar{s}_2(x_2) + \bar{s}_1(x_1)s_2(x_2) \right] F_2(\theta_W) \right]
\]

where \( F_1(\theta_W) = \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W + \frac{8}{9} \sin^4 \theta_W \)

\( F_2(\theta_W) = \frac{1}{4} - \frac{1}{3} \sin^2 \theta_W + \frac{2}{9} \sin^4 \theta_W \)

obtained from the products of the neutral current couplings given above for the charge 2/3 and charge 1/3 quarks respectively. The total cross section is given by

\[
\sigma_\mu = \int \int \frac{d^2\sigma}{dx_1dx_2} \delta(x_1x_2 - \frac{M^2}{s}) dx_1dx_2
\]

with a similar expression for \( \sigma_\nu \), where \( s \) is the total centre of mass energy squared.

Figure 46 shows the W and Z cross sections calculated by Paige\(^{(64)}\). The dashed curves are for scaling quark distributions \( q(x) \) and the solid curves for non-scaling distributions \( q(x,M^2) \) where the \( q^2 \) variation is taken into account. At the CERN collider (\( \sqrt{s} = 540 \text{ GeV} \)) there is little difference, but at the Fermilab collider (\( \sqrt{s} = 2 \text{ TeV} \)) the non-scaling functions give a factor 5 higher cross section. Notice that in proton-proton collisions the antiquarks are from the sea which means that proton-antiproton collisions are more favourable close to threshold.

The predicted \( W^+ \) cross section at 540 GeV is 2-3 nb, or about 0.2 nb for \( W^+e^-\).\n
6.5 Higher order corrections to the cross sections

The above calculations are to lowest order in QCD. Higher order processors not only increase the cross sections but also give transverse momentum to the \( W \) or \( Z \). Examples of first order diagrams (in \( \alpha_s \)) are:

![First order diagrams](image-url)
Their effect has been calculated(65). The cross sections are increased by approximately a factor of 2 - the K factor, observed experimentally in Drell-Yan(63). p_{T} values, for the W and Z, averaging several GeV are expected (see later) as well as observable recoil jets. The p_{T} distribution of the electron in W-decay is further smeared out, but not seriously.

6.6 Experimental considerations (W-reconstruction from W + e + ν and W + νν events)

In UA1 and UA2 the momentum of the electron (or muon) is measured - in fact, for the electron, it is the energy and direction since the calorimeter measurement is much more precise (±2%). However, for the neutrino, only the transverse energy (i.e. the missing transverse energy in the event) is measured because part of the total energy (∼100 GeV) escapes along the beam pipe. For ordinary hadronic events one expects the transverse components to be balanced. The decay lepton from the W is expected to be isolated (i.e. not part of a jet).

The W mass cannot be calculated but the transverse mass m_{T} (< m_{W}) can be found where

\[ m_{T}^{2} = 2p_{T}^{e}p_{T}^{ν}(1 - \cos \phi_{νe}) \]

and \[ \phi_{νe} \] is the angle between the ν and e directions in a plane perpendicular to the beam direction (transverse plane). It is easily shown that m_{T} is independent of p_{T}^{W} to first order in p_{T}^{W}. Nevertheless it is necessary to assume a p_{T} distribution for the W (given by QCD) and a decay angular distribution in the W centre of mass in order to find m_{W} which is done by a Monte Carlo fitting procedure.
If $p_t^W = 0$, $p_t^e = \frac{m_W}{2} \sin\theta^w$: the decay angular distribution is $(1+\cos\theta^w)^2$. The calculation is then straightforward(66).

The measurement of $p_t^e$ only determines $\theta^w$, the centre of mass decay angle, with a sign ambiguity; i.e. $\cos\theta^w = \frac{1}{\sqrt{1-\sin^2\theta^w}}$. In the majority of the cases it is possible to solve this 2-fold kinematic ambiguity.

For a given value of $\theta^w_{\text{lab}}$ for the electron, which determines $p_t^e$, there are two values of $x_W = 2p_t^W/\sqrt{s}$ corresponding to the two signs for $\cos\theta^w$, or equivalently there are two possible directions for the neutrino. In about 70% of the cases one of the solutions for $x_W$ is unphysical ($> 1$) and the other solution is unique. This is relevant in the study of the decay asymmetry discussed later. Furthermore $x_1$ and $x_2$ are then also determined using the fact that

$$m_W = x_1 x_2 s; \quad x_W = x_1^{-1} x_2$$

6.7 Backgrounds to $W$ events

There are several sources of background for $W + e\nu(\mu\nu)$

1) misidentified electrons (or muons), i.e. a pion giving an energy deposition in the electromagnetic calorimeter like an electron (figure 47) (or a hadron "punching through" the calorimeters).

2) photon overlap with a charged particle simulating an electromagnetic response in a calorimeter.
   - UA1 compare the central detector momentum for the track with the energy in the calorimeter
   - UA2 compare the impact point of the track and the photon (UA1 also does this).

3) Asymmetric Dalitz pairs and conversions in the apparatus.

4) Genuine electron or muon from heavy quark decays (primarily). In the case of muons, also $\pi^\pm \mu$ and $K^\pm \mu$ decays.

5) Genuine $W + \tau \nu$ events; $\tau \rightarrow e\nu\bar{\nu} \tau$ or $\tau \rightarrow \mu\nu\bar{\nu} \tau$. This gives electrons or fake electrons and missing energy but with lower $p_t$.

All of the processes 1) to 4) give an "electron" spectrum that decreases with increasing $p_t$. Therefore a $p_t$ cut on the electron helps to remove the background. The striking feature of the data, however, is the large missing energy in $W$ events that is not present in the background processes and proves to be an extremely powerful way of rejecting background events, resulting in quite small backgrounds.

Figure 48 shows the missing transverse energy for 55 single electron events with $p_t^e > 15$ GeV/c in UA1(67). The dashed curve is the resolution function for missing energy, obtained from ordinary events, normalised to the three lowest missing energy events. Empirically(67), each component of missing transverse energy has an error $\Delta E_y, z = 0.4 \sqrt{E_T}$. To avoid the effect of 'cracks' at the top and bottom of the magnet, regions where the missing energy 'vector' is within $\pm 15^\circ$ of the vertical are excluded reducing the sample to 43 events for further study.

If they originate from $W$ decays, there should be a strong correlation between the
Figure 48. A plot of the missing transverse energy for single electron events with $p_T > 15$ GeV/c (UA1). The dashed curve is the resolution function for missing energy normalised to the three lowest missing energy events.

Figure 49. A plot of the electron transverse energy against the parallel component of neutrino transverse energy showing a striking correlation as would be expected for $W$ decays.

Figure 50. The transverse mass distribution for $e\nu$ events. The solid curve is a fit for $m_W = 80.9$ GeV/$c^2$. The dashed curve is the expected shape for an object decaying into $e\nu\nu$. 
Figure 51 An example of a $W + \nu \nu$ event. The struck muon drift tubes are indicated and the hadron calorimeter cells. The dashed arrow is the direction of the missing transverse energy.

Figure 52. UA2 experiment a) the $p_T$ distribution of the electron in the $W + e \nu$ sample b) the same with strict cuts on the central detector. The full curves are background estimates and the dashed curve background $+ W + e \nu + W + \tau \nu (\tau + e \nu)$. The fitted mass is 83.1 GeV/c$^2$. 
Figure 53. The fractional energy $x_w$ carried by the $W$. The curve is the prediction assuming the $W$ has been produced by valence quarks and antiquarks.

Figure 54. The $x$-distributions of $u(\bar{u})$ and $d(\bar{d})$ quarks in the proton (antiproton) for events where the $W$ charge is determined.
Figure 55. Production and decay of $W^+$ and $W^-$ showing the behaviour of the particle spins. For example the $W^+$ is formed from left-handed quarks (proton) and right-handed antiquarks (antiproton). It decays to a right-handed $e^+$ and a left-handed $\nu_e$ which are constrained to travel in the directions shown to conserve angular momentum giving rise to a $(1 + \cos \theta^*)^2$ angular distribution.

Figure 56. The $W$ decay asymmetry observed in the UA1 experiment.

Figure 57. The asymmetry for $W$ decays in UA2 using the forward toroidal spectrometers to measure the electron charge.
electron and neutrino transverse energies which should balance on average. Figure 49 shows that this behaviour is observed. Note that $W + \tau \nu_{\tau}$ events lie on the same line. It is estimated that the plot contains $0.5$ event where $\tau + \pi \nu_\tau$ and $2$ events where $\tau + e\nu_\tau$ which necessarily have low $p_T$ values for the electrons. Figure 50 shows the transverse mass distribution for the $43$ events. The solid curve is the best fit to the $W$ mass and the dashed curve is the expectation for a hypothetical particle of the same mass decaying to $e\nu\nu$. Actually, the mass was determined from events with $p_T^e, p_T^{\nu} > 30$ GeV/c to eliminate $\tau$-backgrounds, and yielded a value

$$m_W = 80.9 \pm 1.5 \text{ (stat.)} \pm 2.4 \text{ (syst.) GeV/c}^2$$

The systematic error arises from the uncertainty on the absolute energy calibration of the electromagnetic calorimeters ($\pm 3\%$).

UA1$^{(68)}$ has also observed 14 examples of $W + \nu\bar{\nu}$ (see figure 51) from which $m_W = 81.0_{-5}^{+5}$ GeV/c. The statistical error is larger because momenta have to be measured by the magnetic field and the number of events smaller because of lower geometrical acceptance and overall running time with a muon trigger. For full details see reference $68$. Figure 52 shows the $W + e\nu$ sample from UA2 in which the electron transverse momentum is plotted$^{(69)}$. One can clearly see the falling background spectrum. The dashed curve is a fit for background + ($W + e\nu$) + ($W + \tau\nu_{\tau}$, $\tau + e\nu\nu$). The fit gives $32.1 \pm 6.0$ $W + e\nu$ decays yielding

$$m_W = 83.1 \pm 1.9 \pm 1.3 \text{ GeV/c}^2$$

The systematic error is lower than in UA1.

6.8 Longitudinal motion of the $W$

As already discussed the Feynman $x$ of the $W$ can be found with a 2-fold ambiguity. In 70% of the cases one solution is unphysical. In the remaining 30% the lower value is used. Figure 53 shows $x_W$ for the 43 events$^{(67)}$ compared to a curve obtained using $u$ and $d$ quark distributions varying as $x(1-x)^3$ and $x(1-x)^4$ respectively. In 29 of the events the electron charge is determined to $3\sigma$ and one can actually obtain $x_u$ and $x_d$ which are shown in figure 54, together with the curves mentioned above. It is assumed that only valence quarks participate, otherwise the analysis would be impossible.

6.9 Decay asymmetry of $W + e\nu$

The full reconstruction of the $W$ production and decay is a necessary first step in determining the decay asymmetry which obviously requires a unique knowledge of $\cos\theta^*$. The left-handed coupling of the $W$ arising from the pure $V-A$ form of the theory leads to an angular distribution of $(1+\cos\theta^*)^2$. This is easily understood through a helicity argument (figure 55). The $W$s are longitudinally polarised provided only valence quarks are involved. Figure 56 shows the UA1 result which is in excellent agreement with expectations. Figure 57 shows the results from UA2, obtained from their forward toroidal spectrometers$^{(69)}$. Note that in $pp$ collisions there is no asymmetry since the antiquark must come from the sea.
Figure 58. Mass plot of $l^+l^-$ pairs from UA1 and UA2 showing a clear $Z^0$ mass peak containing 5 $\mu^+\mu^-$ and 12 $e^+e^-$ events. The background is obviously negligible.

Figure 59. The $Z^0$ decay angular distribution in the UA1 experiment. The expected asymmetry is very small because of the small vector coupling. The data are consistent with this behaviour.
Table 4

$W^+$ and $Z^0$ parameters from the UA1 and UA2 experiments

<table>
<thead>
<tr>
<th></th>
<th>UA1</th>
<th>UA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(W + e\nu)$</td>
<td>52a)</td>
<td>37b)</td>
</tr>
<tr>
<td>$m_W (\text{GeV}/c^2)$</td>
<td>80.9±1.5±1.4</td>
<td>83.1±1.9±1.3</td>
</tr>
<tr>
<td>$T_W (90%\text{CL})$</td>
<td>&lt; 7 GeV</td>
<td>-</td>
</tr>
<tr>
<td>($\sigma$) (nb)</td>
<td>0.53±0.08±0.09</td>
<td>0.53±0.10±0.10</td>
</tr>
<tr>
<td>$N(W + \mu\nu)$</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>$m_W (\text{GeV}/c^2)$</td>
<td>81.0±6.9</td>
<td>-</td>
</tr>
<tr>
<td>($\sigma$) (nb)</td>
<td>0.67±0.17±0.15</td>
<td>-</td>
</tr>
<tr>
<td>$N(Z^0 + e^+e^-)$</td>
<td>3±1c)</td>
<td>7±1c)</td>
</tr>
<tr>
<td>$m_Z^* (\text{GeV}/c^2)$</td>
<td>95.6±1.4±2.9</td>
<td>92.7±1.7±1.4</td>
</tr>
<tr>
<td>$T_Z^* (90%\text{CL})$</td>
<td>&lt; 8.5 GeV</td>
<td>&lt; 6.5 GeV</td>
</tr>
<tr>
<td>($\sigma$) (nb)</td>
<td>0.05±0.02±0.009</td>
<td>0.11±0.04±0.02</td>
</tr>
<tr>
<td>$N(Z^0 + \mu^+\mu^-)$</td>
<td>4±1c)</td>
<td>-</td>
</tr>
<tr>
<td>$m_Z^* (\text{GeV}/c^2)$</td>
<td>85.6±6.3</td>
<td>-</td>
</tr>
<tr>
<td>($\sigma$) (nb)</td>
<td>0.105±0.05±0.15</td>
<td>-</td>
</tr>
<tr>
<td>$\sin^2\theta_W = (38.65/m_W)^2$</td>
<td>0.228±0.008±0.014</td>
<td>0.216±0.010±0.007</td>
</tr>
<tr>
<td>$\rho = [m_W/m_Z\cos\theta_W]^2$</td>
<td>0.97±0.05</td>
<td>1.02±0.06</td>
</tr>
</tbody>
</table>

a) $p_T > 15$ GeV/c
b) $p_T > 25$ GeV/c
c) $Z^0 \rightarrow e^+e^-\gamma (E_\gamma > 20$ GeV)

[Averaging the electron and muon results for UA1 gives $m_W = 80.9±1.5±2.4$ GeV/c$^2$; $m_Z = 93.5±1.5±2.9$ GeV/c$^2$.]
6.10 \( Z^0 \) production

As two charged leptons result from the decay of the \( Z \), the events are extremely clean as can be seen from a simple mass plot (figure 58). Consequently, no background discussion is necessary. UA1 has observed 4\( e^+e^- \) and 5 \( \mu^+\mu^- \) events\(^{(70,75)} \) while UA2\(^{(71)} \) has 8 \( e^+e^- \) events. However 3 of them have hard \( \gamma \)-rays at appreciable angles to the leptons which cannot easily be accounted for as bremsstrahlung and will be discussed later. The results for \( Z \) masses are summarised in table 4 together with the \( W \) masses from the two experiments.

The \( Z \) decay angular distribution is expected to be almost symmetric because of the small vector coupling of the \( Z \) to electrons, \( (1-4 \sin^2\theta_W) \). The almost pure axial vector nature of the decay leads to equal couplings to left and right handed electrons which, according to the helicity argument, gives no significant asymmetry (figure 59).

6.11 Comparison with the Standard Model

Table 4 summarises the results of the two experiments for the \( W \) and \( Z \) masses, widths and cross sections. The values of \( \sin^2\theta_W(m_W) \) obtained from \((38.65/m_W)^2 \) (6.2) and the values of \( \rho = (m_W/m_Z \cos\theta)^2 \) are also given for each experiment. The results for \( \sin^2\theta_W \) are in excellent agreement with the prediction of \( 0.217 \pm 0.016 \) and \( \rho \) is compatible with unity as expected for Higgs doublets. Furthermore, the cross sections (\( x \) branching ratios) are in satisfactory agreement with QCD calculations\(^{(65)} \), in view of the theoretical uncertainties, the predicted values being \( \sigma(W + e^-) = 0.39\text{nb} \) and \( \sigma(Z + e^+e^-) = 0.04\text{nb} \).

An alternative way of comparing the results with the standard model is to assume that \( \rho = 1 \) in which case \( \sin^2\theta_W(m_W) = 1 - m_W^2/m_Z^2 \). One may then test radiative corrections using

\[
m_W = \frac{A}{\sin^2\theta_W}
\]

where \( A = 37.2810 \pm 0.0003 \) GeV

(1-\( \Delta r \))\(^{1/2} \)

i.e. the low energy value with a correction term \( \Delta r \). Hence

\[
\Delta r = 1 - \frac{37.2810^2}{m_W^2(1-m_W^2/m_Z^2)}
\]

The results are the following:-

\[
\begin{align*}
\Delta r & \\
\text{Theory} & + 0.070 \pm 0.002 \\
\text{UA1} & + 0.16 \pm 0.11 \pm 0.05 \\
\text{UA2} & - 0.03 \pm 0.24 \pm 0.03
\end{align*}
\]

assuming that the systematic errors cancel on \( m_W/m_Z \). There is agreement within statistics but the test is clearly not definitive.

What might one expect in the future? The improved collider, equipped with the new antiproton collector ring ACOL, is expected to give an improvement of a factor 10 in antiproton flux (\( \sim 10^{11} \) per bunch).
Together with the higher energy of around 330 GeV per beam one can expect perhaps a factor 20 increase in W and Z yields. Three years of running (1987-9) should then produce $-10^4 W + e^- e^-$ and $-10^3 Z + e^- e^-$ in the two experiments together. Systematic errors on the W and Z masses should be < 1% in both experiments which corresponds to accuracy of ± 0.005 or sin$^2 \theta_W$ such as is planned in the CHARM II experiment(72). The error on m$_W$/m$_Z$ should be < 0.2% giving

$$\Delta r = 0.070 \pm 0.013 \text{(stat.)} \pm 0.018 \text{(syst.)}$$

where we have inserted the theoretical value for $\Delta r$ (whose error is ± 0.002). However a more accurate measurement of m$_Z$ from SLC or LEF would greatly reduce the systematic error. Thus one can hope to check radiative corrections to about ± 20%.

We note that the error from the collider on $\Gamma_Z$ should be ± 250 MeV corresponding to Z extra neutrinos at 90% confidence level.

6.12 Z-width and number of neutrino families

Each additional neutrino family increases the Z-width by 0.18 GeV but the direct measurements are not at this level of sensitivity (UA1 finds N$_\nu < 31$ and UA2 N$_\nu < 22$ with 90% confidence level).

A better result is obtained indirectly from

$$\sigma_{Z}^{\gamma} + e^+ e^- / \sigma_W + e^- = \frac{\sigma_{Z}}{\sigma_W} \left( \frac{\Gamma_Z + e^+ e^-}{\Gamma_W + e^-} \right) \frac{\Gamma_W}{\Gamma_Z}$$

which requires a knowledge of $\sigma_{Z}/\sigma_W$ from QCD and $\Gamma_Z + e^+ e^-$, $\Gamma_W + e^-$ and $\Gamma_W$ from theory.

This assumes no heavier charged lepton that the W can decay into. $\frac{\sigma_{Z}}{\sigma_W} = 0.24 \pm 0.05$ from QCD and should be fairly reliable as most corrections are common to both cross sections.

From UA1

$$\sigma_{Z}^{\gamma} / \sigma_W = 0.098 + 0.052 - 0.035$$

giving $\Gamma_Z = 1.8 \pm 5.5$ GeV and N$_\nu < 18$ (90% c.l.)(58). In a similar analysis UA2(69) finds $\Gamma_Z < 2.6 \pm 0.3$ GeV and N$_\nu < 3$ (90% c.l.), the low value stemming from their rather high Z cross section. However with the present statistics neither result is very interesting. Furthermore there are the questions of whether the radiative decays of the Z should be included or not and the value of m$_Z$ that should be used. The limit from cosmological constraints(58) is N$_\nu < 4$.

6.13 Transverse momentum of the W and Z

As discussed earlier (6,5) QCD corrections to the simple Drell-Yan type diagram produce appreciable $p_T$-values for the W and Z as well as observable jets. Several authors have calculated the expected $p_T$-distributions which are compared with the data in figures 60 and 61. Good agreement is found. Furthermore UA1 have compared the jet $E_T$ distributions for W and Z events to those from multijet data(74), which shows them to be consistent with initial state gluon bremsstrahlung (figure 63). However there is some
Figure 60 $p_T$ distributions of the W compared to QCD predictions\(^{(73)}\).

Figure 61 $p_T$ distribution of the Z compared to a QCD prediction\(^{(73)}\).
Figure 63  Jet $E_T$ distributions for
a) 66 $W \rightarrow \ell \nu$ events,
b) 9 $Z \rightarrow \ell^+\ell^-$ events.

Figure 64  a) Comparison of $1/E$ and $1/p$ for $W$ decay electrons and the expected behaviour.

Figure 64  b) Energy deposition of $W$ decay muons in the calorimeters compared to expectation.
### Table 5

Properties of the $e^+e^-$ events

<table>
<thead>
<tr>
<th></th>
<th>$e^+e^-$ (UA1)</th>
<th>$e^+e^-\gamma$ (UA2) a)</th>
<th>$\mu^+\mu^-\gamma$ (UA1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_\gamma$ (GeV)</td>
<td>38.8 ± 1.5</td>
<td>24.4 ± 1.0</td>
<td>28.3 ± 3</td>
</tr>
<tr>
<td>$E_{e^+}$ (GeV)</td>
<td>61.0 ± 1.2</td>
<td>69.9 ± 1.8</td>
<td>53.6 ± 5.8</td>
</tr>
<tr>
<td>$E_{e^-}$ (GeV)</td>
<td>9 ± 1</td>
<td>11.5 ± 0.7</td>
<td>42.2 ± 44.0</td>
</tr>
<tr>
<td>$\Delta\alpha(e^+,\gamma)(^o)$ b)</td>
<td>132.0 ± 4.0</td>
<td>129.9</td>
<td>7.9</td>
</tr>
<tr>
<td>$\Delta\alpha(e^-,\gamma)(^o)$ b)</td>
<td>14.4 ± 4.0</td>
<td>31.8</td>
<td>59.0</td>
</tr>
<tr>
<td>$m(e^+e^-)$ (GeV/c²)</td>
<td>42.7 ± 2.4</td>
<td>50.4 ± 1.7</td>
<td>70.9 ± 37.2</td>
</tr>
<tr>
<td>$m(e^+e^-\gamma)$ (GeV/c²)</td>
<td>98.7 ± 5.0</td>
<td>90.6 ± 1.9</td>
<td>88.4 ± 46.1</td>
</tr>
<tr>
<td>$m(e^+\gamma)$ (GeV/c²)</td>
<td>88.8 ± 2.5</td>
<td>74.7 ± 1.8</td>
<td>5.0 ± 0.4</td>
</tr>
<tr>
<td>$m(e^-\gamma)$ (GeV/c²)</td>
<td>4.6 ± 1.0</td>
<td>9.1 ± 0.3</td>
<td>52.5 ± 27.5</td>
</tr>
</tbody>
</table>

a) The $e^+$ is identified since it goes into the forward region; the charge of $e^-$ is unknown.

b) $\Delta\alpha$ is the angular difference in space.
evidence for a higher jet multiplicity for $Z$ events (figure 63) though this is not seen in
UA2 and therefore needs confirmation.

6.14 Summary of $W$ and $Z$ Physics compared to the Standard Model

In summary the following features have been observed within experimental errors:

i) correct $W$ and $Z$ masses ($\rho = 1$)

ii) correct $W$ and $Z$ rates

iii) correct decay asymmetries

iv) $\nu\bar{\nu}$ universality

v) agreement with QCD for $p_T$-distributions.

On the other hand there are two unexpected features requiring confirmation:

- Evidence for radiative $Z$-decays. These will be discussed further in section 7.1.

- Possible anomalous jet activity on $Z$-events.

7. NEW PHENOMENA

7.1 Introduction

In this section we shall discuss a number of miscellaneous topics, some of which may
indicate phenomena that are not explicable within the framework of QCD and the electroweak
model. They are:

1. Radiative $Z$ decays

2. Charm production in jets (UA1)

3. Dimuon events (UA1)

4. High mass $e\nu$ Jet events (UA2)

5. Monojets or $\gamma\gamma$ with missing $E_T$ (UA1)

6. Evidence for the top quark (UA1)

Finally, we discuss the future prospects at the $pp$ collider in the search for Higgs
mesons.

7.2 $Z + e^+e^-\gamma$ events.

Two events in UA1 ($e^+e^-\gamma$ and $\mu^+\mu^-\gamma$)(70),(75) and one in UA2 ($e^+e^-\gamma$)(71) have been
observed in which a hard $\gamma$ carries off a large fraction of the energy of one of the
leptons and is at an appreciable angle $\Delta\theta$ to it. The details are presented in Table 5.
All three events are consistent with $Z$ decays. The most obvious explanation would be that
the $\gamma$-rays are bremsstrahlung, either internal, or due to radiation in the apparatus.
However, the probability of observing a $\gamma$ with $F = E_{\gamma} / (E_{\gamma} + E_e)$ greater than that
observed and $\Delta\theta$ greater than that observed is $< 1\%$ in each case. Even when multiplied by
the total number of lepton tracks, each of which could in principle radiate a $\gamma$, the
probability of observing three such events is still less than $1\%$. In fact Barger
et al.(76) estimate that (for $E_{\gamma} > 15$GeV, $\Delta\theta > 10^\circ$)

$$\Gamma(Z^0 + e^+e^-\gamma) / \Gamma(Z^0 + e^+e^-) \sim 1.6\%$$

for inner bremsstrahlung using the Sterman-Weinberg formula(77). This has to be compared
with three events out of seventeen making the bremsstrahlung explanation quite unlikely ($<
0.3$ event expected).

A similar study has been performed by Berends et al.(78) which takes into account the
experimental conditions and gives an upper limit of 0.1 event (95% confidence level) for
the expected bremsstrahlung rate with $F > F_{\text{obs}}$ and $\Delta \alpha > \Delta \alpha_{\text{obs}}$.

7.3 Search for $W \rightarrow \ell \nu \gamma$

Both UA1\textsuperscript{(79)} and UA2\textsuperscript{(80)} have carried out careful searches for $W \rightarrow \ell \nu \gamma$. UA1 has found no events out of a sample of 55 $W \rightarrow e \nu$ and 18 $W \rightarrow \mu \nu$. UA2 has one $W \rightarrow e \nu \gamma$ event out of 37, but this occurs in the forward region and has a 4.5% probability of being a bremsstrahlung on the observed event. Furthermore the characteristics of the UA1 events, in terms of momentum-energy comparisons for electrons and energy loss in the calorimeters for muons, are entirely consistent with conventional behaviour (figure 64).

7.4 Possible explanations of the $Z \rightarrow \ell^+ \ell^- \gamma$ events

Many authors\textsuperscript{(81-83)} have considered alternative explanations for the events which, including bremsstrahlung, fall into four main classes

1) Bremsstrahlung
2) Composite $W$, $Z$\textsuperscript{(81)}
3) Excited leptons\textsuperscript{(82)}
4) Anomalous $ZZ \gamma$ or $Z\gamma\gamma$ couplings\textsuperscript{(82)}.

7.4.1 Composite $W$, $Z$\textsuperscript{(81)}

The basic idea is that the $W$ and $Z$ are not elementary but are composed of spin 1/2 or spin 0 constituents $\alpha$, $\beta$ giving rise to

$$W^+, W^0, W^- = \frac{\alpha \alpha - \beta \beta}{\sqrt{2}}, \frac{\alpha \beta}{\sqrt{2}}$$

$$\nu^0 = \frac{\alpha \beta}{\sqrt{2}}$$

For spin 1/2 constituents these would be $3S_1$ states, mathematically analogous to $\rho$, $\omega$. The $Z^0$ would be a mixture of $W^0$, $\nu^0$. $1S_0$ states $X^+$, $X^0$, $X^-$ and $U^0$ would also occur. The hard $\gamma$ rays would then result from transitions between $Z^0$ and $U^0$, the $U^0$ subsequently decaying into $\ell^+ \ell^-$. The non-observation of $U^0$ at PETRA places its mass greater than 47 GeV.

```
\begin{center}
\begin{tikzpicture}
\node at (0,0) {$W^-$};
\node at (1.5,0) {$Z^0$};
\node at (3,0) {$W^+$};
\node at (0,-1) {$U^0$};
\node at (0,-2) {$X^-$};
\node at (2,-2) {$X^0$};
\node at (4,-2) {$X^+$};
\draw (0,0) -- (1.5,0);
\draw (1.5,0) -- (3,0);
\draw (3,0) -- (4,0);
\draw (0,-1) -- (1.5,-1);
\draw (1.5,-1) -- (3,-1);
\draw (3,-1) -- (4,-1);
\end{tikzpicture}
\end{center}
```

Decays of $W^+ \rightarrow X^- \gamma$ and $Z^0 \rightarrow X^0 \gamma$ are approximately forbidden in this scheme.

For $M_{\gamma} = 50$ GeV an 8.4% $e^+e^-\gamma/e^+e^-$ ratio is expected with spin 1/2 constituents and 6.7% with spin 0 constituents. The scheme would also predict $\gamma\nu\nu$ and $\gamma\pi J$ decays each at a several times higher rate. The difficulty is that the $\gamma$ is not correlated with either
Figure 65 Dalitz plots according to various hypotheses for $Z \rightarrow l^+l^-\gamma$
events a) composite $Z$, b) excited lepton, c) anomalous couplings,

d) Bremsstrahlung, e) actual events.
lepton. The idea is in principle easily tested by a peak in the \( \ell^+\ell^- \) invariant mass.

7.4.2 **Excited leptons**\(^{(82)}\)

This idea is easily understood, i.e.

\[
Z \rightarrow \ell^+\ell^-
\]

\[\rightarrow \ell^+\ell^-, \; \ell^+\ell^-\]

It could imply composite leptons. However in this case the \( \ell^+\ell^- \) invariant mass should have a peak. Furthermore experiments at PETRA have ruled out \( m_e < 50 \text{ GeV} \)\(^{(84)}\), so only a high mass combination would be allowed. In this case the direction of the \( \gamma \) would not be strongly correlated with the direction of either lepton.

7.4.3 **Anomalous Z\(\gamma\), Z\(\gamma\) couplings**\(^{(83)}\)

Anomalous Z\(\gamma\) or Z\(\gamma\) couplings are most easily explained if the Z has a strong coupling to some constituents which would then couple with normal electromagnetic coupling to photons. This would also give a natural explanation to the large value of \( \sin^2 \theta_W \). The two processes can be distinguished through neutrino decays of the Z, which occur for Z\(\gamma\) only

\[
Z \rightarrow \gamma 'Z'
\]

\[\rightarrow e^+e^-, \; \nu\bar{\nu} \text{ etc.}\]

\[
Z \rightarrow \gamma '\gamma'
\]

\[\rightarrow e^+e^-, \; \mu^+\mu^-\]

Again there is no explanation of why the \( \gamma \) should be close to one of the leptons.

7.4.4 **Conclusion on Z + \( \ell^+\ell^- \) decays**

A Dalitz plot Monte Carlo study has been carried out by Barger et al.\(^{(76)}\) (figure 65) who also plot the actual data for the three events. It is obvious that topologically they are most consistent with bremsstrahlung for the reasons alluded to above. However, the rate remains a factor 10 - 20 too high and one must await further experimental results before any conclusion can be drawn.

7.5 **Charm production in jets**

UA1\(^{(85)}\) have followed the now standard procedure of using the mass difference between D* and D-mesons in order to isolate events containing charm, i.e.

\[
D^{*+} + D^{0} \rightarrow K^-\pi^+\pi^+
\]

and the charge conjugate process. The mass difference

\[
\Delta M = M(K^-\pi^+\pi^+) - M(K^-\pi^+)
\]

peaks at 147 MeV. The data sample (113 nb\(^{-1}\)) comes from two triggers

1) "electron" trigger \( E_T > 10 \text{ GeV} \)

2) global \( E_T \) trigger \( E_T > 60 \text{ GeV} \)

Figure 66 shows the relevant mass plots. 22 D* events on a background of 7 are observed out of 3 \( \times 10^3 \) jets with the following cuts.
Figure 66  a) $M(K\pi)$ distribution in the region of $M(D^0)$ for all events.  

b) $M(K\pi)$ distributions for $146 < M < 148$ MeV/c$^2$.

Figure 67  Mass of dimuon pairs with $p_T^\mu > 5$ GeV/c excluding $Z$ events.
\[ 16 \leq E_{T,jet} \leq 20 \text{ GeV} \]
\[ |\eta| < 1 \]
\[ \phi > 45^\circ \text{ from the horizontal.} \]

Applying branching ratios etc. gives
\[
\frac{N(p^{\pm})}{N(\text{jet})} = 1.2 \pm 0.2 \pm 0.7
\]

with a rather soft observed fragmentation function\(^{(85)}\). The result is, of course, sensitive to the branching ratio used but nevertheless implies that a very large fraction of what should be mainly gluon jets decay into charmed particles. In one sense this is bad news because it means that the observation of charm is not a distinctive signature, i.e. it does not act as a clean tag for primary heavy quark decays.

7.6 Dimuon events in UA1

From a sample of 108 nb\(^{-1}\) UA1\(^{(86)}\) have observed 15 dimuon events with \(p_t^\mu > 5 \text{ GeV/c}\) of which 5 are \(Z + \mu^+\mu^-(\gamma)\). Of the ten remaining events 7 have unlike sign and 3 like sign for the two muons. Furthermore they have an unusually high number of accompanying strange particles. Roughly, their characteristics are as follows:

- 7 \(\mu^+\mu^-\) with jets and \(\Lambda^0\) or \(K^0\)
- 1 without jet, with \(\Lambda^0\) \text{ and } \(K^0\)
- 3 with jets

- 3 \(\mu^+\mu^-\) with jets
- \(\mu^+\mu^-\) without jets, with \(\Lambda^0\)
- \(\mu^-\mu^+\) without jets, with \(\bar{\Lambda}^0\).

The mass plot for the ten events is given in figure 67.

The main sources of dimuons expected are

i) Drell-Yan (~ 2 events)

ii) Heavy flavour decays (semi leptonic)
   a) \(\overline{c}c + Q\bar{Q}X; \ Q + t, b, c\)
   b) \(pp' + ggX; \ g + b\bar{b}, c\bar{c}\)
   c) W and Z decays e.g. \(W + t\bar{b}\)

\(W + t\bar{b}\) decays would give like sign dimuons but can easily be estimated to be much less than one event on the basis of the observed \(W + \mu\nu\) rate (14 events). Similarly, Z decays are negligible. Process a) gives unlike sign dimuons as primary decays, but like signs can be produced as a result of cascade decays. Process b) gives both like sign and unlike sign dimuons and there is already evidence of a high yield of charm decays in jets. It is therefore possible that the observed events have a number of different origins. However, it is interesting to consider \(B-B\) mixing in more detail as an origin of the like-sign pairs, particularly the ones with \(\Lambda^0\) and \(\bar{\Lambda}^0\).

7.7 \(B^0 - \bar{B}^0\) mixing

\(B^0 - \bar{B}^0\) mixing has been studied by Ali and Jarlskog\(^{(87)}\). Neutral \(B\)-mesons are of two types
\[ B^0_d = \bar{b}d \quad \bar{B}^0_d = \bar{b}d \]
\[ B^0_s = \bar{b}s \quad \bar{B}^0_s = \bar{b}s \]

As doubly weak transitions can occur between \( B^0 \) and \( \bar{B}^0 \) (as in \( K^0, \bar{K}^0 \)) the mass eigenstates are non-degenerate

\[ B^{0, 2} = \frac{1}{\sqrt{2}} (B^0_d \pm \bar{B}^0_d) \]
\[ B^{0, L} = \frac{1}{\sqrt{2}} (B^0_s \pm \bar{B}^0_s) \]

The fact that \( b + u \) transitions are strongly suppressed (88) compared to \( b + c \) transitions leads to a larger mass difference \( \Delta M \) between \( B^0 \) and \( B^0 \) than between \( B^{0, 2} \) and \( B^{0, L} \). In fact, \( \Delta M / \Gamma \) is estimated to be \( \sim 2.5 \) meaning that several oscillations occur during the lifetime. As the lifetime is very short all \( B \)-mesons decay in the apparatus and one integrates from \( t = 0 \) to \( t = \tau \) to find the amount of mixing which is given by

\[ r = \text{prob.} (B + \bar{B}) = \text{prob.} (\bar{B} + B) \sim \frac{(\Delta M / \Gamma)^2}{2 + (\Delta M / \Gamma)^2} \]

giving \( r \sim 0.75 \). Consequently we expect

\[ \frac{B^0}{B^0} + \bar{\pi}^+ \pi^- \]
\[ \frac{\bar{B}^0}{B^0} + \bar{\pi}^- \pi^+ \sim 0.75 \]

and similarly for \( \bar{B}^0 \). Hence, after allowance is made for other \( B \) meson decays Ali and Jarlskog find that

\[ \frac{b\bar{b} + u \bar{u} \pm X}{b\bar{b} + d \bar{d} \pm X} \sim 0.2 \]

Furthermore, the \( \bar{u} \pm \bar{u} \) are expected to be accompanied by \( \Lambda^0 \) (\( \bar{\Lambda}^0 \)) spectator hyperons, and Monte Carlo calculations indicate little jet activity within the experimental cuts (87).

Thus it is tempting to identify two of the \( U \Lambda \) like sign events as examples of \( B \bar{B} \) mixing. However, gluons decaying into \( \bar{c}c \) pairs can also give like sign dimuons as well as strange particles (89). Furthermore the rates are of the right order of magnitude in both cases (87, 89).

7.8 Conclusions on dimuon events

In conclusion, there are possibly at least two sources of dimuon events in addition to Drell-Yan, namely \( bb \) production and \( gg \) production with subsequent fragmentation into \( cc \).
Figure 68 Topology of the four events from UA2.

Figure 69 Mass plot for events A, B and C assuming the eν come from a W.
pairs, but the detailed features are not easily explained. Further data are clearly required. However, it is already clear that flavour tagging is not likely to be an easy way to identify primary processes. UA1 have more dimuon data with $p_T^u > 3$ GeV/c which show a similar fraction of like sign events but will not be discussed here. Such events will be an area of great interest in future runs.

7.9 High mass $e\nu$ Jet events (UA2)

UA2 have observed 4 events with the following characteristics:

$$p_T^e > 15 \text{ GeV/c}; \quad p_T^\nu > 25 \text{ GeV/c}; \quad E_T^J > 30 \text{ GeV}$$

They are shown in figure 68, plotted in the transverse plane. An estimate of the background due to misidentified "electrons" is 0.45 event. Event D could be a $qq$ pair where one quark decays semi-leptonically. The remaining events have large transverse masses for the $e\nu$ system (assuming that the missing transverse energy is carried by a single neutrino).

<table>
<thead>
<tr>
<th>Event</th>
<th>$M_{e\nu}^T$</th>
<th>$GeV/c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>56$\pm$4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>81$\pm$3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>82$\pm$4</td>
<td></td>
</tr>
</tbody>
</table>

Furthermore, the total transverse mass of the three events peaks around 170 GeV/c² (figure 91), approximately twice the W mass. In fact they are all consistent with $W^+W^-$ associated production where one W decays to jets. However, there is no known mechanism that could produce such a high rate. The rate is also too high to be simply a high $p_T^W$ accompanied by a QCD recoil jet. UA1 has not detected any events of the same type. However, the statistics are low and more data are clearly needed. If confirmed, the events would be an indication of physics beyond the standard model.

7.10 Monojets and $\gamma s +$ missing $E_T$ (UA1)

A sample of 113 nb$^{-1}$ was used to search for events with missing transverse energy using the following selection criteria:

$$E_T^{\text{Jet}} > 25 \text{ GeV}; \quad E_T^{\gamma} > 10 \text{ GeV}$$

$$E_T^{miss} > 15 \text{ GeV} \quad \text{and} \quad > 4\sigma$$

Here $E_T^{miss}$ is a "vector" quantity and its empirical standard deviation $\sigma = 0.7\sqrt{\sum T}$ where $\sum T$ is the scalar sum of the calorimeter transverse energies. The result of the search is:

50 W + $e\nu$
2 $\gamma +$ $\nu$
17 J "$\nu$"
5 2J "$\nu$"
3 2J "$\nu$"

The 2 $\gamma$-events and 17 single jet events are considered further here. To reduce QCD
Figure 70  Distribution of missing transverse energy squared for monojet events.

Figure 71  Electron polar vs azimuthal angle plot for $W + e\nu$ events (solid circles) and the two photon candidates (open circles). The dashed line is the limit for > 20 points on a track.
background, a further cut is applied to the 17 single jet events which removes events with
\( \cos \Delta \phi < 0.8 \), where \( \Delta \phi \) is the azimuthal angle between the jet and the \( E_T \) vector of
the rest of the event. Of those that remain, six have \( E_T^{\text{miss}} > 30 \text{ GeV} \) and are labelled
A, B, C, D, E, F in figures 70 and 72. Figure 70 shows the missing \( E_T \) for the
six events in a plot where the cut has been reduced to 2\( \sigma \). The solid curve shows the
expected background from jet fluctuations and the dashed curve the calculated background
from \( W \rightarrow \tau\nu \) decays where the \( \tau \) produces a "jet". Event F is consistent with \( W \rightarrow \tau\nu \) and is
not considered further. The remaining events A-E all have large missing \( E_T \) and all have
'jet-\nu' masses greater than \( m_W \). Event A is particularly spectacular in having a high
energy muon in the jet. Their features are summarised in the following table.

<table>
<thead>
<tr>
<th>Event</th>
<th>Jet ( E_T ) (GeV)</th>
<th>Missing ( p_T ) (GeV)</th>
<th>( m_T^{(\text{jet}, p_T^{\text{missing}})} ) (GeV)</th>
<th>Charged Multiplicity</th>
<th>Invariant mass of charged particles (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25 (71 inc ( \mu ))</td>
<td>24 \pm 4.8 (66.48 inc ( \mu ))</td>
<td>130 \pm 16 inc ( \mu )</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>B</td>
<td>48</td>
<td>59 \pm 7</td>
<td>106 \pm 12</td>
<td>3</td>
<td>0.79 \pm 0.12</td>
</tr>
<tr>
<td>C</td>
<td>52</td>
<td>46 \pm 8</td>
<td>97 \pm 17</td>
<td>1</td>
<td>Other unreconstructed tracks?</td>
</tr>
<tr>
<td>D</td>
<td>43</td>
<td>42 \pm 6</td>
<td>85 \pm 12</td>
<td>4</td>
<td>3.14 \pm 0.38</td>
</tr>
<tr>
<td>E</td>
<td>46</td>
<td>41 \pm 7</td>
<td>87 \pm 14</td>
<td>2</td>
<td>Other unreconstructed tracks</td>
</tr>
<tr>
<td>F</td>
<td>39</td>
<td>34 \pm 7</td>
<td>73 \pm 14</td>
<td>2</td>
<td>0.52 \pm 0.06</td>
</tr>
<tr>
<td>H</td>
<td>54(( \gamma ))</td>
<td>40 \pm 4</td>
<td>93 \pm 5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The azimuthal and polar angles of the "photons" in the 2\( \gamma \)-events are shown in
figure 71. The '\( \gamma \)' from event C falls in a region of the central detector which is
insensitive to charged tracks and therefore could be an electron from \( W \) decay. The dashed
curves outline regions with at least twenty points per track so event H is definitely
neutral and could be due to one or more photons with missing energy. Thus we are left
with 5j'\nu' and 1 \( \gamma'\nu' \) events all with \( m > m_W \). The missing \( E_T \) is plotted against the \( E_T \) of
the jet or \( \gamma \) in figure 72. A more detailed discussion has been given by J. Rohlf(92). A
typical event is shown in Figure 73.
Figure 72 Scatter plot of missing $E_T$ vs jet or photon(s) $E_T$ from events A - H.

Event B

Figure 73 One of the monojet events. The arrow shows the direction of the missing transverse energy.
7.10.1 Possible interpretations of the events

Two basic interpretations have been considered for the J'ν' events:

a) Excited quarks\(^{(93)}\)

b) Supersymmetry\(^{(94)}\)

The first hypothesis has been considered by Kuhn and Zerwas\(^{(93)}\) and implies composite quarks. The process is

\[
\begin{align*}
\bar{p}p + q^* &\rightarrow X \\
&\rightarrow qZ; Z \rightarrow \nu\bar{\nu}
\end{align*}
\]

The fact that \(Z + \nu\bar{\nu}\) is six times more probable than \(Z + e^+e^-\) could explain why no \(e^+e^-\) event has been seen. Similarly the process could explain the UA2 events as \(q^* + q\bar{\nu}; \bar{W} + e\nu\). However, one would also expect \(q^* + q\gamma\) events. More statistics would obviously test this hypothesis. Such an explanation would require \(m_{q^*} > 2m_W\), similar to the masses of the UA2 events. Under this hypothesis the events are indeed consistent with a common mass of \(\sim 170\) GeV/c\(^2\) \(^{(92)}\).

The explanation based on supersymmetry has been considered by a number of authors\(^{(94)}\). The events could be either gluino or squark decays. The basic process for gluino production is

\[
\begin{align*}
\bar{p}p + \tilde{g} &\rightarrow X \\
&\rightarrow q\tilde{q} \gamma
\end{align*}
\]

The gluino is assumed to decay to \(q\bar{\nu}\gamma\) where the photino escapes without interaction, giving rise to the missing energy. The events have several jets but yield predominantly single jet events after the application of experimental cuts. If the 5 events are taken as an upper limit on gluino production, then \(m_{\tilde{g}} > 40\) GeV/c\(^2\) to give the observed cross section (0.06 nb).

The production of squarks is by

\[
\begin{align*}
\tilde{g} + \tilde{q} &\rightarrow \tilde{q}\tilde{q} \\
\tilde{q} + \tilde{g} &\rightarrow \tilde{q}\tilde{q} \\
\tilde{q} + \tilde{g} &\rightarrow \tilde{q}\tilde{g}
\end{align*}
\]

In order to explain the events as squark (\(\tilde{q}\)) decays it is assumed that \(\tilde{q} \rightarrow q\gamma\). This will only be the main decay mode if \(m_{\tilde{g}} > m_q\). Again the five events (σ<0.06 nb) are consistent with \(m_{\tilde{g}} \gtrsim 40\) GeV/c\(^2\). Furthermore, the narrowness of the jets favours this interpretation.

The type of explanation offered for the \(\gamma\) event is an unusual fragmentation of a jet. However, \(Z + \nu\nu\gamma\) could not be entirely ruled out. More exotic explanations for the events have also been considered\(^{(95)}\).

7.10.2 Conclusion on UA1 monojets

The event are not explicable within the standard model. Excited quarks should have other decay modes which would be observed with more data. Supersymmetric particles would
have to have masses in the 40 GeV/c² region to be consistent with the observed cross sections. Squarks are favoured over gluinos by the narrowness of the jets (and absence of other jets) but this hypothesis only works if \( m_R > m_\tilde{q} \) so that the squark decays into a quark and photino. However, the events may have a completely different origin and obviously provide a topic of great excitement for forthcoming collider runs.

7.11 Evidence for the Top Quark (UA1)

Data from the 1983 run (120 nb⁻¹) have been used to study events with an electron + jets or a muon + jets in which the lepton is isolated from the jets. Such events should include candidates for \( t \rightarrow tVb \) as the large t mass (> 22.5 GeV/c² from PETRA) would cause the three decay products to be produced with large relative transverse momenta.

More specifically the decay \( W \rightarrow t\bar{b} \) would give 2 jets, a lepton and missing energy with a total mass consistent with that of the W. The experimental task is to demonstrate that any such 'top' candidates are distinguishable from background due to QCD processes, e.g. \( pp \rightarrow b\bar{b}X \) where the b or \( \bar{b} \) decays semi-leptonically. Only the general principles of the background study will be described here. For full details the reader is referred to the published work. Unlike in the W search, the full power of the apparatus proved to be needed to isolate the candidates.

7.11.1 Electron selection

Electron candidates were selected by the following cuts

1) \( E_{\text{T}}^{\text{em}} > 12 \) GeV with \( E_{\text{T}}^h < 0.2 \) GeV,

2) \( p_T > 7 \) GeV/c for matching CD track,

3) isolated i.e. the 'electron' having > 90% of the \( E_{\text{T}} \) in \( \Delta R < 0.7 \) around the track \( (\Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2}) \).

The search yielded 49 W + eν events which were used as a calibration sample for electrons and 152 'e' + jets. Of those 152 events 43 with \( \gamma + e^+e^- \) conversions were removed by detecting the second electron. Next a momentum-energy comparison \( |1/p - 1/E| < 3\sigma \) and a \( \chi^2 \) test on the shower profile through the four layers of the electromagnetic calorimeters were applied to remove \( \pi^\pm/\pi^0 \) overlaps. Together with a further isolation cut, requiring \( \sum p_T \) (charged tracks) < 1.0 GeV/c and \( \sum E_{\text{T}} < 1.0 \) GeV in \( \Delta R < 0.4 \) around the electron, this reduced the sample to 19 events on which the final analysis was performed. Using as jet definition \( E_{\text{T}} \) (1st jet) > 8 GeV, \( E_{\text{T}} \) (other jets) > 7 GeV the events fall into the following classes.

- e + 1 jet : 14 events
- e + 2 jets : 3 events
- e + 3 jets : 2 events

As the principal source of background is expected to be residual \( \pi^\pm/\pi^0 \) overlaps a study of the background shape was made using \( \pi^0 \) + jets events with similar selection criteria for
b) Equivalent plot for the electron or muon + ≥ 2 jets events.

Figure 74. a) Measured shape of the QCD background from μ + ≥ 2 jets events.
the '$\pi^0$'. Secondly, from the measured flux of $\pi^\pm (+ m\pi^0) + \text{jets}$ events, the probability of an overlap simulating an electron was estimated. It turns out that the 'background' events populate a two-dimensional plot of $E_T^{\text{out}}$ vs $\cos \theta^*_{J_2}$ (where $E_T^{\text{out}}$ is the transverse momentum of the electron relative to a plane containing the beam and jet$_1$, and $\cos \theta^*_{J_2}$ is the centre of mass angle of the second jet with respect to the beam direction) in quite a different way from the candidate events (Figure 74). The background peaks at $\cos \theta^*$ close to $\pi$, as expected for gluon bremsstrahlung, and low values of $E_T^{\text{out}}$. The resulting background estimate is $< 0.1$ event in the region occupied by the electron + 2 jet events ($E_T^{\text{out}} > 8 \text{ GeV}, |\cos \theta^*_{J_2}| > 0.73$).

### 7.11.2 Muon selection

The selection of muon + jets events is somewhat simpler. Using the same jet definition, events with a muon + 1 jet having $p_T^\mu > 12 \text{ GeV/c}$ were selected with an isolation criterion that the muon should carry $> 90\%$ of the $p_T$ (charged tracks) and $> 80\%$ of $E_T$ in a radius $\Delta r < 0.4$. The 14 $W + \mu \nu$ events were used as a calibration sample. Twelve $\mu + \text{jets}$ events are found:

- $\mu + 1 \text{ jet} : 7 \text{ events}$
- $\mu + 2 \text{ jets} : 4 \text{ events}$
- $\mu + 3 \text{ jets} : 1 \text{ event}$

$\pi + \mu$ and $K + \mu$ decay background dominates but this is calculable from the observed hadron spectrum, assuming 25% $Ks$ and 50% $\pi s$. The resulting background estimate is 0.4 event for the $\mu + 2 \text{ jet}$ events. One of them has $\cos \theta^*_{J_2} = 0.93$ and is most likely background. It has therefore been removed. The background for the remaining three is estimated to be $< 0.1$ event.

### 7.11.3 Consistency with the $W + t\bar{t}$ hypothesis

The topological features of one of the 3 electron and 3 muon events with 2 jets are shown in figure 75. Figure 76 is a two-dimensional plot of the mass of $\ell W_1 J_2$ versus $\ell W_2$. Here the transverse momentum of the neutrino is used and the lower energy jet ($J_2$) is assumed to come from the $t$-decay. The alternative assumption produces a much broader mass peak for $\ell W_1$. Corrections have been applied to the jet energies using a Monte Carlo method as described in section 5.6. The events peak at a mass consistent with the $W$ (see curve) and give $m_t = 40 \pm 10 \text{ GeV/c}^2$ assuming they come from $W + t\bar{t}$. Furthermore the rates agree with expectation after allowance for the leptonic branching ratio of the $t$ ($\sim 12\%$) and the experimental cuts ($96\%$).

### 7.11.4 Conclusion on top candidates

While it is not possible with the present statistics to be totally certain that the top quark has been isolated, the following remarks can be made.

1) A signal has been observed for an isolated lepton + $> 2 \text{ jets}$.
2) The 2-jet events cluster around the $W$-mass.
3) They are consistent with $W + t\bar{t}$ with $t + \ell\nu\bar{b}$. 
iv) If so then $30 \text{ GeV/c}^2 < m_{t} < 50 \text{ GeV/c}^2$.
In addition the events with a lepton + 3 jets could be examples of \texttt{tt QCD production}.

7.12 Prospects for observing Higgs scalars at the collider

Higgs mesons couple to heavy objects preferentially. However, the prospect for observing Higgs particles is not particularly promising unless the Higgs mass is rather low\(^{(96)}\). For $m_H = 10 \text{ GeV}$, $\frac{\sigma(W + \text{Higgs})}{\sigma(W)} \approx 10^{-2}$ and the experimental signature rather difficult to interpret. Events involving Z's would be cleaner

$$\frac{\sigma(Z + t^+ t^- H)}{\sigma(Z + t^+ t^-)} \approx 10^{-2} \text{ for } m_H = 10 \text{ GeV}$$
$$\approx 10^{-4} \text{ for } m_H = 40 \text{ GeV}$$

The diagram is

The initial $Z$ may be virtual and the intermediate $Z$ real, or vice versa. Given that only $10^3 Z + t^+ t^-$ decays are expected at the upgraded collider the sensitivity is however, very low. If $m_H > 2m_t$ but $< 2m_t$ the Higgs will decay to $bb$. A microvertex detector in UA1 could detect decay vertices of $b$ or $c$ in a reasonable proportion of the events; e.g. 2 vertices would be detected in 37% of events for $m_H = 30 \text{ GeV/c}^2$ according to a Monte Carlo calculation\(^{(98)}\). However, we have already noted that gluon jets fragment into charmed particles at high rates which could give a high background.

8. CONCLUSIONS AND FUTURE PROSPECTS

The CERN proton-antiproton collider has proved to be an extremely rich source of physics. Not only has it fulfilled all the expectations based on the standard model of the electroweak interaction in the discovery of the $W$ and $Z$ particles with the predicted masses and properties and the probable discovery of the top quark, but has turned out to be an excellent testing ground for QCD as clear jet events are easily observed.

In fact, the ease with which the various processes can be isolated has exceeded even the most optimistic forecasts. It is a fitting tribute that Carlo Rubbia and Simon van der Meer have been awarded the 1984 Nobel Prize for Physics, an honour that is fully deserved for the realisation of this adventurous project. However, perhaps it will also prove to be the basis for the next major step in the study of elementary particles as there are already hints of physics beyond the standard model. The UA1 monojets and UA2 high mass electron-jet events with missing energy have no conventional explanation and may be the first signs of supersymmetry or compositeness. In addition there is a variety of other phenomena - dimuons, radiative Z-decays, high jet multiplicity with Zs, etc. - which may also have exotic explanations.
What can we look forward to in the next few years? The collider is already scheduled
to run in 1984, 1985 and 1986 with a luminosity at least comparable to the best obtained
in 1983 and an increased energy of 630 GeV. In fact, $2.5 \times 10^{29}$ cm$^{-2}$s$^{-1}$ has already been
achieved giving 10 nb$^{-1}$ per day. The scheduled running should therefore produce at least
a five-fold increase in statistics, sufficient to confirm many of the phenomena for which
there are hints. UA1 has improved muon detection and a micro-vertex detector which will
help in unravelling the new physics. A special ramped collider mode, cycling up to 900
GeV, has been tested and will run in 1985 with modest luminosity ($10^{26}$ cm$^{-2}$s$^{-1}$) enabling
UA1 and UA2 to search for phenomena that might have a higher threshold energy such as the
Centauro events.

In the longer term, the antiproton yield is to be increased by a factor of ten in
1987 by the addition of an antiproton collector ring, ACOL. Both UA1 and UA2 are to
have major upgrades to their calorimeters. UA2 will close the end regions, thus
improving the missing energy determination and UA1 will replace the central
electromagnetic calorimeters by a much more finely divided set of uranium calorimeters.
Uranium provides compensation for the different response to hadrons and electromagnetic
particles through the fission process and, since it will absorb most of the hadron energy,
will give a factor of two improvement in jet and missing energy resolution. The improved
granularity will help the study of electrons in jets, extending the physics capability.
Thus we can look forward with great excitement to 50 times the present data by 1990,
approximately the time when LEP will be fully operational.

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References


579; A. Salam, Proc. 8th Nobel Symposium, Aspenasgarden, Almqvist and Wiskell;
Stockholm (1968) 367.
3. UA6 proposal; CERN/SPSC/80-63/P148.


7. UA4 proposal; CERN/SPSC/78-105/P114.

8. UA5 proposal; CERN/SPSC/78-70/P108.


P. Kroll ibid. p295. (See also refs. 11, 17).
33. P. Carlson, 4th Topical Workshop on pp collider physics, Bern, 5-8 March, 1984; CERN 84-09.
34. A. Capella and A. Krzywicki, Orsay preprint LPTHE 83/12.
47. F. E. Paige and S. D. Protopopescu, ISAJET, BNL 31987.
48. see ref. 40.
        G. Cohen-Tannoudji et al., preprint sPh.T/70 (1982).
55. see reference 51.
56. G. Jarlskog, Lectures at the school.
57. D. Haidt, lectures at this school.
58. E. Rademacher, CERN EP/84-41 to be published in Progress in
        Particle and Nuclear Physics.


   Recent calculations have been done by:


64. F. Paige, BNL-27066 (1979).


   Submitted to Z. Phys. C.


72. CHARM II experiment at the CERN SPS, WA79.

   see also ref. 65.


76. V. Barger, H. Baer and K. Hagiwara, Wisconsin preprint MAD/PH/176.


and others.


F. M. Renard, Phys. Lett. 139B (1984) 449,

84. S.L. Wu, DESY 84-028, To be published in Physics Reports.


Other relevant papers are by:
F. Halzen et al., Wisconsin preprint, MAD/PH/171
Durham preprint, DTP/84/8.

89. F. Halzen and F. Herzog, Wisconsin preprint MAD/PH/985.


    J. Ellis and M. Sher, CERN-TH 3968/84.


97. E. Eisenhandler et al., UA1 technical note, TN 84-64.


99. Design study of an Antiproton Collector for the Antiproton Accumulator, edited by
    E.J.N. Wilson, CERN 83-10.

100. Proposal to improve the performance of the UA2 detector, CERN/SPSC 84-30, SPSC/P93
     Add. 2.

101. Technical report on the design of a new combined electromagnetic/hadronic calorimeter
     for UA1, CERN/SPSC 84-72, SPSC/P92 Add. 5.


103. J. Ellis, Proc. 4th Topical Workshop on pp collider physics, Bern, 1984; CERN EF
     84-09.
EXPERIMENTAL TESTS OF GAUGE THEORIES

D. Haidt

Lecture 1: Phenomenology
Lecture 2: Purely leptonic interactions
Lecture 3: Semileptonic neutral current interactions
Lecture 4: Semileptonic charged current interactions (selected topics)
Lecture 5: Strong interactions

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Appendix
Lecture 1: Phenomenology

1. Introduction
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   - The forces
   - The fermion representation
3. The electroweak Lagrangian
4. The currents
5. The current x current form
6. Tests of the electroweak Standard Model
7. The first test
8. Free parameters in the Standard Model
9. Choice of basic couplings
1. **INTRODUCTION**

The experimental and theoretical efforts of the last two decades merged into a comprehensive picture of elementary particle physics. A central rôle played the theoretical concept of local gauge theories. The forces acting between the elementary particles are then arising through the exchange of spin 1 gauge bosons. It became gradually clear that the leptons and quarks constitute the elementary building blocks of all matter and that they appear systematically ordered in families. The concepts 'flavor' and 'color' proved particularly useful. The experimental observation of weak neutral currents - just a decade ago - was the discovery giving substance to the idea that weak and electromagnetic phenomena have a common origin. It was one of the driving forces in the research program highlighted by the observation of interference effects between the weak and electromagnetic currents and finally the observation of the weak gauge bosons with the predicted mass. The other main research line centered on the investigation of the strong force. This field progressed considerably since, also a decade ago, the rôle of the exact local color gauge group was recognized.

This series of five lectures is intended to provide the experimental basis to the theoretical courses on gauge symmetries delivered by C. Jarlskog\(^1\) and R. Petronzo\(^2\). The framework will be the standard model. The experimental material is taken mainly from lepton-hadron and \(\text{e}^+\text{e}^-\)-experiments. Results from the CERN \(\overline{\text{p}}\text{p}\)-collider are presented in the lectures by J. Dowell\(^3\). The same subject "Test of Gauge Symmetries" was also treated in previous CERN schools\(^4\) and other schools\(^5\). There is necessarily substantial overlap. Choosing the Standard Model as framework offers the possibility of a simple and organized presentation of the rich material. But it should not be forgotten that the present picture grew up step by step and remarks here and there shall illustrate this. The other advantage of the present form of the standard model concerns the formulation of the critical questions leading beyond the tested ground, for instance why are weak interactions lefthanded or is the multitude of quarks and leptons hinting at yet another substructure.

2. **THE STANDARD MODEL**

The standard model is characterized by the

- group structure \(\text{SU}(2) \times \text{U}(1) \times \text{SU}(3)\)
- fermion representations
- spontaneous symmetry breaking and HIGGS representation
a) THE BUILDING BLOCKS

On a distance scale of $10^{-15} \text{ cm}$ the fundamental building blocks of matter are the spin 1/2 fermions. There are many of them. They can be ordered in a periodic system, as shown in table 1. All known particles are classified vertically in families or generations, of which the first one is explained in more detail.

Table 1: Periodic System of the Spin 1/2 Fermions

<table>
<thead>
<tr>
<th>PARTICLES</th>
<th>FORCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$</td>
<td>only weak</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>weak and electromagnetic</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>weak, electromagnetic</td>
</tr>
<tr>
<td>$\mu$</td>
<td>and strong</td>
</tr>
<tr>
<td>$\tau$</td>
<td></td>
</tr>
</tbody>
</table>

in table 3, and horizontally in groups of equal electric charge. Most striking are the symmetry between the leptons and quarks and the regular mass pattern. Neutrinos are the particles of lowest mass in each family, in fact it is still an open question whether they are exactly massless, as will be assumed in these lectures. The properties of the $\nu_\tau$ are inferred from decays. So far no reaction induced by a $\nu_\tau$ has been observed.

The first fundamental fermion was the electron discovered at the end of the last century. In order to explain the nuclear $\beta$-decay experiments PAULI postulated 1929 the neutrino. It took some time to recognize that nucleons and mesons were in fact composite particles. Only in the 70ies the periodic structure of leptons and quarks became evident. As a consequence of the periodicity a new quark was anticipated at the marked place in the third family with predicted properties - except for its mass. The postulated top-quark was announced to exist just at the beginning of this School. It is amusing to think of the historical parallel, when 1871 an element, called eka-Silicinium, was predicted to occupy a yet empty place in the periodic table of MENDELEYEV and the subsequent discovery 15 years later of this element called then Germanium.
Sofar there are no experimental indications of fermions beyond the ones in the three families. The Standard Model does not tell how many families exist.

b) THE FORCES

As indicated in table 1 three types of forces between the pointlike, spin 1/2 fermions are distinguished according to their strength. Within the Standard Model all three forces are assumed to arise from a local gauge symmetry. The forces are then mediated by vector gauge bosons. The crucial feature is the locality of the gauge symmetry fulfilled by the Lagrangian.

Table 2: The types of forces

<table>
<thead>
<tr>
<th>Local Gauge Symmetry</th>
<th>Force</th>
<th>Intermediate Vector Boson</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2)</td>
<td>weak</td>
<td>$W^+ \ W^- \ Z^0$</td>
</tr>
<tr>
<td>U(1)</td>
<td>electromagnetic</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>SU(3)</td>
<td>strong</td>
<td>$g_1, g_2, \ldots g_8$</td>
</tr>
</tbody>
</table>

Contrary to U(1) the algebrae SU(2) and SU(3) are nonabelian, i.e. not commutative. Therefore, the 3 respectively 8 intermediate spin 1 bosons have the property of coupling to themselves. The electromagnetic force is of infinite range, because the photon is massless. The weak force, on the other hand, has short range requiring massive mediators and thus a broken symmetry. The situation is again different in the case of the strong force which is assumed to be mediated by massless gluons and nevertheless of finite range (confinement).

c) THE FERMION REPRESENTATION

The three generations are replicas of each other regarding their symmetry properties. It is therefore sufficient to show the multiplet structure of the first generation under the gauge groups SU(2) and SU(3).
Table 3: MULTIPLE STRUCTURE UNDER SU(2) AND SU(3)

<table>
<thead>
<tr>
<th>(V^e_e)_L</th>
<th>(U^u^f_d)_L</th>
<th>(U^d^w_d)_L</th>
<th>(U^d^b_d)_L</th>
<th>SU(2) doublet</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_R</td>
<td>u^R_u^w_u^b_R</td>
<td>d^R_d^w_d^b_R</td>
<td>SU(2) singlet</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SU(2) singlet</td>
<td>SU(2) singlet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SU(3) singlet</td>
<td>SU(3) triplet</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Left- and righthanded fermions behave differently under SU(2). Each fermion can be decomposed uniquely into a left- and a righthanded spinor:

\[ \psi_L \equiv \frac{1}{2} (1 + \gamma_5) \psi \quad \text{and} \quad \psi_R \equiv \frac{1}{2} (1 - \gamma_5) \psi \]

with

\[ \psi = \psi_L + \psi_R \quad ; \text{note:} \quad (1 + \gamma_5)(1 - \gamma_5) = 0 \]

Should experiments prove the neutrino to be massive, then there would be also a \( \nu_R \).

3. THE ELECTROWEAK LAGRANGIAN

In the Standard Model weak and electromagnetic phenomena are treated on the same footing. Both phenomena exhibit a local gauge structure. However, a new principle had to be introduced: spontaneous symmetry breaking and the HIGGS mechanism. The Lagrangian can be decomposed into a kinetic term, into a term responsible for electromagnetic interactions \((\gamma)\) and three terms for weak interactions \((W^+, W^-, Z)\), and finally into a term containing the HIGGS sector:

\[ L_{\text{GSW}} = L_\omega + L_{\text{em}} + L_{CC} + L_{NC} + L_H \]

free \( \gamma \quad W^\pm \quad Z \quad \text{HIGGS} \)

with

\[ L_{\text{em}} = g \sin \theta \quad J^\text{em}_\lambda A^\lambda \]

\[ L_{CC} = \frac{g}{2\sqrt{2}} \quad (J^\text{CC}_\lambda W^\lambda + \text{h.c.}) \]

\[ L_{NC} = \frac{g}{4\cos \theta} \quad J^\text{NC}_\lambda Z^\lambda \]
All three interaction Lagrangians have the same structure:

COUPLING CONSTANT x CURRENT x GAUGE BOSON FIELD

One identifies $gs \sin \theta$ with the electromagnetic coupling $e$:

$$e \equiv g s \sin \theta$$

This may be called the unification equation. It is evident, that weak and electromagnetic phenomena are not truly unified, since then only a single coupling constant would appear. Nevertheless, it is justified to talk about "electroweak" phenomena, since the three coupling constants can be expressed in terms of two, e.g. the electromagnetic coupling constant $e$, which is precisely measured, and the weak angle $\theta$, which is not yet precisely measured.

Two terms of the Lagrangian appearing in the simplest, nontrivial HIGGS sector

$$L_H = \left( \frac{g Y}{2} \right)^2 W_\lambda^+ W_\lambda \lambda + \frac{1}{2} \left( \frac{g Y}{2 \cos \theta} \right)^2 Z_\lambda Z_\lambda + \ldots$$

are given explicitly. They show which quantities are to be identified with the masses of the weak gauge bosons:

$$m_W \equiv \frac{g Y}{2} \quad \text{and} \quad m_Z = \frac{m_W}{\cos \theta}$$

The constant $v$ is the vacuum expectation value of the HIGGS field $\phi$ defined as $v \equiv \frac{1}{\sqrt{2}} \langle 0 | \phi | 0 \rangle$. All the other terms appearing in $L_H$ describe the HIGGS couplings to itself, to $Z$, $W^\pm$ and to the fundamental fermions.

4. THE CURRENTS

The explicite structure of the three currents is summarized in the following three equations:

$$J^{em}_\lambda = \sum_f Q_f \bar{\psi}_f \gamma_\lambda \psi_f$$

$$J^{CC}_\lambda = \sum_{\bar{q}, q} \bar{\psi}_\bar{q} \gamma_\lambda \left( 1 + \gamma_5 \right) \psi_q + \sum_{\bar{q}, q', q} \bar{\psi}_\bar{q} \gamma_\lambda \left( 1 + \gamma_5 \right) U_{qq'} \psi_{q'}$$

$$J^{NC}_\lambda = \sum_f \bar{\psi}_f \gamma_\lambda \left( g^e_v + g^e_A \gamma_5 \right) \psi_f$$
To the notation:

- \( f \) is the flavor index, i.e. runs over \( \nu_e, \, e, \, \nu_\mu, \, \mu, \, \nu_\tau, \, \tau, \, u, \, d, \, c, \, s, \, t, \, b \)
- \( \varepsilon = (e, \, \mu, \, \tau) \quad q = (u, \, c, \, t) \quad q' = (d, \, s, \, b) \)
- \( Q_f \) is the electric charge in units of \( e > 0 \)
- \( U = \) flavor mixing quark matrix (KOBA YASHI - MASKAWA matrix)
  - \( U \) is unitary, i.e. \( U U^\dagger = 1 \)
- \( g_{V,A}^f \) = vector and axialvector coupling constants of fermion \( f \) to the neutral weak gauge boson \( Z \), which depends only upon \( \sin^2 \theta \) (c.f. appendix)

The electromagnetic current is a pure vector current (V), whereas the weak charged current is of pure \( V - A \) type (the minus sign is conventional), i.e. only the lefthanded component of \( \psi_f \) is active in interactions due to \( W^\pm \) (c.f. p. 6). The weak neutral current is in general neither pure \( V \) nor pure \( A \).

5. THE CURRENT \times CURRENT FORM

Low energy weak phenomena are known to be well described by the following effective Lagrangian:

\[
\mathcal{L}_{\text{eff}} = \frac{G}{\sqrt{2}} \left( J^{CC}_\lambda (J^{CC}_\lambda)^\dagger + \frac{\rho}{2} J^{NC}_\lambda J^{NC}_\lambda \right)
\]

where \( G \) is the FERMI coupling constant and \( \rho \) a parameter measuring the overall strength of the weak neutral current with respect to the weak charged current. This effective form can be derived in the Standard Model in the limit \( |q^2| \ll m^2 \) (\( m \) is the relevant gauge boson mass). Consider the \( \tau^\text{nd} \) order process \( \nu_\mu e^- + \mu^+ \nu_e \):

![Diagram](attachment:image.png)
The amplitude of this process contains besides the weak currents the W-propagator:

\[-i \frac{\delta_{\lambda\kappa} q^\lambda q^\kappa}{m_W^2} \quad \frac{|q^2| \ll m_W^2}{q^2 + m_W^2 - i \varepsilon} \quad -i \frac{\delta_{\lambda\kappa}}{m_W^2}\]

Thus one obtains:

\[L^{CC} \rightarrow (\frac{g}{2\sqrt{2}})^2 \frac{1}{m_W^2} J^{CC}_\lambda (J^{CC})^*_\lambda\]

\[L^{NC} \rightarrow (\frac{g}{2\cos\theta})^2 \frac{1}{m_Z^2} J^{NC}_\lambda J^{NC}_\lambda\]

As a consequence, one reads off

\[\frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2} \quad \text{and} \quad \frac{\rho}{Z} = \frac{G}{\sqrt{2}} = \frac{g^2}{16\cos^2\theta m_Z^2}\]

thus \(\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta}\)

This consideration makes clear what the terms "low energy" and "weak" really mean. The masses of the weak gauge bosons set the scale for weak phenomena. So, all previous neutrino experiments are to be considered low energy experiments, since e.g. for a wide band neutrino experiment at the CERN SPS \(<q^2> \approx (7 \text{ GeV})^2\) which is small compared to \(m_W^2 \approx (80 \text{ GeV})^2\). Furthermore, it becomes clear, that weak interactions get "weak" due to the suppression factor \(1/m^2\), which is the remainder of the W or Z propagator. At sufficiently high energies weak and electromagnetic phenomena occur at comparable rate. In the Standard Model the parameter \(\rho\) equals 1 in lowest order as a consequence of the simplest choice or the HIGGS representation. This parameter is experimentally accessible.

6. TESTS OF THE ELECTROWEAK STANDARD MODEL

For practical purposes the most important claim is the renormalizability of the theory. In recent years processes have been predicted in next to leading order. The radiative effects turn out to be finite, but for the time being too small to be detected. However, a promising test seems to be
possible, provided a precision measurement of $\sin^2 \theta$ can be performed. This has been discussed in the context of the SPS Fixed Target Workshop\textsuperscript{11}). The test consists in comparing the mass of the weak gauge bosons predicted in terms of $\sin^2 \theta$ (including 1-loop corrections) with the value measured directly in experiments at the CERN SppS collider (see sect. 9 below).

The tests of the Standard Model fall under two heads:

a) the gauge sector
   - existence of 4 gauge bosons with masses predicted
   - gauge couplings to fermions
   - gauge self couplings
   - fermion representations, existence of t-quark, $\ell$-q symmetry, universality

b) the HIGGS sector
   - existence of a neutral spin 0 boson
   - $\rho$-parameter
   - $H$ couplings to fermions, $W, Z$ and itself.

All tests described in lectures 2, 3 and 4 are based on "low" energy experiments ("low" in the sense mentioned above). This implies that many aspects listed here remain untested. Nevertheless, precisely the low energy experiments played the crucial rôle in establishing the current form of the standard model. The recent tests performed at the $\bar{p}p$ collider, in particular those related to the discovery of the intermediate vector bosons $W^\pm, Z$, are discussed in J. Dowell's lectures\textsuperscript{3}). Before entering into the description of the experimental tests it may be worthwhile to sketch the starting-point of the Standard Model and in particular the discovery of the weak neutral currents in the GARGAMELLE neutrino experiment.

7. THE FIRST TEST

The situation of weak interaction physics in the 1960ies can be summarized as follows:
- successful description of low energy ($\sqrt{s} < 4$ GeV) weak phenomena (V-A theory, CABIBBO theory)
- calculations of higher order processes are divergent
Thus, the outstanding theoretical problem was to investigate solutions to such divergences, i.e. to understand the high energy behaviour of weak processes. The first step consisted in postulating - in analogy to the photon in QED - intermediate vector bosons $W^+$, $W^-$. They would mediate the weak force as the photon mediates the electromagnetic force. However, the $W^\pm$ should be massive, contrary to the photon, in order to agree with the short range behaviour of weak forces. The postulated $W^\pm$, indeed, led to some taming of the divergences. As a further step, other new particles were proposed, which give rise to new phenomena such that their contributions cancel the dangerous infinities, as for instance in $\nu\bar{\nu} + W^+W^-$. One such proposal was the introduction of weak neutral currents, another one new heavy leptons.

Since these speculations involved genuinely new weak phenomena, available experimental data were scrutinized to get evidence for at least upper limits. Around 1970 upper limits on the existence of weak neutral currents came from two sources:

a) decays: strangeness changing transitions ($\Delta S \neq 0$) and $\Delta Q = 0$ are strongly suppressed, e.g.

\[
\begin{align*}
K^+ \to & \{ \pi^+ e^+ e^- \} < 0.26 \cdot 10^{-6} \\
\pi^+ \to & \{ \pi^+ \mu^+ \mu^- \} < 2.4 \cdot 10^{-6} \\
\pi^+ \to & \{ \pi^+ \nu \bar{\nu} \} < 0.6 \cdot 10^{-6}
\end{align*}
\]

\[
\begin{align*}
K^0_L \to & \{ e^+ e^- \} < 1.6 \cdot 10^{-9} \\
K^0_S \to & \{ \mu^+ \mu^- \} < 35 \cdot 10^{-5} \\
K^0_S \to & \{ e^+ e^- \} < 0.3 \cdot 10^{-5}
\end{align*}
\]

b) neutrino experiments:

\[
\begin{align*}
\# \nu p \to & \nu p \quad < 0.12 \pm 0.06 \\
\# \nu n \to & \mu^- p \quad < 0.08 \pm 0.04
\end{align*}
\]

from CERN HLBC$^{12}$
The main experimental problem in the neutrino experiments using the CERN heavy liquid bubble chamber was the treatment of the neutron background. Elastic $\bar{v}_\mu$ and $\nu_\mu$ interactions appear in a bubble chamber as a short track due to the recoil proton and nothing else. Therefore, only upper limits could be quoted (actually in agreement with measurements\textsuperscript{14} performed later).

These upper limits were quite discouraging. One of the highlights at that time was the observation of BJORKEN scaling in ep-experiments at SLAC and, indeed, the investigation of this new phenomenon got highest priority in the first neutrino proposal\textsuperscript{15}) for the new heavy liquid bubble chamber GARGAMELLE, whereas the search for neutral currents ranged lowest in priority. At first sight the observed strong suppression of strangeness changing neutral currents appeared desastrous for models based precisely on weak neutral currents. This shortcoming is related to the particular structure of the hadronic weak charged current as determined from experimental studies of semileptonic weak interactions. In the CABIBBO theory the $u$-quark couples weakly to $d$- and $s$-quarks only in the combination:

\[ d_c = d \cos \theta_c + s \sin \theta_c \]

where $\theta_c$ is the CABIBBO angle. Recent measurements\textsuperscript{16}) gave

\[ \sin \theta_c = 0.231 \pm 0.003 \]

Thus, two types of weak neutral currents are expected

\[ u + u \quad \text{and} \quad d_c + d_c \]

With the shorthand notation \((d_c d_c)\) for the amplitude

\[ \bar{\psi}_d \gamma_\lambda (g_V + g_A \gamma_5) \psi_{d_c} \]

one gets \((d_c d_c) = (d \cos \theta_c + s \sin \theta_c \quad d \cos \theta_c + s \sin \theta_c)\)

\[ = (dd) \cos^2 \theta_c + (ss) \sin^2 \theta_c + ((sd)+(ds)) \cos \theta_c \sin \theta_c \]

\[ \Delta S = 0 \quad \Delta S \neq 0 \]
It is now evident that the neutral current $d_c \rightarrow d_c$ would give rise to unsuppressed strangeness changing processes $d \rightarrow s$ or $s \rightarrow d$ in evident contradiction to the experiment. A way out of this dilemma has been put forward 1970 by GLASHOW, ILIPOULOS, MAIANI\(^{17}\). They postulated in addition to the three known quarks $u$, $d$, $s$ a forth quark $c$ (called $u'$ at that time) with electric charge $2/3$ like the $u$-quark. Then, a new charged current

$$(cs_c) = \bar{\psi}_c \gamma_\lambda (1 + \gamma_5) \psi_s$$

with $s_c = -d\sin\theta_c + s \cos\theta_c$

could be introduced relating the charmed quark $c$ to the combination $s_c$ being orthogonal to $d_c$. It then follows, that $(d_c d_c) + (s_c s_c) = (dd) + (ss)$, i.e. the dangerous strangeness changing currents drop out.

1971 the model of GLASHOW, SALAM, WEINBERG\(^{18}\) received a decisive theoretical support by 't'HOOFT proving its renormalizability\(^{19}\). Since no further experimental information, except for the known upper limits, became available the subject "neutral currents" played a rather minor rôle at the Tirrenia workshop 1972, where the physicists prospects of the forthcoming CERN SPS were discussed\(^{20}\).

The situation changed dramatically, when the GARGAMELLE neutrino collaboration found in december 1972\(^{21}\) a candidate for the reaction

$$\bar{\nu}_\mu e^- + \bar{\tau}_\mu e^-$$

and had collected by spring 1973 a sizeable sample of muonless hadronic events in their neutrino and antineutrino runs (see table 4).

Table 4: Event rates in the GARGAMELLE experiment\(^{22}\)

<table>
<thead>
<tr>
<th>Event type</th>
<th>$\nu$-Expt.</th>
<th>$\bar{\nu}$-Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td># Events without $\mu$</td>
<td>102</td>
<td>64</td>
</tr>
<tr>
<td># Events with $\mu$</td>
<td>428</td>
<td>148</td>
</tr>
</tbody>
</table>

The vertex distribution of events with and without muons is displayed in fig. 1 as a function of the position along the chamber axis. They look similar. It was very tempting to conclude that the muonless events are neutrino induced and not dominated by neutron interactions, since
the neutron interaction length in the bubble chamber liquid is only 70 cm and an exponentially decaying distribution at the beginning of the chamber volume would have to be seen. One of the specific features of GARGAMELLE was its longitudinal extent of almost 5 m. Unfortunately, the above argument proved to be fallacious, since neutrons do not only enter at the front but also along the side of the cylindrical chamber with the consequence that also neutron induced events would have a rather flat vertex distribution along the beam direction. Therefore, it was crucial to perform an absolute calculation of the neutron background in order to find out whether the observed muonless events are evidence for a new phenomenon or simply neutron (or more generally neutral hadron) induced background.

The bubble chamber GARGAMELLE came into operation 1970, it was filled with the heavy liquid CF$_3$Br and was exposed to the CERN PS neutrino and antineutrino beams. The average neutrino energy was 2 GeV, useful event rates could be obtained up to 10 GeV. The fiducial volume of a bit more than 3 m$^3$ was sufficient to have typically 1.5 m potential path for each track such that a good distinction of muons from charged hadrons and a good efficiency for neutron interactions were ensured. Since muons could not be identified the search for the new process

$$\nu N \rightarrow \nu + \text{hadrons}$$

was restricted to those events which consisted only of final state particles identified as hadrons, with total visible event energy in excess of 1 GeV. These events, called NC, were compared to a corresponding charged current event sample, called CC, where apart from the presence of a muon candidate with the appropriate charge the hadrons satisfied the same selection criteria as in the NC sample. Early 1973 the analysed NC sample consisted of about 100 events (cf table 4). Without going into the details of the analysis the essential features of the neutron, or more generally the neutral hadron, background calculation is sketched next$^{23,24}$. The origin of neutrons having enough (namely more than 1 GeV) energy to simulate a NC event are neutrino interactions themselves. As indicated by the two sketches in fig. 2 the neutron source, i.e. upstream neutrino interaction, can occur in two configurations. The neutral hadron interaction (n*) is consequently said to be associated (AS) or not associated (B). It was the aim of the background calculation to get the number of B-events and to compare it with the observed number of NC-events. This was done in the following way:

$$\#B = \frac{B}{AS} \times \#AS$$
i.e. the number of background events (#B) is obtained from the number of observed associated events (#AS) by means of the calculated ratio B/AS. With this trick only a ratio had to be calculated. The great worry in attempting such a calculation was how to treat the neutron cascade. Obviously, AS events are trivial, as they represent just the first cascade step. But in the case of B events, the source is in the heavy shielding material and the cascade can consist of many steps before finally a neutral hadron enters the chamber to produce a star simulating a NC candidate. Furthermore, the density of the shielding material was about 5 times higher than the density of the chamber liquid, thus the neutrino induced neutral hadron flux was potentially high. Obviously,

\[ \text{mesons} \]
\[ \text{nucleons} \]

the understanding of the cascade was the crucial part for the evaluation of the neutral hadron background and thus for the interpretation of the whole experiment. The basic ideas of the cascade calculation were:

i) the meson component is inactive

ii) at each step at most one nucleon carries the cascade further

In other words, the cascade is linear. This reflects the dynamics of neutrino reactions as well as the dynamics of nucleon and meson interaction in the few GeV region together with the 1 GeV requirement mentioned above. The initially complex structure of the cascade has been reduced to the extent that a single quantity, the elasticity, can characterize it. This quantity could be extracted from published data. In conclusion, the ratio B/AS could be safely calculated with the result (for the neutrino experiment):

\[
\begin{align*}
\frac{B}{AS} &= 0.6 \pm 0.3 \\
#AS &= 15
\end{align*}
\rightarrow #B = 9 \pm 4.5
\]

to be compared with \#NC = 102

showing that really a new effect has been observed. A similar conclusion could be drawn for the antineutrino data. By spring 1974 - after some turbulent months - the new effect, interpreted as weak neutral currents, has been observed in three experiments\(^{25}\). The GARGAMELLE experiment itself corroborated its first results by three further investigations\(^{26}\).
- BARTLETT analyses of the spatial distribution of the events in the NC and CC samples confirm that the neutron contamination of the NC sample is small.

- the charge distributions of pions in NC-events and in neutron induced events are different.

- in a separate run protons of 4, 7, 12 and 19 GeV/c have been sent into GARGAMELLE thus allowing a direct check of the cascade calculation\(^{24}\). Fig. 3 shows an example. In fig. 4 the measured and the calculated - following the method described above - cascade length is displayed. The agreement is good.

Once established, the weak neutral currents initiated a new and rich activity in physics and even astrophysics. The aim of the experiments was to measure the properties of the weak neutral current and to compare them with the variety of theoretical models. One of these models, the CSW model, received particular popularity, because only one unknown parameter, the weak angle, \(\theta\), was involved and because of its success. Very soon, this model was simply called the Standard Model.

8. FREE PARAMETERS IN THE STANDARD MODEL

Here and in the next three lectures only the electroweak part is dealt with. The above discussion has shown that there are three groups of free parameters:

- couplings \(g, \sin^2\theta, v\)
- KM-matrix 3 angles, 1 phase
- masses Higgs and all fermions

All these free parameters must be determined from experiment. Once this is done, all electroweak phenomena can be predicted and consistency with measurements can be checked. For instance, all measured \(Z\gamma\) couplings must be shown to agree with a universal value of \(\sin^2\theta\) (cf. lecture 3 sect. 2).

9. CHOICE OF BASIC COUPLINGS

The outstanding property of a renormalizable theory, as for instance the Standard Model, is that any observable can be calculated in any order in terms of a finite set of parameters. Each such parameter must be defined by a suitable experimental procedure.
In the literature various choices are adopted. For the purpose of these lectures dealing with low energy electroweak phenomena the following choice is appropriate: \(e, G, \sin^2 \theta\).

1. The fine structure constant: The positron charge is defined as the electromagnetic coupling at very low energies.

Using the JOSEPHSON effect

\[
\frac{1}{\alpha} \equiv \left( \frac{e^2}{4\pi} \right)^{-1} = 137.035963 \ (\pm 15)
\]

has been obtained\(^{27}\).

2. The FERMI coupling constant \(G\): The lifetime of positive muons can be measured very accurately. \(G\) is by definition obtained from:

\[
\tau_{\mu}^{-1} = \frac{G^2 m_{\mu}^5}{192\pi^3} \left( 1 - 8 \frac{m_{e}^2}{m_{\mu}^2} \left( 1 + \frac{\alpha}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right) \right)
\]

\[\rightarrow G = 1.166365 \ (\pm 16) \ 10^{-5} \text{ GeV}^{-2} \quad ^{28}\]

The term proportional to \(\alpha\) represents the electromagnetic radiative correction.

3. The weak angle \(\theta\): At low energies:

\[
\frac{\sigma(\bar{\nu}_e \rightarrow \bar{\nu}_e e)}{\sigma(\nu_e \rightarrow \nu_e e)} = \frac{1 - \xi + \xi^2}{1 + \xi + \xi^2} \quad \text{with } \xi \equiv 1 - 4 \sin^2 \theta
\]

This definition involves only leptons. The CHARM collaboration obtained so far \(\sin^2 \theta = 0.24 \pm 0.04 \pm 0.015\)\(^{29}\). An improved measurement with the anticipated precision of \(\pm 0.005\) is underway. In the meantime the more precise value coming from measurements of the NC/CC ratio in neutrino nucleon experiments is used.

Electroweak observables can be expressed as functions of \(e, G, \sin^2 \theta\). For illustration, the prediction of the \(W^\pm\)-mass may be considered. In terms of \(e\) (or \(\alpha\)) \(G, \sin^2 \theta\) - the renormalized quantities - one derives from the Standard Model

\[
m^\text{BORN}_W = \left( \frac{\pi \alpha}{\sqrt{2} G \sin^2 \theta} \right)^{1/2}
\]

prediction in lowest order (BORN approximation)

\[
m^\text{CORR}_W = m^\text{BORN}_W (1 + \delta)
\]

prediction including next to leading order
The shift \( \delta \), predicted by corrections due to 1-loop graphs, has recently been calculated and amounts to about 3.5\%\(^{30}\), i.e. about 3 GeV. The presence and size of this correction tests the renormalization aspect of the Standard Model. In order to make this test significant the measurement precision of \( \sin^2 \theta \) must be increased to \( \pm 2\% \), i.e. \( \Delta \sin^2 \theta \leq 0.005 \). Among other choices of 3 basic couplings the one of MARCIANO and SIROLIN\(^{30}\) may be mentioned.

With the operation of the CERN SppS collider the energy regime beyond the \( W, Z \)-masses is accessible and thus a natural choice are the physical masses \( m_W, m_Z \) of the weak bosons together with the fine structure constant \( \alpha \).

In this scheme \( \sin^2 \theta \) is a derived quantity, which can be related to \( \sin^2 \theta_M \equiv 1 - \left( \frac{m_W}{m_Z} \right)^2 \). Note that: \( \sin^2 \theta \neq \sin^2 \theta_M \).
Fig. 1: The vertex distribution of events without muon (above) and with muon candidate (middle) along the chamber. The fiducial volume starts at -200 cm. Below is the raw NC/CC ratio (ref. 23).
Fig. 2: Sketch of the simplified, but realistic setup. The chamber filled with the liquid freon \((\rho=1.5 \text{ g/cm}^3)\) is imbedded in a dense medium (chamber well, magnet coils, iron shielding etc.). The neutrino beam enters from left and has a broad energy dependent radial distribution. A neutrino event with a subsequent neutron star is shown in two topologies: case above is an associated neutron star (AS), case below a nonassociated neutron star (B)(ref. 24.).
Fig. 3: Photo of a 6.1 GeV proton entering GARGAMELLE from below. The insert sketches the cascade (ref. 24).
Fig. 4: Comparison of the measured and calculated cascade length as a function of the incoming proton momentum (ref. 24).
Fig. 5: Neutrino induced neutral current event observed in GARGAMELLE. All final state hadrons are indentified. Note the charge exchange reactions of the $\pi^-$. 
Lecture 2: Purely Leptonic Interactions

1. Introduction
2. Elastic neutrino-electron scattering
3. The processes $\nu^+ e^- \rightarrow \nu^+ e^-$
4. Muon decay
5. Inverse muon decay
6. The tau lepton
7. Higher order QED processes
8. Search for new heavy leptons
1. INTRODUCTION

Conceptually, but not necessarily experimentally, the simplest interactions are those involving only leptons. Their interpretation is theoretically clean. Leptons are pointlike and interact only weakly, like neutrinos, or electroweakly, like charged leptons. No complications due to strong interactions occur.

Table 5 shows some reactions, the currents involved and the couplings they are sensitive to. A prominent role play $\nu_{\mu} e$ and $\overline{\nu}_{\mu} e$ elastic reactions. Their occurrence proves weak neutral current interactions, since $\nu_{\mu}$ and $e$ belong to different generations. Charged lepton interactions, as observed at $e^+ e^-$ colliders for instance, were for a long time prototypes of electromagnetic processes. Recently, with the advent of high energy colliders like PETRA and PEP it could be demonstrated that charged leptons interact also weakly.

Table 5: Some purely leptonic reactions

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Currents</th>
<th>Sensitive to</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{\mu} e + \nu_{\mu} e$</td>
<td>$(\nu_{\mu} \nu_{\mu})(ee)$</td>
<td></td>
</tr>
<tr>
<td>$\overline{\nu}<em>{\mu} e + \overline{\nu}</em>{\mu} e$</td>
<td>$(\overline{\nu}<em>{\mu} \overline{\nu}</em>{\mu})(ee)$</td>
<td>$g^e_e, g^e_A$</td>
</tr>
<tr>
<td>$\overline{\nu}<em>{e} e + \overline{\nu}</em>{e} e$</td>
<td>$(\nu_{e} \nu_{e})(ee)$</td>
<td></td>
</tr>
<tr>
<td>$\nu_{\mu} e \rightarrow \nu_{e} e$ (e = $\tau, \mu$)</td>
<td>$(\nu_{\mu} \nu_{\mu})(\nu_{e} e)$</td>
<td>$V - A$</td>
</tr>
<tr>
<td>$e^+ e^- + X^+ X^-$</td>
<td>$(ee)(\xi \xi)$</td>
<td>$g^\mu_A, g^{\tau}_A$</td>
</tr>
</tbody>
</table>

2. ELASTIC NEUTRINO-ELECTRON SCATTERING

Neutrino beams at accelerators are derived from $\pi^+$ and $K^+$ decays and are thus basically $\nu_{\mu}$ beams with small contaminations of $\nu_{e}, \overline{\nu}_{e}, \overline{\nu}_{\mu}$. The use of magnetic horns ensures a good separation of particles from
antiparticles. Since the early GARGAMELLE runs at the PS up to the recent
CHARM runs at the SPS a tremendous development has taken place from
1 event/run to about 50 - 100/run. This gain comes from better and more
intense neutrino beams, higher neutrino energies (\(\sigma \sim E_l\)) and the use of
very massive target calorimeters.

The kinematics and dynamics of neutrino and antineutrino interactions off
electrons is easy to work out:

\[
\begin{align*}
\frac{d\sigma(\nu \mu e)}{d\omega} &= \sigma_o(e_L^2 + (1-y)^2 e_R^2) \ dy \\
\frac{d\sigma(\bar{\nu} \mu e)}{d\omega} &= \sigma_o(e_R^2 + (1-y)^2 e_L^2) \ dy
\end{align*}
\]

where \(y\) measures the final state electron energy in terms of the initial
neutrino energy and \(\sigma_o = \frac{G^2 q^2 s}{\pi} = 1.72 \cdot 10^{-41} \text{ cm}^2 \ \frac{E}{\text{GeV}}\).

Neutrinos, which are left-handers, interacting with a left-handed electron,
lead to an isotropic distribution, whereas those interacting with a right-
hand electron to a distribution \(\sim (1-\cos \theta)\) in the rest frame or \((1-y)^2\) in
the laboratory frame. The two contributions are proportional to the square of
the respective weak couplings \(e_L\) and \(e_R\) (cf. appendix).

For antineutrino interactions \(e_L^*\) and \(e_R^*\) are to be exchanged. In the presently
accessible energy regime is \(q^2 < m_Z^2\); therefore, the total elastic cross
section rises linearly with neutrino energy. Since \(\sigma \sim m_e\) the expected rates
are very small.

Fig. 6 shows all the experimental results so far. The CHARM collaboration\(^{29}\)
has investigated in the same apparatus \(\nu \mu\) and \(\bar{\nu} \mu\) interactions and noted that
the cross section ratio

\[
R = \frac{\sigma(\nu \mu e)}{\sigma(\bar{\nu} \mu e)} = \frac{1 + \xi + \xi^2}{1 - \xi + \xi^2} \ \text{with} \ \sin^2 \theta = \frac{1}{4} (1-\xi)
\]

can be obtained with small systematic error. The sensitivity of \(R\) to \(\sin^2 \theta\)
is high due to the fact that \(\sin^2 \theta\) is around 0.22 (and not e.g. 0.5), as
seen in fig. 7. From \(\sigma(\nu \mu e)\) the couplings \(e_L, e_R\) or \(\nu_e, \bar{\nu}_e\) can be
deduced. Fig. 9 shows the two ellipses of all data combined\(^{31}\):

a) Assume \(\rho = 1\):

\[
\begin{align*}
\mu_e &= -0.521 \pm 0.034 \\
\nu_e &= 0.002 \pm 0.058
\end{align*}
\]

This solution is in agreement with the Standard Model. The constraint
equations contain the coupling constants squared, thus there is another
solution which can however be excluded on the basis of other experiments (e.g. \( e^+e^- \rightarrow \mu^+\mu^- \)).

b) Assume \( a_e = -0.5 \), \( v_e = -\frac{1}{2} (1-4 \sin^2 \theta) \)

get \( \sin^2 \theta = 0.251 \pm 0.029 \), \( \rho = 1.04 \pm 0.07 \)

c) Assume \( a_e, v_e \) and \( \rho = 1 \), then \( \sin^2 \theta = 0.244 \pm 0.029 \)

d) Use R (CHARM) alone\(^{29}\)

\[ \sin^2 \theta = 0.215 \pm 0.040 \pm 0.015 \] (independent of \( \rho \))

Results on \( \bar{\nu}_e e \) scattering are also indicated in fig. 9. They get contributions both from \( Z^0 \) and \( W^- \) exchange. The interference reduces the sign ambiguities by a factor 2. Data on \( \nu_e e \) scattering are expected from LAMPF. In a dedicated experiment the CHARM Collaboration\(^{34}\) is aiming at more than 1000 \( \nu_e e \) and 1000 \( \bar{\nu}_e e \) elastic events. Compared to their previous set-up the fiducial mass will be increased from 70 to 436 tons. The background rejection (see fig. 8) will be improved by reducing the angular resolution from 32 mr/\( \sqrt{E} \) to 16 mr/\( \sqrt{E} \). The relative \( \nu, \bar{\nu} \) flux monitoring should be controlled to \( \pm 2\% \). Measuring then \( \sigma(\nu_e e)/\sigma(\bar{\nu}_e e) \) to an accuracy of \( \pm 0.05 \) translates into \( \Delta \sin^2 \theta = \pm 0.005 \). This precision is enough to test whether the difference

\[
m_{Z}^{PHYS} - m_{Z}^{\text{TREE}} = m_{Z}^{PHYS} - \frac{e}{2\sqrt{2}G} \frac{1}{\sin \theta \cos \theta} = m_{Z}^{PHYS} - \frac{37.2810(\pm 3)}{\sin \theta \cos \theta} \text{ GeV}
\]

is equal to the 1-loop weak correction calculated to be about 3 GeV within the renormalizable Standard Model. \( m_{Z}^{PHYS} \) is the physical mass of the \( Z \) as measured at the \( S \bar{p}pS \) collider. This test on the presence and predicted size of higher order weak corrections is crucial to the Standard Model.

4. THE PROCESSES \( e^+e^- \rightarrow \xi^+\xi^- \)

Consider the Lagrangian for \( e^+e^- \rightarrow \mu^+\mu^- \):

\[
L = -g \sin \theta (\bar{\nu}_e \gamma_\lambda \psi_e + \bar{\psi}_\mu \gamma_\lambda \psi_\mu) A^\lambda
\]

\[
+ \frac{g}{4\cos \theta} (\bar{\nu}_e \gamma_\lambda (g^{e}_V + g^{e}_A) \gamma_5 \psi_e + \bar{\psi}_\mu \gamma_\lambda (g^{\mu}_V + g^{\mu}_A) \gamma_5 \psi_\mu) Z^\lambda
\]
In tree or BORN approximation the two relevant graphs are:

![Graphs for electron and muon transitions](image)

The electromagnetic contribution is of order \( \frac{(gs\sin\theta)}{s} = \frac{4\pi\alpha}{s} \)
and dominates the weak contributions of order \( \frac{1}{4\cos^2\theta} \left( \frac{1}{2} \right) \frac{G}{m^2_Z} = \frac{G}{2\sqrt{2}} \)
as long as \( s \ll m^2_Z \). In the standard model \( \rho^2 = 1 \) in tree approximation and gets slightly reduced by 1-loop corrections. For most of the following discussion it is sufficient to consider only the leading order.

The angular distribution is:

\[
\frac{4s}{\alpha^2} \frac{d\sigma}{d\Omega} = R_\mu \left| (1 + \cos^2\theta) + \frac{8}{3} A_\mu \cos\theta \right|
\]

with \( R_\mu = Q^2_\mu - 2Q_\mu g^eV^\mu e + ((g^e_V)^2 + (g^e_A)^2) ((g^\mu_V)^2 + (g^\mu_A)^2) \kappa^2 \)

\[
A_\mu = -\frac{3}{2} \frac{Q^2_\mu g^eV^\mu e - 2Q_\mu g^eV^\mu g^\mu V^\mu A}{R_\mu}
\]

\[
= \frac{1}{(2\sin^2\theta)^2} \frac{s}{s - m^2_Z} \frac{G m^2_Z}{8\sqrt{2} \pi \alpha} \frac{s}{s - m^2_Z} = 0.45 \cdot 10^{-4} \frac{s}{1 - \frac{s}{m^2_Z}}
\]

These formulae apply equally well to \( e^+e^- \rightarrow f\bar{f} \) provided \( f \neq e \) and \( g^V_A, Q_\mu \) replaced by \( g^f_V, Q_f \). The existence of \( Z^0 \)-exchange in addition to photon exchange entails a significant modification of the differential cross section of \( e^+e^- \rightarrow \mu^+\mu^- \) (fig. 10).

- Angular asymmetry \( A_\mu \): Right- and lefthanded contributions are no longer equally strong, the angular distribution gets a term proportional to \( \cos\theta \).
The $\gamma - Z^0$ interference ($V,A$ type) gives rise to a forward-backward angular asymmetry:

$$A_\mu = \frac{F - B}{F + B} = \frac{3}{2} \frac{g_A^e g_A^\mu}{Q^2} \kappa + O(k^2) \approx \begin{cases} -10\% \text{ PETRA} \quad (\sqrt{s} = 35 \text{ GeV}) \\ -6\% \text{ PEP} \quad (\sqrt{s} = 29 \text{ GeV}) \end{cases}$$

For small $s/m_Z^2$, $A_\mu$ decreases with increasing $s$. In the adopted scheme with $\alpha, G, \sin^2\theta$ as basic couplings $A_\mu$ depends only upon $G/\alpha$, which is accurately known, and weakly upon $m_Z$, which is calculable using $\alpha, G, \sin^2\theta$ up to a few GeV (due to uncertainty in $\sin^2\theta$). The $Z$-mass enters through the propagator term and has little influence, since $s/m_Z^2 < 0.2$ in the PETRA energy range. The measurement of $A_\mu$ and its $s$-dependence is a crucial test of the standard model. The results (excluding the results at the Leipzig 1984 conference) are summarized in Table 6.35:

**Table 6: Results from asymmetry measurements**

<table>
<thead>
<tr>
<th></th>
<th>PETRA</th>
<th>PEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A^e g_A^\mu$</td>
<td>$1.16 \pm 0.10$</td>
<td>$1.03 \pm 0.14$</td>
</tr>
<tr>
<td>$g_A^e g_A^\tau$</td>
<td>$0.88 \pm 0.20$</td>
<td>$0.98 \pm 0.18$</td>
</tr>
<tr>
<td>$g_A^\tau/g_A^\mu$</td>
<td>$0.76 \pm 0.19$</td>
<td>$0.95 \pm 0.21$</td>
</tr>
</tbody>
</table>

The measurements agree with the predictions of the Standard Model ($g_A^e g_A^\mu = 1$ and $g_A^e g_A^\tau = 1$) within 1 to 2 standard deviations. This is a remarkable achievement, extending its validity up to $Q^2 = s = 2000 \text{ GeV}^2$ (time like). The ratio of the axial couplings of $\mu$ and $\tau$ confirms lepton universality within 20%.

Also the process $e^+e^- \rightarrow e^+e^-$ gets modified through the $Z$-graph. However, the angular distribution is already strongly asymmetric due to the $\gamma$-exchange in the $t$-channel. No group succeeded yet in demonstrating a significant deviation from the QED-prediction of the measured angular distribution.

- **Ratio** $R_\mu = \sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma_{\text{QED}}$ is predicted to deviate from 1, the more so the higher the energy. Putting in the weak couplings $g_A^e = g_A^\mu = -1$ and $g_A^e = g_A^\tau = -1 + 4 \sin^2\theta \quad |R_\mu - 1| \leq 0.02$ up to highest PETRA energies. The data, both for $e^+e^- \rightarrow \mu^+\mu^-$ (fig. 11) and $e^+e^- \rightarrow \tau^+\tau^-$ (fig. 12), agree well with 1. Since there is a normalisation error of about 5%, these
measurements provide a poor limit on the weak vector couplings. Electromagnetic radiative corrections at $\sqrt{s} = 35$ GeV are about 30%, in other words the effective fine structure constant is $\alpha^{-1}_{\text{eff}} \approx 120$ compared to $\alpha^{-1} = 137$.

$R_\mu$ can also be used to set limits on the pointlikeeness of charged leptons by interpreting the deviation from 1 in terms of a formfactor:

$$F_\pm(q^2) = 1 \mp \frac{q^2}{g^2 - \Lambda_F^2}$$

Present data constrain $\Lambda_F \gtrsim 200$ GeV, which means that leptons are structureless on the distance scale $10^{-16}$ cm.

- Z-propagator: It is intriguing to find out whether the existing data on the muon angular asymmetry $A_\mu$ exhibit the $Z^0$-propagator. $A_\mu$ is proportional to the ratio of the $Z$- and $\gamma$-couplings and to the ratio of the $Z$- and $\gamma$-propagators. Since the photon propagator contributes a term $1/s$, the quantity $\frac{A_\mu(s)}{s}$ is proportional to $\frac{1}{s - m_Z^2} (A_\mu)^{-1}$. The most general test consists in just looking at the slope of $\frac{A_\mu(s)}{s}$ for a massive propagator. Fig. 13 shows all data on $A(e^+\mu^- \rightarrow e^+\mu^-)$. The uncertainties are still too big to conclude on a significant nontrivial $s$-dependence. However, the test can be sharpened by requiring a straight line through the fix point $\frac{2}{3\alpha}$ at $s = 0$ as given by the Standard Model. The line drawn in fig. 13 is the prediction of the Standard Model ($m_Z = 93$ GeV). The agreement with the data is fair. The highest energy point includes new data:

$A_\mu = \pm (17.6 \pm 2.5)\%$ at average $\sqrt{s} = 42.5$ GeV (expected: $-14.7\%$).

MUON DECAY

The study of the decay $\mu^+ \rightarrow \mu^- e^+ \nu_e$ or $\mu^- \rightarrow \mu^- e^- \bar{\nu}_e$ had an essential impact on the development of weak interaction physics. Although $q^2$ in this process is very small, high precision measurements provide valuable information. Four results will be quoted:

a) The MICHEL parameter $\rho_M$:

$$\frac{\rho_{\text{meas}}}{\rho_{\text{M}}^V} = 1.0024 \pm 0.0035$$
b) The lifetime of the $\mu^+ \ (37)$: $\tau_\mu = (2.19695 \pm 0.00005) \times 10^{-6}$ sec

$$\frac{1}{\tau_\mu} = \frac{G^2 m_\mu^5}{192 \pi^3} (1 - 8 \frac{m_e^2}{m_\mu^2}) (1 + 0.6 \frac{m_\mu^2}{m_W^2}) - \frac{\alpha}{2\pi} (\pi^2 - \frac{25}{4})$$

follows $G = 1.166365 (\pm 16) \times 10^{-5}$ GeV$^2$ which is the most precise measurement of the FERMI coupling constant.

c) Search for the right handed currents in polarized $\mu^+$-decay: by looking at the electron energy spectrum near the end point the assumption of a righthanded $W$ is only consistent with the data if its mass exceeds 380 GeV$^38$.

d) New upper limit $\Gamma(\mu^+ \to e^+ e^-)/\Gamma(\mu^+ \to e^+ \nu_e \overline{\nu}_\mu) < 1.6 \times 10^{-10}$ (90% CL)$^{39}$

**INVERSE MUON DECAY**

The process $\nu_\mu e^- + \mu^- \nu_e$ is induced by a lefthander and contains more information than the $\mu$-decay (with its two neutrinos in the final state). This reaction was only some years ago observed, since a threshold at $E_\nu = 11$ GeV is involved. The results from GARGAMELLE (1979) and CHARM (1980) give$^{40}$

$$\frac{\sigma(\nu_\mu e^- + \mu^- \nu_e)}{\sigma_{V-A}} = 0.98 \pm 0.18$$

**THE TAU LEPTON**

The heavy lepton $\tau^+ \ (41)$ was the first member of the third generation, discovered 1975. It is produced in $e^+ e^- \to \tau^+ \tau^-$ for $\sqrt{s} > 2m_\tau$. All observations confirm that the $\tau$ behaves like a sequential lepton. Up till now the inverse process: $\nu_\tau N \to \tau^- + X$ has not yet been observed. Fig. 14 summarizes measurements of the $\tau$-lifetime$^{42})$. It agrees within 10% with the expected V-A prediction assuming $\mu$-$\tau$ universality. Another check is provided by comparing the purely leptonic decays:

$$\frac{\tau \to \nu_\tau + \mu^- \overline{\nu}_\mu}{\tau \to \nu_\tau + e^- \overline{\nu}_e} = 0.97 \pm 0.08$$

where the standard model predicts 0.973.
As already mentioned above the weak neutral current ($\tau\tau$) is tested by measuring the angular asymmetry of $e^+e^- \to \tau^+\tau^- \cdot g_A^\tau$ agrees within 15% with the theoretical expectation (cf. table 6).

HIGHER ORDER QED-PROCESSES

The next figures shall demonstrate that even at $\sqrt{s}$ up to 40 GeV QED works very well. The processes considered are\(^{(43)}\):

\[ e^+e^- \to \gamma\gamma\gamma \quad \text{(fig. 15)} \]
\[ e^+e^- \to e^+e^-\gamma \quad \text{(fig. 16)} \]
\[ e^+e^- \to e^+e^-\mu^+\mu^- \quad \text{(fig. 17)} \]

SEARCH FOR NEW HEAVY LEPTONS

Interactions of $e^+e^-$ are ideal to look for new heavy leptons. The charged member of a forth sequential lepton doublet ($\nu_L$) would be produced like

\[ e^+e^- \to L^+L^- \quad \sqrt{s} > 2m_L > 2m_\tau \]

with a cross section

\[ \sigma_{\text{QED}} \frac{d\sigma}{d\cos\theta} = \frac{3}{8} \beta (1 + \cos^2\theta + (1-\beta^2) \sin^2\theta) \]

where $\beta$ describes the threshold behaviour.

The signature of such events is characterized by missing momentum and non-collinear, non-coplanar topologies as a consequence of

\[ L \to \nu_\tau + \text{anything} \rightarrow \begin{cases} \nu_\tau + \bar{\nu}_e \cr \nu_\tau + q\bar{q}'-\text{jets} \end{cases} \]

The experiments at PETRA and PEP exclude such sequential charged leptons up to 21 GeV\(^{(44)}\) (fig. 19).
Another type of heavy leptons has been searched for:

\[
\begin{array}{c}
e^+ & \bar{\nu}_e \\
\downarrow & W^+ \\
e^- & \nu_e \nu_e \bar{e}^0 \\
\end{array}
\]

If produced, its decay is assumed to proceed according to

\[
\nu_e \rightarrow \nu_e \nu_e \bar{e}^0 \\
\nu_e \rightarrow \nu_e \nu_e \bar{e}^0
\]

again with characteristic topologies depending upon the mass assumed for the \( \nu_e \). From the absence of such event topologies a lower limit, \( m_\nu > 22.5 \text{ GeV} \) with 95\% confidence level, can be deduced.
Fig. 6: Compilation of $\nu_e$ - and $\bar{\nu}_e$ scattering experiments (ref. 32 and 33).
Fig. 7: The CHARM analysis (ref. 29).

Fig. 8: The background subtraction in the CHARM experiment.
Fig. 9: Fit of all $\nu_e$ and $\bar{\nu}_e$ data (ref. 31).
Fig. 10: Angular asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ (ref. 35).
Fig. 11: Production cross section of $e^+e^-\to\mu^+\mu^-$ normalised to the QED cross section as a function of $s$. 

$\Lambda_\sigma = 200 \text{ GeV}$

$\Lambda_- = 200 \text{ GeV}$
Fig. 12: Same as fig. 11 but $e^+e^- \rightarrow \tau^+\tau^-$
Fig. 13: Z-propagator effect in the energy dependence of the angular asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$. The fixpoint at $s = 0$ depends upon $G/\alpha$. The line represents the prediction of the Standard Model for $m_Z = 93$ GeV as calculated from $G$, $\alpha$ and $\sin^2\theta = 0.217$. 
Fig. 14: Compilation of τ-lifetime measurements (ref. 42).
Fig. 15: Photon energy spectrum of $e^+e^- \to \gamma\gamma\gamma$ and comparison with QED.
Fig. 16: Photon energy spectrum of $e^+e^- \rightarrow e^+e^-\gamma$ and comparison with QED.
Fig. 17: Invariant $\mu^+\mu^-$ mass and $P_T^2$ distribution of $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ and comparison with QED.
Fig. 18: Lepton masses vs generation number.
Fig. 19: The acoplanarity distribution of the data (crosses) compared with expectation from multihadron events (solid line). The two histograms refer to simulated $L^2$ events (solid) and $E^0$ (dotted).
Fig. 20: Neutrino interactions seen in the CDHS apparatus.\(^{46}\)

a) Charged current event (CC), b) neutral current event (NC).
Lecture 3: SEMILEPTONIC NEUTRAL CURRENT INTERACTIONS

1. Introduction
2. The Zqq couplings
3. Electroweak results from e⁺e⁻-experiments
4. Weak effects in charged lepton-nucleon scattering
5. Parity violation in Caesium
6. Conclusions on Zqq
1. **INTRODUCTION**

The lepton vertices are well understood. This means, that the gauge bosons ($\gamma, W^\pm, Z$) can be used as probes for the electroweak and the strong properties of quarks. Typical experiments to investigate these two aspects are:

- neutrino-nucleon scattering ($\nu N \rightarrow \nu$ or $\mu$ + anything)
- charged lepton-nucleon scattering ($l^\pm N \rightarrow l^\pm$ + anything)
- $e^+e^-$ - annihilation into hadrons ($e^+e^- \rightarrow$ hadrons)

These deep inelastic scattering experiments revealed the weak neutral current structure of light quarks, to some extent also for heavy quarks, and the structure of nucleons appearing as if made of quasi free pointlike constituents. The color is a distinguishing property of quarks and causes a deep difference between quarks and leptons: leptons exist as free particles, whereas quarks, when leaving the interaction region, develop strong forces with the consequence of getting confined. Since the gauge bosons $W^\pm, Z, \gamma$ couple only to the electroweak properties of the quarks, they are ideal tools to study the color aspects of quarks under well defined conditions. The results from deep inelastic scattering experiments are important input for the interpretation of hadron-nucleon experiments, including the recent SppS collider experiments.

2. **THE ZqQ COUPLINGS**

When the weak neutral currents were discovered 1973, it was immediately clear that a new chapter in physics was opened and that an extensive research program would start. Early contributions came from various neutrino experiments:

- **GARGAMELLE bubble chamber at CERN-PS**
- **AACHEN-PADOVA setup at CERN-PS**
- **HPWF calorimeter**
- **CITF calorimeter**
- **12' ANL bubble chamber**
- **7' BNL bubble chamber**
- **15' FNAL bubble chamber**
1975 an experiment at SLAC with polarized electrons scattering off deuterons reported the observation of a parity violating asymmetry, interpreted as a \((\gamma, Z^0)\)-interference effect, and leading to a 10% measurement of \(\sin^2 \theta\) in agreement with neutrino data.

1977 the first neutrino experiments were carried out using the CERN-SPS thus reaching neutrino energies up to 200 GeV. Compared to the CITF- and HPWF-apparatus running already since a few years in this energy regime the new CDHS calorimeter at CERN was a second generation apparatus (fig. 20). The big european bubble chamber BEBC, of similar size as the 15' bubble chamber, started - upstream in the same neutrino beam as CDHS - its 8 years lasting research program. Soon after, the fine grain marble calorimeter of the CHARM collaboration joined the other two. GARGAMELLE ran for a short while at the SPS before it broke down. At Serpuchov two experiments were operating: the bubble chamber CKAT and a counter apparatus. Dedicated experiments were performed on \(\nu, \bar{\nu}\)p at BNL and \(\nu, \bar{\nu}\)e scattering at CERN and BNL. At FNAL the new calorimeter of the CCFRR collaboration came into operation. The big bubble chambers got upgraded with an external muon identifier (EMI), which ensured an efficient distiction of charged current from neutral current induced events (fig. 21).

With the advent of the high energy \(e^+e^-\) colliders PETRA and PEP electroweak effects got accessible in a new energy regime. The latest achievement was the observation of weak phenomena at the \(\sqrt{s}p\)S collider, culminating 1983 in the discovery of the weak gauge bosons.

All data available up to 1979 have been analysed by KIM et al.\(^{48}\). Three of their results are quoted here:

a) Use all neutrino data: 4 parameters are fitted

\[
\begin{align*}
  u_L &= 0.340 \pm 0.033 \\
  \alpha &= 0.589 \pm 0.067 \\
  d_L &= -0.424 \pm 0.026 \\
  \beta &= 0.937 \pm 0.062 \\
  u_R &= -0.179 \pm 0.019 \\
  \gamma &= -0.272 \pm 0.081 \\
  d_R &= -0.017 \pm 0.058 \\
  \delta &= 0.101 \pm 0.093
\end{align*}
\]

\[
\sqrt{u_L^2 + d_L^2} = 0.544 \pm 0.007
\]

\[
\sqrt{u_R^2 + d_R^2} = 0.180 \pm 0.015
\]

fit quality: \(\chi^2/d.o.f. = 13.5/24\)
b) Use all data: 2 parameters are fitted

\[ \sin^2 \theta = 0.234 \pm 0.013 \quad \rho = 1.002 \pm 0.015 \]

c) Use all data: 1 parameter is fitted

\[ \sin^2 \theta = 0.233 \pm 0.009 \quad \chi^2 / \text{d.o.f.} = 33.1 / 45 \]

It was an important achievement that already after 5 years of research, mainly in neutrino-nucleon scattering, the host of models describing weak neutral currents basically reduced to what is called today the Standard Model. The chiral couplings of the light quarks u and d could be uniquely determined from the data. In the Standard Model these 4 couplings are expressed in terms of only one parameter, namely \( \sin^2 \theta \), and precisely this fact is borne out by the data, although still with sizeable uncertainty in the righthanded sector (\( d_R^2 \)). The measurements of \( u_R^2 + d_R^2 \) gave a value significantly different from 0 and demonstrated that the weak neutral gauge bosons couple also to righthanded quarks.

An unsatisfactory feature in the determination of the chiral couplings consisted in the fact that many experimental results with rather different systematic errors got combined. Furthermore, the interpretation of inclusive pion production data required a distinction of pions associated to current fragments from pions associated to target fragments which is not trivial at low energies. Also the single pion data, for example \( \pi^0 \) or \( \pi^+ \), can only be interpreted within models describing weak isobar production\(^{49}\). In a recent bubble chamber experiment in BEBC\(^{50}\) the double role of deuterium, being an isoscalar target and providing quasi-free proton and neutron targets, was successfully used to get a simultaneous measurement of 4 neutral to charged current ratios (cuts: \( E_H > 5 \text{ GeV}, \ T_T > 1.5 \text{ GeV/c} \):

\[ R_{vp} = \frac{\sigma(\nu p + \nu x)}{\sigma(\overline{\nu} p + \mu^- x)} = 0.49 \pm 0.05 \]

\[ R_{vn} = \frac{\sigma(\nu n + \nu x)}{\sigma(\overline{\nu} n + \mu^- x)} = 0.25 \pm 0.02 \]

\[ R_{vp} = \frac{\sigma(\nu p + \nu x)}{\sigma(\overline{\nu} p + \mu^- x)} = 0.26 \pm 0.04 \]

\[ R_{vn} = \frac{\sigma(\overline{\nu} n + \nu x)}{\sigma(\overline{\nu} n + \mu^- x)} = 0.57 \pm 0.09 \quad \text{(first measurement)} \]
Each ratio involves a different combination of the 4 chiral couplings squared. In valence quark approximation the first 2 ratios would be:

\[ R_{\nu p} = \left( 2u_L^2 + d_L^2 \right) + \frac{1}{3} \left( 2u_R^2 + d_R^2 \right) \]

\[ R_{\nu n} = \left( u_L^2 + 2d_L^2 \right) + \frac{1}{3} \left( u_R^2 + 2d_R^2 \right) \]

It was thus possible to obtain from the 4 ratios all 4 chiral couplings:

\[ u_L^2 = 0.133 \pm 0.026 \pm 0.015 \]

\[ d_L^2 = 0.192 \pm 0.026 \pm 0.015 \]

\[ u_R^2 = 0.020 \pm 0.019 \pm 0.004 \]

\[ d_R^2 = 0.002 \pm 0.019 \pm 0.004 \]

If only \( \sin^2 \theta \) is fitted to the data a 10% measurement results:

\[ \sin^2 \theta = 0.20 \pm 0.02. \]

The prediction of the Standard Model is illustrated in fig. 22 for the lefthanded sector. A significant test of the righthanded sector is limited by the precision in the \( \nu \)-data.

Recent \( \nu \) and \( \bar{\nu} \) experiments using isoscalar targets have provided accurate measurements of

\[ R_{\nu} = \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu^- X)} = (u_L^2 + d_L^2) + \frac{1}{3} (u_R^2 + d_R^2) \]

\[ R_{\bar{\nu}} = \frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\bar{\nu} N \rightarrow \bar{\mu}^+ X)} = (u_L^2 + d_L^2) + 3(u_R^2 + d_R^2) \]

The actual evaluation of these ratios is quite complicated by the nucleon structure. For instance, the contributions of charged and neutral current interactions with sea quarks have to be evaluated; then, in the case of flavor changing transitions quark masses and the KOBAYASHI-MASKAWA matrix elements have to be taken into account. Finally, quarks are not really free. This illustrates, that although a ratio is measured its interpretation in terms of \( u_L^2 + d_L^2 \), \( u_R^2 + d_R^2 \) or in terms of \( \rho \) and \( \sin^2 \theta \) will necessarily be affected by small systematic uncertainties. The present status is shown in fig. 23\(^{51}\) and fig. 24\(^{52}\). It is interesting to note that the \( \rho \)-parameter
is measured with an accuracy of ± 0.02 and agrees with 1, as expected if the 
HIGGS representation is a doublet (assumed in the Standard Model). There are 
now three determinations of \( \sin^2 \theta \) which have each an accuracy comparable to 
the previous average of all experiments in KIM et al. (1981). It is appropriate 
to apply 1-loop corrections to \( R_\nu \) and \( R_{\bar{\nu}} \) to obtain the renormalized value 
of \( \sin^2 \theta \). The uncorrected value is then lowered by typically \( 5\% \). All 
recent data on isoscalar, when combined, give \(^{54}\)

\[
\sin^2 \theta_{\text{corr}} = 0.223 \pm 0.007.
\]

Not all data are published yet.

A careful study of the systematic limitation in determining \( \sin^2 \theta \) from 
\( R_\nu = \frac{\sigma(\nu N \rightarrow \nu N)}{\sigma(\nu N \rightarrow \mu N)} \) has been carried out in the context of the SPS fixed target 
workshop 1981 \(^{11}\) and is believed to be \( \Delta \sin^2 \theta < 0.005 \). The ratio \( R_{\bar{\nu}} \) 
is not suited, since for values of \( \sin^2 \theta \approx 0.22 \) \( \Delta R_{\bar{\nu}} \approx 0 \Delta \sin^2 \theta \). Both the 
CHARM and the CDHS collaborations have investigated the possibility 
to decrease the experimental uncertainties to match the above limit and came 
to an affirmative conclusion \(^{55,56}\). The importance to really reach the 
accuracy in \( \sin^2 \theta \) of 0.005 has already been discussed in the second lecture.

Once accurate measurements of the squares of the weak neutral current 
couplings (for the light flavors) exist, the sign ambiguities can be 
removed even with experiments of minor precision. For instance, the observation 
(fig. 25) of a prominent \( \Delta^*(1236) \) resonance in \( \nu p \rightarrow \nu p \pi^0 \) in the GARGAMELLE 
propane experiments allows to conclude:

\[
u_L \pi_L = \frac{1}{\sqrt{6}} \left[ (-\alpha + \beta)^2 + (\gamma + \delta)^2 \right] < 0
\]

since the isovector term dominates the isoscalar term. The other ambiguity 
is solved by a form factor analysis of the elastic scattering experiments 
\( \nu p \rightarrow \nu p \) and \( \bar{\nu} p \rightarrow \bar{\nu} p \) \(^{58}\) yielding \( u_L u_R < 0 \).
The recent observation of coherent \( \pi_0 \) production in neutrino nucleus 
scattering provides a direct test of the axial vector coupling \(^{59}\)

\[
|\beta| = 0.93 \pm 0.12
\]

where 1 is predicted by the Standard Model.
3. **ELECTROWEAK RESULTS FROM $e^+e^-$ EXPERIMENTS**

The processes to be considered here are $e^+e^- \rightarrow$ hadrons. The final state is dominated by a forward and a backward jet at PETRA and PEP energies. These jets are induced by $u, d, s, c, b$ - quarks (resp. antiquarks) in the relative proportion $4 : 1 : 1 : 4 : 1$. The interference between the electromagnetic ($\gamma$) and the weak ($Z^0$) amplitude leads to observable effects in the hadron production rate and the quark angular asymmetries.

The first quantity is defined

$$ R = \frac{\sum_q \sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{QED}}} $$

$$ = 3 \sum_q \left( Q_q^2 - 2Q_q x e_{\nu_q} + O(x^2) \right) \left( 1 + \delta_{\text{QCD}} \right) $$

with $x$, defined in lecture 2, of order 10% at PETRA/PEP energies. The electroweak contribution is proportional to

$$ \nu_e (\frac{4}{3} \nu_u - \nu_d) $$

and therefore small, since $\nu_e \approx 0$ for $\sin^2 \theta = 0.22$. The factorized form of the weak couplings reflects the assumption of just one massive neutral gauge boson. Results are given in table 7 and fig. 26.

<table>
<thead>
<tr>
<th>EXPT</th>
<th>$\sin^2 \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JADE</td>
<td>$0.23 \pm 0.05$</td>
</tr>
<tr>
<td>MARK J</td>
<td>$0.28 \pm 0.08 - 0.05$</td>
</tr>
<tr>
<td>TASSO</td>
<td>$0.30 \pm 0.23 - 0.07$</td>
</tr>
</tbody>
</table>
The other quantity, the angular asymmetry, measures at these energies only the product \( a_e a_q \), provided the quark flavor \( q \) can be isolated:

\[
A_q = -7.5\% \left( \frac{\sqrt{s}}{35 \text{ GeV}} \right)^2 \frac{g_A^q}{Q_q}
\]

Note that the fractional quark charge enhances the asymmetry. Two methods have been applied to isolate \( e^+ e^- \rightarrow c \bar{c} \) and \( e^+ e^- \rightarrow b \bar{b} \):

i) reconstruction of \( D^0 \) and \( D^{*\pm} \) (fig. 40, 28)
ii) use of semileptonic c- and b-decays (fig. 40, 28)

Results from PETRA at \( \sqrt{s} = 34.5 \text{ GeV} \) from JADE, MARK J, TASSO and from PEP at \( \sqrt{s} = 29 \text{ GeV} \) from HRS, MARK II, MAC are summarized \(^{61}\) in table (\( g_A^c = -1 \) is assumed):

<table>
<thead>
<tr>
<th>MACHINE</th>
<th>( g_A^c )</th>
<th>( g_A^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETRA</td>
<td>1.22 ± 0.40</td>
<td>-1.00 ± 0.30</td>
</tr>
<tr>
<td>PEP</td>
<td>2.40 ± 1.70</td>
<td>-1.10 ± 0.50</td>
</tr>
</tbody>
</table>

The axial couplings agree in magnitude and sign with the prediction of the standard model. The study of \( e^+ e^- \)-interactions gives access to the weak couplings of c and b quarks which are members of the 2\(^{nd}\) and 3\(^{rd}\) fermion generation.

4. WEAK EFFECTS IN CHARGED LEPTON-NUCLEON SCATTERING

There are results from two experiments on electroweak asymmetries involving the product of charged lepton coupling and quark coupling. The effect is of the order

\[
\frac{G}{e^2} Q^2 \approx 10^{-4} \frac{Q^2}{\text{GeV}^2}
\]

and requires a good control of systematic errors.
a) The SLAC experiment\textsuperscript{62}): polarized $e^- + D \rightarrow e^- + \text{anything}$.

A fairly detailed description of this classic experiment is given in the 1981 CERN summer school lectures\textsuperscript{4b}). The quantity measured is the parity violating asymmetry

$$A = \frac{d\sigma(e^-_R) - d\sigma(e^-_L)}{d\sigma(e^-_R) + d\sigma(e^-_L)} = \frac{G_F Q^2}{\sqrt{2} e^2} \frac{18}{5} \left\{ a_1 + a_2 f(y) \frac{q(x) - \bar{q}(x)}{q(x) + \bar{q}(x)} \right\} \approx 1 \text{ for } x > 0.2$$

$$\frac{A}{Q^2} = -(0.57 \pm 0.27) \times 10^{-4 \text{ GeV}^{-2}}$$

$$a_1 = 0.30 \pm 0.08 = \frac{2}{3} a_e (2 \nu_u - \nu_d) = -\frac{1}{2} \left( 1 - \frac{20}{9} \sin^2 \theta \right)$$

$$a_2 = 0.15 \pm 0.25 = \frac{2}{3} \nu_e (2 a_u - a_d) = -\frac{1}{2} \left( 1 - 4 \sin^2 \theta \right)$$

Results from 2 parameter fit: $\rho = 1.74 \pm 0.36 \quad \sin^2 \theta = 0.25 \pm 0.03$

and from 1 parameter fit ($\rho \equiv 1$): $\sin^2 \theta = 0.224 \pm 0.020$.

The $y$-dependence (cf. fig. 29) is incompatible with the assumption, that the righthanded electron is a member of a weak doublet. This experiment with its precise determination of $\sin^2 \theta$ supported strongly the Standard Model. At that time there was no other individual experiment with such a precision.

b) The NA4 Experiment\textsuperscript{63}): $\mu^- C \rightarrow \mu^- + \text{anything}$.

The naturally polarized $\mu$ beam from the CERN-SPS is scattered off carbon. The asymmetry

$$B = \frac{d\sigma(\mu_R^+) - d\sigma(\mu_L^-)}{d\sigma(\mu_R^+) + d\sigma(\mu_L^-)} = \frac{G_F Q^2}{\sqrt{2} e^2} \frac{18}{5} f(y) \frac{q(x) - \bar{q}(x)}{q(x) + \bar{q}(x)} \left\{ b_1 |\lambda| + b_2 \right\}$$

is measured using data at 200 GeV and 120 GeV. The $\mu$-helicity is denoted by $\lambda$. The measurement of $B$ leads to

$$b_1 |\lambda| + b_2 = \frac{2}{3} (\nu_\mu |\lambda| - a_\mu) (2 a_u - a_d) = 0.45 \pm 0.11 \quad 0.05$$

and from this $\sin^2 \theta = 0.23 \pm 0.07 \pm 0.04$. Assuming $\sin^2 \theta = 0.23$ the weak isospin of the righthanded muon comes out to be $I_3^R(\mu) = 0.00 \pm$
0.06 \pm 0.04 in agreement with the standard assignment of righthanded muons in SU(2)-singlets.

If 2a_u - a_d = \frac{3}{2} is assumed, then one gets

\nu_e = 0.15 \pm 0.25 \text{ from the SLAC experiment}
\nu_\mu = -0.06 \pm 0.14 \text{ from the NA4 experiment}

This can be compared with the combined results from \nu_\mu-e-experiments giving \nu_e = 0.002 \pm 0.058 (cf. lecture 2).

5. PARITY VIOLATION IN CAESIUM

Significant parity violating effects have been observed in bismuth, lead, thallium and caesium\(^{64}\). The recently published results of the experiment of M.A. BOCCHIAT et al.\(^{65}\) will be mentioned in this section. A circularly polarized laser beam (direction \(\vec{K}\)) is used to excite the Cs 6S \(F = 3\) state to 7S \(F = 4\) in a constant STARK field with \(\vec{E}\) perpendicular to \(\vec{K}\). The effective dipole operator is given by

\[
\vec{d} = -\alpha \vec{E} - i\beta \vec{E} \times \vec{E} + M_{\Gamma} \gamma_0 \vec{K} - i \text{ Im } E_1 \vec{\sigma}
\]

where the first 2 terms are STARK induced, the third term is due to the magnetic dipole and the last term is the weak neutral current induced electric dipole. One looks for an interference between the \(E_1\) and \(\beta\)-term.

Two measurements were carried out:

\[
\text{Im} E_1/\beta = -(1.78 \pm 0.26 \pm 0.12) \text{ mV/cm} \quad \Delta F = 1
\]
\[-(1.34 \pm 0.22 \pm 0.11) \text{ mV/cm} \quad \Delta F = 0
\]

It follows:

\[
Q^{\text{exp}}_W = -66.5 \pm 7.2 \pm 5.1 = -0.574 (N - (1-4 \sin^2 \theta)Z)
\]

thus \(\sin^2 \theta_{\text{corr}} = 0.205 \pm 0.034 \pm 0.024\) in good agreement with other measurements (cf. fig. 30).
CONCLUSIONS

1. In the first decade after the discovery of weak neutral currents
   the electro-weak parameter $\sin^2 \theta$ has been measured over a big range
   of space- and timelike-momentum transfers squared. This is illustrated
   in fig. 30. All determinations are compatible with each other.
   The data from the neutrino-quark sector are the most precise measurements.

2. On the basis of $\alpha, G, \sin^2 \theta^{\nu q}$ the masses of the weak gauge bosons
   can be predicted within the Standard Model:

   $$m_W = \frac{37.2810 \pm 0.0003}{\sqrt{1-0.0696 \pm 0.002 \sin \theta}} \text{ GeV} \quad \text{and} \quad m_Z = \frac{m_W}{\cos \theta}$$

   Using the corrected value from KIM et al (1981) the masses
   can be calculated (including 1-loop corrections):

   $$m_W = 83.0 \pm 2.9 \text{ GeV}$$
   $$m_Z = 93.8 \pm 2.4 \text{ GeV}$$

   These values anticipated on the basis of the Standard Model have been
   measured 1983 by the UA-experiments at the CERN SppS collider and are
   (average over values from UA1 and UA2):

   $$m_W = 82.2 \pm 1.8 \text{ GeV}$$
   $$m_Z = 93.2 \pm 1.5 \text{ GeV}$$

   in good agreement with the predicted masses within the Standard Model.
   This is a great success.
   Sofar, all tests described refer to leading order only. The next step
   will be tests sensitive to 1-loop corrections which are in progress.
Fig. 21: Top view of the bubble chamber BEBC in its hybridized version with veto plane, picket fence, and 2-plane muon identifier. The neutrino beam enters from right (47).
Fig. 22: Constraints of the BEBC $\nu, \bar{\nu}$, $D_0$ measurements on the lefthanded couplings and comparison with Standard Model.
Fig. 23: Comparison of $R_{\nu}$ and $R_{\bar{\nu}}$ with the prediction of the Standard Model. The CDHS data in this plot are preliminary. In the meantime the final numbers are available ($R_{\nu}$ increased from $0.357 \pm 0.015$ to $0.363 \pm 0.015$).
Fig. 24: Comparison of simultaneous $\rho$ and $\sin^2 \theta$ fits from recent experiments. The final point, indicated KIM et al. 48), is the combined fit of all data up to 1979.
Fig. 25: Observation of the $\Delta^0 (1236)$ resonance induced by weak neutral currents (ref. 57). For comparison the exclusive $p\pi^0$ state in charged current events.
Fig. 26: The ratio $R$ compared to predictions of Standard Model with $\sin^2 \theta = 0.23$ and $\alpha_s = 0.20$. The dotted curve is the expectation of the naive quark-parton model.
Fig. 27
Fig. 28: Asymmetries in c- and b-enriched samples from various $e^+e^-$ experiments (ref. 61).
Fig. 29: Asymmetry vs $y$ compared to the Standard Model and a hybrid model, where the righthanded electron is assumed to be a member of a weak doublet.
Fig. 30: Comparison of measurements of $\sin^2 \theta$ in various types of low energy experiments.
Lecture 4: SEMILEPTONIC CHARGED CURRENT INTERACTIONS  
(Selected Topics)

1. The naive quark-parton model
2. The strange sea in the nucleon
3. Limits on right-handed currents
4. $\nu_e$-interactions
5. The KOBAYASHI-MASKAWA matrix
6. Determination of the $B$-lifetime
7. Search for top quarks
8. $B$-decays
1. THE NAIVE QUARK-PARTON MODEL

The deep inelastic scattering experiments $eN \rightarrow e + \text{anything}$ at SLAC and later on $\nu N \rightarrow \mu^- + \text{anything}$, $\bar{\nu} N \rightarrow \mu^+ + \text{anything}$, $\mu N \rightarrow \mu + \text{anything}$ have led to a simple picture of the nucleon. At sufficiently high $Q^2 = -q^2$ (the 4-momentum transfer squared) the intermediate vector bosons interact incoherently with quasifree, pointlike partons identified with quarks and antiquarks. Since the scattering process is purely spacelike a frame can be found in which the gauge boson carries only 3-momentum, i.e. no energy. This particularly simple frame is called the BREIT frame:

The $W$ interacts either with a lefthanded quark-parton or a righthanded antiquark-parton according to the V-A structure of charged weak currents. Angular momentum conservation implies a helicity $-1$ $W$ to couple to a quark and a helicity $+1$ $W$ to an antiquark. This determines the angular distribution: an isotropic contribution (from $W^+q$)

and a contribution $4\cos^2\theta - (1-y)^2$ from $W^\pm\bar{q}$, where $1-y$ is the muon energy in units of the energy of the incoming neutrino. In the approximation of spin 1/2 quark-partons with zero intrinsic transverse momentum a helicity 0 $W$ has nothing to couple to, therefore the angular distribution has no
term \((1-y)^1\). It is now trivial to write down the differential cross section of deep inelastic \(\nu p\)-scattering:

\[
d^2\sigma(\nu p) = \sum_f P_f(x) dx \cdot d\sigma(\nu f)
\]

\[
= \frac{G^2}{\pi} 2ME_\nu \left[ x P_d(x) 1 + x P_u(x) (1-y)^2 \right] dx dy
\]

where \(x = \frac{2 \sqrt{-q^2}}{p}\) is the fractional momentum of the parton in the proton, \(P_f(x)\) is the probability density to find a quark-parton with flavor \(f\) in the proton and \(d\sigma(\nu f)\) the elastic cross section of the subprocess. In the above formula only the flavors of the first generation are taken into account. The \(\nu p\) cross section depends only on the scaling variables \(x\) and \(y\). This is called BJORKEN scaling. Precise measurements have shown that scaling is not strictly fulfilled (see lecture 5), but for many applications a quite good approximation.

Due to the V-A structure neutrino and antineutrino experiments have the unique feature of differentiating between quarks and antiquarks in the nucleon and thus between valence and sea quarks. For instance, a proton is composed of \((uud)^{\text{valence}}\) and \((w \bar{w} + d\bar{d} + s\bar{s} + \ldots)^{\text{sea}}\). This implies sumrules for the quark-parton densities \(P_f(x)\):

\[
\int_0^1 dx \left( P_u(x) - P_d(x) \right) = 2
\]

\[
\int_0^1 dx \left( P_d(x) - P_u(x) \right) = 1
\]

\[
\int_0^1 dx \left( P_f(x) - P_{\bar{f}}(x) \right) = 0 \quad \text{for} \ f = s, c, b, t
\]

In fig. 31 recent measurements of the fractional momentum distributions by the CDHS group\(^{66}\) are shown. Note the substantial rise of the strange sea. There is no direct evidence of the charmed sea yet.

The comparison of charged lepton nucleon (coupling \(Q_f^2\)) with neutrino-nucleon scattering (V-A coupling) shows that quarks are fractionally charged (fig. 35).
2. THE STRANGE SEA IN THE NUCLEON

Forgetting about the quarks in the third generation there are 4 charm changing charged current interactions:

\[νd + μ−c \quad \text{rate} \sim |U_{cd}|^2\]
\[νs + μ−c \sim |U_{cs}|^2\]
\[\bar{ν}s + μ^+c \sim |U_{cs}|^2\]
\[\bar{ν}d + μ^+c \sim |U_{cd}|^2\]

Only the first reaction takes place on a valence quark, but is suppressed by the mixing matrix element squared: \[|U_{cd}|^2 \approx \sin^2θ_c \quad (θ_c = \text{CABIBBO angle}) \approx 0.05.\] Reaction 4 is negligible compared to reaction 3. Reactions 1, 2, 3 are of similar strength. All subprocesses are flat in y and are sizeable, where the quark momentum distributions \((xd(x), xs(x), \bar{x}s(x))\) are sizeable, i.e. for small values of x. In a recent analysis the CDHS group\(^{67a}\) has detected the final state charmed quark by its semileptonic decay:

\[c \rightarrow μ^+ν_μ + \text{anything}\]
\[\bar{c} \rightarrow μ^−\bar{ν}_μ + \text{anything}\]

Therefore, the opposite sign dimuon events in the \(\bar{ν} - \text{run}\) allow to extract the strange sea \((x\bar{s}(x))\). The result is shown in fig. 32 and compared to the shape of the distribution \(x\bar{u}(x) + x\bar{d}(x) + 2x\bar{s}(x)\) as measured directly in normal \(\bar{ν}\) interactions. There is a slight dependence on how the charm threshold is treated. The strange sea and the nonstrange sea have the same form. The neutrino dimuon data get two contributions, one from the valence quarks and one from the strange sea. A good fit is obtained (fig. 32).

The dimuon data confirm the GIM construction\(^{17}\), which is a special case of the KOBAYASHI-MASKAWA scheme. The matrix elements \(|U_{cd}| = 0.24 ± 0.03\) and \(|U_{cs}| > 0.59\) (90% CL) have been derived from the dimuon data. Early neutrino experiments\(^{69}\) have given already indirect evidence of semileptonic charm decay in correlation with strange particle production. In the bubble chamber BEBC filled with hydrogen, i.e. free protons, some rare cases of fully reconstructable charmed particle decays did occur. Fig. 36 shows a famous event\(^{70}\). The 3 C-fit resulted in a very accurate mass measurement of \(D^0\) and \(D^{*+}\). It should be mentioned that there are \(ν, \bar{ν}\) induced dimuon events where the two muons have the same charge. No satisfactory explanation of their anomalous rate has so far been proposed\(^{71}\).
3. **LIMITS ON RIGHT-HANDED CURRENTS**

The effective Lagrangian for the semileptonic charged current process \( \nu_\mu d \rightarrow \mu^- u \) is

\[
L_{\text{eff}}^{\text{CC}} = \frac{G_f}{\sqrt{2}} \left( \bar{\nu}_u \gamma_\lambda (1 + \gamma_5) \nu_d \right) \left( \bar{\psi}_u \gamma_\lambda (1 + \gamma_5) \psi_\mu \right)
\]

and excludes by construction righthanded weak current. It is convenient to generalize this Lagrangian in the following way:

\[
L_{\text{eff}}^{\text{CC}} = \frac{G_f}{\sqrt{2}} \left( \bar{\psi}_u \gamma_\lambda (1 + \gamma_5) \nu_d \right) \left[ \bar{\psi}_u \gamma_\lambda \left( C_L (1 + \gamma_5) + C_R (1 - \gamma_5) \right) \psi_d \right]
\]

and to test on \( C_R \neq 0 \) by comparison with the measured \( y \)-distributions at large \( x \). The differential cross section is now modified:

\[
\frac{1}{\sigma^0} \left( \frac{d^2 \sigma(N)}{dx dy} \right) = \left[ q(x) + \left( \frac{C_L}{C_R} \right)^2 \bar{q}(x) \right] + (1-y)^2 \left[ q(x) + \left( \frac{C_R}{C_L} \right)^2 \bar{q}(x) \right]
\]

\[
\equiv q_L(x) + (1-y)^2 q_R(x)
\]

\[
\frac{1}{\sigma^0} \left( \frac{d^2 \sigma(N)}{dx dy} \right) = q_R(x) + (1-y)^2 q_L(x)
\]

where \( q(x) \) and \( \bar{q}(x) \) are the relevant quark and antiquark distributions. The \( \nu \) and \( \bar{\nu} \) induced differential cross sections are accurately measured (fig. 37), thus the CDHS Collaboration\(^\text{72}\) could conclude

\[
\left( \frac{C_R}{C_L} \right)^2 < 0.005 \text{ at } 50\% \text{ CL.}
\]

4. **\( \nu_e \)-INTERACTIONS**

Neutrino beams at accelerators are derived from \( \pi^- \) and \( K^- \)-decays and are therefore predominantly \( \nu_\mu \) or \( \bar{\nu}_\mu \) beams depending on the selected parent charge. Nevertheless, these beams contain a contamination of \( \nu_e \) and \( \bar{\nu}_e \) at the 1% level and the GARGAMELLE experiments at the PS did collect quite a few \( \nu_e, \bar{\nu}_e N \) interactions. Fig. 38 shows the total cross sections vs the neutrino energy\(^\text{73}\). The linear rise, characteristic for low energy experiments, manifests scaling behaviour. The slopes agree well with the ones measured
in $\nu_\mu$, $\overline{\nu}_\mu$ experiments and constitute a check of e - $\mu$ universality at the level of 30%.

The very first antineutrino induced interaction was observed in a reactor which produces an $\overline{\nu}_e$-beam\cite{74}.

Physics in a $\nu_e$-beam derived via the cascade $\pi^+ \to \mu^+ \nu_\mu$ and $\mu^+ \to \overline{\nu}_\mu e^+ \nu_e$ is considered at Los Alamos\cite{75}.

5. THE KOBAYASHI-MASKAWA MATRIX

The KM-matrix\cite{76} is the extension of the GIM matrix from 2 to 3 quark generations. Instead of

$$
\begin{pmatrix}
\cos \theta & \sin \theta \\
-sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
d \\
s
\end{pmatrix}
$$

with 1 angle, the CABIBBO angle, there is now

$$
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
= 
\begin{pmatrix}
c_\beta c_\theta & c_\beta s_\theta & s_\beta \\
c_\gamma s_\theta - s_\gamma b_\theta e^{i\delta} & c_\gamma c_\theta - s_\gamma b_\theta e^{i\delta} & s_\gamma e^{i\delta} \\
c_\gamma b_\theta - s_\gamma b_\theta e^{-i\delta} & c_\gamma b_\theta - s_\gamma b_\theta e^{-i\delta} & c_\gamma c_\theta
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
$$

with 3 angles $\theta$, $\beta$, $\gamma$, 1 phase $\delta$ and $c$, $s$ standing for $\cos$, $\sin$.

For example:

$$J^{ud'}_{\lambda} = \overline{\psi}_u \gamma_\lambda (1-\gamma_5) (c_\beta c_\theta \psi_d + c_\beta s_\theta \psi_s + s_\theta \psi_b).$$

Instead of the above notation due to MAIANI\cite{77} an alternative notation has recently been proposed by WOLFENSTEIN\cite{78} who expands the unitary, nearly unit KM matrix in terms of a small parameter.

There are detailed analyses of experiments relevant for the determination of the KM matrix elements\cite{79}. Here, only a few ingredients will be mentioned:
Nuclear $\beta$-decay

$K_{e3}$-decays

$Y$-decays (WA2)

$\nu, \bar{\nu} + \mu \bar{\nu} \chi$ (CDHS)

CLEO, CUSB $\frac{b+u}{b+c}$

$\frac{U_{ub}}{U_{cb}} < 0.15$ (90% CL)

$U_{ud} = 0.9735 \pm 0.0015$

$U_{us} = 0.221 \pm 0.002$

$U_{cd} = 0.23 \pm 0.03$

$|U_{cs}| > 0.59$ (90% CL)

$b$-lifetime

\[ \frac{\sin^2 \beta}{a_\beta} + \frac{\sin^2 \gamma}{a_\gamma} = 1 \]

$0.5 < \tau_b < 1.4$ psec

$a_\beta = 0.598 a_\gamma; a_\gamma = \frac{0.058}{\sqrt{\tau_b / \text{psec}}}$

The constraints are shown in fig. 39. Only a small region remains.
The $B$-lifetime measurements determine essentially $|\sin \gamma|$. The statistical and systematic uncertainties in measuring the $B$-lifetime are for the time being quite large. Therefore, the upper limit of the JADE result\(^\text{80}\) and the lower limits of the MARK II\(^\text{81}\) and MAC\(^\text{82}\) results are used.

6. DETERMINATION OF $B$-LIFETIME

In $e^+e^-$ interaction at PETRA and PEP energies the quark-antiquark jets occur $b$-flavored at a rate 1 : 11. The natural mixture can be made $b$-rich or $c$-rich applying certain selection criteria. First of all a fast muon is requested in a multihadron event. The second criterion uses the fact that $B$-hadrons are heavy and fast objects, giving rise to high transverse momentum particles and a characteristic event shape. For instance fig. 40 shows the composition of the transverse momentum distribution of prompt muons, i.e. those coming from $b$- and $c$-decays. It is obvious that a cut in $P_T(\mu)$ around 1 GeV/c generates a $b$-rich ($P_T > 1$ GeV/c) and a $b$-poor sample ($P_T < 1$ GeV/c). The background due to hadrons misidentified as muons or $K_{\mu2}$, $\pi_{\mu2}$ decays in flight, which is of the order 20 - 30%, has been subtracted.
A schematic $b \bar{b}$ event with $b \rightarrow \mu + c$ is displayed in fig. 41. The muon track originating from the $B$-vertex is extrapolated from the inner detector to the interaction region. The measured distance $\delta$ of closest approach to the $e^+ e^-$-interaction point (fig. 41) is related to the lifetime of $B$-hadrons:

$$\delta = \ell \sin \alpha = \frac{\beta \gamma c \tau}{\frac{\ell}{L}} f(\alpha^*)$$

where $\ell$ is the actual decay length, $L = \beta \gamma c \tau$ the average decay length of the $B$ with velocity $\beta$ and lifetime $\tau$, $\alpha^*$ the decay angle of the $\mu$ in the $B$ rest frame. It is assumed that the $B$ flight direction is well approximated by the reconstructed event axis. Assuming $\tau = 1$ psec the typical average $\delta$ is around 150 $\mu$m (depending on $P_T$-criterion). This is to be compared with the track uncertainty $\sigma$ after extrapolation, which is 450 $\mu$m for JADE, 250 $\mu$m for MARK II, 800 $\mu$m for MAC. The statistical precision using $n$ $B$-events is $\sigma / \sqrt{n}$. With the help of an extensive Monte Carlo simulation the observed $\delta$-distribution is expressed in terms of $b \rightarrow \mu$, $c \rightarrow \mu$, and hadrons $\rightarrow \mu$. The published results are

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$#b$ (estimated)</th>
<th>lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JADE</td>
<td>12</td>
<td>$&lt; 1.4$ psec</td>
</tr>
<tr>
<td>MARK II</td>
<td>67</td>
<td>$1.20^{+0.45}_{-0.36} \pm 0.3$ ps</td>
</tr>
<tr>
<td>MAC</td>
<td>112</td>
<td>$1.8 \pm 0.6 \pm 0.6$ ps</td>
</tr>
</tbody>
</table>

If the lifetime is indeed high, say 1.5 psec, it should be possible to measure the $B$-decay vertex distribution. In this case the average length is

$$L = \beta \gamma c \tau \approx 3 \times 0.45 \text{ mm} \approx 1.4 \text{ mm},$$

this means effectively about 1 mm in the plane perpendicular to the beam. In an apparatus with a vertex detector a fair fraction of the $B$-decay vertices will be detectable and open new perspectives for $B$-physics.
7. SEARCH FOR TOP-QUARKS

A substantial amount of time at PETRA was devoted to search for t quarks (fig. 42). The machine was operated in a scanning mode. The step size of $\Delta E = 30$ MeV corresponds to the machine energy resolution. For each point each of the 4 running experiments collected 60 nb$^{-1}$. This gives just enough multihadron events to detect a resonance in

$$R = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma_0(e^+e^- \rightarrow \mu\bar{\mu})}.$$  

Assuming a BREIT-WIGNER resonance shape one obtains

$$f\sigma(E)dE = \frac{6\pi^2}{M^2} \frac{\Gamma_e}{\Gamma} \frac{\Gamma_h}{\Gamma}$$

with $M =$ mass of $t\bar{t}$ and $\Gamma_e$ resp. $\Gamma_h$ being the leptonic resp. hadronic widths. Fitting a GAUSS curve width $\Delta E$ and varying position along the energy axis ($\sqrt{s}$) on top of the constant continuum due to $u$, $d$, $s$, $c$, $b$ events the limit $\Gamma_e/\Gamma < 1$ keV (50% CL) is obtained to be compared with $\Gamma_e(t\bar{t}) \approx 5$ keV as extrapolated from $\phi(s\bar{s})$, $J/\psi(c\bar{c})$, $Y(b\bar{b})$. In conclusion, there is no indication of a resonance due to $t\bar{t}$ with $Q_t = 2/3$ and its mass must exceed 22.6 GeV.

At the time of these lectures the UA1-group$^8$ claimed the observation of a few events containing the flavor top so long searched for. No precise mass was quoted, but is well above the PETRA limit.

8. B-DECAYS

Hadrons containing b-quarks are abundantly produced at $e^+e^-$ machines (CESR, DORIS, PETRA, PEP). The CLEO group$^{84}$ at CORNELL succeeded in reconstructing B-mesons starting from a sample of identified $D^0$, $D^{*\pm}$ and adding 1 or 2 charged pions. They obtained for the masses:

$$m(B^-) = (5270.8 \pm 2.3 \pm 2.0) \text{ MeV}$$

$$m(B^0) = (5274.2 \pm 1.9 \pm 2.0) \text{ MeV}.$$  

There are many investigations using continuum events, of which 1 : 11 contain B-hadrons.

a) Semileptonic B-decay: In the free quark model this decay proceeds like

$$b \rightarrow (c,u) + \{ l\bar{\nu}_l \quad l = e, \mu, \tau$$

$$q\bar{q}' \quad q = d, s$$
Thus, the branching ratio into $\mu\bar{\nu}_\mu$ is expected to be

$$BR_\mu = \frac{1}{(1+1+0.3) + 3(1+0.3)} = 16\%$$

The two terms in the denominator count the lepton and quark flavors. Heavy flavors are suppressed (phasespace). The average over all PETRA/PEP measurements\(^{85}\) is $(11.8 \pm 0.6)\%$, which indicates that the use of the free quark model is too naive and hadronic corrections must be taken into account.

b) The electron spectrum in $b$-decays near the kinematic endpoint is sensitive to the mass of the accompanying flavor. Measurements of CUSB and CLEO\(^{86}\) show that $b \rightarrow u + e\bar{\nu}_e$ is suppressed against $b \rightarrow c + e\bar{\nu}_e$ (fig. 44).

for founding the KOBAYASHI-MASKAWA matrix elements $U_{cb}$ and $U_{ub}$ (see sect. 5).

c) The standard model forbids at tree level flavor changing neutral weak currents. Searches have been made for $b \rightarrow (s, d) + \bar{\nu}\nu$.

4 experiments reported\(^{85}\)

$$\frac{\# B \rightarrow \bar{\nu}\nu + X}{\# B \rightarrow \text{all}} < 0.7\% \text{ at } 50\% \text{ CL.}$$

d) The CLEO group compared $B \rightarrow D^0 + X$ with $b \rightarrow c + \bar{\nu}\nu$ and got good agreement. This is support for the $V - A$ structure and dominance of the spectator model.
Fig. 31: Flavor composition of the nucleon

Fig. 32: The strange sea in the nucleon (ref. 67a)
Fig. 33: The nonstrange quark distributions (ref. 67b)

Fig. 34: The d/u ratio (ref. 68)
Fig. 35: Comparison of $\nu$ and $\mu$ deep inelastic experiments (ref. 66)
Fig. 36: Charm production in BEBC
Fig. 37: $y$-distributions in $\nu$- and $\bar{\nu}$-Fe charged current interactions (ref. 72)
Fig. 38: Total $\nu_e$ and $\bar{\nu}_e N$ cross sections observed in GARGAMELLE

- $\sigma_{\nu_e} = (0.7 \pm 0.2)E$
- $\sigma_{\bar{\nu}_e} = (0.25 \pm 0.07)E$
Bounds on weak angles

Fig. 39: Bounds on weak angles
Fig. 40: Transverse momentum distribution of prompt muons in multihadron events. The dotted histograms are Monte Carlo predictions of the contributions from semileptonic c- and b-quark decays. The nonprompt background is subtracted.
Fig. 41a:
Sketch of an event $e^+e^- \rightarrow b\bar{b}$ and semileptonic $b$-decay. The extrapolated $\mu$-track fails the $e^+e^-$ interaction point. This point is either determined from the event itself or assumed a priori using external information.

Fig. 41b: The observed impact parameter distribution of 104 muons in the $b$-enriched event sample of MARK II (ref. 81). The average is $(106 \pm 29)$ $\mu$m. The dotted curve is a GAUSSian distribution with width 250 $\mu$m.
$\Delta \sqrt{s} = 30 \text{ MeV}, \int L dt \sim 50 \text{ nb}^{-1} / \text{step.exp.}$

$\sqrt{s}_{\text{max}} = 43.15 \text{ GeV}$

$\langle R \rangle = 3.94 \pm 0.06$

Fig. 42: Search for top in the highest PETRA energy range
Fig. 43: The mass spectrum for B meson candidates with restrictive cuts on the D mass.

Fig. 44: Electron momentum spectrum and comparison with theory (Altarelli et al.)
Lecture 5: STRONG INTERACTIONS

1. The claims of QCD
2. Asymptotic freedom (qualitative)
3. The basic problem
4. Gluons
5. Scaling violation in deep inelastic scattering (DIS)
6. The structure function $x F_3(x, Q^2)$
7. Analysis of $F_2$ and $\eta^V$
8. $\alpha_s$ from $e^+e^-$-interactions
1. THE CLAIMS OF QCD

Quantum chromodynamics, together with the existence of flavored quarks, describes the structure of all hadrons. Hadrons are composites of quarks. The force between quarks is derived from the local SU(3) gauge group. The analog role of electric charge and photon in QED is played by color and gluon in QCD. There is a crucial difference: the color gauge group is nonabelian. Gluons, unlike photons, will interact with themselves. At decreasing distance the force gets smaller and smaller. This is called asymptotic freedom and allows for a perturbative treatment. On the contrary, at increasing distance the force gets stronger and stronger, hindering quarks to leave the interaction region as free particles. This aspect is called confinement and is an unsolved problem. An account of the development of QCD can be found in ref. 88.

2. ASYMPTOTIC FREEDOM (QUALITATIVE)

a) Deep inelastic scattering: The SLAC-MIT experiment 1967 has initiated an important research line. At high \( Q^2 \) it turned out that the nucleon structure function \( F_2(x, Q^2) \) is essentially independent of \( Q^2 \). This phenomenon was called scaling or BJORKEN-scaling. A simple interpretation due to FEYNMAN is to think of the nucleon as being made of quasi-free partons later on identified with quarks. The eN experiments were paralleled by the neutrino experiments in GARGAMELLE. The comparison of eN and \( \nu N \) experiments (cf fig. 35) confirmed the assignment of fractional charges to quarks. Furthermore, it turned out that quarks and antiquarks carry only about 50% of the nucleon momentum. This was taken as evidence for the existence of another type of partons in the nucleon which are not "seen" in reactions induced by \( \gamma \) or \( W^\pm, Z \). These inert partons were called gluons. More detailed experimental investigations, again at SLAC, have shown 1974 that scaling did not hold exactly. The small scaling violation attracted a strong theoretical interest and was the beginning of a big experimental effort both in \( eN \) and \( \nu, \bar{\nu}N \) experiments extending \( Q^2 \) up to 200 GeV^2.

b) \( e^+e^- \rightarrow q\bar{q} \): One of the most fundamental observable is the total hadronic cross section normalized to the QED \( \mu^+\mu^- \) production cross section:

\[
\frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{QED}}} = 3 \left[ \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 + \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right] = \frac{11}{3} = 3.67
\]
The factor 3 takes the three color degrees of freedom of quarks into account. The five terms represent the electric quark charges squared for the flavors $u, d, s, c, b$ assuming $\sqrt{s}$ to be above the threshold for $b\bar{b}$ and below the threshold for $t\bar{t}$ production. A glance at fig. 45 shows that this simple theoretical calculation is a quite good approximation provided the resonance regions are avoided. This means that away from thresholds strong interactions between quarks are small. Theoretically expected deviations due to $\gamma, Z^0$ interference have been considered in lecture 3, deviations due to strong interactions will be discussed below.

3. THE BASIC PROBLEM

As mentioned above quarks are quasifree pointlike particles only at small distance scale. In this regime the force due to strong interactions is weak and perturbation calculations can be performed. However, observations are made at large distance scale. The transition from quarks and gluons to the observed hadrons should in principle be described by nonperturbative QCD, however so far without success. For this reason more or less sophisticated models for the hadronisation process have been invented and investigated. They contain free parameters which are fixed by fitting the model predictions to experimental data. Various models give a reasonably good phenomenological description of all experimentally investigated distributions.

The hadronic final states in $e^+e^-$-interactions, for instance, are interpreted with the help of such models to deduce the properties of perturbative QCD, in particular the strong coupling constant $\alpha_s(Q^2)$. At the present time the extraction of the running coupling constant $\alpha_s(Q^2)$ is model dependent. A better understanding of the hadronisation process must be attained.

4. GLUONS

More than 10 years ago the determination of $\int_0^1 F_2(x)dx \approx 0.5 + 1$ in lepton nucleon deep inelastic scattering experiments led to the conclusion that there must be partons in the nucleon which do not couple to $\gamma$ and $W^\pm$. The carriers of the missing momentum were named gluons or more vaguely glue. A few months after the start up of PETRA clear 2-jet and, at about 10 times lower rate, 3-jet events have been observed. At these high energies the process $e^+e^- \rightarrow q\bar{q}$ with the subsequent hadronisation of the
q and \( q \) running off in opposite directions appears as two jets. The 3-jet events (fig. 46, 47) were then interpreted as \( e^+e^- \rightarrow \bar{q}qg \) with a hard gluon giving rise to the third jet. The gluon bremsstrahlung spectrum disfavors hard gluons. Therefore, in most of the hadron events jets will only be broadened. The same kind of broadening is also observed in the forward jet in lepton nucleon scattering. The observed \( p_T \) behaviour in \( \pi^0N \) and \( \mu N \) experiments agree well (fig. 48)\(^9\).

Structure function analyses favor the spin 1 assignment to gluons. In events \( e^+e^- \rightarrow 3\)-jets the angular distribution contains information about the gluon spin and agrees with the assumption of vector gluons (fig. 46).

ISR data (fig. 49) and \( \overline{S}\overline{P}pS \) data\(^3\) support this.

A crucial feature of QCD is the selfinteraction of gluons giving rise to processes like \( g + g + g \) (triple gluon vertex). In reactions induced by \( \gamma \) and \( Z, W \) the triple gluon vertex occurs in second or higher order in \( \alpha_s \).

In 3-jet events, as observed in \( e^+e^- \)-interactions, the gluon-jet should be different than the quark- or antiquark-jet. The JADE collaboration at PETRA has investigated this question for some time and indeed noticed differences\(^9\)\(^1\).

The most direct way of looking for gluon selfinteraction is in hard scattering processes in pp (ISR) or \( pp \) (CERN collider) collisions. In understanding the behaviour of the ratio \( K^-/\pi^- \) as opposed to \( K^+/\pi^+ \) for various energies as a function of \( x_T = 2p_T/\sqrt{s} \) (fig. 49) the ABCDHW collaboration\(^9\)\(^0\) at the ISR is led to the conclusion that the hard scattering process \( qg \rightarrow qg \), which includes also the triple gluon vertex, contributes significantly. The qualitative argument is this: \( K^- \) being composed of \( \bar{u}s \) cannot be formed in first generation by a knocked out valence quark (u,d) of the colliding protons. Most likely is the assumption that a hard gluon splits into \( s\bar{s} \) with the \( \bar{s} \) picking up a \( u \) quark from the vacuum to from a \( K^- \).

5. SCALING VIOLATION IN DI-SCATTERING

The inclusive processes

\[
\begin{align*}
\nu_N & \rightarrow \mu^- + \text{anything} \\
\bar{\nu}_\mu N & \rightarrow \mu^+ + \text{anything} \\
\ell N & \rightarrow \ell + \text{anything} \quad (\ell = e, \mu)
\end{align*}
\]
can be written in terms of 3 resp. 2 structure functions:

\[ d^2\sigma(vN) = \sigma_0^{v} dx dy \left[ \frac{1}{2} y F_1^v (x, Q^2) + (1-y) F_2^v (x, Q^2) + y(1-y) F_3^v (x, Q^2) \right] \]

\[ d^2\sigma(\bar{v}N) = \sigma_0^{\bar{v}} dx dy \left[ \frac{1}{2} y F_1^{\bar{v}} (x, Q^2) + (1-y) F_2^{\bar{v}} (x, Q^2) + y(1-y) F_3^{\bar{v}} (x, Q^2) \right] \]

\[ d^2\sigma(2N) = \sigma_0^{2} dx dy \left[ \frac{1}{2} y F_1^2 (x, Q^2) + (1-y) F_2^2 (x, Q^2) \right] \]

\( N \) is assumed to be an isoscalar target. The heavy target calorimeters are nearly isoscalar. From the measured outgoing charged lepton and the hadronic energy \( v \) the three variables

\[ x = \frac{Q^2}{2M_v} \quad y = \frac{E}{E'} \quad Q^2 = 2E'E''(1-\cos \theta) \]

can be computed. \( E \) is the energy of the incoming lepton. In neutrino experiments this quantity must be obtained from the final state. In narrow band beams there is however a correlation between the radial position of a \( v \) event and its energy.

The aim of the experiments is to extract from the data

\[ xF_3(x, Q^2) \]
\[ F_2(x, Q^2) \]

\[ R \equiv \frac{F_2(x, Q^2)(1 + \frac{Q^2}{M_v^2}) - 2xF_1(x, Q^2)}{2xF_1(x, Q^2)} = \frac{F_L(x, Q^2)}{2xF_1(x, Q^2)} \]

The \( v \) and \( \bar{v} \) experiments are unique in getting access to the structure function \( xF_3 \).

In the quark-parton model the structure functions have a simple interpretation:

\[ xF_3(x) = x(q(x) - \bar{q}(x)) \]
\[ F_2(x) = x(q(x) + \bar{q}(x)) \]
\[ R(x) = 0 \]

with the abbreviation \( q(x) \equiv u(x) + d(x) + s(x) + c(x) \). \( R = 0 \) is called the CALLAN-GROSS relation and reflects the fact that quarks have spin 1/2. It is interesting to note that \( xF_3 \) depends only on the valence quarks in the nucleon, since due to the difference \( q(x) - \bar{q}(x) \) the sea contribution drops out. Not so for \( F_2 \), which has both a valence and a sea contribution.
6. THE STRUCTURE FUNCTION $x F_3(x,Q^2)$

From the difference of the differential cross sections of $\nu N$ and $\bar{\nu} N$ data the structure functions $x F_3$ is obtained as follows:

$$x F_3(x,Q^2) = \frac{\pi}{G^2 M^2 E} \frac{d^2\sigma(\nu N) - d^2\sigma(\bar{\nu} N)}{(1-(1-y)^2) dx dy} = x q(x,Q^2) - \overline{x q(x,Q^2)}$$

High statistics is needed since the difference of the cross sections is involved. The valence distributions are measured up to $x \approx 0.65$. The QCD-interpretation of $x F_3$ is given in terms of the ALTARELLI-PARISI equation\textsuperscript{92}:

$$\frac{3}{3\pi Q^2} x F_3(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 P_{qq} \left( \frac{x}{z} \right) z F_3(z,Q^2) \frac{x dz}{z^2}$$

The left hand side is directly measured (the slopes in fig. 50). The integral on the right hand side is a convolution of $x F_3$, which is measured up to 0.65, with a theoretically known function $P_{qq}$. Thus, the running coupling constant can be deduced.

In practice, a parametrisation (cf. fig. 51)

$$x F_3(x,Q^0_0) = a_3 (1+b_2 x)(1-x)^3 \quad \text{for } Q^0_0 = 4.5 \text{ GeV}^2$$

is assumed, furthermore the GROSS-LLEWELLYN-SMITH sumrule (fig. 52)

$$\int_0^1 F_3 dx = 3(1- \frac{\alpha_s}{\pi})$$

is used to constrain $a_3$. In order to avoid the high twist region cuts are applied: $Q^2 > 2$ GeV$^2$ and $W^2 > 11$ GeV$^2$. Under these conditions the CDHS collaboration\textsuperscript{95} has obtained the result

$$\Lambda_{MS} = 0.2^{+0.2}_{-0.1} \text{ GeV}$$

This result does not depend upon the gluon distribution function. It is per construction insensitive to the gluon selfcoupling. However, it is sensitive to the spin of the gluon and supports in fact its vector nature.
The QCD mass scale parameter $\Lambda_{\overline{\text{MS}}}$ ($\overline{\text{MS}}$ = modified minimal subtraction) is related to $\alpha_s(\mu)$ through

$$\Lambda_{\overline{\text{MS}}}^{(n)} = \mu \exp\left(\frac{1}{b_0 \alpha_s(\mu)} - \frac{b_1}{b_0} \ln \frac{\mu^2}{\Lambda_{\overline{\text{MS}}}^2}\right)$$

with

$$b_0 = \frac{1}{2\pi} (11 - \frac{2}{3} n), \quad b_1 = -\frac{1}{4\pi^2} (51 - \frac{19}{3} n), \quad n = \# \text{ flavors with mass} < \mu.$$  

7. ANALYSIS OF $F_2$ AND $\overline{q}^\nu$

The structure function $F_2$ is extracted from the sum of the $\nu$ and $\overline{\nu}$ nucleon data:

$$\frac{1}{\sigma_0} \frac{d^2}{dxdy} (\sigma(\nu N) + \sigma(\overline{\nu} N)) = (1 + (1 - y)^2) F_2(x, Q^2) - \Delta F$$

with $\sigma_0 = \frac{G_{\text{ME}}^2}{\pi}$ and the term $\Delta F = y^2 F_L - 2x(s-c)(1 - (1 - y)^2)$, which is small for $x > 0.3$. Nevertheless, the extraction of $F_2$ needs assumptions about $R = \frac{F_L}{2xF_1}$ and about the nonstrange sea. Results from 3 experiments are shown in fig. 53. The QCD interpretation of $F_2$ is more complicated than the one of $xF_3$. The change of $F_2$ w.r.t. to $\ln Q^2$ receives two contributions, one due to the process $q \to q + g$ and another due to $g \to q + \overline{q}$ (fig. 54). This means that the gluon distribution function is a necessary input. In order to get a handle on the shape of the gluon distribution function the CDHS collaboration considered a suitably chosen combination of the differential $\nu$ and $\overline{\nu}$ cross section, namely

$$\overline{q}^\nu \equiv x(\overline{u} + \overline{d} + 2\overline{s}) = \frac{1}{1 - (1 - y)^4} \frac{1}{\sigma_0} \left| \frac{d^2}{dxdy} (\sigma(\overline{\nu} N) - (1 - y)^2 \sigma(\nu N)) \right| + \delta$$

where $\delta$ depends upon the strange and charmed sea $x(s-\overline{c})$ and the structure function $F_L$ with the property that $\delta \to 0$ as $y \to 1$. The term $\sigma(\nu N)$ subtracts the amount of scattering off quarks. At $y = 0.5$ this represents about 50%. Thus the specific sea quark combination $x(\overline{u} + \overline{d} + 2\overline{s})$ is directly measurable. It has an important property: at large $x$, say $x > 0.4$, the sea $\overline{q}^\nu$ is negligible over the measured $Q^2$-range. This implies a constraint on the shape of the gluon distribution function. Qualitatively speaking its shape cannot be too broad, otherwise the $Q^2$-evolution of $\overline{q}^\nu$ would generate a nonnegligible contribution at large $x$ due to $g \to q + \overline{q}$. 
The simultaneous evaluation of $F_2$ and $q^V$ yields $\alpha_{\text{MS}}$ and the gluon distribution function $G(x)$ (fig. 55). Also the CHARM collaboration has obtained $G(x)$ from their data and has a new analysis in progress. The measured $q^V$ distribution can be used to obtain

$$F_2^{\text{MS}}(x, Q^2) = F_2(x, Q^2) - 2(q^V(x, Q^2) - xs(x, Q^2)) \quad x > 0.3$$

which is now independent of the sea like $x F_3$. Thus a nonsinglet analysis is possible and has been performed in both $\nu$ and $\mu$ experiments. Two results may be quoted:

$$\alpha_{\text{MS}} = 0.17^{+0.17}_{-0.11} \quad \text{EMC} \quad (\text{ref. 99})$$

$$0.30 \pm 0.15 \quad \text{CDHS} \quad (\text{ref. 95})$$

In conclusion, there is good agreement between the 5 highest statistics experiments, i.e. CDHS, CFFRR, CHARM, EMC, NA4. All agree in the observation of substantial scaling violation. If interpreted within QCD the strong interaction coupling constant is small in the $Q^2$-range from 5 till 100 GeV$^2$. However, the systematic uncertainties are for the time being too big to conclude about the running of $\alpha_s(Q^2)$.

There is a second type of QCD analyses based upon the moments of structure functions (fig. 57). A detailed account may be found in ref. 4b.

8. $\alpha_s$ FROM $e^+e^-$ - INTERACTIONS

Over the past 5 years many QCD analyses have been performed by the experimental groups at PETRA and PEP. Early analyses were done in 1st order perturbation theory. When the 2nd order calculations became available more refined analyses were done. This section restricts to three groups of results.

a) R-measurements: The R-value

$$R = \frac{\sigma(e^+e^\rightarrow \text{hadrons})}{\sigma_{\text{QED}}} = R_0 \left(1 + \frac{\alpha_s}{\pi} (1 + c \frac{\alpha_s}{\pi}) + \ldots\right)$$

with $R_0$ as calculated in the quark parton model including electroweak effects, $c = 0.08$ in the $\overline{\text{MS}}$ scheme (see lectures by R. Petronzio) is an inclusive quantity and is supposed to offer a clean way of measuring
\( \alpha_s \). "Clean" refers to the belief that for \( \sqrt{s} > 15 \text{ GeV} \) nonperturbative effects are negligible. Indeed, \( R \) at a given energy \( \sqrt{s} \) could be used to define \( \alpha_s \).

Unfortunately, \( R \)-measurements are not easy. The average over 5 results obtained by JADE, MARK J, TASSO and MAC, MARK II\(^{100} \) at \( \langle s \rangle = 1170 \text{ GeV}^2 \)

\[ \langle \alpha_s \rangle = 0.190 \pm 0.015 \pm 0.047 \]

The error is dominated by systematics.

b) R. FIELD\(^{101} \) has compared 4 observables with calculations in 2\(^{nd} \) order at the parton level. The argument is this:

\[ \text{Obs (W)}^{\text{exp}} = \text{Obs (W)}^{\text{parton}} + \text{Had (W)} \]

the experimentally observed quantity is written as a sum of the calculated quantity at parton level plus the unknown nonperturbative contribution (Had). In 2\(^{nd} \) order perturbation in terms of \( \alpha_s \):

\[ \text{Obs (W)}^{\text{parton}} = c_0 \alpha_s(W) (1 + c_1 \alpha_s(W) + ...) \]

with theoretically known \( c_0 \) and \( c_1 \). It follows in 1\(^{st} \) approximation:

\[ \alpha_s^{(1)}(W) = \frac{\text{Obs(W)}^{\text{exp}} - \text{Had(W)}}{c_0} \]

2\(^{nd} \) approximation:

\[ \alpha_s^{(2)}(W) = \frac{\alpha_s^{(1)}}{1 + c_1 \alpha_s^{(1)}} \]

\( \alpha_s^{(1)} \) can only be obtained provided the term Had(W) is known. This is, of course, not the case. Nevertheless, the sign of Had(W) decides on whether \( \alpha_s^{(2)}(W) \) is a lower (Had(W) > 0) or an upper bound (Had(W) < 0)

of the true value.

R. FIELD remarked that for the quantities 1-T (T = thrust of an event \( e^+e^- \rightarrow \text{hadrons} \)) and \( M_h^2/s \) (in each multihadron event the so-called CLAVELLI masses \( M_h \) and \( M_h \) can be calculated with \( M_{h^2} < M_h \)) the sign of the nonperturbative effects is positive, whereas for \( (M_h^2 - M_{h^2})/s \) and \( A_{EE} \) (the asymmetry of the energy-energy correlation, see below) the sign is negative.

Various Monte Carlo hadronization models differ in the absolute value of Had (W), but seem to agree in the sign. The data of 3 groups are used (fig. 58a) to obtain \( \alpha_s^{(1)} \) and \( \alpha_s^{(2)} \) (fig. 58b) with the result

\[ 0.10 \leq \alpha_s \leq 0.14 \quad \sqrt{s} = 30 \text{ GeV} \]
c) Energy-Energy-Correlations: Calling \( x_i = E_i / \sqrt{s} \) the fractional energy of particle \( i \) in \( e^+e^- \rightarrow i + j \) anything the normalized energy-weighted angular distribution is defined as follows:

\[
\frac{d\sigma}{d\theta} = \frac{1}{\sigma_{\text{tot}}} \sum_{i,j} \int x_i x_j \frac{3}{\sigma_i \sigma_j \theta} \, dx_i \, dx_j
\]

From this quantity an asymmetry can be derived:

\[
A(\theta) = \frac{d\sigma(\pi-\theta)}{d\theta} - \frac{d\sigma(\theta)}{d\theta}
\]

The idea of forming the asymmetry consists in reducing the effects from hadronisation and in suppressing the contribution of 2-jet events which cluster near \( \theta = 0 \) and \( \pi \). Three groups at PETRA (CELLO, JADE and TASSO)\(^{102} \) have presented fully corrected asymmetries (fig. 59) for \( \sqrt{s} \approx 34 \text{ GeV} \). Within the quoted errors all data are consistent. When analysing the data in terms of QCD the three groups find different values for \( \alpha_s \) depending on the hadronisation model used ranging from 0.11 to 0.16. The JADE group gets a good representation of their data with the LUND string model and \( \alpha_s = 0.165 \pm 0.01 \pm 0.01 \) (fig. 60a), however an unsatisfactory representation when using the independent jet fragmentation model (fig. 60b). It is clear that the extraction of \( \alpha_s \) and its precision are not limited by statistics and quality of the experiments.

OUTLOOK

50 years after FERMI's theory of weak interactions the Standard Model of electroweak interaction based on SU(2) \( \times U(1) \) and of strong interactions based on SU(3) provides a framework of all the elementary particle phenomenology. This is an important achievement as it constitutes a solid basis for future developments. It is, however, clear that even crucial aspects of the Standard Model are yet untested. Experiments aiming at tests at the 1-loop level are underway. Tests of the nonabelian structure of electroweak interactions require very high energies. The HIGGS sector, and thus the nature of electroweak symmetry breaking is essentially unexplored.

The tests of QCD are qualitatively successful, but crucial quantitative tests are still missing.

The Standard Model summarizes known facts and is a source of new fundamental questions ensuring an exciting future.
Fig. 45: The total hadron cross section in units of the QED-cross section $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. The dotted line in the upper figure is the prediction of the naive quark parton model. The figure below shows two resonances in the Y-region (CLEO 1981).
Fig. 46: TASSO 3-jet event and ELLIS-KARLINER analysis (ref. 89).
JADE

$\sqrt{s} = 27.7, 30.0$ GeV
$Q_2 - Q_1 > 0.07$

Data

Fig. 47: The JADE analysis of planar events. The 3-jet character is recognized by investigating the "fat" jet in its own rest frame (ref. 89).
Fig. 48: Transverse jet broadening in $\nu_e$ (BEBC) and $\nu_P$ (EMC).
Fig. 49:
Results from an ISR experiment.
Fig. 50: The nucleon structure function $x F_3^N$ from 3 high statistics neutrino experiments (Ref. 93).
Fig. 51: The structure functions $F_2$, $q^2$, and $G(x)$ for fixed $Q^2$ obtained from a QCD fit to $F_2$ and $q^2$ (ref. 95).

\[ \int_0^1 F_3 \, dx = 3 \left[ 1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right] \]

- ABCDLOS (BEBC)
- CCFRR (prelim)

Fig. 52: The GROSS-LLEWELLYN-SMITH sumrule (ref. 96).
Fig. 53: The nucleon structure function $F_2$ from 3 high statistics neutrino experiments (ref. 93).
Fig. 54a: QCD interpretation of the slopes $dF_2/d\ln Q^2$ (ref. 95).

Fig. 54b: Comparison between CDHS ($\nu$Fe) and EMC ($\mu$Fe) (ref. 95).
Fig. 55: The gluon distribution function

Fig. 56: Compilation of data on $\frac{\sigma_L}{\sigma_T}$ (ref. 95).
Fig. 57: Log moment plots of BEBC and CDHS neutrino data and comparison of slopes with QCD predictions (ref. 98).
Fig. 58:  

a) Data on 4 quantities vs c.m. energy $W$ - input for analysis of R.D. FIELD (ref. 101)  
b) Interpretation in 1$^{st}$ approximation ($\alpha_S^{(1)}$)  
c) Interpretation in 2$^{nd}$ approximation ($\alpha_S^{(2)}$)
Fig. 59: Comparison on energy-energy correlations in $e^+e^- \rightarrow$ hadrons (ref. 103).
The data of all three groups are fully corrected.
Symbols: + JADE, □ TASSO, o CELLO.
Fig. 60: JADE data (ref. 102b) on energy-energy correlations compared to two types of models.
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LITERATURE

1) C. Jarlskog, Introduction to Gauge Theories, Lectures at this School.
2) R. Petronzio, Theoretical Aspects of QCD, Lectures at this School.
3) J. Dowell, Physics at the pp Collider, Lectures at this School.
5) H. Pietzschmann, Elementary Introduction to Gauge Theories, Acta Physica
   Austriaca, Suppl. XIX (1978) 5 - 46.
   E. Paschos: Introduction to Electroweak Theory, María Laach School 1981,
   DESY Preprint 82-49.
   K. Mess and B. Wik, Recent Results in e+e- and ℓh Interactions,
   DESY Preprint 82-11.
   L. Jauneau, Introduction to new Physics, Kupari Lectures 1982,
   Orsay Preprint LAL 82/34.
   G. Ecker, Introduction to Gauge Theories of Electroweak Interactions,
6) H. Fritzsch, Composite Quarks and Leptons and their Flavor Mixing,
   Max Planck Institut MPI-PAE/PTh 31/84.
7) G. Myatt, Contribution to the Workshop on SPS Fixed Target Physics 1982,
   CERN 83-02, Yellow Reports Vol. I and II.
9) D.A. Ross and M. Veltman, NP B 95 (1975) 135.
10) D. Haidt, Summary Talk on Neutrino Physics at the SPS Fixed-Target Workshop
    loc.cit.
11) C. Llewellyn-Smith, Contribution to the SPS Fixed-Target Workshop, loc. cit.
14) W. Lerche et al., NP 142 (1978) 65-76.
    M. Pohl et al., NC leett. 24 (1979) 540.
18) S. Glashow, NP 22 (1961) 579.
    A. Salam, 1968 in Elementary Particle Theory: Relativistic Groups
    and Analyticity (Nobel Symposium No. 8), edited by N. Svartholm,
    Almquist and Wiksell, Stockholm.
20) 300 GeV Working Group, Proceedings of the 2nd Tirrenia Study Week 1972, CERN/ECFA/72/4 Vol II.
   B. Aubert et al., PRL 32 (1974) 1454 and 1457.  
   B.C. Barish et al., PRL 34 (1975) 538.
27) E. Williams and P. Olsen, PRL 42 (1979) 1575.
    A. Sirlin, PR D22 (1980) 571.  
31) W. Krenz, private communication (May 1984), TH Aachen.
32) J. Blietschau et al. (GARGAMELLE), NP B144 (1976) 189.  
    H. Faissner et al. (AC-PD), PRL 41 (1978) 213.  
    A.M. Cnops et al., PRL 41 (1978) 357.  
    N. Armenise et al. (GARGAMELLE-SPS), PL 86 B (1979) 225.  
    R.H. Heisterberg et al., PRL 44 (1980) 635.  
36) C. Bowdery, Private Communication, DESY.
39) W. Bartl et al. (SINDRUM) SIN Preprint PR-84-01.
40) N. Armenise et al., PL 84B (1979) 137.
41) M. Perl, The Tau Lepton, Ann. Rev. of Nucl. and Part. Science, 
      H.J. Behrend et al. (CELLO), NP B211 (1983) 369.
      J.A. Jaros et al. (MARK II), PRL 51 (1983) 955.
      M. Althoff et al. (TASSO), DESY 84-17 (1984)
45) W. Bartel et al. (JADE), PL 123B (1983) 353.
46) F. Dydak, private communication, CERN.
47) H. Wenninger, private communication, CERN.
48) J.E. Kim, P. Langacker, M. Levine and H.H. Williams, 
      A Theoretical and Experimental review of the Weak Neutral Current, 
      CDHS 83 : Ref. 51, New Value Ref. 54.
      BEBC D83: Ref. 50
55) F. Dydak et al., Proposal to Measure sin²θ in Semileptonic νFe 
      Interactions with High Precision, CERN/SPSC/83-49.
56) J.V. Allaby et al., Proposal to Measure the Ratio σ_{ν} (NC)/σ_{ν} (CC) 
57) W. Krenz et al. (GARGAMELLE), NP B135 (1978) 45.
60) W. Bartel et al. (JADE), PL 129 (1983) 145.
63) A. Argento et al., PL 120B (1983) 245.
69) H. Deden et al., PL 58B (1975) 361.
70) J. Blietschau et al. (BEBC), PL 86B (1979) 108.
74) F. Reines and C.L. Cowan, PR 90 (1953) 492 and 113 (1959) 273.


81) N.S. Lockyer et al. (MARK II), PRL 51 (1983) 1316.

82) E. Fernandez et al. (MAC), PRL 51 (1983) 1022.


86) C. Klopfenstein et al. (CUS8), PL 130B (1983) 444,
A. Chen et al. (CLEO), PRL 52 (1984) 1084.

87) J. Green et al. (CLEO), PRL 51 (1983) 347.


W. Bartel et al. (JADE), PL 91B (1980) 142.
J.J. Aubert et al. (EMC), PL 95B (1980) 306.

90) D. Drijard et al. (CDHW), PL 121B (1983) 433.


100) G. Wolf, The Determination of $\alpha_s$ in $e^+e^-$ Annihilation, DESY 83-96.

102) H.J. Behrend et al. (CELLO), DESY 83-127.
    W. Bartel et al. (JADE), DESY 84-50 (1984).
    M. Althoff et al. (TASSO), DESY 84-57 (1984).

103) A. Petersen, private communication, DESY.

104) Weak Interactions - Formulae, Results and Deviations,
     Springer Verlag, 1983.
Some definitions and relations are collected here (cf ref. 104).

1. The Interaction Lagrangians

\[ L^\gamma = g \sin \theta \, \mathcal{J}^\text{em}_\lambda \gamma^\lambda \]
\[ L^W = \frac{g}{2\sqrt{2}} \left( \mathcal{J}^\text{CC}_\lambda \gamma^\lambda W^\lambda + \text{hc.} \right) \]
\[ L^Z = \frac{g}{4\cos \theta} \mathcal{J}^\text{NC}_\lambda \gamma^\lambda Z^\lambda \]

Identify \( e = g \sin \theta \)

2. Current \times Current Form

\[ L^{\text{weak}} = \frac{G}{\sqrt{2}} \left( \mathcal{J}^\text{CC}_\lambda \gamma^\lambda \mathcal{J}^\text{CC}_\lambda \right) + \frac{\rho}{2} \mathcal{J}^\text{NC}_\lambda \gamma^\lambda \mathcal{J}^\text{NC}_\lambda \] for \( Q^2 \ll m_w^2, m_Z^2 \)

Identify \( \frac{G}{\sqrt{2}} = \frac{g^2}{8m_w^2} = \frac{1}{2v^2} \quad \rho = \frac{m_w^2}{m_Z^2 \cos^2 \theta} \)

The masses of \( W^\pm, Z \) are related to the HIGGS expectation value \( \langle 0 | \phi | 0 \rangle = \frac{1}{\sqrt{2}} v \)

\[ m_w = \frac{gv}{2} \quad \text{and} \quad m_z = \frac{gv}{2 \cos \theta} = \frac{m_w}{\cos \theta} \]

3. The Currents

\[ \mathcal{J}^\text{em}_\lambda = \sum_f Q_f \bar{\psi}_f \gamma_\lambda \psi_f \quad Q_f = \text{electric charge of fermion } f \]

in units of \( e > 0 \)

\[ \mathcal{J}^\text{CC}_\lambda = \sum_{\lambda} \bar{\psi}_\lambda \gamma_\lambda (1 + \gamma_5) \psi_\lambda + \sum_{q, q'} \bar{\psi}_q \gamma_\lambda (1 + \gamma_5) U_{qq'} \psi_{q'} \]

\( \lambda = (e, \mu, \tau) \quad q = (u, c, t) \quad q' = (d, s, b) \)

\( U = \text{unitary quark flavor mixing matrix} \)
\[ J_{\lambda}^{NC} = \sum_f \bar{\psi}_f \gamma_\lambda \left( g_V^f + g_A^f \gamma_5 \right) \psi_f \equiv 2 \sum_f \bar{\psi}_f \gamma_\lambda \left( \epsilon_L^f (1 + \gamma_5) + \epsilon_R^f (1 - \gamma_5) \right) \psi_f \]

\[ g_{V,A}^f \] weak coupling constants

\[ \epsilon_{L,R}^f \] chiral weak coupling constants

Sometimes the shorthand notation

\[ f_L^f \equiv \epsilon_L^f \quad f_R^f \equiv \epsilon_R^f \]

for the left- resp. right-handed weak coupling of a fermion with flavor \( f \) is used. Often \( V \) and \( A \) couplings \( v_f \) and \( a_f \) are introduced instead of \( g_{V,A}^f \). The relation is:

\[ v_f \equiv \frac{1}{2} g_V^f = f_L + f_R \equiv \epsilon_L^f + \epsilon_R^f \]

\[ a_f \equiv \frac{1}{2} g_A^f = f_L - f_R \equiv \epsilon_L^f - \epsilon_R^f \]

There is a third type of notation which proved particularly useful when investigating the isospin structure of the weak neutral quark current of the first generation:

\[ J_{\lambda}^{iso} = \alpha(\bar{\psi}_u \gamma_\lambda \psi_u - \bar{\psi}_d \gamma_\lambda \psi_d) + \beta(\bar{\psi}_d \gamma_\lambda \gamma_5 \psi_u - \bar{\psi}_u \gamma_\lambda \gamma_5 \psi_d) \\
IV - V \quad IV - A \]

\[ + \gamma(\bar{\psi}_u \gamma_\lambda \psi_u + \bar{\psi}_d \gamma_\lambda \psi_d) + \delta(\bar{\psi}_u \gamma_\lambda \gamma_5 \psi_u + \bar{\psi}_d \gamma_\lambda \gamma_5 \psi_d) \\
IS - V \quad IS - A \]

\[ \equiv \bar{\psi}_u \gamma_\lambda \left( u_L(1+\gamma_5) + u_R(1-\gamma_5) \right) \psi_u + \bar{\psi}_d \gamma_\lambda \left( d_L(1+\gamma_5) + d_R(1-\gamma_5) \right) \psi_d \]
The isovector-vector (IV - V), isovector-axialvector (IV - A), isoscalar-vector (IS - V) and isoscalar-axialvector (IS - A) pieces are marked. The relations between α, β, γ, δ and \( u_L, R, d_L, R \) follow from the above identity:

\[
\begin{align*}
    u_L &= \frac{1}{4} (\alpha + \beta + \gamma + \delta) \\
    u_R &= \frac{1}{4} (\alpha - \beta + \gamma - \delta) \\
    d_L &= \frac{1}{4} (-\alpha - \beta + \gamma + \delta) \\
    d_R &= \frac{1}{4} (-\alpha + \beta + \gamma - \delta)
\end{align*}
\]

α = \( u_L + u_R - d_L - d_R = 1 - 2 \sin^2 \theta \)

β = \( u_L - u_R - d_L + d_R = 1 \)

γ = \( u_L + u_R + d_L + d_R = -\frac{2}{3} \sin^2 \theta \)

δ = \( u_L - u_R + d_L - d_R = 0 \)

Note that β and δ do not depend upon \( \sin^2 \theta \).

In the following table the chiral Zff couplings are listed for the fermions of the first generation together with their predicted value assuming \( \sin^2 \theta = 0.22 \).

<table>
<thead>
<tr>
<th>PARTICLE</th>
<th>( T_3 - Q \sin^2 \theta )</th>
<th>( \sin^2 \theta = 0.22 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_L )</td>
<td>( + \frac{1}{2} )</td>
<td>0.500</td>
</tr>
<tr>
<td>( e_L )</td>
<td>( - \frac{1}{2} + \sin^2 \theta )</td>
<td>-0.280</td>
</tr>
<tr>
<td>( u_L )</td>
<td>( + \frac{1}{2} - \frac{2}{3} \sin^2 \theta )</td>
<td>0.353</td>
</tr>
<tr>
<td>( d_L )</td>
<td>( - \frac{1}{2} + \frac{1}{3} \sin^2 \theta )</td>
<td>-0.427</td>
</tr>
<tr>
<td>( \nu_R )</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>( e_R )</td>
<td>( \sin^2 \theta )</td>
<td>0.220</td>
</tr>
<tr>
<td>( u_R )</td>
<td>( - \frac{2}{3} \sin^2 \theta )</td>
<td>-0.147</td>
</tr>
<tr>
<td>( d_R )</td>
<td>( \frac{1}{3} \sin^2 \theta )</td>
<td>0.073</td>
</tr>
</tbody>
</table>
Q C D

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ABSTRACT

This a rather general survey of the properties of quantum chromodynamics in the perturbative approach and in the non-perturbative one of lattice QCD.

1. INTRODUCTION

Quantum chromodynamics is a theory of strong interactions based on local colour invariance\(^1\). The colour is an internal quantum number which can be classified according to the symmetry group SU(3): the locality means the freedom of choosing independently the colour at each point of the space-time. This invariance requirement needs the presence of fields "transmitting" the information about the colour degrees of freedom at different points. They are called gluons. The situation is rather similar to what happens in quantum electrodynamics: there, the photons guarantee the freedom of choosing arbitrarily at each point of the space-time the phase of the electron fields. This is realized by substituting for the ordinary derivative in the fermion's Lagrangian, the covariant derivative, defined as:

\[
D_\mu = \partial_\mu - ie A_\mu
\]

where \(e\) is the electric charge and \(A_\mu\) is the Lorentz covariant vector potential representing the photon field in terms of which the electric (E) and magnetic (B) fields can be defined:

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix}
0 & E_x & E_y & E_z \\
-E_x & 0 & -B_z & B_y \\
E_y & B_z & 0 & -B_x \\
-E_z & -B_y & B_x & 0
\end{pmatrix}
\]

In QCD, the covariant derivative reads:

\[
D_\mu^{ab} = \partial_\mu \delta^{ab} - ig \sum_i \lambda_i \ A_i^a \ A_i^b
\]

where \(g\) is the strong charge, \(\lambda_i^{ab}\) are the generators of the SU(3) group (the Gell-Mann
matrices for example) and $A^i_\mu$ are eight fields representing the gluons. The indices $(a,b)$ are colour indices and, for quarks*, run from one to three. Similarly to QED an antisymmetric tensor can be defined:

$$
\Gamma^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g \tilde{f}^{ijk}_{\mu\nu} A^j_\mu A^k_\nu
$$

$$
= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g [A^i_\mu, A^j_\nu]^i
$$

(1.4)

The last term in Eq. (1.3) is peculiar to QCD: it represents a self-interaction of the gluons. Differently from the photons in QED, they carry a non-zero charge and therefore must interact. This is a consequence of the fact that the symmetry group under which a local invariance is required, $SU(3)$, is non-Abelian, i.e., the commutator appearing in the second line of Eq. (1.3) is different from zero. The gauge invariance in this case is not just a shift of the field, like in QED

$$
A^\gamma_\mu (x) = A^\gamma_\mu (x) + \partial_\mu \chi (x)
$$

(1.5)

but also a "rotation" in colour space:

$$
A^i_\mu (x) = A^i_\mu (x) + \partial_\mu \Omega^i (x) - \frac{g}{4} [\Omega, A^i_\mu] ^i
$$

(1.6)

where $\Omega^i$ are the set of eight independent phases that one can choose in $SU(3)$ at each point of the space-time. The square bracket means, of course, the commutator. From Eqs. (1.4) and (1.5), it follows that while the QED $F_{\mu\nu}$ is invariant under (1.4), the $F^i_{\mu\nu}$ of QCD is covariant, i.e., transforms like

$$
(F^i_{\mu\nu})' = F^i_{\mu\nu} [\hat{\Omega}, F_{\mu\nu}] ^i
$$

(1.7)

The Lagrangian is always invariant in both cases:

* I assume that some elementary notions, like the concept of quark, are acquired.
\[ \mathcal{L} \sim -\frac{1}{4} F_{\mu \nu}^2 \] (QED) \[ \mathcal{L} \sim -\frac{1}{4} \sum_a \tilde{F}_{\mu \nu}^a F^{\mu \nu \lambda}_a \] (QCD) (1.8)

In this last case, it is like forming the square of a vector which is invariant under the "rotations" like in Eq. (1.7). By means of the covariant derivative defined in Eq. (1.3), one forms the part of the Lagrangian concerning the coupling of (anti)quarks to gluons

\[ \mathcal{L}_{\text{quarks}} \sim i \bar{\psi}^a D_{\mu}^{ab} \psi^b \] (1.9)

where \( \psi^a \) is a Dirac spinor, "a" is its colour index which runs from one to three [quarks transform as the fundamental representation of SU(3)].

From the Lagrangian density one can obtain the equations of motion which govern the classical limit of the theory:

\[ \partial_{\mu} F_{\mu \nu}^i = 0 \] (1.10)

in the absence of quarks. The quantum fluctuations around the classical limit can be calculated by means of a functional integral over the possible field configurations\(^*\) of the type:

\[ \mathcal{Z} = \int d[A] \ e^{-iS} \] (1.11)

where \( S \) is the action defined as the four-dimensional integral of the Lagrangian density:

\[ S = \int d^4x \ \mathcal{L}(x) \] (1.12)

Do not be frightened by the appearance of the integral in Eq. (1.9) which must remain mysterious to most of you. Any long digression into formal manipulations is outside the purpose of these lectures. It is enough for you to believe that the whole quantum theory can be derived from these integrals: in particular, the time-ordered Green function (again a mysterious word ...) of two gluon fields at different points of the space-time is given by:

\(^*\) "Functional" means that one is integrating over the possible functions \( A_\mu(x) \), which can represent the fields at each point of the space-time \( \).
\( \langle 0 | T (A^a_\mu(x) A^b_\nu(o)) | 0 \rangle = \frac{\int e^{-S} d[A]}{\int d[A]} T (A^a_\mu(x) A^b_\nu(o)) \) \quad (1.13)

This two-point Green function is nothing but the "propagator" of the gluon field between the point "o" and "x": it gives you the amplitude for a gluon with colour b, polarization v to propagate from the point "o" to the point "x" with colour a and polarization \( \mu \). In terms of Feynman diagrams it is represented as in Fig. 1.

![Feynman diagram](image)

\( v, b \quad \mu, a \)

\( 0 \quad X \)

Fig. 1 : A gluon propagating from the point o to the point x.

Unfortunately, we are not able to perform in the most general case the integrals like in Eqs. (1.11) or (1.13): the only feasible functional integrals are the "Gaussian" ones when the action \( S \) is quadratic in the fields that we want to integrate. Actually, with the imaginary unit in the exponent, these integrals never look very 'Gaussian': to make them more "Gaussian-like" one has to perform a "Wick rotation" from the real time \( t \) to an imaginary time \( i t \). This brings an extra "i" into the exponent so that, when all these tricks are done, one obtains a "Euclidean" functional integral. Now the metric is no longer Minkowski like but Euclidean-like in four dimensions, i.e., the product of two Lorentz vectors \( a_\mu \), \( b_\mu \), in terms of their components, is:

\[
\begin{align*}
    a_\mu b^\mu &= a_0 b_0 + a_\nu b_\nu - a_\nu b_\nu + a_3 b_3 \\
    \quad & \text{Euclidean}
\end{align*}
\]  
\quad (1.14)

instead of the usual

\[
\begin{align*}
    a_\mu b^\mu &= a_0 b_0 - a_\nu b_\nu \\
    \quad & \text{i.e.}
\end{align*}
\]  
\quad (1.15)

The "Euclidean" expression for \( Z \) is

\[
Z_E = \int d[A] \exp[-S] \]  
\quad (1.16)

If \( S = \int d^4x [A_0 A] \), i.e., is quadratic in \( A \), where \( 0 \) is some operator acting on \( A \), the integral assumes a Gaussian form. These integrals are feasible: in fact

\[
\int dy e^{-\sigma y^2} \sim \frac{1}{\sqrt{\sigma}}
\]  
\quad (1.17)

and also
\[
\frac{\int dy e^{-\sigma y^2} y^2}{\int dy e^{-\sigma y^2}} \sim \frac{\lambda}{\sigma}
\]  \hspace{1cm} (1.18)

Of course, the last two integrals are ordinary integrals. However, if we imagine the space-time discretized, the functional integral becomes

\[
\int d[A] \rightarrow \prod_{x_i} \int dA_{x_i}
\]  \hspace{1cm} (1.19)

where \(A_{x_i}\) is the value of the field in the point \(x_i\): making the product over all the points of the integrals of the corresponding values of the fields, one obtains an estimate of the integral over all possible functions of the points. Gaussian functional integrals become then products of ordinary Gaussian integrals. They factorize in the product of independent integrals only in the case where the operator \(0\) in Eqs. (1.6)-(1.18) is diagonal (i.e., local). The simplest example of a Gaussian functional integral is given by the photon propagator. The Lagrangian density is quadratic in the photon field \(A^\mu\); according to Eq. (1.16), the photon propagator is just the inverse of the "coefficient" (actually it is an operator because it contains derivatives) of the quadratic term of \(F^\mu_\nu\). However, there is a complication: this inverse does not exist unless a gauge is fixed which removes the unphysical (or part of) degrees of polarization of the photon. Examples of the resulting propagator are (in momentum space):

\[
\Pi_{\mu \nu}(q) = \frac{i}{q^2} \left[ -g_{\mu \nu} + (4-\lambda) q_{\mu} q_{\nu} \right]
\]  \hspace{1cm} (1.20)

in a Lorentz-covariant gauge with the gauge fixing condition

\[
\partial^\mu A^\mu = 0
\]  \hspace{1cm} (1.21)

and with \(\lambda\) a parameter varying from one (Feynman gauge) to zero (Landau gauge) and

\[
\Pi_{\mu \nu}(q) = \frac{i}{q^2} \left[ -g_{\mu \nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{(n \cdot q)} \right]
\]  \hspace{1cm} (1.22)

in a non-covariant (light-like) gauge with the gauge fixing condition:

\[
\mathcal{N}_\mu A^\mu = 0 \quad \left(\mathcal{N}^2 = 0\right)
\]  \hspace{1cm} (1.23)

In the latter case, in order to implement the gauge condition, one has to introduce an auxiliary light-like four vector which breaks the Lorentz covariance of the propagator (it is like choosing a given reference frame). Notice that, in the latter case,
The propagator is fully "transverse"; it propagates only the "physical" transverse degrees of freedom of the photon. For this reason, these gauges are sometimes called "physical" gauges. In the case of QED the action $S$ is purely quadratic in the photon fields: a theory like that is said to be "free". In fact, if there were no electrons, QED with only photons would be a theory without interactions: photons propagate freely without doing anything. In the case of QCD the action, due to the non-vanishing commutator in Eq. (1.3), contains also cubic and quartic terms. The integrals containing such terms in the exponents cannot be done and one needs methods of approximation. If $g$ is sufficiently small, one can make a series expansion in $g$ of the cubic and quartic terms: this is the perturbative expansion:

$$e^{-S} = e^{-S(g=0)} \left[ 1 + \int g \left[ A_{\mu} A_{\nu} \right]^2 \left[ D_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \right]^2 + \int g^2 \left[ A_{\mu} A_{\nu} \right]^3 \left[ A^{\mu} A^{\nu} \right]^3 + \cdots \right]$$

which can be translated, for the propagator, in the series of Feynman diagrams of Fig. 2.

![Fig. 2: Perturbative corrections to the gluon propagator.](image)

Another method is the numerical evaluation of the functional integral after having reduced the space-time to a discrete set of points. This method is non-perturbative because it keeps the anharmonic terms in the exponent: this is the approach followed by lattice QCD.

The outline of the lectures is as follows. Section 2 contains a discussion of the renormalization and the property of asymptotic freedom. Section 3 deals with the factorization theorem and with the evolution equations for parton densities. Scaling violations which are power-behaved, as opposed to logarithmically-behaved, are discussed in Section 4. To the lattice approach are devoted the two last sections, 5 and 6.
2. - RENORMALIZATION AND ASYMPTOTIC FREEDOM

Imagine a "laboratory" of a size of ~ 0.1 Fermi and, inside, the creation of a quark-antiquark pair by a virtual photon having an invariant mass of the order of 10 GeV (and, therefore, by the uncertainty principle, a resolution of about 0.02 Fm's). By applying the perturbative expansion, one obtains for the amplitude, which has then to be squared to get the rate for the process, the diagrams in Fig. 3. Among them, there are some, like the ones in Fig. 3b, whose value is divergent, being proportional to:

Fig. 3a: Perturbative corrections to the a quark pair production from a photon.

![Diagram](image)

Fig. 3b: Some diagrams giving rise - in the Feynman gauge - to ultraviolet divergences.

$$\int_0^\infty \frac{dk}{k^4} \sim -\mu E \int_0^\infty E_{\mu}$$  \hspace{1cm} (2.1)

This divergence cancels between the two diagrams in Fig. 3b, but at order $g^n$ one is left with a net divergence of the type (2.1) in the result. The ratio $R$ for the rate of $q\bar{q}$ creation over, say, the one for $\mu^+\mu^-$ creation, takes the form

$$R = \left( \sum_\pi \epsilon^{z}_\pi \right) \left[ \lambda + \frac{1}{\pi} (\frac{a^2}{4\pi}) + c \left( \frac{a^2}{4\pi} \right)^2 \mu_{\infty} E_{\mu} \right]$$  \hspace{1cm} (2.2)

which, of course, is not acceptable.

The divergence in Eq. (2.2) is built up from the co-operative contribution of all energy scales entering the integral in Eq. (2.1). In fact, since the original Lagrangian
does not contain any intrinsic scale, the physics at distances of order $1/E_Y$ feels the quantum fluctuations down to distance zero. A "brute force" remedy to the divergences is the introduction of an ultra-violet cut-off $M_{uv}$ which represents the maximum energy up to which one extends the integral in Eq. (2.1). The value of $R$ [Eq. (2.2)] will now depend upon the choice of $M_{uv}$: its presence can be reabsorbed to any order of perturbation theory into an effective, cut-off dependent, bare coupling constant:

$$g_{\text{effective}}^{\text{BARE}}(M_{uv}/E) = g + b g^2 \ln(M_{uv}/E) + \ldots$$

(2.3)

This might appear to be just an aesthetical refinement (it is, up to now); however, the coupling $g_{\text{BARE}}$ satisfies the following equation (trivially true for the lowest order in perturbation theory):

$$\frac{d g_{\text{BARE}}^{\text{effective}}(M_{uv})}{d \ln M_{uv}} = - \left[g^2\right] b + O(g^5)$$

$$= - \left[g_{\text{BARE}}^{\text{effective}}(M_{uv})\right]^2 b + O(g^5)$$

(2.4)

The solution of this equation, by neglecting the $O(g^5)$ terms in the right-hand side, contains all the terms in the expansion of Eq. (2.3) which are "leading" in the quantity $g^2 \ln(M_{uv}/E)$: in other words all the terms of the type $g^2 \ln(M_{uv}/E)^N$ where $N$ is an arbitrary integer. This approximation is called "leading log". The solution of Eq. (2.4) looks like:

$$g_{\text{effective}}^{2}(M_{uv}) = \frac{g_0^2}{1 - b g_0^2 \ln M_{uv}/E}$$

(2.5)

where $g_0$ is some boundary value for $g$ at $M_{uv} = E$. The content of Eq. (2.5) shows that the various energy scales which enter into the determination of the quantum corrections are locally coupled: the variation of $g_{\text{BARE}}^{\text{effective}}$ by an infinitesimal variation of the cut-off around some value is governed by the value of $g_{\text{BARE}}^{\text{effective}}$ at the same value of the cut-off, i.e., at the same energy scale. The need for a cut-off is strictly related to the fact that we have been dealing up to now with a bare coupling constant and that we have been trying to describe the physics at a given energy $E_Y$ in terms of that at distance zero (or at distance $1/M_{uv}$).

The cut-off disappears from the results when we reparametrize (renormalize) the theory in terms of a new coupling constant which is normalized by some physical input at some energy scale\(^4\). This will lead us to the replacement...
where $M_{\text{ren}}$ is the energy scale chosen for normalizing the new coupling constant. Hopefully, I can make more clear (or less obscure ... ) what I said by an explicit example. Let us consider the quantity $R$ of Eq. (2.2) and define:

$$\frac{\alpha_s^{\text{REN}}(Q^2)}{\pi} = \frac{R^{\text{Exp}} - R_0}{R_0}$$

(2.7)

$R^{\text{Exp}}$ is the experimental value of

$$\frac{\sigma^{\text{TOT}}(e^+e^\to\text{hadrons})}{\sigma^{e^+e^\to\mu^+\mu^-}}$$

and $R_0 \equiv \sum_i e_i^2$ where $e_i$ are the quark charges and the sum runs over the quark species. Of course, having defined $\alpha_s^{\text{REN}}(Q^2)/\pi$ via Eq. (2.7), there are no predictions for $R^{\text{Exp}}(Q^2)$ which has been used as an input. However, one can now make cut-off independent predictions for other processes or the same one but at a different energy scale. In fact, consider the Eq. (2.2) at two different values of $Q^2$:

$$R\big|_{Q^2_0} = R_0 \left[ 1 + \frac{\alpha_s}{\pi} + C \left( \frac{\alpha_s}{\pi} \right)^2 \epsilon \mu \frac{Q^2_0}{M_{\text{UV}}^2} + \ldots \right]$$

(2.8)

$$R\big|_{Q^2} = R_0 \left[ 1 + \frac{\alpha_s}{\pi} + C \left( \frac{\alpha_s}{\pi} \right)^2 \epsilon \mu \frac{Q^2}{M_{\text{UV}}^2} + \ldots \right]$$

(2.9)

In terms of $\alpha_s^{\text{REN}}(Q^2)$, $R^{Q^2}$ reads:

$$R\big|_{Q^2} = R_0 \left[ 1 + \frac{\alpha_s^{\text{REN}}(Q^2_0)}{\pi} + C \left[ \frac{\alpha_s^{\text{REN}}(Q^2_0)}{\pi} \right]^2 \epsilon \mu \frac{Q^2}{Q^2_0} + \ldots \right]$$

(2.10)

As promised, any cut-off dependence has disappeared at the expense of buying a new energy scale, the (re)normalization scale $Q^2$.

If $\alpha_s^{\text{REN}}(Q^2) \times \ln Q^2/Q^2_0$ is of order $1$, the series on the right-hand side of Eq. (2.10) converges very slowly; however, all the powers of $\alpha_s^{\text{N}}(Q^2) \ln M_{\text{UV}}^2/Q^2$ can be resummed by solving the renormalization group equation which relates a change of $\alpha_s(Q^2)$ with a change of $\ln Q^2$.
\[
\frac{\alpha_s(Q^2)}{\ln Q^2} = \beta_0 \left[ \alpha_s(Q^2) \right]
\]

(2.11)

The function $\beta[\,\,]$, appearing in Eq. (2.11), is the "beta function" of the theory: it can be calculated in perturbation theory and its lowest order approximation

\[
\beta(\alpha_s) = -b \alpha_s^2 + O(\alpha_s^3)
\]

(2.12)

allows the resummation of all the leading logarithms of $[\alpha \ln Q^2/\Lambda^2]^N$.

If we knew the whole perturbative development, the result would not depend upon the choice of the normalization scale, provided one were to readjust the value of $\alpha_s(Q^2)$ (the "running coupling constant") according to Eq. (2.11). The same physics can be described with an expansion in terms of different renormalization scales and running coupling constants. This freedom implies that the parameters $\alpha(Q^2)$ and $\alpha_s(Q^2)$ are redundant: for each choice of $Q^2$ there is a corresponding choice of $\alpha(Q^2)$ which leads to the same physics. This is depicted in Fig. 4 where the curves in the plane $\alpha(Q^2)$, $Q^2$ are curves of "constant physics". The parameters $\Lambda_1, \Lambda_2, \Lambda_3$ are the ones characterizing the curves: the explicit form of $\alpha(Q^2)$ in terms of $\Lambda$ and $Q^2$ in lowest order perturbation theory [Eq. (2.12)] is:

\[
\alpha(Q^2) = \frac{1}{b \ln Q^2 / \Lambda^2}
\]

(2.13)

By inverting Eq. (2.13), one gets:

\[
\Lambda^2 = Q^2 e^{-b / \alpha_s(Q^2)}
\]

(2.14)

which satisfies the equation

\[
\left[ \frac{\partial}{\partial \ln Q^2} + \left( \frac{\partial Q^2}{\partial \ln Q^2} \right) \cdot \frac{\partial}{\partial \alpha_s} \right] \Lambda = 0
\]

(2.15)

In the equation above, the variation of $Q^2$ and the variation of $\alpha$ compensate each other, leaving $\Lambda$ (the real "parameter" of the theory) invariant.

Fig. 4 : The behavior of the running coupling constant $\alpha(Q^2)$ for various values of $\Lambda$. 

From Eq. (2.13) or Fig. 4, one can see the unique feature of non-Abelian gauge theories [i.e., with a non-vanishing commutator in Eq. (1.4)]: when $Q^2$ gets large, the corresponding $a(Q^2)$ decreases\(^5\). Differently from QED, the vacuum fluctuations make the "effective charge" weaker at smaller distances. For example, asymptotically $R$ tends to $R_0$, the free $qq$ production rate.

When we describe a given process characterized by a scale $Q^2$ (like $Q^2 = 4E^2$ for $R$), we have to choose which one is the optimal scale entering in the running coupling constant: $a(Q^2)$, $a[Q^2/4]$ ...? Different choices will actually influence the result only at next-to-leading level, by altering the coefficient $c$ in Eq. (2.10). It might be surprising for you that, after having said that different values of $Q^2$ and $a(Q^2)$ correspond to the same physics, I am now saying that this choice does actually alter a physical result. The point is that the above-mentioned invariance is a property of the entire perturbation series and not of an expansion truncated by human (and machine) limitations to a few terms. Different choices are always influencing terms which are formally of higher orders in the perturbative expansion: a "good" or "bad" choice is then equivalent to "good" or "bad" guesses about the uncalculated terms of the expansion. Stevenson\(^6\) has made a natural suggestion to guide the choice: his "principle of minimal sensitivity" states that the choice should be such as to make the answer minimally sensitive to it. For example, one can express the quantity of Eq. (2.2) in terms of $a[xQ^2]$ where $x$ is a parameter to be "optimized". The optimization criterion is to find a value of $x$ for which the result has an extremum as a function of $x$. The reason for this is simple; the whole perturbative expansion is totally insensitive to the choice: a good approximation is to make the truncated expansion locally insensitive.

We have learnt that in order to obtain finite predictions for the processes which can take place in our imaginary laboratory introduced at the beginning of the section, we have to renormalize the theory and relate a physical process at a given scale to a reference process at another (but not infinite) energy scale. Only then does the ultra-violet cut-off dependence, which was necessarily introduced during the intermediate steps of the calculation, drop out. The perturbative expansion in terms of the running coupling constant having as its argument the "typical" energy scale of the process (seen at quark-gluon level) converges better and better as the energy increases, thanks to the asymptotic freedom property of QCD.

The renormalization procedure made the results independent upon the ultra-violet cut-off: however, our imaginary laboratory possesses also a natural infra-red cut-off, the box size. If we increase it too much, we must follow the dynamics developing at larger and larger distances which is governed by low momentum transfers and therefore by a strong coupling constant. We would then be faced with the non-perturbative properties of QCD which will be tackled with a different approach explained in the last two sections. If we want to remain in the perturbative domain, we have to ask the question whether the prediction for a given process is sensitive or not to the infinite volume limit or, more precisely, if such a limit is smooth or not. Consider a generic cross-section $\sigma$, characterized by an energy scale $Q$ and form the dimensionless quantity:
\[ Q^2 \sigma = \frac{\alpha}{Q^2} \left( \frac{m_i^2}{Q^2} \right) \frac{\mu^2}{Q^2} \alpha(\mu^2) \]  \hspace{1cm} (2.16)

which is a function of the dimensionless variables obtained as ratios of the possible energy scales of the process: \( Q, m_i \) (a set of quark masses which represent the influence of the infra-red cut-off: \( m_i \sim 1/l_{\text{BOX}} \)) and the renormalization scale \( \mu \). If the limit \( m_i^2/Q^2 \to 0 \) is smooth (i.e., non-singular), one gets

\[ Q^2 \sigma \to \frac{\alpha}{Q^2} \left[ O, \mu^2/Q^2, \alpha(\mu^2) \right] \to \frac{\alpha}{Q^2} [0, 1, 0] \]  \hspace{1cm} (2.17)

by exploiting the freedom of choosing an arbitrary value for the renormalization scale, we can set \( \mu^2 = Q^2 \) and obtain

\[ Q^2 \sigma \to \frac{\alpha}{Q^2} [O, 1, \alpha(Q^2)] \to \frac{\alpha}{Q^2} [0, 1, 0] \]  \hspace{1cm} (2.18)

The last equality follows from the asymptotic freedom and shows that in this case, one obtains the "scaling limit" where a dimensionless cross-section can only depend upon the ratio of the external kinematical invariants of the process. In the case we are considering we assumed only one invariant, \( Q^2 \): there are no ratios which can be formed and the scaling limit is just a constant \([R \to R_0]\) as we have seen in Eq. (2.12).

Sometimes the zero mass limit of the theory is singular: the dependence upon the laboratory size becomes critical, as before was critical the dependence upon the ultraviolet cut-off. In the latter case, the renormalization procedure got rid of the ultraviolet cut-off; in the case of the zero mass limit a new procedure, logically very similar to the renormalization, must be introduced. It is called factorization and it will be discussed in the next section.

3. - FACTORIZATION

We want to discuss a process where the zero mass limit is singular and which therefore requires a "factorization" procedure \(\dagger\). The simplest example is the deep inelastic scattering of a quark by a virtual space-like photon carrying a high momentum transfer. The corresponding lowest order Feynman diagram is given in Fig. 5a. The rate is obtained by

\[ \begin{array}{c}
\mu \\
\rightarrow \gamma \rightarrow (p+q) \\
\downarrow \\
p
\end{array} \]

Fig. 5a : The lowest order amplitude for the photon-quark scattering.
taking the square of this amplitude or, equivalently, the imaginary part (the dashed line) of the diagram in Fig. 5b. By projecting the photon's polarization by $g_{\mu\nu}$ one obtains for the rate:

$$g_{\mu\nu} T^{\mu\nu}_{\nu}(p, p) \propto F^I(x) = \delta(A - x)$$  \hspace{1cm} (3.1)

where $x \equiv q^2/2p \cdot q$ is the usual Bjorken scaling variable in terms of the quark momentum $p$ (there are no nucleons up to now).

The calculation of the corrections of order $a_8$ to the rate in Eq. (3.1) gives:

$$F^I(x) = \delta(A - x) + \frac{4}{3} \frac{\alpha_s}{\pi} \int \frac{d^2 q}{m_q^2} \frac{Q_r^2}{A - x} \left( \frac{A + x^2}{A - x} \right)$$  \hspace{1cm} (3.2)

where

$$\left( \frac{A + x^2}{A - x} \right) = \frac{A + x^2}{A - x} - \delta(A - x) \int_0^1 dy \left( \frac{1 + y^2}{1 - y} \right)$$

As anticipated, the limit $m_q \to 0$ cannot be taken because of a logarithmic singularity.

The identification of the diagrams responsible for the singularity among the ones providing the order $a_8$ corrections is, in general, gauge dependent. By changing the gauge fixing condition, one can shift the appearance of a singularity from one diagram to another. However, some more physical intuition can be gained by adopting a gauge where gluons propagate only the physical degrees of polarizations: it is the light-like gauge defined in Eq. (1.23) and leading to the transverse gluon propagator of Eq. (1.22). In this gauge the amplitude for the emission of a gluon collinear to a massless fermion goes to zero with the opening angle between the gluon and the fermion. Indeed, given that the gluon-fermion coupling conserves the fermion's helicity, there is no way to emit a physical gluon (which carries helicity $-1$) collinear to a massless \footnote{Note, that if the fermion is massive, kinematics forbids a totally collinear emission.} fermion and to conserve at the
same time the angular momentum along the direction of the momenta (see Fig. 6). This leaves (at order $\alpha_s^k$) only one class of diagrams responsible for the logarithmic dependence in Eq. (3.2), the ones of Fig. 7. For these diagrams, when gluon are emitted collinearly, the above-mentioned suppression of the amplitude is not powerful enough to prevent a divergence for $m_q \to 0$ arising from the poles developing in the fermion propagators of Fig. 7 carrying momentum $(p-\lambda)_\mu$. Physically, when the quark mass goes to zero, the gluons can be emitted collinearly allowing the propagation of intermediate "on-shell" quarks: the divergence is related to the "lifetime" of these particles which tends to infinity when they go on-shell. In fact, an on-shell particle can propagate indefinitely.

Mass singularities spoil the possibility of getting rid of the infra-red cut-off (the quark mass or the box size) in a painless way. But, like the scales building up the ultraviolet divergence, also the "infra-red" scales are locally coupled: in other words, one can write an equation for an infinitesimal change of scale which depends only upon the scales nearby. The equation is:

$$\frac{d}{d \ln m^2} F(x, Q^2/m^2) = \int_x^1 \frac{dy}{y} F_0(y) P(x/y)$$

(3.3)

if we take $F_0(y) \equiv \delta(1-y)$ and

$$P(x) = \frac{4}{3} \left[ \frac{\alpha_s}{\alpha_s + \ln \frac{Q^2}{m^2}} \right]_+$

(3.4)

we see that Eq. (3.3) can be obtained by differentiating with respect to $\ln m^2$ the Eq. (3.2). Actually, up to the first order in perturbation theory one can replace $F_0(y)$ with $F(x, Q^2/m^2)$. The resulting equation is an example of the "Altarelli-Parisi evolution equation". The solution of the equation, as was the case for the solution of Eq. (2.11), contains arbitrarily high power of $\alpha_s$: if the right-hand side is of order $\alpha_s$, as is the case for Eq. (3.3) with the choice (3.4) for the "probability" $P(x)$, one obtains the resummation to all orders in $\alpha_s$ of terms $[\alpha_s \ln Q^2/m^2]^N$: i.e., the "leading log" approximation for the $Q^2$ evolution of the parton structure function $F(x, Q^2/m^2)$. 
The evolution of hadron structure functions satisfies similar equations. We start from squaring the amplitude shown in Fig. 8a where one of the partons contained in a nucleon of momentum P hits the virtual photon of momentum q. The square of the amplitude is shown in Fig. 8b which takes the form of a “hand-bag” diagram. Now, a little formalism: let us choose a reference frame as follows:

\[
\begin{align*}
\frac{P_\mu}{\mu} &= (P_0, P_x, P_y, P_z) = (1, 0, 0, 1) \\
\nu_\mu &= \frac{1}{2} (1, 0, 0, -1) \\
K_{\lambda\mu} &= (0, 0, 1, 0) \\
\end{align*}
\]  
(3.5)

In terms of the parametrization (3.5), a general vector can usually be decomposed as:

\[
\kappa_\mu = \frac{k \cdot n}{p \cdot n} p_\mu + \frac{k \cdot p}{p \cdot n} p_\nu + k_{\mu} = x_P p_\mu + (k \cdot p) n_\mu + k_{\mu}
\]  
(3.6)

where \( p \cdot n = 1, p^2 = n^2 = 0 \). The momentum q of the photon is:

\[
q_\mu = -X_B p_\mu + \nu_\mu \frac{Q^2}{2X_B}
\]  
(3.7)

where

\[
X_B = -\frac{q^2}{2p \cdot q}, \quad Q^2 = -q^2
\]

The hadronic squared amplitude \( T_{\mu\nu}(q, p) \) (after projecting the photon polarizations with \( g_{\mu\nu} \)) can be written as:

\[
q_{\mu\nu} T^{\mu\nu} \propto \int \! d^4 k \, \hat{C}(q, k) \hat{H}(k, p)
\]  
(3.8)
where one integrates over the four momentum $k$ of the quark which is struck by the photon.

The "parton model" approximation consists in expanding this four momentum around the projection along the proton's momentum $P$

$$
\hat{\mathcal{C}}(q_k) \approx \hat{\mathcal{C}}_0(q_k)_{\mid_{k \cdot xP}} + (k\cdot xP) \mu \left[ \frac{d}{dk_\mu} \hat{\mathcal{C}}(q_k) \right]_{\mid_{k \cdot xP}} + \ldots \ldots (3.9)
$$

If one retains only the first term of the Taylor expansion in Eq. (3.9)** and substitutes

$$
\int d^4k \hat{\mathcal{C}}_0(q_k xP) \hat{H}(kP)
$$

and, if the structure function $G_{\mu}^{\nu} \mathcal{M}^{\mu\nu}$ has been constructed dimensionless:

$$
\mathcal{G}_{\mu\nu} = \int d^4k \hat{\mathcal{C}}_0 \left[ - \frac{q_\mu q_\nu}{2} \delta(xqP) \right] \hat{H}(kP) =
$$

$$
\int d^4k \hat{\mathcal{C}} \left[ \frac{xB}{x} \right] \int d^4k \delta \left[ x - \frac{k_\mu}{m} \right] \hat{H}(kP)
$$

where in the last equality we used the fact that $\mathcal{C}_0$ depends only upon $x$. If we define:

$$
F(x) \equiv \int d^4k \delta \left( x - \frac{k_\mu}{m} \right) \hat{H}(kP)
$$

we get

$$
\mathcal{G}_{\mu\nu} = \int dx \mathcal{C}_0 \left( \frac{xB}{x} \right) F(x)
$$

i.e., a convolution between the hadron structure function $F(x)$ and the "parton cross-section" $\mathcal{C}_0(xB/x)$. In lowest order:

$$
\mathcal{C}_0(xB/x) \sim \delta \left[ 1 - \frac{xB}{x} \right]
$$

so that

$$
\int dx \mathcal{C}_0 \left( \frac{xB}{x} \right) F(x) \rightarrow \hat{F}(xB)
$$

*) Equation (3.8) contains also summation over spinor indices which are suppressed here for simplicity.

**) We will see later that higher order terms correspond to "higher twists". In Eq. (3.8), one gets:
At order $\alpha_s$ we get:

$$F'(x_B) + \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{\ln Q^2}{m^2} \int_{x_B}^1 \frac{dy}{y} \mathcal{P}(\frac{X_F}{y}) F'(y)$$

Again we find a singular behaviour when $m$ goes to zero. However, the relation between the structure functions at two different scales are cut-off ($m$) independent

$$F(x_B, Q^2) = F'(x_B, Q_0^2) + \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{\ln Q^2}{Q_0^2} \int_{x_B}^1 \frac{dy}{y} \mathcal{P}(\frac{X_B}{y}) F'(y, Q^2)$$

This fact, which at order $\alpha_s$ is just a consequence of the basic properties of the logarithm, is actually valid to all orders in perturbation theory: by introducing a set of parton densities normalized from the experiment at some reference momentum transfer $Q_0$, and by expressing the prediction for any hard process in terms of these structure functions, the singular dependence upon the infra-red cut-off drops out. The factorization procedure is very similar to the renormalization technique where one gets rid of a cut-off (the ultra-violet one) at the expense of introducing an extra scale $z$ at which the coupling constant is normalized at some experimental input. The evolution equations for the hadron structure functions follow from Eq. (3.17) and have the classical form:

$$\frac{d\mathcal{F} (x, Q^2, Q_0^2)}{d\ln Q^2} = \int \frac{dy}{y} \mathcal{P}(\frac{x}{y}) F'(y, x, Q^2)$$

Actually, one gets in general a set of coupled equations for the quark and the gluon densities which contain the probabilities $P$ for a quark to go into a quark or a gluon and for the gluon to go into a quark or a gluon. The detailed form of these equations and the functional dependence of the probabilities is given, for example, in Ref. 8). The terms of order $\alpha_s^2$ in the probabilities $P$ allow us to resum the next-to-leading logarithms, i.e., the terms of order $\alpha_s (\alpha_s \ln Q^2)^N$: their explicit form can depend upon the specific factorization procedure. The evolution equations can be extended to the "time-like" case, i.e., to the $Q^2$ variation of hadron fragmentation functions. The specification "time-like" refers to the positive invariant off-shell mass of the partons in the jet cascade.  

For the time-like case, and in particular for the evolution of gluons into gluons, the order $\alpha_s^2$ terms in the probability are very important when the fraction of momentum carried by the secondary gluons gets very small. Indeed, the order $\alpha_s$ term behaves like:

$$\mathcal{P}^D_{gg}(x) \sim c \frac{\alpha_s}{x}$$

which, after performing the convolution integrals of the type in Eq. (3.18), gives

*) In other words, the "time-like" does not refer to the sign of $Q^2$: for the Drell-Yan muon pair production, for which the dimuon mass is time-like, the parton off-shellness is space-like.
The leading contribution of the second order term in the same region of small $z$ is:

\[ \int_{z}^{\infty} P_{g g}^0(x) \, dx \sim c \alpha_s l_n \frac{z}{x} \tag{3.20} \]

We see that the ratio of the second order term [Eq. (3.21)] to the first order one [Eq. (3.20)] is:

\[ \alpha_s \frac{l_n}{z} \tag{3.22} \]

Therefore, when $\alpha l_n^2 z$ is of order 1 (small values of $z$), the second (and higher) order corrections cannot be neglected. One needs a specific resummation of these "doubly logarithmic" terms: this has been performed. As an application of these calculations, one can predict the $Q^2$ behaviour of the parton's (hadron's) multiplicity which takes the form:

\[ \langle n(Q^2) \rangle = A \left[ \alpha_s(Q^2) \right] b \exp \left\{ \frac{C}{\sqrt{\alpha_s(Q^2)}} \right\} \left\{ 1 + \mathcal{O} (\sqrt{\alpha_s}) \right\} \tag{3.23} \]

The coefficients $b$ and $c$ are the result of the resummation procedure: the determination of the overall normalization is a problem which cannot be solved with perturbation theory techniques. In Fig. 9 a numerical simulation of the parton branching sequence according to

---

**Fig. 9**: Behaviour of the hadron's multiplicity in $e^+e^-$ annihilation compared with the prediction from Ref. 10.
QCD is reported: the final conversion into hadrons is made in a phenomenological way. The $Q^2$ dependence, which remains essentially unaffected by the details of the hadronization procedure, fits well with the data taken from $e^+e^-$ annihilation into hadrons.

Terms which are "doubly logarithmic" like in Eq. (3.22) occur in general when the radiated gluons are soft. Indeed, besides the "mass singularities", due to the kinematical configurations when the emitted radiation is collinear, there are also potential "infra-red singularities" which cancel between the emission of real gluons and the self-interactions with virtual gluons. If, by the kinematics, the real emissions are restricted to be soft, there is a mismatch between the real and virtual contributions leading to terms of the type

$$\alpha_s \frac{\ln^2 Q^2}{\alpha_s Q_s^2}$$

(3.24)

where $Q_s^2$ is the scale restricted by the kinematics to be much lower than $Q^2$, the maximum allowed one. The example that I want to discuss is the form of the transverse momentum ($p_T$) distributions of $W$'s and $Z$'s produced at the collider. Of course, this also covers the transverse momentum distributions of lepton pairs inclusively produced in hadron-hadron collisions. Let us take the $p_T$ differential cross-section suitably made dimensionless by multiplying it by $(Q^2)^2$:

$$\left[ \alpha_s^2 \right] \frac{d\sigma}{dp_T^2} = \int \left[ \frac{Q^2}{\mu^2}, \alpha_s, S, m_T^2, M_{\text{UV}}^2, Q_0^2 \right]$$

(3.25)

On the right-hand side of Eq. (3.25) there are the possible parameters upon which the cross-section depends: the transverse momentum $p_T$, the $W/Z$ mass $Q$, the total centre-of-mass energy squared $s$, the bare strong coupling constant, the bare parton densities $q_i^0$, and finally the quark mass which is kept finite to avoid mass singularities and the ultraviolet cut-off $M_{\text{UV}}$ which prevents the divergences arising from short wavelength fluctuations. After the renormalization and factorization procedure, the $m_T, M_{\text{UV}}$ scales disappear in favour of a normalization scale $\mu$, taken for simplicity to be the same for both renormalization and the factorization. Then Eq. (3.25) becomes:

$$\left( \frac{Q^2}{\mu^2} \right) \frac{d\sigma}{dp_T^2} = \int \left[ \frac{Q^2}{\mu^2}, S, \alpha_s(\mu^2), q_i(\mu^2) \right]$$

(3.26)

where $\alpha_s(\mu^2)$ and $q_i(\mu^2)$ are "normalized" at $\mu^2$. If we choose $\mu^2 = Q^2$:

$$\left( \frac{Q^2}{s} \right) \frac{d\sigma}{dp_T^2} = \int \left[ \frac{Q^2}{\alpha_s}, 4p_T^2, \alpha_s(Q^2), q_i(Q^2) \right]$$

(3.27)

When the variables $Q^2/s \equiv \tau$ and $4p_T^2/s \equiv x_T^2$ are kept fixed, one obtains a "scaling" behaviour, modulo the logarithmic corrections due to the $Q^2$ dependence of the running coupling
constant and parton densities. However, if \( p_{\perp}^2/Q^2 \) is very small, the leading perturbative terms look like:

\[
\delta(p_{\perp}^2) + C \alpha_s \frac{1}{p_{\perp}^2} \ln \frac{p_{\perp}^2}{Q^2} + C \alpha_s \frac{1}{p_{\perp}^2} \ln^4 \frac{p_{\perp}^2}{Q^2} + \ldots
\]

so that the integrated cross-section:

\[
\sigma(p_{\perp}) = \int \frac{Q^2}{p_{\perp}^2} d\sigma \, dp_{\perp}^2
\]

goes like

\[
1 - \alpha_s \frac{\ln^2 Q^2}{p_{\perp}^2} + \frac{\alpha_s^2}{2} \frac{\ln^4 Q^2}{p_{\perp}^2} (+\text{next-to-leading terms})
\]

The resummation of the leading terms in Eq. (3.30) to all orders of perturbation theory can be done by a "Block-Nordsieck" technique and gives:

\[
\sigma(p_{\perp}) \propto \exp \left\{ - \alpha_s \frac{\ln^2 \left( \frac{p_{\perp}^2}{Q^2} \right)}{2} \right\}
\]

However, the right-hand side of Eq. (3.31) acts like a form factor and kills the cross-section at \( p_{\perp} = 0 \). This is wrong: the experimental value for the cross-section is definitely different from zero! We can understand why the "leading log" approximation (or better "misleading" in this case) leads us to a wrong result. In fact, in the kinematical configuration which gives rise to the terms in Eq. (3.30), the gluons emitted, see Fig. 10, have transverse momenta which are "strongly ordered". The transverse momentum of the W/Z is higher than the ones of the emitted gluons: therefore, a small \( p \) for the W/Z implies, in the configuration of "strong ordering", a correspondingly smaller \( p_{\perp} \) for each gluon. At \( p_{\perp} \sim 0 \), one is practically inhibiting all the real gluon emission and the cross-section is driven to zero by the virtual emissions.

---

**Fig. 10**: Gluon bremsstrahlung from a q-\( \bar{q} \) pair annihilating into a massive photon.
This approximation is obviously bad, because at small $p_\perp$ of the $W$, many real gluons can still be emitted if there is a compensation of their transverse momenta. A better resummation technique which remedies this problem can be constructed. I will only give a few details of this technique and discuss the result. Define by $\nu(k_\perp)$ the probability for a single gluon emission $^*$; the probability $\sigma^n(p_\perp)$ for producing $n$ gluons whose total transverse momentum sums up to $p_\perp$ is given by:

$$\sigma^n(p_\perp) \sim \int d^2k_1 d^2k_2 ... d^2k_n \nu(k_1) ... \nu(k_n) \delta^2 \left[ \sum_{i=1}^n k_i - p_\perp \right]$$  \hspace{1cm} (3.32)

The expression in Eq. (3.32) is a convolution: it can be more easily calculated by introducing the Fourier transform of the probability $\nu(k_\perp)$:

$$\tilde{\nu}(b) \equiv \int d^2k \ e^{-i \hat{b} \cdot \hat{k}} \nu(k)$$  \hspace{1cm} (3.33)

Then

$$\sigma^n(p_\perp) \sim \int d^2b \ e^{-i \hat{b} \cdot \hat{p}_\perp} \left[ \tilde{\nu}(b) \right]^n$$  \hspace{1cm} (3.34)

where $b$, the "impact parameter", is the two-dimensional variable conjugate to $k_\perp$.

The utility of using the representation in the impact parameter space is that it allows us to reduce to a power series the calculation of the soft gluon emissions taking into account the compensation of their transverse momenta, one of the terms being like the one in Eq. (3.34). The series can then be resummed very much like the one in Eqs. (3.28)–(3.31) and the final result is:

$$\frac{d\sigma}{dq_\perp^2 dy} = N \left\{ \int \frac{d^2b}{4\pi} e^{-i \hat{q}_\perp \cdot \hat{b}} \left[ \exp \left( \mathcal{R}(b,q_\perp^2,y) \right) + \nu(q_\perp^2,\alpha_s,y) \right] \mathcal{R}(b,q_\perp^2,y) \right\}$$  \hspace{1cm} (3.35)

where the resummed "soft" emissions are given by the exponential and the function $\nu(q_\perp^2,\alpha_s,y)$ contains the contribution of "hard" gluons with $p_\perp \sim 0(\sqrt{q_\perp^2})$. The details of the $p_\perp$ distribution calculated according to Eq. (3.35) depend upon various parameters: their influence is shown in Figs. 11$^{12}$. In particular, Fig. 11a refers to two different choices of $A$ and of gluon distributions and Fig. 11b shows the effects of using $\alpha_s(q_\perp^2)$ versus $\alpha_s(q_\perp^2)$. According to Eq. (3.35), all values of $b$ are integrated out, in particular those corresponding to large distance physics which is not governed by perturbative QCD ($|b|$.

$^*$The possibility of defining such a probability is related to the softness of the emitted gluons.
Fig. 11a: Predictions obtained from Ref. 12 using different parton densities for the $W p$ distribution normalized to the total cross-section.

Fig. 11b: Influence on the predictions of the use $\alpha(Q^2)$ or $\alpha(q^2_{\perp})$.

Fig. 11c: Predictions confront the experimental data.
large). This forces the introduction of a parameter "a" which is completely phenomenological: it rules the "freezing" of the running coupling constant which is transformed according to:

\[
\alpha (Q^2) = \frac{1}{b \ln \frac{Q^2}{\Lambda^2}} \rightarrow \alpha (Q^2) = \frac{1}{b \ln \left( \frac{Q^2 + a\Lambda^2}{\Lambda^2} \right)}
\]  

(3.36)

For values of \( Q^2 \) below \( a\Lambda^2 \), the coupling constant ceases to increase and flattens out. In spite of these spurious parameters, when \( Q^2 \) (or \( s \) with \( Q^2/s \) fixed) becomes large enough, the prediction becomes insensitive to the large distances: indeed, the exponential in Eq. (3.35) acts like a form factor which kills the contributions coming from large values of \( b \). Physically, this can be understood as follows: the cross-section when the \( W \)'s have a small \( p_\perp \) gets most of its contributions from processes where many gluons are emitted with sizeable \( p_\perp \)'s which compensate each other. Therefore, the actual \( W \) production occurs most of the time when the annihilating partons are very off-shell and interact with each other at small distances. Fig. 1lc contains the comparison of theoretical predictions with experimental data: as one can see, it is not so bad.

The general picture for hard processes is then the following. There are reactions which are free of mass singularities; i.e., for which the infinite volume limit is smooth: they can be expressed as a series expansion in \( \alpha (Q^2) \) of a function depending upon scaling variables. Instead, those processes which are affected by mass singularities need a "factorization" procedure which leads to the introduction of running parton densities and running fragmentation functions\(^\ast\) \( q(Q^2), D(Q^2) \). Sometimes, the kinematical configurations are such that they enhance the relevance of soft bremsstrahlung: then the perturbative series contains double logs which must be resummed with special techniques. "Asymptotically" simple scaling laws must hold. A nice check has been performed by the UA1 collaboration on the high \( p_\perp \) jet cross-section\(^\ast\). This process in general decomposes into a sum of many subprocesses which are convoluted with quark or/and gluon structure functions. However, for a sufficiently large angular acceptance, i.e., when small angles are accessible, the cross-section as a function of the jet energy fractions \( x_1, x_2 \) and the scattering angle \( \cos \theta \), takes a factorized form:

\[
\frac{d^3\sigma}{dx_1 dx_2 d\cos \theta} \sim \frac{F_1(x_1)}{x_1} \frac{F_2(x_2)}{x_2} \frac{d\tilde{\sigma}}{d\cos \theta}
\]  

(3.37)

where

\[
F_1(x) = Q(x) + \frac{4}{g} \left[ Q(x) + \bar{Q}(x) \right]
\]

\(^\ast\) We have not discussed the case of fragmentation functions: it is conceptually the same stuff as the structure functions.
Fig. 12: The experimental combination of parton distributions $G(x) + 4/f(Q(x) + \bar{Q}(x))$ extracted from jet-jet data of UA1 is compared with the one evolved from low energy neutrino experiments CDHS (full line) and CHARM (dashed line). The dashed area represents the gluon’s contribution.

$G$ and $Q$ being the gluon and quark distributions respectively. The resulting $F(x)$ can be successfully compared, as is done in Fig. 12, with the values for the same quantities extrapolated from the "low energy" data of fixed target neutrino experiments.

4. - POWER LOW CORRECTIONS

Before the "asymptotic" regime where the scaling laws are affected by simple logarithmic corrections is reached, one has to deal with violations which have a stronger energy dependence, dying as some inverse power of the typical scale of the hard process $Q$, i.e., like $Q^2/Q^2$. They are often referred to as "higher twists" and can be divided into two classes:

1) inclusive, referring to $O(1/Q^2)$ corrections to inclusive hard processes;

2) exclusive, referring for example to the behaviour of meson/baryon form factors, to the "prompt" resonance production at large transverse momentum in hadronic collisions or to the angular distribution of Drell-Yan lepton pairs close to the kinematic limit of the lepton's invariant mass.

The simplest "laboratory" to discuss the inclusive higher twists remains the deep inelastic scattering of hadrons against external electroweak currents. The following discussion is
rather technical, but necessary. The "lowest twist" approximation for the hadron structure function is expressed in Eq. (3.13) as the convolution of a function referring to the "short-distance" interaction, denoted by $\hat{\mathcal{C}}$ and one referring to the "long distance" physics, denoted by $F$. The physical picture of the process in the leading twist approximation is the following. One observes the interference pattern of amplitudes for the interaction with the external current at points which are separated by a light-like distance. In order to see this, it is enough to take the Fourier transform of the quantity $\hat{H}(k,p)$ entering the Eqs. (3.10)-(3.12):

$$\hat{H}(k) = \langle \cdots \rangle \int d^4\eta \ e^{-i\kappa \cdot \eta} H(\eta)$$

(4.1)

then

$$F'(x) = \int dk \int d^4\eta \ e^{i\eta \cdot k} \delta \left[ x - \frac{kn}{p \cdot n} \right] H(\eta)$$

(4.2)

the integral over $k$ can be performed in the usual frame defined in terms of the vectors $\eta_\mu$, $p_\mu$ [see Eqs. (3.5) and (3.6)]. In this frame, the four vector $\eta_\mu$ (remember that it defines the distance conjugate to the momentum $k$) is parametrized as:

$$\eta_\mu = \lambda \gamma_\mu + \frac{\eta_+^2 \eta_-^2}{4\lambda^2} \gamma_\mu + \eta_{\perp \mu}$$

(4.3)

so that

$$k \cdot \eta = \lambda x + \frac{(k^+ + k^-) (\eta^+ + \eta^-)}{4\lambda x} - \eta_{\perp} \cdot k_{\perp}$$

(4.4)

Remembering that

$$d^4k \sim dk^2 \frac{dx}{x} d^2k_{\perp}$$

(4.5)

one obtains from the integral over $k^2$:

$$\int dk^2 e^{i k^2 (\eta^2 + \eta_{\perp}^2)} \Rightarrow \delta (\eta^2 + \eta_{\perp}^2)$$

(4.6)

and from the one over $k_{\perp}$:

$$\int d^2k_{\perp} e^{i \eta_{\perp} \cdot k_{\perp}} \sim \delta^2 (\eta_{\perp})$$

(4.7)

Equations (4.6) and (4.7) imply: $\eta^2 = 0$, $\eta_{\perp} = 0$. Equation (4.2) then reads:

$$F'(x) = \int d\lambda \ e^{i\lambda x} H(\lambda \eta_\mu)$$

(4.8)

where

$$H(\lambda \eta_\mu) = \langle \bar{\psi}^\mu (0) \eta_\mu \gamma^\mu \psi (\lambda \eta_\mu) | \bar{p} \rangle$$

(4.9)
The structure function $F(x)$ is the Fourier transform of a bilocal operator $\tilde{\psi}(0)n^\mu\gamma^\nu\psi(\lambda n)$ defined on points (\"$0$\" and \"$\lambda n$\") which are at a relative light-like distance ($\lambda^2 n^2 = 0$). The bilocal operator can be expanded into a series of local operators:

$$F(x) = \int d\lambda d\xi \sum_{K=0}^{\infty} \frac{\lambda^k}{k!} \eta_\mu \eta_{\mu A} \cdots \eta_{\mu N} \left< \bar{\psi}(0) \right| \lambda^n \partial_{\mu A} \cdots \partial_{\mu N} \psi(0) \left| \bar{P} \right>$$  \hspace{1cm} (4.10)

The single local operator of the series in Eq. (4.10) can be isolated by taking the moments of the function $F(x)$:

$$\int d\lambda d\xi F(x) \propto \eta_\mu \eta_{\mu A} \cdots \eta_{\mu N} \left< \bar{\psi}(0) \right| \lambda^n \partial_{\mu A} \cdots \partial_{\mu N} \psi(0) \left| \bar{P} \right>$$  \hspace{1cm} (4.11)

where the ordinary derivatives $\partial_\mu$ appearing in Eq. (4.10) have been replaced by covariant derivatives $D_\mu \equiv \partial_\mu + igA_\mu$, given that the above picture holds in the axial gauge where $n^\mu A_\mu = 0$ (\$n^\mu = n^\mu D_\mu\$). From Eq. (4.11) we see that different moments of the structure function $F$ measure the matrix elements of the operator having the same

$$\text{Dimension} - \text{spin} = \text{twist}$$  \hspace{1cm} (4.12)

The leading (lowest) twists are selected in Eq. (4.11) because the operator is \"projected\" by the light-like vector $n^\mu$, a different projection is needed in order to isolate the higher twists. The general representation of the proton matrix element of a given operator is:

$$\left< \bar{P} \right| \bar{\psi}(0) \gamma_{\mu A} \partial_{\mu A} \cdots \partial_{\mu N} \psi(0) \left| \bar{P} \right> =$$

$$= A(N) \partial_{\mu A} \cdots \partial_{\mu N} + B(N) \frac{\partial_{\mu A} \cdots \partial_{\mu N}}{P^i P^j} + O(\alpha^2 \cdots \alpha^2)$$  \hspace{1cm} (4.13)

where $g_{\mu\nu}$ is the usual metric tensor and the neglected terms contain at least two of them.

From simple dimensional analysis follows:

$$\text{dim} (A) = \text{dim} (B) - 2$$  \hspace{1cm} (4.14)

If $A$ is dimensionless, $B$ has dimension two (in units of mass), i.e., $B$ is of order of $\Lambda^2$ and is the matrix element giving rise to the \"higher twists\" contributions of the order of $\Lambda^2/Q^2$. It is clear that if the operator matrix element in Eq. (4.13) is \"projected\" by a light-like vector $n^\mu$ ($n^2 = 0$), one can isolate its leading twist part (A).
Next-to-leading twists are related to the terms $B$ in the expansion (4.13): in terms of diagrams, they involve processes where more than a single parton from the proton participates in the hard scattering, like in Fig. 13. There we see the interference between the amplitude where a state of one quark and one gluon scatters against the external currents and the amplitude with one quark only. The hadronic matrix element $H_\mu(x_1,x_2)$, which now depends on two momentum fractions $x_1,x_2$, involves a new operator:

$$H_\mu(x_1,x_2) \simeq \int d\lambda_1 d\lambda_2 e^{i\lambda_4 x_4 + i\lambda_2 x_2}$$

\[ \langle P | \bar{\psi} (0) \gamma_\mu (\lambda_1 n_\mu) \psi (\lambda_2 n_\mu) | P \rangle \]  

(4.15)

where the notation is in terms of positions on the light cone $(\lambda_1 n_\mu, \lambda_2 n_\mu)$, like in Eq. (4.9). The diagrammatic interpretation of higher twists is not unique: diagrams with a different number of partons coming from the hadronic box in Fig. 13 can be related by the equation of motion like:

$$\bar{\psi} \gamma^\mu \partial_\mu \psi = ig \bar{\psi} \gamma^\mu A_\mu \psi$$

(4.16)

The operators appearing on the two sides of Eq. (4.16) correspond to the two possible definitions of the hadronic blob drawn in Fig. 14.

---

**Fig. 13:** Higher twist diagrams.

**Fig. 14:** The box drawn with a full line and the one drawn with a dashed one represents higher twist operator matrix elements which are related by the equations of motion.
The parametrization of the structure function in terms of operators corresponding to different multiparton densities (the generalization of the usual quark and gluon densities) depends upon the chosen operator basis: for a detailed (overdetailed?) description of all the technicalities, I suggest Ref. 13. Here I will only report the result for the longitudinal \( F^L_\perp \) and the transverse \( F^T_\perp \) structure function

\[
F^L_\perp(x, \frac{Q^2}{x}) = \frac{H^2}{Q^2} A(x) + \frac{\Lambda^2}{Q^2} \left\{ \frac{4}{x} T^T_4(x) - x \sum_{x_1, x_2} d x_2 d x_2' \left[ \frac{\delta(x_2' - x) - \delta(x_2 - x)}{x_2' - x_2} \right] \right\} \tag{4.17}
\]

\[
F^T_\perp(x, \frac{Q^2}{x}) = A(x) + \frac{\Lambda^2}{Q^2} \left\{ \frac{4}{x} T^T_4(x) - x \sum_{x_1, x_2} d x_2 d x_2' \left[ \frac{\delta(x_2' - x) - \delta(x_2 - x)}{x_2' - x_2} \right] \right\} \tag{4.17}
\]

The functions \( T_1 \) and \( T_2 \) are related to multiparton densities: I will only report the explicit form of \( T_1 \):

\[
\frac{\Lambda^2}{Q^2} L^T_4(x) = \int d \alpha \ e^{i \lambda x} \left\{ \bar{\psi}(0) \gamma_\mu \psi(0) \right\} \left\{ \bar{\psi}(0) \gamma^\mu \psi(0) \right\} \tag{4.18}
\]

where \( D^\perp_\mu \) is the covariant derivative on the transverse components only. In the limit of zero strong coupling constant, \( D^\perp_\mu + \delta^\perp_\mu \) and \( T_1 \) reduces to:

\[
\frac{\Lambda^2}{Q^2} L^T_4(x) \rightarrow \int d \alpha \ e^{i \lambda x} \left\{ \bar{\psi}(0) \gamma_\mu \partial^\perp_\mu \psi(0) \right\} \tag{4.19}
\]

which measures the amount of "intrinsic \( k_t \)" carried by the quarks. This is a well-known result in the naive parton model where the longitudinal structure function is proportional to the parton's "intrinsic \( p_t \)." Besides the terms of the order of \( \Lambda^2/Q^2 \), Eq. (4.17) contains also terms of order \( M^2/Q^2 \) where \( M \) is the proton mass. I call these "kinematical higher twists": their normalization is fixed in terms of the same function \( A(x) \) in Eq. (4.17) which parametrizes the leading twist behaviour. In Fig. 15 are reported the data for \( F_\perp \) from the CHERM collaboration \(^{15}\), and, superimposed, the QCD prediction including only the leading twist contribution, the same and the "kinematical" higher twists, the latter and a phenomenological parametrization of dynamical higher twists. The experimental tests of the contributions of inclusive higher twists are difficult: one has to extract from the data an entirely new set of multiparton densities. If we think about the uncertainty by which the gluon distribution is known, we can imagine how poor the determination of further and more complicated parton distributions would be.

The tests of power law corrections seem more feasible for exclusive processes \(^{16}\). The simplest example is provided by the electromagnetic meson form factor. In Fig. 16a is reported the simplest diagram where one of the quarks forming the meson is hit by the photon and it reverses (in a suitable frame) its momentum while the other acts as a
Fig. 15: The longitudinal structure function from CHARM data is compared with the prediction based on O(\(a_s\)) calculations (dashed-dotted line), the one including \(N_c^3/q^6\) corrections (full line) and the one including a simple parametrization of dynamical power corrections (dashed line).

Fig. 16a: A meson hit by a photon in the simplest (and wrong) picture.

Fig. 16b: Lowest order contribution to the meson form factor.

Fig. 17: As in Fig. 16b, specifying the positions where the photon and the gluon interact with the \(q\bar{q}\).

"spectator". The final state would consist of a \(q\bar{q}\) pair where the fermions get further and further away from each other: It is very unlikely that they would turn out as a single meson. In order to achieve this, one needs to pay the price of exchanging a hard gluon like in Fig. 16b, which transmits to the "spectator" quark part of the momentum transfer of the photon. The corresponding diagram in the configuration space (Fig. 17) contains the positions where the photon ("0") hits the meson and where the gluon is emitted (\(z_1\)) and reabsorbed (\(z_2\)). In formulae, one has:

\[
\langle \Pi (f) | \bar{J}^\mu | \Pi (\text{init}) \rangle = \\
= \int d\xi_1 d\xi_2 \langle \Pi (f) | \bar{\psi}^{(\xi_1)}_\kappa \psi^{\delta_3}_\delta (\xi_2) | 0 \rangle \cdot C^{\kappa \delta_3 \lambda \lambda \delta_2} (q) \cdot \langle 0 | \bar{\psi}^{(\xi_2)}_\delta \psi^{\delta_2}_L (0) | \Pi \text{init} \rangle
\]

(4.20)
The expression in Eq. (4.20) is wrong by inspection: in fact it contains a bilocal operator matrix element

$$\langle 0 | \psi^\alpha_i(x_1) \psi^\beta_j(x_2) | \Pi \rangle$$

(4.21)

which is not gauge invariant. We need some operator made of gluon fields which transports the information about the colour at points \(z_1\) and \(z_2\). Such an operator is the following:

$$\exp \left\{ \frac{i g}{\Lambda} \int_{z_1}^{z_2} d\sigma^\mu \ A^\mu \right\}$$

(4.22)

where the line integral in Eq. (4.22) is taken along a trajectory joining the points \(z_1\) and \(z_2\). The form of the operator in Eq. (4.22) is easy to understand; for, remember that the operator which performs the infinitesimal translation is \(i \partial_\mu\). Out of it one can form the corresponding operator for a finite translation which reads

$$\exp \left\{ i a_\mu \partial_\mu \right\}$$

(4.23)

In fact

$$\phi(x_\mu + a_\mu) = \exp \left\{ i a_\mu \partial_\mu \right\} \phi(x_\mu)$$

(4.24)

where \(\phi\) is a generic scalar field. (It works for ordinary functions.) At the beginning of these lectures we have learned that the gauge invariance for the matter fields (i.e., for non-gauge fields) can be obtained simply by replacing in the Lagrangian the ordinary derivative by the covariant one:

$$\partial_\mu \rightarrow \partial_\mu + i g A_\mu$$

(4.25)

Therefore, when one needs the "colour transporter" for a finite distance between two points, one has to exponentiate the "infinitesimal" transporter \(i g A_\mu\) over a trajectory joining the two points macroscopically separated. The operator matrix element in Eq. (4.21) is then replaced by:

$$\langle 0 | \psi^\alpha_i(x_1) e^{i g \int_{x_1}^{x_2} d\sigma^\mu A^\mu} \psi^\beta_j(x_2) | \Pi \rangle$$

(4.26)

The function "C" in Eq. (4.20) represents the "short-distance" kernel describing the absorption of the photon and the exchange of the hard gluon as in Fig. 16b. In configuration space, it is proportional to the quark and the gluon propagator:

$$C(x_1, x_2) \propto S_{\text{quark}}(x_1, 0) S_{\text{gluon}}(x_2, x_1)$$

(4.27)
When \( q^2 = 0 \), for the position vector \( z_1^\mu \) only the components on the light-cone survive and the four vector \( z_1^\mu \) becomes proportional to the light-like vector \( n_\mu = z_1^\mu = z_1^0 n_0 \). If we take the Fourier transform with respect to the \( z_1 \) variables of the operator in Eq. (4.26) we get:

\[
\langle 0 | \mathcal{O}_\mathcal{Q}^{(z_1 n_\mu)}(z_1 n_\mu) \mathcal{O}_\mathcal{Q}^{(z_2 n_\mu)}(z_2 n_\mu) | \Pi \rangle =
\]

\[
= \alpha \left( \gamma^5 \gamma^5 \right) \mathcal{P}_g \int dx_1^0 dx_2^0 \delta (x_1 - x_2) e^{i \mathbf{z}_1 \cdot \mathbf{x}_1 - i \mathbf{z}_2 \cdot \mathbf{x}_2}
\]

(4.28)

where I have specified the flavours of the fermions appearing in Eq. (4.26). The variables \( x_1, x_2 \), conjugate to the light cone positions \( z_1, z_2 \), are collinear momentum fractions in a \( p_\perp \) frame. The function \( \Phi_\pi(x_1, x_2) \) then represents the pion wave function in the same frame.

The total answer is:

\[
\langle \Pi (p_2) | \mathcal{J}(q) | \Pi (p_1) \rangle = (p_1 + p_2)_\mu F_\pi^\mu (q^2)
\]

(4.29)

where

\[
F_\pi^\mu (q^2) = \frac{32 \pi \alpha_s}{g} \frac{A}{q^2} I_\Pi
\]

with

\[
I_\Pi = \int d^2 x d^2 y \Phi_\Pi (y) \frac{e_u}{x_2 y_2} - \frac{e_d}{x_1 y_1} \int \Phi_\pi (x)
\]

with \( e_u, e_d \) the electromagnetic quark charges. The variable \( x(y) \) is related to the variables \( x_1, x_2 \) by:

\[
x \equiv x_1 - x_2 \\
d^2 x = \delta^4 \mathbf{x} dx_1 dx_2 \delta (\mathbf{1} - x_1 - x_2)
\]

(4.30)

The only unknowns in Eq. (4.29) are the pion wave functions \( \Phi_\pi \) which are related, as we have seen, to the Fourier transform of some matrix elements of bilocal operators like:

\[
\langle 0 | \mathcal{D}(0) \gamma^5 \gamma^5 \exp \left\{ i g \int d^2 x \mathcal{A}_\mu \right\} \mathcal{O}(z_1 n_\mu) | \Pi \rangle
\]

(4.31)

These have the same form of the bilocal operators found in deep inelastic scattering, although the matrix element is different. The evolution with \( q^2 \) of these matrix elements,
Fig. 18: Higher order corrections (leading log in a non-covariant gauge).

or better of the short-distance kernel C, due to the exchange of a gluon ladder between the quark lines as in Fig. 18, is calculable by means of evolution equations similar to the ones used for deep inelastic scattering.

The methods applied to the analysis of the pion form factor apply in general to all the cases where a resonance interacts "promptly" or "coherently" in a hard process. Examples of such processes are given in Figs. 19a,b. In the first case there is a "prompt" hadron (meson) production at large momentum transfer in a hadron-hadron collision; in the second, there is the "coherent" hard interaction of a meson in a Drell-Yan lepton pair production. In general, these "exclusive" processes are suppressed by one or more inverse powers of \( q^2 \): perhaps they represent an easier test of power-behaved non-scaling effects in QCD.

Fig. 19a: "prompt" meson production at large \( p_\perp \) in hadron-hadron scattering.

Fig. 19b: Lepton pair production in the scattering of a quark from a hadron \( h_2 \) against a meson \( M \).

Let us summarize the situation concerning perturbative QCD:

1) "generalized scaling" (with logarithmic corrections) is well understood and for most processes also next-to-leading contributions have been calculated.

11) There are important systematic limitations [at present (SPS) energies] coming from the ignorance of the low energy, non-perturbative dynamics

   a) the precise form of gluon distribution
   b) the need of hadronization models in jet physics
   c) the uncontrollable effects of \( p_\perp \) smearing coming from quark Fermi motion.

The situation improves at higher (collider or supercollider) energies: the low energy uncertainties on the gluon distribution are washed out by the long evolution of the gluon
distribution which is eventually determined by the low energy behaviour of quark distributions. Hadronization models will still affect events with only a few jets, but should not matter too much in the study of rare multijet events\(^{18}\). The effects of "intrinsic \(p_{\perp}\) smearing" sometimes become milder, like for small \(Z/W\) production at small \(p_{\perp}\).

iii) The general behaviour of inclusive power law corrections is understood, but a systematic analysis is rather prohibitive, due to the complete ignorance of the new multi-parton distribution functions which parametrize these effects.

An interesting approach for estimating the long-distance behaviour of QCD and in particular the values of various hadron matrix elements of quark/gluon operators is given by the "QCD sum rules". A short description of them was part of my lectures: however, the presentation proved to be too technical, especially after the previous already technical sections. I then decided to suppress this part in the written version of my lectures. The interested reader can address her(him)self to the original literature on the subject\(^{19}\), I prefer to go directly to the last subject of my lectures: lattice QCD. Within this approach the only parameters which are needed to describe any QCD processes are the quark masses and \(A\): modulo the uncertainties of any numerical approach, it is the best candidate for the correct evaluation of non-perturbative effects.

5. - LATTICE QCD

In order to estimate the effects of quantum fluctuations, we have said in Section 1 that one has in general to perform exactly a non-Gaussian functional integral. When one is facing an integral that cannot be done analytically, one can evaluate it numerically by estimating its value on a finite set of points. In the case of quantum field theories, this amounts to discretizing the space-time by introducing an imaginary lattice whose nodes identify a finite set of co-ordinates\(^{19}\).

If we want to simulate the behaviour of QCD on a finite region of space-time, it means that we are calculating the properties of hadrons in a box. The question is: how big should it be to avoid finite size problems? We know from nuclear physics that the energy shift for nucleons squeezed in a region with a size of about 2 Fermis is of \(10^{-15}\) MeV. This means that one can expect an effect of the order of two per cent on the nucleons' mass if the box size \(l_{\text{BOX}}\) is \(\sim 2\) Fermi. This is true for hadrons (like the nucleons) made of "light" quarks (i.e., up or down quarks). Hadrons made of heavier quarks (the strange one) will also tolerate smaller box sizes. Indeed, from the experimental data on total cross-sections which can be used for estimating the hadrons' size, one would deduce for the pion a dimension of about 1.5 Fermi and for the kaon of about 1 Fermi. To summarize, \(l_{\text{BOX}} \sim 2\) Fermi and \(l_{\text{BOX}} \sim 1\) Fermi can be suitable values for hadrons made of light and strange quarks respectively.

\(^{18}\) Although, in this case, the contamination from other multiparton subprocesses may be relevant.
In order to have a finite number of degrees of freedom one has to introduce a finite resolution "a": the lattice spacing. How small should it be? The answer to this question comes from practical and theoretical considerations. The practical ones are dictated by the limitations of present computers capabilities. Indeed, the total number of degrees of freedom per node is very high: there are (complex) variables - the fermions - which are defined on the nodes

$$\psi_{\alpha}^a(i)$$

(5.1)

where $a$ is a colour index ($a = 1-3$ for quarks) and $\alpha$ is the ordinary Dirac index running from one to four; this makes in total 24 (real) variables per node. There are variables which are defined on the links associated to the gauge fields:

$$U_{ij}^{AB}$$

(5.2)

where $i$ and $j$ are neighbouring points and $A, B$ are colour indices. $U$ is a unitary $3 \times 3$ matrix, a total of 9 complex variables times the four possible directions; this makes 72 real variables per node. The relation between the matrices $U$ and the gauge field $A_{\mu}^{\alpha}$ is the following

$$U_{ij}^{AB} = \left[ e^{i A_{\mu}^{\alpha} d x_{\mu}^{\alpha}} \right]^{AB}_{ij} = \sum_{i} \lambda_{i}^{A} \lambda_{i}^{B}$$

(5.3)

i.e., $U$ is the exponential of the line integral of the gauge field $A_{\mu}$ along the link associated with a given matrix. The action on the lattice is written in terms of the variables $U$'s. One starts from the action on the continuum:

$$S = \int d^4x \left[ - \frac{1}{4} F^{\mu \nu}_{\alpha} F^{\mu \nu}_{\alpha} \right]$$

(5.4)

where

$$F^{\mu \nu}_{\alpha} = \partial_{\mu} A_{\nu}^{\alpha} - \partial_{\nu} A_{\mu}^{\alpha} + g \epsilon^{\alpha \beta \gamma \delta} A_{\mu}^{\beta} A_{\nu}^{\gamma} A_{\delta}^{\delta}$$

and rescales the fields with the coupling constant:

$$g A^{\alpha \Delta} = A_{\mu}^{\alpha \Delta}$$

(5.5)

then

$$F^{\mu \nu}_{\alpha}^{\text{new}} = \frac{1}{g} \left\{ \partial_{\mu} A_{\nu}^{\alpha} - \partial_{\nu} A_{\mu}^{\alpha} + \epsilon^{\alpha \beta \gamma \delta} A_{\mu}^{\beta} A_{\nu}^{\gamma} A_{\delta}^{\delta} \right\}$$

(5.6)
then
\[ S = \frac{1}{g^2} \int d^4 x \left[ -\frac{1}{4} G_{\mu \nu} G^{\mu \nu} \right] \] (5.7)

On the lattice,
\[ S \propto \sum_{\text{plaquettes}} \frac{1}{g^2} U_{\mu}^{\nu}(n) \] (5.8)

where the sum is performed over the plaquettes defined by a pair of directions \( \hat{\mu}, \hat{\nu} \) and an origin "n" as in Fig. 20; the symbol \( U_{\mu} \) denotes the trace of the product of the four links around the plaquette. This product will involve the matrices \( U_{\mu}^a \) or their Hermitian conjugates \( U_{\mu}^+ \), depending on the relative orientation of the curve defining the plaquette and of the link. \( U_{\mu}(n) \) describes a link from the point \( n \) to the point \( n+\hat{\mu} \), while \( U_{\mu}^+(n) \) describes a link from \( n+\hat{\mu} \) to the point \( n \). One has:
\[ U_{\mu}^{\mu \nu}(n) = U_{\mu}(n) U_{\nu}(n+\mu) U_{\mu}^+(n+\nu) U_{\nu}^+(n) \] (5.9)

The action on the lattice in terms of \( U_{\mu}^{\mu \nu}(n) \) can be written as follows:
\[ S_{\text{Lattice}} = \frac{6}{g^2} \sum_{\text{plaquettes}} (U_{\mu}^{\mu \nu} - 1) \] (5.10)

The reduction of the action in Eq. (5.10) to the one of the continuum theory in the limit \( a \to 0 \) can be understood as follows. Remember that
\[ U_{\mu} \sim \exp \left( iA_{\mu} A_{\nu} \right) = \exp \left( iA_{\mu} A_{\nu} \cdot A_{\lambda} \right) \] (5.11)

If the matrices \( \lambda_{\mu}^a \) were commuting among themselves, one would form, by combining the four \( U_{\mu} \)'s around a plaquette, the exponential of the curl of the field \( A_{\mu} \) on a closed curve. The Stokes theorem then tells us that this is equivalent to the flux of \( \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \) through a surface bounded by the curve. The expansion in powers of "a" gives:
\[ U_{\mu}^{\mu \nu} \simeq 1 + i a^2 G_{\mu \nu} + \frac{1}{2} (i a)^2 G_{\mu \nu} G_{\mu \nu} + \ldots \] (5.12)

The term quadratic in \( a \) vanishes when summed over \( \mu \) and \( \nu \) indices because of its (anti) symmetry properties; one is left with
\[
\left( \frac{1}{2} \right) \langle \partial^\mu \rangle^2 (p^a)^2 \nabla^{\mu} \nabla^{\nu} + O(a^6)
\]

i.e., the continuum action once the correspondence \( d^4x \to S \) plaquettes \( a^4 \) is made. Non-commuting \( \lambda \) matrices (as is the case for QCD) produce extra terms which conspire to form the complete expression for \( G_{\mu\nu} \) of Eq. (1.4). The reduction to the continuum action works to order \( a^4 \): the terms of order \( a^6 \) are irrelevant in the limit \( a \to 0 \). In fact, different actions on the lattice may lead to the same "continuum" limit. This is a non-trivial statement in the interacting theory: for the moment, I just mean that the naive limit \( a \to 0 \) leads to the same action up to terms of order \( a^6 \).

The practical considerations about the smallness of "\( a \)" are related, as I already mentioned, to the present computers' capabilities. If for light hadrons one needs an overall size of about 2 Fermi and one wants a resolution of 1 Fermi, one needs 20 points per direction: this amount to about 30 megawords of (fast) memory, less without fermions or with some tricks on the storage of the unitary matrices. For strange hadrons with the same resolution but an overall size of about 1 Fermi, one can do it with 2 megawords. Present computer capabilities are closer to the 2 megaword than to the 30 megaword case and allow a reasonable estimate of the "strange hadrons" spectrum.

Of course, the answer "we choose "\( a \)" at our best" is not satisfactory. One must have a criterium for judging, from the results themselves, whether the value of \( a \) is adequately small. Here come the theoretical considerations. The question is: what guarantees that, for "\( a \)" small enough, the physical results will not be affected?

You must realize that "\( a \)" acts as an ultra-violet cut-off: wavelengths shorter than \( a \), i.e. the high momenta, are cut off. The independence upon \( a \) of physical results is a consequence of the renormalizability of the continuum theory. According to this property, the lattice spacing \( a \) (i.e. the ultra-violet cut-off) and the bare coupling constant \( g \) are redundant parameters. The same physics is described by any of the pair of values of \( g \) and \( a \) which lies on a curve (the renormalization curve) parametrized by a single dimensionful parameter \( \Lambda \). For any given value of \( \Lambda \) (\( \Lambda = \bar{\Lambda} \)), there is a curve like the one of Fig. 21;

\[ \text{Fig. 21 : The behaviour of the bare coupling constant } g(a) \text{ as a function of the lattice spacing } a \text{ for a fixed value of } \Lambda: \Lambda = \bar{\Lambda}. \]

\(^*\text{Within some approximation which we will discuss later.} \)
if \( \Lambda \) is known and if the theory on the lattice behaves like the one on the continuum, by fixing \( g(a) \), one fixes \( a \Lambda \), i.e., value of \( a \) in physical units. The signature that the theory is behaving like the continuum theory, i.e., that physical results are only dependent on \( \Lambda \) and not on \( a \), is the scaling test. It can be obtained by studying the behaviour of a dimensionful quantity \( M \) (with the dimension of a mass, for example) as a function of the lattice coupling constant (remember that it is a bare coupling). On the lattice, actually from the computer output, one always obtains "dimensionless" numbers, i.e., any dimensionful quantity \( M \) naturally comes in lattice spacing units: \( M = Ma \). The curve in Fig. 21 is parametrized by the equation:

\[
\frac{a^2}{\sigma^2(a)} = \frac{1}{b \beta \mu \left( 1/a^2 \right)}
\]

which can be inverted:

\[
a^2 \Lambda^2 = e^{-1/b \sigma^2}
\]

Therefore, for any dimensionful quantity in units of \( a \), one expects a definite scaling pattern as a function of \( \beta = 6/\sigma^2 \)

\[
M^2 \alpha^2 = M^2/\Lambda^2 \cdot P \cdot e^{-C\beta} \left( 1 + O(1/\beta) \right)
\]

where the constants \( P \) and \( C \) are calculable perturbatively as well as the higher order \( O(1/\beta) \) corrections. When the scaling law defined by Eq. (5.16) is at work, one can say that the lattice approximation is simulating well the continuum theory and that the influence of the cut-off in the relations between physical quantities is negligible.

The next question is: how does one simulate the dynamics? One could think of solving a set of coupled Schrödinger-like equations to find the energy levels: this is by far too hard, given that the amount of independent unknowns is of the order of \( 10^6 \). The "natural" way is through the functional integral: after all, as I said at the beginning of this section, the lattice approximation can be seen as a way of performing numerically a difficult integral. Remember from Section 1 that the "propagator" of the gauge field can be expressed as a suitable average:

\[
\langle A_\mu(x) A_\nu(0) \rangle \sim \frac{\int d[A_\mu] e^{\frac{-i}{\hbar} S(A)}}{\int d[A_\mu] e^{\frac{-i}{\hbar} S(A)}}
\]

(5.17)

On the lattice, instead of the field \( A_\mu \), one uses the exponentiated form:

\[
U_\mu \sim e^{i A_\mu}
\]
The analogue of Eq. (5.17) is then an average over some function of the fields $U$:

$$
\langle \mathcal{P}(U) \rangle = \frac{\int d[U] \ e^{-\frac{i}{\hbar} S(U)} \mathcal{P}(U)}{\int d[U] \ e^{-\frac{i}{\hbar} S(U)}}
$$

(5.18)

where $S(U)$ is the lattice action in terms of $U$, Eq. (5.10).

The dynamics is simulated by producing a set of "configurations", i.e., a set of values of the $U$'s for all the links on the lattice (remember that the $U$'s are defined on the links) with the following probability distribution:

$$
\tilde{\mathcal{P}}(U) d[U] \sim \frac{d[U] \ e^{-\frac{i}{\hbar} S}}{\int d[U] \ e^{-\frac{i}{\hbar} S}}
$$

(5.19)

The distribution in Eq. (5.19) is not reasonable. It contains an imaginary part which is hard to interpret as a probability. To remedy this problem, one performs the "Wick" rotation, already discussed in Section 1, of the time axis from the real to the imaginary axis:

$$
t_M \rightarrow i t_E
$$

(5.20)

As we have seen, this transforms the matrix of the space from a Minkowski to a Euclidean metric. In more simple terms, the product of two four-dimensional vectors $(a_0, \mathbf{a}), (b_0, \mathbf{b})$ is defined by:

$$
a_0 b_0 - \mathbf{a} \cdot \mathbf{b}
$$

(5.21)

in a Minkowski space and by

$$
a_0 b_0 + \mathbf{a} \cdot \mathbf{b}
$$

(5.22)

in a Euclidean space. With the Wick rotation, the probability distribution is

$$
\mathcal{P}(U) d[U] \sim \frac{d[U] \ e^{-\frac{i}{\hbar} S}}{\int d[U] \ e^{-\frac{i}{\hbar} S}}
$$

(5.23)
i.e., the imaginary unit in the exponent has been compensated by the "i" appearing in the infinitesimal four-volume:

$$i \mathcal{L} = i \int d^4 x \, \mathcal{L}(x) = \int dt \int d^3 x \, \mathcal{L}(x)$$

(Euclidean)

$$\equiv \mathcal{S}_E$$

(5.24)

The values of the correlation function \( f(U) \) in Eq. (5.18) are obtained by averaging its value over the set of "equilibrium" configurations collected.

$$\langle f(U) \rangle_E = \frac{\int d[U] \, e^{-\frac{1}{\beta} \mathcal{S}_E(f(U))}}{\int d[U] \, e^{-\frac{1}{\beta} \mathcal{S}_E}}$$

(5.25)

A learned reader can recognize (it is not dramatic if he does not) a strong similarity between Eq. (5.25) and the expression for the average taken on a statistical system in four dimensions: lattice QCD appears, from the point of view of statistical mechanics, as a theory defined on a four-dimensional crystal.

The "equilibrium" configurations are produced most of the time by a Monte Carlo algorithm introduced by Metropolis \(^{20}\).\(^{x})\). One:

1) starts from an arbitrary configuration (for example, randomly distributed);
2) looks at the value of the field (call it \( \phi \)) at any given point and tries to change it: \( \phi \rightarrow \phi' \);
3) calculates the variation of the action:

$$\delta \mathcal{S} = \mathcal{S}(\phi') - \mathcal{S}(\phi)$$

(5.26)

then if \( \mathcal{S}(\phi') < \mathcal{S}(\phi) \), one replaces the old value of \( \phi \) with the new value \( \phi' \), if \( \mathcal{S}(\phi') > \mathcal{S}(\phi) \), one accepts the new value with probability \( e^{-\delta \mathcal{S}} \).

The above procedure produces:

$$W(\phi \rightarrow \phi') = e^{-[\mathcal{S}(\phi') - \mathcal{S}(\phi)]}$$

$$W(\phi' \rightarrow \phi) = e^{-[\mathcal{S}(\phi) - \mathcal{S}(\phi')]}$$

(5.27)

\(^{x})\)There are other methods which I will not discuss.
i.e., the ratio of the probability $W$ for going from the value $\phi$ to $\phi'$ over the probability for the opposite transition is said to respect the "detailed balance". Let us check Eq. (5.27) according to the rules stated at the point iii) above: we can take, for example, $S(\phi') < S(\phi)$. Then

$$
\begin{align*}
W (\phi \to \phi') &= 1 \\
W (\phi' \to \phi) &= e^{S(\phi) - S(\phi')}
\end{align*}
(5.28)
$$

and Eq. (5.27) is satisfied. The "detailed balance" law on the right-hand side of Eq. (5.28) is nothing but the ratio of the equilibrium distributions:

$$
\frac{e^{-[S(\phi') - S(\phi)]}}{P_{\text{eq}} (\phi')} = \frac{P_{\text{eq}} (\phi)}{P_{\text{eq}} (\phi)}
(5.29)
$$

given that:

$$
P_{\text{eq}} (\phi) d[\phi] = \frac{d[\phi]}{[d[\tilde{\phi}]} \frac{e^{-S(\phi)}}{e^{-S(\tilde{\phi})}}
(5.30)
$$

The detailed balance rule leads "asymptotically", i.e., after many applications of the same algorithm, to the equilibrium distribution $P_{\text{eq}} (\phi)$. Indeed, let us calculate the variation of the probability distribution out of equilibrium

$$
\delta P (\phi) = \sum_{\phi' \neq \phi} W (\phi' \to \phi) P (\phi')
- \sum_{\phi' \neq \phi} W (\phi \to \phi') P (\phi)
(5.31)
$$

The minus sign in Eq. (5.31) comes from the fact that the "population" of the distribution for a certain value of $\phi$ decreases if there is a non-zero probability $W(\phi \to \phi')$ to migrate out of that value. The equilibrium configuration is reached when the variation is zero: $\delta P = 0$. This implies

$$
P_{\text{eq}} (\phi') W (\phi' \to \phi) = P_{\text{eq}} (\phi) W (\phi \to \phi')
(5.32)
$$
which is true from Eqs. (5.27) and (5.29). To reach an equilibrium configuration, one needs in general many iterations over the lattice. This procedure is called "thermalization" by analogy with the approach to equilibrium of a statistical system having a given temperature.

I summarize what I have said so far as follows. An overall lattice size of about 1 Fermi seems appropriate for studying the dynamics of hadrons containing strange quarks. The value of the lattice spacing a can be judged as reasonably small when one can check the scaling law based on the renormalizability of the theory. The dynamics is simulated by taking field averages over a set of "configurations" properly thermalized.

I will now give you some examples of what one can calculate at present. The subject can be divided into two main groups: the "pure gauge" sector and the "fermion" sector, depending upon the inclusion or not of quarks. In the first sector, I will discuss the calculation of the string tension and of the "deconfining" temperature (we will see later what that means); in the second, the calculation of the hadron spectrum. We start with the string tension.

The general shape of the potential between two coloured quark charges in QCD is the one in Fig. 22. One can identify two different regimes in its behaviour: at small distances, say below 0.2 Fermi, one has a "Coulomb" region where

$$V(r) \sim \frac{\alpha(r)}{r}$$  \hspace{1cm} (5.33)

while for $r \gtrsim 0.4$ Fermi, one expects the linearly rising confining shape:

$$V(r) \sim Kr$$  \hspace{1cm} (5.34)

The constant $K$, which has the dimension of a mass squared, is called in Eq. (5.34) the string tension. In a simple model where a $q\bar{q}$ pair is bound by a string, $K$ is the string tension. How does one calculate the string tension? By the "Wilson loop". Consider a $q\bar{q}$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{potential.png}
\caption{The $q\bar{q}$ potential $V(r)$ and its two regimes: the Coulomb-like ($V(r) \sim \alpha(r)/r$) and the confining ($V(r) \sim Kr$).}
\end{figure}
pair at a distance $d$. In quantum mechanics, the time evolution of such a system is governed by:

$$ e^{-\frac{i}{\hbar} \mathcal{H}_{\text{qq}} T} $$

(5.35)

where $T$ is the time. In the Euclidean space-time, one has

$$ e^{-\frac{1}{\hbar} \mathcal{H}_{\text{qq}} T} $$

(5.36)

In the expression of $\mathcal{H}_{\text{qq}}$, if one considers only very (infinitely) massive quarks, one gets only the potential part of the energy (the kinetic one goes to zero as $1/m$). If,

$$ V_{\text{qq}} \sim kd $$

(5.37)

i.e., if one is in the confining regime of the potential, one gets:

$$ e^{-\frac{\mathcal{H}_{\text{qq}} T}{\hbar}} \rightarrow e^{-k\left[\mathcal{A} T\right]} - k\mathcal{A} $$

(5.38)

where $\mathcal{A}$, see Fig. 23, is the area spanned by the $\bar{q}q$ system during its time evolution. The slope of the exponential is the string tension. To define the correlation whose exponential behaviour gives the string tension, we have to answer the question: what is the effect of the gluon field along the quark ($\bar{q}q$) trajectory? It performs a "colour rotation" on the quark fields: in the discussion about the form factors in Section 4, we have seen that such a rotation is obtained by integrating the exponential of the gluon field along the quark trajectory:

$$ e^{-t \int_{\text{trajectory}} dx_k A_k} $$

(5.39)

The Wilson loop is defined as the trace of the product of the gluon matrices $U$ along the "world line" (or more simply the trajectory in the space-time) of the $q(\bar{q})$ as in Fig. 24. The expectation value of the Wilson loop $W(U)$ for large values of $T$ and $d$ behaves like:

$$ \langle W(U) \rangle \sim e^{-k \cdot \text{(AREA)}} $$

(5.40)

![Fig. 23: The area $A$ spanned by a $\bar{q}q$ pair sitting at a relative distance $d$ as a function of the time $T$.](image)

![Fig. 24: A Wilson loop.](image)
The "area law" is a manifestation of confinement. The string tension is a dimensionful quantity. It can then be used for performing the "scaling" test. Of course, its value on the computer output is a pure number, i.e., the string tension in lattice spacing units (call it \( K_{\text{LATTICE}} \)). From Eq. (5.16) we expect the scaling law:

\[
K_{\text{LATTICE}} = \kappa \alpha^2 = k/\lambda^2 \frac{\rho(\beta)}{f(\beta)}
\]

where \( f(\beta) \sim e^{-c/\beta} \). There are two ways of performing the scaling test:

1) one plots \( K_{\text{LATTICE}} \) as a function of \( \beta \) and looks for the exponential behaviour for \( \beta \) large enough (remember \( a^2 = 1/\Lambda^2 e^{-c/\beta} \to 0 \) when \( \beta \to \infty \));
2) one plots the value of \( [K_{\text{LATTICE}}/f(\beta)] \) as a function of \( \beta \). When the scaling holds it must be independent of \( \beta \).

The results for the latter test are given in Fig. 25. For \( \beta \lesssim 6 \), there are big violations of the scaling law, while a reasonable agreement is obtained for \( \beta \gtrsim 6 \). Notice that the function \( f(\beta) \) which has been used is the one obtained by working to lowest order in perturbation theory, i.e., by neglecting the higher order terms in Eq. (5.16). This scaling regime is called "asymptotic scaling".

---

**Fig. 25**: The string tension in units of \( \Lambda_{\text{lattice}} \) as a function of \( \beta \).
From the "asymptotic" value of the string tension in lattice spacing units that one can extract from Fig. 25, and from the knowledge of its value in physical units based on the angular momentum dependence of the \( \bar{q}q \) resonance spectrum, one obtains a first indication of the value of the lattice spacing in physical units:

\[
\alpha (\beta = 6) \sim 1 \text{ Fermi}
\]

(5.42)

Another example of a physical quantity calculable in the pure gauge sector is the "deconfining" temperature \( T_{\text{confining}} \); when warmed up by a huge amount (to a temperature of the order of \( 10^{11} \) degrees!), the hadrons are supposed to melt into a plasma of quarks and gluons. The questions that one can answer by a lattice calculation are: does the phase transition actually exist? If the answer is affirmative, at which temperature?

A theory at finite temperature can be simulated on a lattice where one of four dimensions, say the time, is much smaller than the others. In the limit where the space dimensions go to infinity, but the time remains finite, the value of the temperature can be related to the time size by

\[
T_{\text{time}} = \frac{1}{K(T_{\text{temperature}})}
\]

(5.43)

where \( K \) is the Boltzmann constant. Finite temperature QCD is then studied on anisotropic lattice; denoting by \( N_1 \) the number of points in the dimension 1:

\[
N_1 \ll N_2 = N_3 = N_4
\]

(5.44)

The temperature in lattice spacing units is given by

\[
T_{\text{lattice}} = \frac{1}{N_{\text{time}}}
\]

(5.45)

How does one identify the phase transition? One needs an "order parameter" which can tell you when the confinement no longer holds because the hadrons have melted into a plasma. This can be constructed as follows. Preliminary, one has to know that the Wilson loop is not the only quantity which can be used to determine the string tension. The Polyakov loop can also be used and it is defined as the trace of the product of the links along a given direction through the whole lattice. The gauge invariance is assured by the periodicity of the boundary conditions which allow us to "close" the loop. In Fig. 26 one can see two

![Fig. 26: Two Polyakov loops closed by the periodic boundary conditions.](image-url)
such loops in the time direction. The points denoted by A(B) are physically the same point due to the periodic boundary conditions. The correlation of the two loops as a function of their separation \( d \) decreases as:

\[
\exp \left\{ - \frac{1}{\tau} \int E_q \left( \frac{d}{d} \right) \right\}
\]

where \( E_q \left( \frac{d}{d} \right) \) is the potential energy of a \( \bar{q}q \) pair at a relative distance \( d \). Imagine now that one separates the two Polyakov loops more and more up to the point where one of the two goes out of the lattice volume, i.e., disappears. In this situation, for volumes large enough, one is measuring: \( E_q \left( \frac{d}{d} \right) \), i.e., the energy of a free quark. Therefore, the expectation value of one Polyakov loop, \( L \), behaves like:

\[
\langle L \rangle \sim e^{-E_q \left( \frac{d}{d} = \infty \right) / \tau}
\]

This is the order parameter of the phase transition: if there is confinement, \( E_q \left( \frac{d}{d} \right) = \infty \) and \( \langle L \rangle = 0 \); if there is the plasma without confinement, \( E_q \left( \frac{d}{d} \right) \) is finite and \( \langle L \rangle \neq 0 \).

The results obtained by various groups so far indicate that:

1) the phase transition exists;
2) it is of the first order (i.e., with a latent heat of fusion);
3) the transition temperature is of about 250 MeV.

The transition temperature can also be used for the scaling tests, much in the same way as the string tension. The results about the value of \( \beta \) where the scaling sets up are very much in agreement with the ones from the string tension.

We are at the last subject of this section and of these lectures: the calculation of the hadron spectrum \(^{23, 24}\). The determination of the mass of a \( \bar{q}q \) bound state goes as follows: one has to consider a bilocal correlation like:

\[
\int d^3 x \langle \mathcal{O}(x) \mathcal{O}^+(0) \rangle
\]

where \( \mathcal{O}(x) \) is in general a local \(^*\) operator carrying the quantum numbers of the resonance that one wants to study. For the pion, one can choose:

\[
\mathcal{O}_\pi(x) = \bar{\Psi}(x) \gamma_5 \psi(x)
\]

By inserting a complete set of states in Eq. (5.48) and by using the translation invariance, one gets:

\(^*\) It can be also an "extended" one, i.e., defined in terms of fields having "on average" the position \( x \).
\[
\int d^3x \left< O(x) O^+(0) \right> = \int d^3x \sum_n < O(x) | n > < n | O(0) > = \\
= \int d^3x \sum_n \frac{-i p_n \cdot x}{m_n} < O(0) | n > < n | O(0) >
\]  
\hspace{1cm} (5.50)

where \( p_n \) is the four momentum of the state \( n \). By performing the volume integral in Eq. (5.50), one obtains a delta function which sets to zero the spatial components of the momentum. The final expression for the correlation is:

\[
\int d^3x \left< O(x) O^+(0) \right> \rightarrow \sum_n e^{-\frac{p_\mu t}{m_n}} c_n
\]  
\hspace{1cm} (5.51)

where \( m_n \) are the masses of the states coupled with the operator \( O \) and \( c_n \) are coefficients positively defined.

For large values of the time, the correlation is dominated by the state having the lowest mass. To summarize, the recipe for calculating the hadron masses is:

i) choose a (local or not) operator \( O \) with the required quantum numbers;

ii) calculate the correlation \( \int d^3x \left< O(x) O^+(0) \right> \);

iii) make a fit to the time behaviour as: \( \sum_n c_n e^{-m_n t} \) or more simply \( e^{-\mu t} c_0 \) for large times.

The correlation \( \left< O(x) O^+(0) \right> \) in terms of a functional integral which involves also the fermion fields reads

\[
\left< O(x) O^+(0) \right> = \frac{\int [\bar{\psi}] d[\bar{\psi}] d[\psi] e^{-\bar{\psi} \Delta(u) \psi} O(x) O^+(0) \int [\bar{\psi}] d[\bar{\psi}] d[\psi] e^{-\bar{\psi} \Delta(u) \psi}}{\int [\bar{\psi}] d[\bar{\psi}] d[\psi] e^{-\bar{\psi} \Delta(u) \psi}}
\]  
\hspace{1cm} (5.52)

The action appearing in the exponent now contains a part bilinear in the fermion fields coupled to gauge fields \( U \) through the matrix \( \Delta(u) \). The functional integration over the \( \bar{\psi}(\psi) \) fields can be performed (believe it) and gives

\[
\left< O(x) O^+(0) \right> = \frac{\int [\bar{\psi}] d[\bar{\psi}] e^{-\bar{\psi} \Delta(u) \psi} \left[ \gamma_5 \Delta^0 (O \rightarrow X) \gamma_5 \Delta^0 (x \rightarrow 0) \right]}{\int [\bar{\psi}] d[\bar{\psi}] e^{-\bar{\psi} \Delta(u) \psi}}
\]  
\hspace{1cm} (5.53)
Fig. 27a:  The quark propagator in the presence of a fixed external gluon field configuration.

Fig. 27b:  A typical diagram contributing to the correlation between two operators 0 at relative distance x.

The inverse matrix $\Delta^{-1}(0 \to x)$ represents the propagator of a quark in the presence of a fixed "background" gauge field configuration, like in Fig. 27a. The figure 27b represents a typical diagram contributing to the correction in Eq. (5.53). The wavy lines represent the gluons, the others are the fermions. The quantity $[\det \Delta(U)]$ in Eq. (5.53) represents the contribution of fermion loops. The calculations that I will discuss use the approximation $\det \Delta(U) \approx 1$, i.e., without fermion loops. Two kinds of justifications can be given: one is phenomenological and relies on the experimental fact that the "Zweig rule" seems to work rather well. According to this rule, the amplitudes involving the creation of a quark pair are suppressed. The second is a theoretical explanation. In the limit where the number of colours $N_c$ [3 for SU(3)] goes to infinity, each fermion loop gives a contribution which is suppressed by a factor $1/N_c$ with respect to the leading diagrams.

How one can calculate diagrams as complicated as the one in Fig. 27b, without fermion loops? First one calculates the fermion propagator $\Delta^{-1}(U)$ for a fixed "configuration" of the U fields. This corresponds to a diagram like the one in Fig. 27a: the crosses at the end of the gluons mean that a given field configuration has been fixed. Then one averages over the set of equilibrium configurations that one has collected:

$$\sum_i \left[ \gamma_5 \bar{\psi}^4(U_i) \gamma_5 \bar{\bar{\psi}}^4(U_i) \right]$$

The expression in the square bracket above is the same appearing in square brackets on the numerator of Eq. (5.53). The functional integral has been "performed" by averaging over the configurations.

The numerical inversion of the matrix $\Delta(U)$ is the most computer-time-consuming part of the calculation given the large size (of the order of 200000 elements!) of the matrix. The method used is the one of GAUSS-SEIDEL: one solves an auxiliary equation

$$\dot{G} = \Delta(U) G - 1$$

where the matrix indices have been suppressed. The derivative (the dot) is taken with respect to the "computer time": asymptotically, for $t \to \infty$, the solution converges to $\dot{G} = 0$, i.e., $G = 1/\Delta(U)$. Equation (5.55) is solved iteratively.
Hadron spectra are calculated for different values of the quark mass: small quark masses need large volumes \(^{29}\). For example, the splitting between the various energy levels (resonance masses) contributing to the sum in Eq. (5.51) reduces when the quark mass goes to zero and makes it more difficult to disentangle the contribution of the lowest mass state. A test of the contribution due to excited states is given by the quantity

\[
\mathcal{M}(t) \equiv -\int d\Omega \left[ \frac{C(t+1)}{C(t)} \right]
\]

where

\[
C(t) = \int d^3x \langle 0(k) 0^+(0) \rangle
\]

If only one state dominates, \(c(t) \sim e^{-mt}\) and

\[
\mathcal{M}(t) \rightarrow \mathcal{M} = \text{constant}\]

(5.57)

In Fig. 28 is shown the quantity \(m(t)\) for the pseudoscalar and for two values of the quark mass. Notice that the lower the quark mass is, the slower is the approach to the regime \(m(t) = \text{constant}\).

![Graph showing the value of \(m(t)\) for two different quark masses as a function of the time in lattice spacing units.](image)

In Fig. 29, I report the results obtained by our group on a \(10^3 \times 20\) lattice, at \(\beta = 6\), where it begins the scaling regime. The hadron mass in lattice spacing units is reported as a function of the quark mass also in lattice spacing units. To determine the physical spectrum, one has two unknowns: the quark mass and the lattice spacing in physical units. One therefore needs two inputs, plus an extra one for each new flavour and therefore each new quark mass value. By using \(M_\rho\) and \(M_\pi\) as inputs, I obtain:

\[
\begin{align*}
\alpha^4 &= 2 \text{ GeV} \\
\mu &= m_d \sim 7 \text{ MeV} \\
M_\rho &\approx 1.05 \text{ GeV} \\
M_\Delta &\approx 1.2 \text{ GeV}
\end{align*}
\]
Fig. 29: The behaviour of various hadron masses as a function of the quark mass: everything is in lattice spacing units.

where the capital letters refer to hadrons and the others to quarks. A different fitting procedure can be obtained by using as inputs the masses of the hadrons containing a strange quark. By using $M_\rho$ and the mass of pseudoscalar made of strange quarks only, I get:

\[ M_{\Delta^-} \sim 1.65 \text{ GeV} \]
\[ M_{\Delta^-(1/2)} \sim 1.55 \text{ GeV} \]
\[ m_{\text{Strange}} \sim 125 \text{ MeV} \]
\[ \alpha' \sim 2 \text{ GeV} \]

By adding the pion mass as an input, I can fit also the spectrum of hadrons containing one strange and one light quark:

\[ M_\pi = m_d + 7 \text{ MeV} \]
\[ M_\rho \sim 0.82 \text{ GeV} \]
\[ M_k \sim 0.5 \text{ GeV} \]
\[ M_{k^*} \sim 0.93 \text{ GeV} \]
\[ M_{\phi} \sim 1.1 \text{ GeV} \]
\[ M_\Delta \sim 1.3 \text{ GeV} \]

(5.60)

I have the following comments on these results:

i) the baryons are rather too heavy.

ii) lower quark masses can be explored with larger lattices allowing us to determine the light hadron spectrum without extrapolations.
iii) Fermion loops have not been included and therefore the results should not be too good. For example, the diagram in Fig. 30 is absent and it corresponds to the emission and reabsorption of a $\bar{q}q$ pair from a state of three quarks. By neglecting fermion loops one is then neglecting important contributions to the proton self-energy due to the pions which can be emitted and reabsorbed.

iv) The scaling tests at different values of $\beta$ on the hadron masses have still to be systematically performed.

v) Last, but not least, there are different ways of defining the lattice action for what concerns the fermions: an agreement between the different formulations should be obtained.

In spite of these comments, I consider the results very encouraging. As usual, the bill comes at the end. In this case it is a computer bill. The following numbers give you an idea of the super\(^a\)-computer requirements. The CPU time for "updating" one link with the Metropolis algorithms takes about 50\(\mu\) seconds. This means that 2000 iterations over a lattice of $10^3\times20$ of the algorithm about 6hs of CPU. The most time-consuming part, the inversion of the quark propagators, takes, on a sample of 14 configurations (very few!) about 7hs of CPU for each value of the quark mass. Memory requirements are the most severe: on a $20^6$ lattice one needs about 10 megawords of fast memory. This is a lot, mainly if one thinks that the inclusion of fermion loops multiplies the figure by a factor three. From all this, it is clear that very precise (order 1%) calculations of lattice QCD are beyond the limits of present computer technology.

Many other problems, besides the hadron spectrum, can be tackled within the lattice approach. The nucleon's magnetic moments, the $G_A/G_V$, the hadron structure functions, the hadronic matrix element of the weak Hamiltonian, the proton lifetime within a given superunification scheme and in general all the "low energy" quantities which cannot be estimated by a perturbative expansion.

The next generation of computers - with a speed of the order of $10^{10}$ floating operations per second and a memory of a hundred megawords - will lead to the "solution" of QCD, or the next generation of theoretical physicists, if faster.

\(^a\)Computers like the CRAY 1S or the CYBER 205.
REFERENCES

H. Fritzsch and M. Gell-Mann, XVI International Conference on High Energy Physics,

2) See also the lectures by C. Jarlskog at this school.

3) If you are interested in the functional formulation of quantum field theories, see for
example:
J. Iliopoulos, J. Itzykson and A. Martin, Rev. of Modern Physics 47 (1975) 165.

M. Gell-Mann and F. Low, Phys. Rev. 95 (1954) 1300.

5) G. 't Hooft, unpublished (1972);


15) CHARM Collaboration in M. Diemoz, F. Ferroni and E. Longo, Nota Interna No. 851
Dipartimento di Fisica, Università di Roma, to appear in Phys. Reports.


18) T. Appelquist and H. Georgi, Phys. Rev. D8 (1973) 4000;
A. Zee, Phys. Rev. D8 (1973) 4038;
M.A. Shifman, A.I. Vainstein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448, 519;
M.B. Voloshin, Yad. Fiz. 29 (1979) 1368;

For a review, see:

21) For a recent review, see:

22) For a review, see:

For a review, see Ref. 21).


PHENOMENOLOGY OF HIGGS PARTICLES

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These lecture notes are organized as follows:
1. Introduction
2. Interactions and decay modes of the Standard Higgs \( \varphi \)
3. Nonstandard Higgs multiplets
4. Upper limits on the Higgs mass
5. Lower limits on the Higgs mass
6. Production mechanisms for the Standard Higgs
7. Outlook
References

1. INTRODUCTION

After the recent discovery of the intermediate vector bosons \( W^\pm \) and \( Z^0 \) at CERN, the only missing ingredient in the Standard Electroweak Model is the Higgs particle. The mass ratio \( m_W/m_Z \) measured by the CERN experiments\(^1\) is in a surprisingly good agreement with the prediction of the minimal Electroweak Model, which introduces one single Higgs doublet \( \varphi \), corresponding to four scalar (spin zero) fields. The Standard Model postulates that this Higgs doublet plays a very fundamental role in Nature. Without it, Nature would have been in a manifestly SU(2) x U(1) symmetric mode. What a grey and boring Universe there would have been, with massless and indistinguishable fermions! It is indeed the Higgs which says "let there be light", by breaking the symmetry to the U(1) of electromagnetism, whereby the concept of electric charge is born and enables us to distinguish a neutrino from an electron, etc. Again the masses are all due to the Higgses.

In the Standard Model the Higgs is a "fundamental scalar", i.e., it is a pointlike particle and not, for example, a bound state. In these notes, which are a continuation of previous lecture notes\(^1\), I shall review some of the properties of the Higgs particle, as predicted by the Standard Model.

The Lagrangian of the Standard Model, in its manifestly invariant mode, may be written in the following form

\[
\mathcal{L} = \mathcal{L}(\psi, W, B) + \mathcal{L}(W, B) + \mathcal{L}(\varphi, W, B) - V(\varphi) + \mathcal{L}(\varphi, \varphi),
\]  

(1)

where \( \psi \) denotes the fundamental fermions (left-handed doublets and right-handed singlets), \( W_r \equiv (W^+_2, W^+_1, W^+_0) \) and \( B \) are the gauge bosons of SU(2) and
$U(1)$ respectively and $\varphi$ is the Higgs multiplet(s). In the minimal version of the Standard Model, one has

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i \varphi_2 \\ \varphi_3 + i \varphi_4 \end{pmatrix} \sim \begin{pmatrix} \varphi^{(s)} \\ \varphi^{(t)} \end{pmatrix},$$

where $\varphi_1$ are real scalar fields. The spontaneously broken mode (or physical mode) of the Standard Model is obtained from (1) by replacing

$$\varphi \to \frac{i}{\sqrt{2}} \begin{pmatrix} \varphi_3 + \nu_r \\ \varphi_4 \end{pmatrix}.$$

After diagonalizing the mass matrices and identifying the physical particles (for details see Ref. 2) one has

$$\mathcal{L} = \mathcal{L}(\psi, G) + \mathcal{L}(G, \varphi_o) - V(\varphi_o) + \mathcal{L}(\psi, \varphi_o),$$

where $G$ denotes the physical gauge fields ($W^{\pm}_u, Z^0, A_\mu$) and $\psi$ stands for the physical fermions (mass eigenstates). The relevant terms for us here are the last three which read

$$\mathcal{L}(G, \varphi_o) = \frac{i}{2} (\partial_{\mu} \varphi_o)(\partial^\mu \varphi_o) + \frac{i}{8} \varphi_o^2 \left[ 2 g^2 \partial^\mu W^{\mu}_{\nu} W_{\nu} + (g^2 + g') Z^\mu Z^\nu \right],$$

$$V(\varphi_o) = \frac{\lambda}{4} \varphi_o^2 \left( \varphi_o^2 + 2 \nu_r \right)^2,$$

$$\mathcal{L}(\psi, \varphi_o) = -(1 + \frac{\varphi_o^2}{\lambda}) \left[ m_u \bar{u} u + m_c \bar{c} c + m_t \bar{t} t + \ldots + m_d \bar{d} d + m_s \bar{s} s + m_b \bar{b} b + \ldots + m_e \bar{e} e + m_{\mu} \bar{\mu} \mu + m_{\tau} \bar{\tau} \tau + \ldots \right].$$

Here we follow the definitions and conventions of the previous lecture notes$^1$; $g$ and $g'$ are coupling constants and $m_j$ refers to the mass of the fermion $j$. From the above formulae we have that

$$M_w^2 = \frac{g^2 v^2}{4} \quad \frac{M_Z^2}{M_w^2} = \frac{g^2}{g'^2} \quad \frac{M_{\nu_e}^2}{M_{\tau}^2} = \frac{g^2}{g'^2}$$

The scale of the symmetry breaking is determined by the constant $v$, which is empirically obtained from the Fermi constant $G_F$. 

\[ \text{(8)} \]
\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \rightarrow \eta = (\sqrt{2} G_F)^{-\frac{1}{2}} = 250 \text{ GeV}
\] (9)

The masses of W and Z are comparable with the scale of symmetry breaking (\(M_W \sim 82\) GeV, \(M_Z \sim 93\) GeV) but the fermion masses span an incredible range of values, viz.,

\[
\frac{m_{\psi}}{m_W} \lesssim 2 \times 10^{-10}, \quad \frac{m_{\nu}}{m_W} \approx 0.2.
\] (10)

How the doublet in (3) manages to produce such a variety of masses is, of course, a great mystery.

2. INTERACTIONS AND DECAY MODES OF THE STANDARD HIGGS, \(\psi_0\)

The interactions of \(\psi_0\) are easily read off formulae (5) - (7). The Higgs has three types of interactions:

a) Interactions with the gauge fields (Eq. (5)) summarized by the following diagrams and the corresponding interaction Lagrangians:

\[
\begin{align*}
\chi_\psi & \rightarrow W^+ W^- \quad gM_W (W^-; \chi_\psi; W^+) \psi_0 \\
\chi_\psi & \rightarrow Z^0 Z^0 \quad gM_Z \frac{1}{2} \cos \theta_W (Z^-; \chi_\psi; Z^+) \psi_0 \\
\chi_\psi & \rightarrow W^+ W^- \quad g^2 \frac{1}{4} (W^-; \chi_\psi; W^+) \psi_0 \\
\chi_\psi & \rightarrow Z^0 Z^0 \quad g^2 \frac{1}{8} \cos^2 \theta_W (Z^-; \chi_\psi; Z^+) \psi_0.
\end{align*}
\] (11)

Note that there is no \(Z^0 \psi_0 \psi_0\) coupling. Such a coupling is forbidden by Bose statistics.

b) Self-interactions of \(\psi_0\), (Eq. (6), gives

\[
\begin{align*}
\chi_\psi & \rightarrow \chi_\psi \quad -h \psi_0^3 \\
\chi_\psi & \rightarrow \chi_\psi \quad -\frac{h}{4} \psi_0^4
\end{align*}
\] (12)

c) Finally, the interactions with the fermions (Eq. (7)) is given by

\[
\begin{align*}
\chi_\psi & \rightarrow f \quad -\frac{\sqrt{2} M_W}{M_F} \bar{f} f \psi_0
\end{align*}
\] (13)
Relations (11) – (13) together with $m_{\phi_0}^2 = 2 \hbar v^2$ summarize the properties of the standard Higgs. From these relations, it is easy to compute the decay rates\(^{3}\) into a pair of gauge bosons. We have

$$\Gamma(\phi_0 \to WW^*) = \frac{G_F m_{\phi_0}^3}{32 \sqrt{2} \pi} \left[ 1 \! - \! 4 \alpha_w + 3 \alpha_w^2 \right] \sqrt{1 - \alpha_w} \sqrt{1 - \alpha_w^2} \Theta(1 - \alpha_w),$$

$$\Gamma(\phi_0 \to ZZ^*) = \frac{G_F m_{\phi_0}^3}{64 \sqrt{2} \pi} \left[ 1 \! - \! 4 \alpha_z + 3 \alpha_z^2 \right] \sqrt{1 - \alpha_z} \sqrt{1 - \alpha_z^2} \Theta(1 - \alpha_z),$$

$$\alpha_w = \frac{4 M_W^2}{m_{\phi_0}^2}, \quad \alpha_z = \frac{4 M_Z^2}{m_{\phi_0}^2}.$$

(14)

where $\Theta(x) = 1, x > 0; \Theta(x) = 0, x < 0$.

The decay rate for Higgs going into a fermion pair is given by

$$\Gamma(\phi_0 \to f\bar{f}) = (N_C) \frac{G_F m_{\phi_0}^2 m_{\phi_0}}{4 \sqrt{2} \pi} \left( 1 - \alpha_f \right) \sqrt{1 - \alpha_f} \sqrt{1 - \alpha_f^2} \Theta(1 - \alpha_f),$$

$$\alpha_f = \frac{4 m_f^2}{m_{\phi_0}^2}.$$

(15)

where the factor $N_C$ (the number of colours) is a reminder that for quarks there is a factor of 3 enhancement as compared to the leptons. From the above formula it is evident that a light Higgs will predominantly decay into the heaviest fermion pair, allowed by phase space. A light Higgs, $m_{\phi_0} < 2M_W^*$ will have a narrow width. For example putting $m_{\phi_0} = 100$ GeV, $m_t = 40$ GeV gives a width of about 60 MeV. The situation changes radically as soon as the Higgs decay into a pair of vector bosons becomes possible, because $W^+W^-$ and $ZZ$ coupling constants are huge (see (11)). For $m_{\phi_0} = 300$ GeV, $\Gamma(\phi_0 \to W^+W^-) \approx 10$ GeV. Thus very heavy Higgses will be hard to establish as bumps in effective mass plots. The Higgs, in addition to having the above direct (lowest order) decay modes, can decay (in higher orders) via other channels such as

$$\phi_0 \to g + g$$

$$\phi_0 \to \gamma + \gamma$$

where $g = \text{gluon}$.

These processes, as shown above, involve a virtual $f\bar{f}$ pair (also $W^+W^-$ pair for the photon mode). The rates have been computed by several authors\(^{3-6}\) who quote
\[
\Gamma(q_0 \to g + g) = \frac{G_F N_c}{36 \sqrt{2} \pi} \left( \frac{\alpha_s(m_{q_0}^2)}{\pi} \right)^2 m_{q_0}^3,
\]
\[
\Gamma(q_0 \to \gamma + \gamma) = \frac{G_F \alpha^2}{8 \sqrt{2} \pi} |I|^2 m_{q_0}^3.
\]

(16)

Here \( N_c \) is the number of heavy flavours and \( I \) is a quantity of order unity for three families. The widths in (16) are very small and negligible as compared to those in (14) and (15).

3. NONSTANDARD HIGGS MULTIPLETS

Here it is perhaps appropriate to briefly discuss some nonstandard Higgses. There are several questions which we may ask ourselves, assuming that the masses are indeed due to fundamental scalars.

a) We know that the existence of a doublet of Higgses (Eq. (2)) predicts that the ratio

\[
\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}
\]

is unity, in agreement with data. Is the converse also true? Does \( \rho = 1 \) imply that the Higgs is a doublet?

b) Is there any reason for going beyond the standard Higgs doublet?

In this section we consider the extension of Higgs multiplets and answer the above questions.

The simplest Higgs multiplet is a singlet, i.e., just a complex scalar field \( \varphi = \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2) \). If we wish to have charge conservation we must require that \( \varphi \) is neutral. Then from \( Q = I_3 + Y \), the hypercharge of \( \varphi \) is zero. Such a singlet (\( I = 0, Y = 0 \)) neither couples to \( W \) nor to \( Z \). Moreover it can't give mass to fermions. It can, however, interact with other Higgses.

Next we consider a Higgs multiplet with isospin \( I \), then \( I_3 = I, I-1, \ldots, -I \). We may denote the corresponding complex fields with \( \varphi(I, I_3) \),

\[
\varphi = \left( \begin{array}{c}
\varphi(I, I) \\
\varphi(I, I-1) \\
\vdots \\
\varphi(I, -I) 
\end{array} \right).
\]

(18)
Assume that one of the components, \( \psi (I, \hat{I}_3) \) is electrically neutral so that it can be shifted \( \psi (I, \hat{I}_3) \rightarrow \psi (I, \hat{I}_3) + \frac{V}{\sqrt{2}} \), as before. Then the mass term for the gauge bosons is given by

\[
\mathcal{L}_{\text{mass}} = \left| (g \hat{T}_+ \hat{W} + g' y B) \frac{\nu}{\sqrt{2}} |I, \hat{I}_3> \right|^2 = \frac{\nu^2}{2} \left| \left[ \frac{g}{\sqrt{2}} (T_+ W'^s \hat{T}_+ W'^t) + (g W - g' B) T_3 \right] |I, \hat{I}_3> \right|^2
\]

(19)

where \( T_\pm = T_1 \pm iT_2 \), are the isospin raising and lowering operators.

Furthermore we have used that \( y = Q - T_3 \) and that \( Q |I, \hat{I}_3> = 0 \), as the component which is shifted is, by assumption, electrically neutral. Now we may use the relations

\[
T_\pm |I, \hat{I}_3> = \left[ \left( I = \hat{I}_3 \right) \left( I = \hat{I}_3 + 1 \right) \right]^{1/2} |I, \hat{I}_3 + 1>
\]

\[
T_3 |I, \hat{I}_3> = I_3 |I, \hat{I}_3>
\]

\[
g W'_r - g' B_r = \frac{g}{\cos \theta_w} E_r
\]

(20)

to compute the gauge boson masses from (19). We find

\[
M_W^2 = \frac{g^2 \nu^2}{2} \left[ I \left( I + 1 \right) - I_3^2 \right],
\]

\[
M^2 = \frac{g^2 \nu^2}{\cos \theta_w} \left[ I_3 \right]^2
\]

where we have dropped the hat on \( I_3 \). If there are several such Higgs multiplets \( \psi_1 \) with corresponding isospins, third components of isospin and the spontaneous breaking scales \( I_1, I_1, I_1 \) and \( \nu_1 \) respectively we get

\[
M_W^2 = \frac{g^2}{2} \sum_1 \nu_1^2 \left[ I_1 \left( I_1 + 1 \right) - I_{13}^2 \right],
\]

\[
M^2 = \frac{g^2}{\cos \theta_w} \sum_1 \nu_1^2 \left[ I_{13} ^2 \right],
\]

\[
\rho = \frac{M_W^2}{M^2} = \frac{\sum_1 \nu_1^2 \left[ I_1 \left( I_1 + 1 \right) - I_{13}^2 \right]}{\sum_1 \nu_1^2 \left[ I_{13} ^2 \right]}
\]

(21)

Clearly, for any number of doublets \( \rho = 1 \) because \( I_1 = \frac{1}{2} \) and \( I_{13} = \pm \frac{1}{2} \). Next consider triplets. There are two possibilities a) \( \hat{I}_3 = \pm 1 \), i.e. \( \nu = (\phi_+^+, \phi_+^0, \phi^-) \) or its antimultiplet and b) \( \hat{I}_3 = 0 \), \( \phi = (\phi_+^+, \phi_+^0, \phi^-) \). Let the \( \nu_3's \) be denoted by \( \nu_a \) and \( \nu_b \) respectively, then the masses are affected as follows

\[
\sum M_W^2 = \frac{g^2}{2} \left( \nu_a^2 + \nu_b^2 \right), \quad \sum M^2 = \frac{g^2}{\cos \theta_w} \nu_a^2
\]

(22)
We see that we can have $p=1$ even with triplets, provided $v^2_a = 2v^2_b$. Thus, we have the answer to our first question: $p=1$ does not imply that Higgs multiplets must be doublets. However with only doublets $p=1$ is obtained automatically, while including, e.g., triplets requires the "miracle" $\Sigma v^2_a = 2\Sigma v^2_b$ in order to insure $p=1$. Furthermore, because $V_L$ is a doublet and the right-handed fermions are singlets ($\bar{V}_L, \bar{\psi}_R$ doublet) only Higgs doublets can couple to fermions and give them masses (this argument does not apply to Majorana mass for neutrinos). Therefore, nondoublet Higgs multiplets are less attractive.

Now we turn to the second question. Historically, there have been several motivations for considering more elaborate Higgs schemes than the one in the minimal Standard Model.

i) With only two families, the Standard Model could not accommodate the observed CP-violation. One solution to this problem was to introduce CP-violation through Higgses. With doublets, one needs at least three of them in order to be able to generate a relative CP-phase in the weak amplitudes. With three families one expects CP-violation even with just one doublet. Nevertheless, some authors feel that CP-violation directly arises through spontaneous symmetry breaking, i.e., it is due to the relative phases in the $v$'s.

ii) It is known that due to nonperturbative effects there can be a term $\sim \theta \epsilon, G_{\mu\nu}^V G^{V*}$ in the Lagrangian of QCD. Here $G_{\mu\nu}^V$ is the nonabelian field tensor for the gluons and $\theta$ is a constant. This so-called $\theta$-term causes both parity and CP-violation in QCD, which is not what is observed in Nature. One solution to this problem is to introduce an extra $U(1)$ symmetry (Peccei-Quinn symmetry) which would allow one to "rotate" $\theta$ to zero. In the Standard Model this requires at least two doublets of Higgses. An important prediction of these considerations is that there is a light pseudoscalar boson, called the axion, which is the Goldstone boson of the broken Peccei-Quinn $U(1)$-symmetry. The axion of the two doublet model is excluded by data but there are more elaborate schemes which make the axion invisible.

iii) The third main reason for introducing more than one Higgs doublet is that in supersymmetric extensions of the Standard Model at least two Higgs doublets are needed.

With two or more doublets of Higgses the conservation of charge is not automatic and depends on the details of the Higgs potential. However, if the components of the Higgs which get shifted are parallel (in the SU(2) x U(1) space) one obtains charge conservation. Another à priori serious problem with multidoublet models is that they generally lead to flavour-changing
neutral currents. Suppose that there are two doublets $\varphi_1$ and $\varphi_2$ and denote their neutral components by $H_1$ and $H_2$. Then the mass matrix of the charge $2/3$ quarks will arise from (see Sections 6 and 7 in Ref. 2)

\[-\mathcal{L} = \sum_{jk} C_{jk}^1 \bar{f}_{jk} (H_1 + \nu_{1} f_{jR} + C_{jk}^{(2)} \bar{f}_{jk} (H_2 + \nu_{2} f_{jR} + h.c. + \ldots \right.

\left. = (1 + \frac{H_1}{\nu_{1}}) \bar{f}_{jk} \nu_{1} f_{jR} + (1 + \frac{H_2}{\nu_{2}}) \bar{f}_{jk} \nu_{2} f_{jR} + h.c. + \ldots \right)

(23)

where $\nu_{1}$ and $\nu_{2}$ are the scales of the spontaneous symmetry breaking, and the $c$'s are constants. We see, from (23) that the mass matrix is

\[ m_{jk} = m_{jk}^{(1)} + m_{jk}^{(2)} \]

(24)

which has to be diagonalized. This diagonalization of the sum in (24) does not, in general, diagonalize the terms $m_{jk}^{(1)}$ and $m_{jk}^{(2)}$. Thus the interactions of $H_1$ and $H_2$ will be flavour nondiagonal and we expect diagrams such as shown in the Fig.

These make large contribution to $K^0 \leftrightarrow \bar{K}^0$ transition which we know, from experiment, happens in second order in $G_F$. Generically, we must require

\[ \frac{g^2}{m_{W}^2} \frac{1}{m_{H}^2} \lesssim G_F^2 \]

i.e., the Higgs mass has to be large ($\mathcal{O}$(TeV)) in order to get enough suppression.

In a 2 doublet model there are 8 real fields. Three of them give masses to $W^\pm$ and $Z^0$ and 5 remain in the physical spectrum. The physical Higgses are two charged ones $H^\pm$ and three neutrals $h_1$, $h_2$ and $h_3$. The physical Higgses are obtained from diagonalizing the Higgs mass matrix appearing in the Higgs potential, which I don't want to go into here. Let me make a final comment. Charged Higgses with masses in the Petra range, $m_{H^\pm} \approx 22$ GeV, would be produced in $e^+e^- \rightarrow \gamma' + H^+H^-$. There is no evidence for such point-like scalar objects from data. Charged Higgs would also be produced copiously in $t \rightarrow b + H^+$ decay. The discovery of top at the Collider, through $t \rightarrow b + \ell^+ + \nu$ is an evidence against the channel $t \rightarrow b + H^+$ which is expected to be much more important (if it is energetically allowed) because it happens in order $G_F^2$ and is a two body decay.
In summary, the multi-Higgs models are generically very different from the single doublet model. However, we cannot exclude them, as they are difficult to pin down. For example the problem with the flavour changing neutral currents is cured by assuming that only one Higgs doublet gives mass to the Q=2/3 quarks and that there is another one responsible for the mass of the Q=-1/3 quarks, etc.

4. UPPER LIMITS ON THE HIGGS MASS

The mass of the Higgs particle $\varphi_0$ in the minimal Standard Model is given by

$$m_{\varphi_0}^2 = 2 h v^2$$

(25)

where $h$ is the coupling constant in the potential (see (6), (12)). We know from experiment that $v \approx 250$ GeV but we don't know $h$ and therefore the Higgs mass is a free parameter in the theory. Very naively, one would expect $h<1$ in order that the $\varphi_0$ would not have strong self-interactions. Because if $\varphi_0$ has strong self-interactions there would be no reason why the simple potential $V(\varphi) = -\mu^2 \varphi^* \varphi + h (\varphi^* \varphi)^2$ should be relevant. Higher order terms $(\varphi^* \varphi)^n$, $n>2$ are generated by self-interactions of the Higgs and their coefficients need not be small. Assuming $h<1$ gives an upper bound for the Higgs mass

$$m_{\varphi_0} \leq \sqrt{2} v = \frac{2 \sqrt{2}}{g} M_W.$$  

(26)

Putting $g=0.7$ gives $m_{\varphi_0} \leq 4 M_W \approx 300$ GeV.

A less naive upper bound on the Higgs mass is obtained by considering $W$-$W$ scattering. Coupling of $W$ to Higgs being large one expects larger sensitivity to the Higgs mass in scattering of gauge bosons than in processes involving quarks and leptons. After all, without the Higgs the Standard Model is not renormalizable; the Higgs cures logarithmic divergences and letting the Higgs mass go to infinity should re-introduce the problem. It is found that $^3$W-$W$ scattering at the three level violates unitarity if the Higgs mass is too large. The limit reads

$$m_{\varphi_0}^2 \leq \frac{16\pi^2}{3} v^2,$$

i.e.,

$$h \leq \frac{8\pi}{3}, \quad m_{\varphi_0} \leq 900 \text{ GeV}.$$  

(27)

Much work has been done$^{13)}$ to try and pin down the Higgs mass. The results are disappointing in the sense that the effects on presently measurable quantities are very small. For example, the two loop corrections to the $\rho$ parameter ($\rho \equiv M_W^2/(M_Z^2\cos^2\theta_W)$) depend on the Higgs mass as follows$^{14)}$
\[ \rho = 1 - \frac{3 \alpha}{16 \pi c_\theta^2} \left( \frac{m_{\phi^3}^2}{M_W^2} \right) + 9.49 \times 10^{-4} \frac{\alpha}{c_\phi^2} \frac{m_{\phi^3}^2}{M_W^2} \]  

(28)

where \( s_\theta = \sin \theta_W \), \( c_\theta = \cos \theta_W \). It turns out that, in addition to coefficients being small, there is a cancellation between the correction terms in (28). Even for huge values of the Higgs mass, \( m_{\phi^3} \lesssim 100 \ M_W \), the correction is very small, \( |\delta \rho| < 0.5\% \).

For nonstandard Higgses, in the Standard Model, it is possible to obtain a result\(^{15} \) similar to (26) for the lightest of the Higgses, i.e., at least one of the neutral Higgs particles should be light, i.e., its mass \( m \) is bounded by

\[ m^2 \lesssim 2 \ h \ \nu^2. \]  

(29)

Take any number of Higgs multiplets (no singlets). Let the corresponding real fields be denoted by \( \varphi_1, \varphi_2, \ldots, \varphi_n \). The mass matrix is then given by

\[ m_{jk}^2 = \frac{\partial^2 V(\varphi)}{\partial \varphi_j \partial \varphi_k} \bigg|\varphi = \varphi^\nu, \]  

(30)

where \( V \) is the potential and \( \varphi^\nu = (\varphi_1^\nu, \varphi_2^\nu, \ldots, \varphi_n^\nu) \) corresponds to the minimum of the potential. The mass eigenstates are denoted by \( h_1, h_2, \ldots, h_n \). These are certain linear combinations of the \( \varphi_i \). \n
\[ \varphi_i = \sum_{r=1}^n O_{ir} \ h_r, \]  

\[ h_r = \sum_{i=1}^n O_{ir} \ \varphi_i, \]  

(31)

where \( O \) is an orthogonal matrix. We can rewrite\(^{15} \) \( V \) in terms of the mass eigenstates \( h_r \). By definition, the quadratic term in \( V \) evaluated at \( \varphi = \varphi^\nu \) or \( h = h^\nu \) is diagonal in the \( h_j \)’s

\[ \frac{\partial^2 V}{\partial h_j \partial h_k} \bigg|_{h_j = h_k^\nu} = m_j^2 \delta_{jk}, \]  

(32)

where \( m_j \) is the mass of the Higgs \( h_j \). \( V \) expressed in terms of the \( h \)'s reads

\[ V(h) = \frac{1}{2} \sum_{jk} m_j^2 \ h_j \ h_k + \frac{1}{3} \sum_{jkl} \sigma_{jkl} \ h_j \ h_k \ h_l + \frac{1}{4} \sum_{jklr} \lambda_{jklr} \ h_j \ h_k \ h_l \ h_r, \]  

\[ \frac{\partial V(h)}{\partial h_j} = m_j \ h_j + \sigma_{jkl} \ h_k \ h_l + \lambda_{jklr} \ h_k \ h_l \ h_r, \]  

(33)
\[
\frac{\partial^2 V(k)}{\partial h_j \partial h_k} = \mu^2 + 2 \sigma \varepsilon_{jkkr} h_k h_r + 3 \lambda \kappa_{jkkr} v_k v_r.
\] (34)

Here, \( \sigma \) and \( \lambda \) are constants. They are completely symmetric in their indices. Let the shift in \( h_j \) be denoted by \( \langle h_j \rangle = v_j \) and define

\[
\nu^2 = \sum_{j=1}^{n} \langle h_j \rangle^2 = \sum_{j} v_j^2,
\]
\[
\nu_j = n_j \cdot v,
\]
\[
\nu_j = \langle h_j \rangle.
\] (35)

Then from (34) and (35)

\[
\delta_{jk} n_j^2 = \mu^2 + 2 \sigma \varepsilon_{jkkr} v_k + 3 \lambda \kappa_{jkkr} v_k v_r.
\] (36)

Multiplying with \( n_j n_k \) and summing over \( j \) and \( k \) gives

\[
\sum n_j^2 n_k^2 = \mu^2 + 2 \sigma \nu^2 + 3 \lambda \kappa^2
\] (37)

where

\[
\mu^2 = \sum_{j,k=1}^{n} \nu_j^2 n_j n_k,
\]
\[
\sigma = \sum_{j,k=1}^{n} \varepsilon_{jkkr} n_j n_k n_r,
\]
\[
\lambda = \sum_{j,k=1}^{n} \kappa_{jkkr} n_j n_k n_r.
\]

Furthermore, at the minimum of the potential we have

\[
\frac{\partial V}{\partial \nu_j} = 0,
\]
\[
\frac{\partial V}{\partial \nu_k} = 0,
\]

i.e.,

\[
\frac{\partial V}{\partial h_k} \bigg|_{h=h^\nu} \nu_k = 0.
\] (38)

From (38) and (33) one has\(^1\)

\[
\mu^2 + \sigma \nu^2 + h \nu^2 = 0.
\] (39)

Since the solutions \( v_\pm = (-\sigma \pm \sqrt{\sigma^2 - 4h\nu^2})/2h \) must be real, we must have \( \sigma^2 \geq 4h\nu^2 \). Substituting \( v_\pm \) in (33) shows that if \( \sigma < 0 \) the solution \( v_+ \) is relevant and \( \sigma > 0 \) the minimum occurs at \( v_- \). In both cases \( \sigma \nu < 0 \). Substituting (39) into (37) gives
\[ \sum_j m_j^2 n_j^2 = 2 \hbar \nu^2 e^{-\nu} \leq 2 \hbar \nu^2, \]  
(40)

i.e.,

\[ \sum_j m_j^2 n_j^2 \geq m_0^2 \sum n_j^2 = m_0^2 \]  
(41)

where \( m_0 \) is the smallest mass. Thus

\[ m_0^2 \leq 2 \hbar \nu^2, \]

i.e., the upper limit for the lightest Higgs is as in the Standard model. This result is important because it indicates that even with many Higgs multiplets we don't have the freedom of pushing all the masses to infinity.

5. LOWER LIMITS ON THE HIGGS MASS

There is a lower limit on the mass of the standard Higgs particle. The limit comes about as follows. The potential for the spontaneously broken mode was

\[ V(\varphi) = -\mu^2 \varphi^4 + h (\varphi^2 \varphi)^2, \]  
(42)

where \( \mu^2 > 0 \). Higher order corrections, such as shown in the Fig. modify this potential,

These corrections have been computed\(^1\) at the one loop level, using the methods of effective potentials. One then computes an effective potential which is a function of the "classical Higgs field", \( \varphi_0 \), i.e., \( \varphi_0 \) is an ordinary function and not a field operator. The computation gives (neglecting the fermions)

\[ V(\varphi_c) = \frac{1}{2} \mu^2 \varphi_c^2 + \frac{1}{4} \hbar \varphi_c^4 + B \varphi_c^4 e^{\varphi_c^2/\nu^2}, \]  
(44)
where

\[
B = \frac{3}{1024 \pi^2} \left[ 2(y^2)^2 + (y^2 + y^2)^2 \right],
\]

\[
\nu^2 = \frac{\mu^2}{\hbar + 2B}.
\]

Note that the minimum of \( V(\phi_c) \) corresponds to \( \phi_c^2 = \nu^2 \) and the mass of the Higgs, \( m_{\phi_c} \), is determined from

\[
\frac{d^2 V(\phi_c)}{d \phi_c^2} = m_{\phi_c}^2,
\]

i.e.,

\[
m_{\phi_c}^2 = 2(\hbar + 6B)\nu^2.
\]

Thus the mass of the Higgs is larger than \((12B)^{1/2} \nu \approx 12 \text{ GeV}\) if \( h \geq 0 \). But \( h \) need not be positive. As \( \phi_c \rightarrow \pm \infty \) the last term in (44) will dominate and there will be a ground state irrespective of the sign of \( h \). We must require, however, that \( V(\nu) < 0 \), otherwise the origin, \( \phi_c = 0 \), will be the minimum and there will be no spontaneous symmetry breaking. We get\(^{17}\)

\[
V(\nu) = \left[ -\frac{\nu^2}{\hbar} (\hbar + 2B) + \frac{1}{\hbar} h \right] \nu^2 < 0,
\]

i.e.,

\[
h > -4B.
\]

Substitution into (46) gives

\[
m_{\phi_c}^2 > 4B\nu^2.
\]

numerically we get

\[
m_{\phi_c} > 7 \text{ GeV}.
\]

Including the fermion contribution in (43) modifies (48) to

\[
m_{\phi_c}^2 > 4B\nu^2 \left[ 1 - \frac{y^2 (m_t)}{(M_W)} \right],
\]

\[
c_B = \cos \theta_w.
\]

The last term in the parenthesis is \( \approx (m_t/M_W)^4 \). This correction is small unless the fermion mass approaches \( M_W \). For \( m_t \approx 50 \text{ GeV}, \left( \frac{m_t}{M_W} \right)^4 \approx 0.14 \).
In summary the mass of the $\phi_0$ of the standard model should lie in the range 7 GeV - 900 GeV. Unfortunately this "prediction" is not very helpful for the discovery of the Higgs. What a difference with the situation for the gauge bosons, where one knew rather precisely what mass to look for!

6. PRODUCTION MECHANISMS FOR THE STANDARD Higgs

In this Section I shall briefly mention the topic Higgs production. Higgs couples predominantly to heavy fermions and to gauge bosons (see (11) and (13)). For example, at the CERN Collider the direct Higgs production through annihilation of light quarks is suppressed by a factor $(m_q/M_W)^2 \approx 10^{-8}$ as compared to the $W$-production, if $m_{\phi_0} \approx M_W$. The favourable production channels are expected to be

a) $Z^0 \rightarrow \phi_0 + \ell^+ + \ell^-$

b) $e^+ e^- \rightarrow Z^0 + \phi_0$

c) $g g \rightarrow \phi_0 + \ldots$

d) quarkonium $\rightarrow \phi_0 + \gamma$

e) $W^+ W^- \rightarrow \phi_0$

f) $Z^0 Z^0 \rightarrow \phi_0$

Here I shall not discuss these processes. The interested reader may consult Ref. 18 for processes a) - d). For very heavy Higgses where the process e) is very important Ref. 19 is an appropriate place to learn more.

Last summer the observation of a particle called $\zeta$ with a mass $M = (8322 \pm 8 \pm 24)$ MeV was reported in the process

$$\gamma(1\sigma) \rightarrow \zeta + \gamma.$$ (50)

This was an example of class d) which is called the Wilczek\textsuperscript{21} mechanism. One can easily calculate the rate $V \rightarrow \gamma + \phi_0$,

$$\Gamma(V \rightarrow \gamma + \phi_0) = \frac{8\alpha G_F^2}{M_V^2} \left( \frac{m_q^2}{\sqrt{2}} \right) |\psi(0)|^2 (1 - \frac{m^2}{M_V^2}) (S_3)^2.$$ (51)

Here $V$ denotes a $^3S_1$ - state with the third component of spin equal to $S_1$; it decays at rest and the quantization axis is along the photon momentum. $Q_q$ is charge of the quark, $V = (q\bar{q})$; $\psi(0)$ is the q-$\bar{q}$ wavefunction at the
origin. We can get rid of the $\psi(0)$ as follows. The rate for $V \rightarrow \ell^+ \ell^-$ is given by

$$\Gamma(V \rightarrow \ell^+ \ell^-) = \frac{8\pi \alpha^2}{M_V^2} q_1^2 |\langle \psi(0) | S_\gamma \rangle|^2,$$

(52)

The ratio of the two rates is independent of the unknown $\psi(0)$, viz.,

$$R = \frac{BR(V \rightarrow \gamma + \nu_\beta)}{BR(V \rightarrow \ell^+ \ell^-)} = \frac{\Gamma(V \rightarrow \gamma + \nu_\beta)}{\Gamma(V \rightarrow \ell^+ \ell^-)} = \frac{G_F m_\ell^2}{\sqrt{2} \pi \alpha} \left( 1 - \frac{m_{\nu_\beta}^2}{M_V^2} \right).$$

(53)

For the process (50) we put

$$M_V = 94 GeV, \quad m_\zeta = 8.3 GeV.$$

Thus $R = 5 \times 10^{-5}$ if $\zeta$ were the Higgs. Experiment gave $R \approx 5 \times 10^{-3}$ which is two orders of magnitude too large. Also $\zeta$ was not seen in $\gamma(2S) \rightarrow \gamma +$ anything. This was a puzzle for the Higgs interpretation as one expects the ratio in (53) to be the same for $V = \gamma(1S)$ and $V = \gamma(2S)$. At the time of the writing of these notes it seems that the particle $\zeta$ has not been confirmed in further runs of the experiment.

7. OUTLOOK

Books on particle physics used to start by telling us that there are four forces in Nature: gravitational, weak, electromagnetic and strong. That was only a few years ago, before we knew that the Standard Electroweak Model does an excellent job in decreasing the number of fundamental forces from four to three. But is it really true that there are only three or less forces? What about the masses? Where do they come from? As long as we only have the Standard Model with arbitrary looking couplings between the Higgs and the fermions there are "fundamental forces" which we don't understand. After all, the Higgs mediates 9-12 Yukawa type forces, with flavour dependent strengths.

It seems unlikely that a fundamental scalar doublet is the origin of all masses. How could it generate 12 different "forces" with strengths in the range $m_\nu \lesssim 2 \times 10^{-10}$ and all the way up to $m_t \approx 0.2$? But we have so far no viable alternatives.

The importance of pursuing the Higgs particle(s) can't be overemphasized. It is the only remaining ingredient in the Standard Model to be dis-
covered. Of course, it also remains to check the nonabelian structure of
the Standard Model but getting to know the Higgs, or its substitute, seems
to be even more essential. After all from the 17-24 arbitrary parameters of
the Standard Model all except $2 (g, g')$, enter via the Higgs. Let's hope that
fundamental discoveries in the "near future" will reveal to us how the pat-
tern of masses is printed.

I am indebted to Mrs Janny Asphaug for typing my lecture notes for
these Proceedings.
REFERENCES

1) For a review of the experimental situation see, for example, D. Haidt, these Proceedings; J.D. Dowell, these Proceedings.

2) C. Jarlskog, these Proceedings.


8) See, for example T. Walsh, these Proceedings.


11) See, for example, C.H. Llewellyn Smith, these Proceedings.


18) G. Barbieri et al., DESY Report 79/27 (1979) and references therein.


THE ELECTROWEAK MODEL

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"The role of these particles, and their properties, being similar to those of the photons, we may perhaps call them 'electro-photons'..."

O. Klein (1948)

In these lectures an introduction to the Electroweak Model is given. The written version of the lectures will be rather short as I have already written such lecture notes before\(^1\),\(^2\) which are available to the interested reader. Furthermore, within the past few years several books have appeared on the subject. However, primarily for the benefit of those who attended the School, I shall summarize the major why's and how's of the Standard Electroweak Model leaving the details as exercises or further reading.

The Standard Model is a very successful theory of combined weak and electromagnetic interactions. Indeed, the recent discovery\(^1\) of the intermediate vector bosons \(W^+, W^-\) and \(Z^0\) at CERN constitutes a great step forward in our understanding of the nature of weak and electromagnetic interactions. An important Chapter in the history of fundamental physics has now been completed. However there are reasons to believe that an even deeper understanding requires reading further "Chapters" which Nature has not shown us yet.

The outline of these notes is as follows.
Section 1 gives a short "history" of the electroweak interactions. It is not a systematical review, however. To write such a review would require perhaps years of studying old journals and books, which I haven't done. Nevertheless, I include a few historical references which I have found very interesting. It is indeed much more instructive and interesting to read the original papers and interpret them oneself than to read historical reviews.
In Section 2 the Standard Model is introduced, in its manifestly symmetric mode. The theory is written for a pair of hypothetical fermions \((f, f')\) which may be visualized as, for example, the quark pair \((u, d)\). The following Sections explain the various terms in the Lagrangian of the Standard Model and take us to the spontaneously broken version of the theory. Finally, the generalization to several pairs of quarks and leptons is discussed and some conclusions are presented.

Now that the gauge bosons of the Standard Electroweak Model \((\gamma, W^+, W^-)\)
and $z^0$) have been demonstrated to exist there is only one ingredient missing: the Higgs particle. Does it exist and if so how would one detect it? These are questions of utmost importance for our understanding of the laws of Nature. The phenomenology of Higgses is reviewed separately in an accompanying set of lecture notes.

1. HISTORICAL REMARKS

The idea of unifying weak and electromagnetic interactions is quite old by now. Already in 1938 Oskar Klein$^1$ introduced two charged vector bosons which he labeled $B^\pm$, as companions of the photon, in order to relate the coupling constant of beta interactions to the electric charge. He introduced an isodoublet for nucleons and another one involving the electron and neutrino. The heavy and light doublets interacted via exchange of the intermediate vector bosons $B^\mp$. In this model $g = \sqrt{2}e$ (instead of $g = \frac{e}{\sin\theta}$ of the Standard Model) and there are of course no axial interactions. He writes$^1$

"It should perhaps be pointed out once more that the Lagrangian $L^0$ may belong either to the pair neutron-proton or to the pair neutrino-electron ... But it is worthwhile to notice that the complete Lagrangian will imply an interaction of heavy and light spinor particles not only through the intermediate of the electromagnetic field but also through the B-field, an interaction which will entail the occurrence of $\beta$-processes, the probability of which may be calculated on the basis of the theory developed in this report."

Ten years lated, Klein even suggested$^5$ a name for the charged intermediate vector bosons, "we may perhaps call them electrophotons", i.e., electrically charged partners of the photon (see the citation in the beginning of this article).

In 1957 Schwinger$^6$ suggested the unification of weak and electromagnetic interactions through a triplet of vector bosons. Salam and Ward$^7$ introduced the vector boson "kinetic terms" à la Yang and Mills$^8$. The $Z^0$ and the angle $\theta_W$ were given in a paper by Glashow$^9$, who introduced two neutral vector bosons $Z^0$ and $Z^\pm$. The linear combinations $Z^\pm \cos \theta + Z^0 \sin \theta$ and $-Z^\pm \sin \theta + Z^0 \cos \theta$ were assumed to correspond to the physical particles (photon and $Z^0$). Glashow noted that

"For no choice of $\theta$ is the interaction of neutral current small compared with weak interactions involving charged currents."

The Standard Model for leptons as we know it today appears first in a paper by Weinberg$^{10}$, where he takes four leptons (of course, the extension to any number of lepton pairs is trivial). The masses are, in this paper, introduced via spontaneous symmetry breaking mechanism. Weinberg introduces
a doublet of Higgs particles and finds the relation $M_W = \cos \theta M_H$. The extension to hadrons as we know it today, is also due to Weinberg, who did it for four quarks, by using the GIM-mechanism\(^2\).

Although the electroweak theory for leptons existed in the literature in 1960's almost no one talked about it, as one can convince oneself by looking into the Proceedings of International Conferences in our field during 1960's. Oppenheimer\(^3\) considered a charged vector boson to be a "horrible object" and many students, during 1960's, had never heard of Yang-Mills fields. Of course one searched for $W$ (as its existence provided the simplest explanation of universality in weak interactions) and ir 1963 there was some evidence\(^4\) for a $W$ with mass in the range 1.3-2.0 GeV. The first time I heard someone take $W$'s very seriously was when I was a student at a Summer School in Copenhagen in late 60's where M. Veltman gave a series of lectures on $W$'s and higher order effects.

The unified theories became suddenly popular when t' Hooft\(^5\) had demonstrated that the spontaneously broken gauge theories are finite (unitary and renormalizable). Since then the development both in theory and experiment has been tremendous. At that time we had no quantum chromodynamic with its several novel ideas: massless mediators and yet short range interactions, asymptotic freedom and confinement. There were no grand unified theories, supersymmetry, supergravity, lattice gauge theories, etc. On the experimental side, there were no neutral currents, charm, beauty, tau lepton and its neutrino. And above all there was only one "dull" gauge boson, the photon, which has no self-interaction. Nowadays we have the gluons, $W^+$, $W^-$ and $Z^0$ who, all of them, promise to be much more subtle than the photon.

2. THE STRUCTURE OF THE STANDARD MODEL

The Standard Model is a Lagrangian field theory. Three types of fields/particles appear in its Lagrangian:

1. Particles of force or the mediators
2. "Fundamental" fermions (quarks and leptons)
3. The Higgs particles

The form of the Lagrangian in gauge theories is severely restricted by the requirement of invariance under local transformations. In the Standard Model, the underlying symmetry is the local weak isospin $SU_W(2)$ combined with the local (weak) hypercharge symmetry $U_Y(1)$. The invariant Lagrangian has the following (symbolic) form

$$\mathcal{L} = \mathcal{L}(\Psi, W, B) + \mathcal{L}(\psi) + \mathcal{L}(\phi) + \mathcal{L}(\psi, \phi) - V(\phi) + \mathcal{L}(\psi, \psi).$$
Here $W^i$, $i = 1, 2, 3$, the three mediators, or gauge bosons, of the weak isospin group. $B$ denotes the gauge boson $B_\mu$ of the hypercharge group. $Y$ is a collective notation for the fundamental fermions (quarks and leptons). The fermions are assumed to come in pairs. In these notes we denote one such pair by $(f, f')$. The left-handed fermions are assumed to transform as doublets under the weak isospin group and the right-handed ones as singlets

$$
\psi_L = \begin{pmatrix} f \\ f' \end{pmatrix}_L, \quad \psi_R = f_R, \quad \psi'_R = f'_R.
$$

There are (at least) 3 such quark pairs and 3 lepton pairs, because we know from experiments that there are three families.

For a lepton pair $(f, f') = (\nu, \lambda)$. Furthermore, the introduction of a right-handed neutrino (i.e., $f'_R$) is optional, i.e., there are two extreme versions of the Standard Model:

i) without right-handed neutrinos

ii) with right-handed neutrinos

Although the two versions, in principle, lead to completely different phenomenology in practice they may be indistinguishable if the neutrino masses and mixings are very small.

Finally in Eq. (1) the symbol $\varphi$ denotes the Higgs doublet

$$
\varphi = \begin{pmatrix} \varphi^{(\rho)} \\ \varphi^{(\omega)} \end{pmatrix}, \quad \varphi^{(\rho)} = \frac{1}{\sqrt{2}} (\varphi_R + i \varphi_A), \quad \varphi^{(\omega)} = \frac{i}{\sqrt{2}} (\varphi_R + i \varphi_A),
$$

where $\varphi_i$, $i = 1-4$ are four scalar (spin zero) real fields. Under the action of the weak isospin group $\varphi$ transforms as a doublet.

The fields $Y$ and $\varphi$ are hypercharged, i.e., they are affected by the action of the hypercharge group.

The Lagrangian described above together with the option i) no right-handed neutrinos, defines the minimal Standard Model. Of course, extension to more complicated schemes is easy to achieve. For example, one may introduce more elaborate Higgs and/or fermion multiplets, etc.

In these notes I shall concentrate on the minimal version of the model, as there is, so far, no convincing experimental evidence for any departure from it. Therefore extensions are only briefly treated.

Note also that I don't discuss the experimental tests of the Standard Model because that subject is extensively reviewed by other authors\(^{14}\).
3. THE TERM $\mathcal{L}(\psi, W, B)$

This term describes the interaction of the fundamental fermions (quarks and leptons) with the gauge bosons ($W, B$). First, we shall write down this term for just one pair of fundamental fermions, denoted by $(f, f')$. These could be e.g. a pair of quarks or leptons. Generalization to several pairs generates mixing angles and CP-violation as described later.

The kinetic term for the pair $(f, f')$ is given by

$$\mathcal{L}_0(f, f') = i \left[ \bar{f} \gamma_5 \frac{\partial}{\partial x} f + \bar{f}' \gamma_5 \frac{\partial}{\partial x} f' \right], \quad (4)$$

where $\gamma_5 = \gamma^\mu \frac{\partial}{\partial x^\mu}$ and the notations are as in Refs. 1 and 2. An essential feature of the kinetic term is that it is helicity conserving, i.e.,

$$\bar{f} \gamma_5 \frac{\partial}{\partial x} f = \bar{f} \gamma_5 \frac{\partial}{\partial x} \left[ \frac{1}{2} (1 - \gamma_5) + \frac{1}{2} (1 + \gamma_5) \right] f =$$

$$= \bar{f}_L \gamma_5 \gamma_4 f_L + \bar{f}_R \gamma_5 \gamma_4 f_R, \quad (5)$$

where

$$f_L = \frac{1}{2} (1 - \gamma_5) f, \quad f_R = \frac{1}{2} (1 + \gamma_5) f,$$

and similarly for the primed fermion. Thus, in the kinetic term, the left-handed and right-handed fermions don't mix and therefore one may treat (gauge) them differently. The Standard Model makes the following assumptions:

1) The Lagrangian is invariant under the local (space-time dependent) symmetry operations SU(2) x U(1), where the first factor denotes rotations in the weak isospin space and the second factor refers to phase transformations.

2) The fermions form the multiplets (collectively denoted by $\psi$)

$$\Psi : \quad \Psi = \begin{pmatrix} f_L \\ f_R \end{pmatrix}, \quad f_L, \ f_R, \ f'_L, \ f'_R. \quad (6)$$

i.e., a left-handed doublet and two right-handed singlets. The hypercharge quantum numbers $y$ are assumed to satisfy the relation

$$Q = I_3 + y, \quad (7)$$

where $Q$ is the observed electric charge of the fermion, $Q = 2/3$ for the up quark, etc. $I_3$ is the third component of the isospin $I_3(f_L) = 1/2, I_3(f_R) = 0$, etc. In the Standard Model there is no reason why the electric charge should be quantized, as the quantum number $y$ is in principle arbitrary and is put in by hand. An attractive feature of the Grand Unified Theories is that they explain why charge is quantized. The relation (7) insures that the interaction of the photon with the fermions is parity conserving, in
spite of the fact that the right-handed and left-handed fermions are treated on different footing.

The Lagrangian (4) is not invariant under the local symmetry group. However we know how to modify it in order to obtain invariance. The details are explained, for example, in Refs. 1 and 2. Here we just sketch how this goes. Under the action of the group

$$\psi \rightarrow \psi' = U \psi$$  \hspace{1cm} (8)$$

where $U$ is a unitary matrix which is local, i.e., it depends on space and time. Then, the rule for obtaining the invariant $\mathcal{L}(\psi, \bar{\psi}, B)$ from $\mathcal{L}_0$ in (4) is to replace the derivative, in the kinetic term, by the covariant derivative, $\partial_{\mu} \rightarrow D_{\mu}$, which is constructed to transform as follows

$$D_{\mu} \rightarrow D'_{\mu} = U D_{\mu} U^{-1}$$  \hspace{1cm} (9)$$

such that

$$\bar{\psi} \gamma^\nu D_{\nu} \psi \rightarrow \bar{\psi'} \gamma^\nu D'_{\nu} \psi' = \bar{\psi} \gamma^\nu U D_{\nu} U \psi = \bar{\psi} \gamma^\nu D_{\nu} \psi.$$

In the Standard Model, the $D$ is given by

$$D_{\mu} = \partial_{\mu} - ig T^1 \bar{\psi} \gamma_{\mu} \psi - ig' \gamma_{\mu} \psi B_{\mu},$$  \hspace{1cm} (11)$$

where $g(g')$ is the coupling constant of the weak isospin (hypercharge) group $T^1 \bar{\psi}_L + T^2 \bar{\psi}_R + T^3 \bar{\psi}_L$, where $T^1$ represent the three generators of the weak isospin group and $W^1_{\mu}$ are the corresponding gauge bosons.

Thus $T^1 \psi_L = \frac{g_1}{2} \psi_L$, because $\psi_L$ is a doublet, and $T^1 \psi_R = T^1 \psi_R = 0$, as the right-handed fermions are singlets.

Finally $B_{\mu}$ is the gauge field of the hypercharge group and $y$ is the hypercharge operator, $y \psi = y(\psi) \psi$. $y \psi = (\gamma_{\mu} - i \frac{g}{2} \bar{\psi} \gamma_{\mu} \psi)$, $y B_{\mu} = \gamma_{\mu} B_{\mu}$.

We may now use Eqs. (5), (6) and (11) to obtain the invariant extension of the kinetic term (4), $\mathcal{L}_0 \rightarrow \mathcal{L}(\psi, \bar{\psi}, B)$,

$$\mathcal{L}(\psi, \bar{\psi}, B) = \frac{1}{2} (\bar{\psi}_L, \bar{\psi}_R) [\frac{2}{2x^\mu} - i \bar{\psi} \gamma_{\mu} \psi + i y B_{\mu}] \frac{f_L}{f'_{L'}} +$$

$$\frac{1}{2} (\bar{\psi}_R, \bar{\psi}_L) [\frac{2}{2x^\mu} - i \bar{\psi} \gamma_{\mu} \psi] f_{R'} + \frac{1}{2} \gamma^\nu (\frac{2}{2x^\mu} - i y B_{\mu}) \bar{\psi} \gamma_{\mu} \psi f_{R'}.$$  \hspace{1cm} (12)$$

This Lagrangian in addition to kinetic terms includes also interactions. The left-handed fermions interact with a triplet of vector bosons $\bar{\psi}$ as well as with the $B$-field. The right-handed fermions, being singlets, interact
only with the B-field. We shall see later on that the left-handed fermions have both charged current as well as neutral current interactions. The right-handed fermions only interact via neutral currents.

4. **THE TERMS \( \mathcal{L}(B) \) AND \( \mathcal{L}(W) \)**

These terms involve the kinetic terms of the gauge bosons. For the Abelian U(1) group, as is well known from quantum electrodynamics, we have

\[
\mathcal{L}(B) = -\frac{1}{4} B_{\mu \nu} B^{\mu \nu}, \quad B_{\mu \nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}. \tag{13}
\]

For a nonabelian group, such as the SU(2) piece of the Standard Model, the "kinetic term" for the gauge fields involves self-interactions among them. The form and the strength of these interactions are dictated by the underlying symmetry. We now sketch how one obtains the invariant kinetic terms. Suppose that the symmetry group were a direct product of unitary groups \( G = G_1 \times G_2 \times \ldots \times G_n \), then

\[
\psi \rightarrow \psi' = U_1 U_2 \ldots U_n \psi, \tag{14}
\]

where \( U_j \) are unitary and commute with each other. The covariant derivative is then given by

\[
D^\nu = \partial^\nu - i \sum_{k=1}^n g_k T_k^\nu \cdot \bar{\psi}^\nu \tag{15}
\]

where \( g_k \) is the coupling constant of the \( G_k \) group, \( T_k^j \), \( j = 1, \ldots, N \) denote the generators of this group and \( \psi_i^j \) are the corresponding gauge bosons. Under the action of the group

\[
D_{\mu} \rightarrow D'_{\mu} = U D_{\mu} U^{-1}, \tag{16}
\]

and this relation determines the transformation properties of the gauge bosons

\[
\partial^\nu - i \sum_{k=1}^n g_k T_k^\nu \cdot \bar{\psi}^\nu \rightarrow \partial^\nu - i \sum_{k=1}^n g_k T_k^\nu \cdot \bar{\psi}^\nu = \partial^\nu + \sum_{j=1}^n U_j (\partial^\nu U_j^{-1} - i \sum_{k=1}^n g_k U_k \cdot \bar{\psi}^\nu U_k^{-1}). \tag{17}
\]

Here \( \bar{\psi}^\nu \) on the R.H.S. comes from the fact that \( \partial^\nu \) is a differential operator and acts on some quantity (such as \( \psi \)) which is not explicitly shown in the above formula. Comparing left and right hand sides of (17) yields

\[
-i g_k T_k \cdot \bar{\psi}^\nu = U_k (\partial^\nu \psi^\nu) - i g_k U_k \cdot \bar{\psi}^\nu U_k^{-1}, \quad k = 1, 2, \ldots, n. \tag{18}
\]
We apply this formula to the U(1) factor of the Standard Model by putting

\[ V_i^\nu \rightarrow B_i^\nu, \]
\[ U_1 \rightarrow \exp \left( i \gamma^\nu \chi \right), \]
\[ T_i \rightarrow \gamma , \quad g_1 \rightarrow g', \] (19)

where \( \Lambda(x) \) is a real function. Substitution of (19) into (18) gives

\[ B_i = B_i - \frac{i}{g'} \beta_i \Lambda(x). \] (20)

which is familiar from QED.

For the SU(2) factor of the Standard Model we make the following identifications

\[ Q_2 \rightarrow ( W'^i, W^2, W^3 ) \]
\[ U_2 \rightarrow \exp \left( i \alpha(x) \gamma^j J^j \right), \]
\[ \alpha(x) \gamma^j = \sum_{j=1}^{3} \alpha^j(x) \gamma^j, \]
\[ T_2 \rightarrow \frac{g}{2}, \quad g_2 \rightarrow g. \] (21)

Here \( \alpha^j(x) \) are three real functions (parameters of the group). In this case Eq. (18) yields

\[ -ig \frac{g}{2} W_\mu^i \rightarrow U_2 (\alpha^i \gamma^j U_2^i) - ig \frac{g}{2} \gamma^i \alpha_\mu \gamma^j U_2^i. \] (22)

The structure of this transformation is better seen if we take infinitesimal \( \alpha \)'s and neglect terms of \( O(\alpha^2) \), we find

\[ W_\mu^i = W_\mu^i + i \frac{g}{2} \beta_\mu \alpha^j - \epsilon^{ijk} \alpha^k \gamma^j W_\mu^i \] (23)

where \( \epsilon \) denotes the structure constants of the group,

\[ [ T^i, T^j ] = i \epsilon^{ijk} T^k. \] (24)

The invariant "kinetic term" is constructed from the covariant derivative as follows. We put

\[ W_\nu^\mu = \frac{i}{g} [ D_\nu, D_\mu]. \] (25)

Under the action of the group
\[ W_{\rho\nu} \to W'_{\rho\nu} = \frac{i}{3} \left[ U D_{\rho} U^\dagger, U D_{\nu} U^\dagger \right] = \frac{i}{3} \left[ U W_{\rho\nu} U^\dagger \right] \]  

(26)

Therefore the quantity

\[ \text{Tr} \left[ W_{\rho\nu} W^{\rho\nu} \right] \]  

(27)

is invariant. This quantity, appropriately normalized is the invariant "kinetic term". It contains the true kinetic term

\[ \mathcal{L}_0(W) = -\frac{i}{4} \left( \partial_{\rho} W_{\nu}^\dagger - \partial_{\nu} W_{\rho}^\dagger \right) \left( \partial^\rho W_{\nu}^{\rho\nu} - \partial^\nu W_{\rho}^{\rho\nu} \right) , \]  

(28)

but that is not all. The remarkable feature of (27) is that it gives self-interacting gauge bosons. To see this we first compute the quantity \( W_{\rho\nu} \)

\[ W_{\rho\nu} = \frac{i}{3} \left[ \sigma_\rho - ig \mathcal{T}_\rho, \sigma_\nu - ig \mathcal{T}_\nu \right] \]

(29)

\[ = \mathcal{T}^j \left( \sigma_\rho \mathcal{T}_\nu \mathcal{T}^j \mathcal{W}^j - \sigma_\nu \mathcal{T}_\rho \mathcal{T}^j \mathcal{W}^j \right) = \mathcal{T}^j \left[ \sigma_\rho \mathcal{W}_\nu^j - \sigma_\nu \mathcal{W}_\rho^j \right] - g e^{ijk} W_{\rho}^{k} W_{\nu}^{j} \right) \equiv \mathcal{T}^j W_{\rho\nu}^j . \]

Thus

\[ \mathcal{L}(W) = -\frac{1}{4} \left( W_{\rho\nu}^j \right) W^{j\rho\nu} = -\frac{1}{2} \text{Tr} \left[ W_{\rho\nu} W^{\rho\nu} \right] , \]  

(30)

where the normalization factor is dictated by the kinetic term (28) and we have used the relation

\[ \text{Tr} \left[ \mathcal{T}^j \mathcal{T}^k \right] = \frac{i}{2} \delta_{jk} . \]  

(31)

Although the above construction involved SU(2), the generalization to any group is, in principle, trivial. Formally, we just have to replace

\[ W_{\rho} \to V_{\rho} , \]

\[ e^{ijk} \to e^{ijk} \]

where

\[ \left[ \mathcal{T}^j, \mathcal{T}^k \right] = i e^{ijk} \mathcal{T}^m \]  

(33)

In Summary

\[ \mathcal{L}(W) = -\frac{i}{4} \text{Tr} \left[ W_{\rho\nu} W^{\rho\nu} \right] = -\frac{1}{4} W_{\rho\nu}^j W^{j\rho\nu} \]

\[ W_{\rho\nu}^j = \partial_{\rho} W_{\nu}^j - \partial_{\nu} W_{\rho}^j - g e^{ijk} W_{\rho}^{k} W_{\nu}^{j} . \]  

(34)
which gives three types of diagrams shown below

\[ \mathcal{L}(B) \text{ is easily obtained from the general case. Since } U(1) \text{ is abelian, } [y, y] = 0, \text{ we have} \]

\[ \mathcal{L}(B) = -\frac{1}{4} \left( \partial_\mu B_\nu - \partial_\nu B_\mu \right) \left( \partial^\mu B^\nu - \partial^\nu B^\mu \right), \]

which is what we wrote down in the beginning of this Section.

The gauge sector of a nonabelian theory, \( \mathcal{L}(V) \), seems to be more fundamental than its other sectors. Note that fermions need gauge bosons in order that the symmetry be respected. But the gauge bosons could alone by themselves make up a self-interacting symmetric Lagrangian. The question is then why do we have fermions at all? Perhaps the answer is to be found in supersymmetry: fermions are needed as partners of the gauge bosons. Unfortunately, however, data indicate that the known fermions and gauge bosons are not partners of each other.

5. THE TERM \( \mathcal{L}(W, B, \varphi) \)

In the Standard Model, the gauge boson masses as well as the fermion masses are assumed to arise from spontaneous symmetry breaking\(^{17}\). One could, of course, add fermion mass terms by hand but adding gauge boson mass terms \(-m^2 W^i W^{i\mu} \) leads to a nonrenormalizable theory (in spite of the fact that a nonzero photon mass in Q.E.D. does not spoil the renormalizability, provided that the "photon" couples to a conserved vector current\(^{13}\)).

The mechanism of mass generation, in the Standard Model, is realized by introducing a doublet (under the weak isospin) of scalar fields

\[ \varphi = \begin{pmatrix} \varphi^0 \\ i\varphi^\omega \end{pmatrix}, \quad \varphi^\omega = \frac{i}{\sqrt{2}} (\varphi_1 + i \varphi_2), \quad \varphi^0 = \frac{i}{\sqrt{2}} (\varphi_3 + i \varphi_4), \]

(35)

where \( \varphi_1 \) are real fields. This doublet is assumed to have a nonzero hypercharge, \( y(\varphi) \neq 0 \). Otherwise the B-field would remain massless and would behave as a "photon" which is parity nonconserving. The kinetic term for \( \varphi \) is

\[ \mathcal{L}_0(\varphi) = (\partial_\mu \varphi)(\partial^\mu \varphi), \]

(36)

which is not invariant under the local SU(2) \( \times U(1) \) transformations
\[ \mathcal{L} (\varphi, W, B) = (D_{\mu} \varphi)^{\dagger} (D^{\mu} \varphi). \]

In order to get spontaneous symmetry breaking one introduces a classical potential (self-interactions) for the \( \varphi \)-field. This potential, which is assumed to be invariant under the symmetry group and give a renormalizable theory, is given by

\[ V(\varphi) = m^2 \varphi^* \varphi + \frac{1}{2} \varphi^* \varphi^2. \]

Clearly, \( \varphi^* \varphi + \varphi^* U^* U \varphi = \varphi^* \varphi \) is invariant and so is any power of it. But terms with \( n \geq 3 \) destroy the renormalizability and are not allowed. The coefficients \( m^2 \) and \( \lambda \) are constants. The quantity \( \lambda \) is assumed to be positive because otherwise \( V(\varphi) \) has no minimum (\( V(\varphi) \to -\infty \) as \( \varphi^* \varphi \to \pm \infty \)) and the theory has no ground state. Note that \( \varphi \) is, so far, considered to be a classical field (i.e., not second quantized). Finally, the quantity \( m^2 \) is assumed to be negative

\[ m^2 = -\lambda^2 < 0, \]

whereby the theory is said to be in the spontaneously broken mode. By this one means that the physical ground state does not respect all the symmetries of the Lagrangian. In our case, \( V(\varphi) \) is invariant under \( SU(2) \times U(1) \) transformations, Eq. (37). If the ground state had been \( \varphi = 0 \) (which is what one gets if \( m^2 \) had been positive) then it too would have been invariant under the above symmetry operations. With \( m^2 < 0 \), the ground state does not correspond to \( \varphi = 0 \). The minimum of the potential determines the ground state. Thus we must look for the solution of the relation

\[ \frac{\partial V}{\partial \varphi} = 0 \implies \varphi = \varphi_{\text{vac}}. \]

We find

\[ (\varphi^* \varphi)_{\text{vac}} = \frac{\mu^2}{2 \lambda} = \frac{1}{2} (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + (\varphi_4^2)_{\text{vac}}. \]

Eq. (41) implies that there are infinite number of possible ground states or vacua. Nature must choose one of them. Conventionally one
assumes that Nature chooses

\[ \varphi_{\text{vac}} = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ \nu \end{array} \right), \quad \text{i.e.,} \quad \left( \varphi_j \right)_{\text{vac}} = 0, \quad j = 1, 2, \nu, \quad \left( \varphi_3 \right)_{\text{vac}} = \nu, \quad \nu \equiv \sqrt{\frac{v^2}{M}}, \tag{42} \]

however any other choice, satisfying (41) would do as well, because by suitable rotations in the SU(2) x U(1) space we may go from any choice to any other.

The potential (39) is invariant under 0(4) transformations, i.e., orthogonal transformations in the four dimensional Euclidian space spanned by \((\varphi_1, \varphi_2, \varphi_3, \varphi_4)\). This group has 6 generators. The vacuum state (41), in which \(\varphi\) is aligned along a direction (for example the \(\varphi_3\)-axis, with the choice (42)) breaks the 0(4) down to 0(3). Rotations in the space spanned by \((\varphi_1, \varphi_2, \varphi_4)\) still leave the vacuum invariant. By Goldstone theorem, the spontaneous symmetry breaking produces one massless boson for each broken symmetry. Thus there are \(6-3 = 3\) massless bosons in \(V(\varphi)\). These three massless degrees of freedom give masses to \(W^+, W^-\) and \(Z^0\). The details are explained, for example, in Ref.s 1 and 2. Here we repeat the main points. One puts

\[ \varphi = \varphi_{\text{vac}} + \hat{\varphi}, \tag{43} \]

where \(\varphi_{\text{vac}}\) is given by (42). Eq. (38), in term of the new variables, reads

\[ \mathcal{L}(\varphi, W, B) = \mathcal{L}(\varphi_{\text{vac}} + \hat{\varphi}, W, B) = \left[ D^\mu(\varphi_{\text{vac}} + \hat{\varphi}) \right]^\dagger D^\nu(\varphi_{\text{vac}} + \hat{\varphi}) \right]. \tag{44} \]

Furthermore

\[ V(\varphi) = V(\hat{\varphi} + \varphi_{\text{vac}}). \tag{45} \]

The last step involves "gauging away" the massless Goldstone bosons, which is equivalent to putting \(\varphi_1, \varphi_2,\) and \(\varphi_4\) to zero\(^{17}\). The only scalar particle which remains in the theory is then \(\hat{\varphi}_3\) where \(\hat{\varphi}_3 = \varphi_3 - \nu\), from (43). This particle is usually called the physical Higgs particle. In the following we shall denote it by \(\varphi_0\). We have from (45), (39) and (40)

\[ V(\varphi_0) = -\frac{\mu^2}{4} (\varphi_0 + \nu)^2 + \frac{h}{4} (\varphi_0 + \nu)^4. \]

Dropping the constant term gives

\[ V(\varphi_0) = \mu^2 \left[ \frac{\varphi_0^2}{\nu} + \frac{1}{\nu^2} \varphi_0^3 \frac{1}{4\nu^2} \varphi_0^4 \right]. \tag{46} \]

Thus the "Standard Higgs" particle has the following properties
The coupling constant is given by $\nu = \frac{\mu^2}{v}$.

Similarly, the relation (44) gives

$$\mathcal{L} (\Phi, W, B) \rightarrow \left[ D \mu \cdot \left( \frac{\Phi \cdot q \nu}{\sqrt{2}} \right) \right]^{*} \left[ D \nu \cdot \left( \frac{\Phi \cdot q \nu}{\sqrt{2}} \right) \right] =$$

$$= \frac{1}{2} \left( \partial_{\mu} - ig \frac{\Phi}{\sqrt{2}} \cdot \nabla \Phi \right) \left( \frac{\Phi \cdot q \nu}{\sqrt{2}} \right) \left( \frac{\Phi \cdot q \nu}{\sqrt{2}} \right).$$

Written out in detail, Eq. (48) reads

$$\mathcal{L} (\Phi, W, B) = \frac{1}{2} \left\{ (\partial_{\mu} \Phi \cdot q \nu \nu) + (\partial_{\mu} \Phi \cdot q \nu \nu) \left[ g \frac{\Phi}{\sqrt{2}} \cdot \nabla \Phi + g' \frac{\Phi}{\sqrt{2}} \cdot \nabla \Phi \right] \left( \frac{\Phi \cdot q \nu}{\sqrt{2}} \right) \right\},$$

where the first term is the kinetic term of the Higgs particle and the second term, which is quadratic in the gauge fields, includes the mass terms for the gauge bosons and their interactions with the Higgs particle. Furthermore, the terms linear in gauge fields have disappeared, as they should. Note that there can not be any vertex of the kind

$$\begin{align*}
\text{due to the Bose statistics. The masses of gauge bosons are obtained from (49) by putting } \gamma = \frac{Q - \sigma}{2} \text{ and using that } \nu \text{ is electrically neutral. In other words, the lower component of the Higgs doublet in (35) is, by definition, electrically neutral and all charges are defined with respect to it. Thus } \nu^{(0)} \text{ is defined to have 1 unit of charge whereby } y_\nu = \frac{1}{2}.
\end{align*}$$

From (48) follows that $W^{+}_{\mu}$ are charged while $W^{3}_{\mu}$ and $B_{\mu}$ are neutral. The piece of (49) involving the vector boson masses reads

$$\mathcal{L}_{\text{mass}} = \frac{v^2}{8} \left\{ (W^{+})^2 \right\} \left( \begin{array}{cc}
g^{2} & -g'g' \nonumber \\
-g'g' & g^{*} \end{array} \right) \left( \begin{array}{c}
w^{3} \\
b \end{array} \right).$$

where we have suppressed the index $\mu$. We must diagonalize the neutral mass matrix to identify the mass eigenstates. The determinant being zero we know that there is a massless vector boson, which we identify with the photon. The massive neutral vector boson is denoted by $Z$. We have
\( A_\mu = \frac{g B_\mu + g' W^3_\mu}{\sqrt{g^2 + g'^2}}, \quad m_A = 0, \)

\( Z_\mu = \frac{-g' B_\mu + g W^3_\mu}{\sqrt{g^2 + g'^2}}, \quad m_Z = \frac{\sqrt{2}}{2} \sqrt{g^2 + g'^2}, \)

\( W^\pm_\mu = \frac{W^+_\mu \mp i W^-_\mu}{\sqrt{2}}, \quad m_W = \frac{g}{\sqrt{2}}. \)  \hspace{1cm} (51)

The charge operator is well known from the minimal substitution principle, 
\( (\gamma_\mu - i e Q A_\mu) \psi^+. \) From (48) and (51) we find

\[ e Q = \frac{g g'}{\sqrt{g^2 + g'^2}} \left[ \frac{e_2}{e} + y \right] = e \left[ \frac{e_2}{e} + y \right], \]

i.e.,

\[ \frac{g g'}{\sqrt{g^2 + g'^2}} = e \quad \Rightarrow \quad \frac{g}{g} + \frac{g'}{g'} = \frac{1}{e^2}. \]

Thus we can introduce the so-called weak and electromagnetic mixing angle, also called the Weinberg angle \( \theta_W, \) where

\[ g \sin \theta_W = e, \quad g' \cos \theta_W = e, \]  \hspace{1cm} (52)

i.e.,

\[ A_\mu = e \cos \theta_W B_\mu + e \sin \theta_W W^3_\mu, \]

\[ Z_\mu = -e \sin \theta_W B_\mu + e \cos \theta_W W^3_\mu. \]  \hspace{1cm} (53)

The marvelous feature of (53) is that there is unification of weak and electromagnetic interactions in the sense that the photon and the \( Z \) have a common origin. However, there are two arbitrary coupling constants \( g, g' \) or \( e, \theta_W \) in the theory and, therefore, the forces are not truly unified. In Grand Unified theories, however, the SU(2) and U(1) coupling constants are related to each by a Clebsch-Gordan coefficient and thereby one achieves unification of forces.

Finally in Klein’s theory, where \( W^+ \) and the photon are members of an isotriplet we have \( g = \sqrt{2} e, \) where \( \sqrt{2} \) is the familiar Clebsch-Gordan coefficient e.g., from the pion nucleon interactions (\( g_{\pi N} = \sqrt{2} g_{\pi N} \)).

I shall discuss further details of the Lagrangian \( \mathcal{L}(\psi, W, B) \) in a subsequent article\(^{13})\).

6. FERMION MASSES AND MIXINGS: THE TERM \( \mathcal{L}(\psi_1, \psi_0) \)

The only term in the Standard Model Lagrangian, Eq. (1) which remains to be discussed is the Higgs-fermion piece \( \mathcal{L}(\psi, \psi) \). Note that, so far there
are only 4 free parameters in the Lagrangian (1). The free parameters may be taken to be \( g, g', \mu^2, h \). We could, if we wish, replace \( (g, g') \) by \( (e, \theta_W) \), and/or introduce \( v^2 = \mu^2/h \). Another possibility is to use \( M_W^2, (M_Z^2 = g^2 \frac{v^2}{4}) \) as one of the free parameters. Furthermore, we have had massless fermions up to now. In the Standard Model the fermion masses arise from the Higgs-fermion "interactions", i.e., the term \( \mathcal{L}(\psi, \phi) \). Unfortunately this term introduces a substantial proliferation in the number of free parameters of the theory. It turns out that for \( n \) quark families there are \( n^2 + 1 \) additional free parameters (2\( n \) quark masses and \( n-1 \))\(^2 \) mixing angles/phases. Including \( n \) family of leptons doubles this number, if neutrinos are massive. For massless neutrinos, the leptonic mixing angles/phases are unobservable quantities and the leptonic sector introduces only \( n \) additional parameters (the masses of the \( n \) charged leptons). Thus the number of free parameters, \( N_p \), for \( n \) families of quarks and \( n \) families of leptons, is given by

\[
N_p = 2n^2 + 6, \quad m_\nu \neq 0, \\
N_p = n^2 + n + 5, \quad m_\nu = 0.
\]  

(54)

Therefore, there are, in the Standard Model, 17(24) free parameters in the case of 3 families of quarks and leptons. All of these parameters except 2 (i.e., \( g \) and \( g' \)) are due to the Higgs field \( \phi \). This is the reason why one generally dislikes fundamental scalars. But up to now no viable alternative to the Higgs mechanism, as origin of masses, has been found.

We shall now briefly review how the fermion masses are generated. The mass term for a fermion \( f \), with mass \( m \), has the form

\[
-\bar{f} m f = -\bar{f}_L (f_L^T f_R + f_R^T f_L) f_R = \bar{f}_{R,L} = \frac{1}{2} (1 - \gamma_5) f.
\]  

(55)

Note that the mass term mixes the left and right handed components of the fermion field \( f \). Gauge interactions (involve vector and axial vector couplings to fermions) are helicity conserving and thus cannot mix \( f_L \) and \( f_R \). However the Higgs particle, which has zero spin can connect them. A term of the kind \( \mathcal{L}_{H} f_L f_R \) is what we are looking for (where \( \phi \) is a scalar neutral field) because then \( \phi \rightarrow \phi + v \) (where \( v \) is a constant) generates a mass term, viz., \( v f_L f_R \). In the Standard Model, we require \( \mathcal{L}(\psi, \phi) \) to be invariant under \( SU(2) \times U(1) \). This invariant is constructed from the fields

\[
\psi: \quad \gamma_L = \left( \begin{array}{c} f_L \\ f_L^T \end{array} \right), \quad \gamma_R = f_R, \quad \gamma_R = \frac{f_r}{f_r}, \quad \gamma_r = \frac{1}{2}.
\]

(56)

and

(57)

Clearly \( \bar{\psi}_L \phi \) is invariant under \( SU(2) \), \( \bar{\psi}_L \phi \rightarrow \bar{\psi}_L \phi' = \bar{\psi}_L U_{\frac{1}{2}} U_{\frac{1}{2}} \phi = \bar{\psi}_L \phi \), but it is
not Lorentz scalar and carries \(y = y_{\psi} - y_{\psi_L} = Q_{\psi} - Q_f\). Multiplying it with \(f_R^*\) gives \(\bar{\psi}_L \psi f_R^*\), which is both Lorentz scalar and invariant under \(SU(2) \times U(1)\). When the spontaneous symmetry breaking takes place, and \(\psi\) gets shifted by a constant, the \(f\) fermion becomes massive. The \(f\) fermion gets its mass from \(\psi^*\) as follows. We know from the ordinary isospin that \((p, n)\) being an isodoublet implies that the antiparticle pair \((\bar{n}, -\bar{p})\) also transforms as an isodoublet. Similarly, \(\psi\) being an isodoublet implies that \(i \sigma^2 \psi^*\) is also an isodoublet, viz

\[
\psi \rightarrow \psi' = \exp\left(i \frac{a}{2} \cdot \vec{E}ight) \psi
\]

\[
\psi^* \rightarrow \psi'^* = \exp\left(-i \frac{a}{2} \cdot \vec{E}^*\right) \psi^*
\]

\[
\vec{E}_2 \psi^* \rightarrow \vec{E}_2 \psi'^* = \exp\left(i \frac{a}{2} \cdot \vec{E}\right) \vec{E}_2 \psi^*
\]

\[
\bar{\psi}_c = i \sigma_2 \psi^* = \frac{i}{\sqrt{2}} \begin{pmatrix} \bar{q}_3 - i \bar{q}_2 \\ -\bar{q}_3 + i \bar{q}_2 \end{pmatrix} = \begin{pmatrix} \psi^* \psi^* \\ -\psi^* \psi^* \end{pmatrix}, \quad y(\bar{q}_c) = \frac{i}{2}.
\]

Thus \(\bar{\psi}_L \psi_c\), which is invariant under \(SU(2)\), when multiplied with \(f_R^*\) makes an invariant quantity both under \(SU(2) \times U(1)\) and Lorentz transformations and generates the mass of the fermion \(f\). The most general form of \(\mathcal{L}(\psi, \phi)\) is given by

\[
\mathcal{L}(\psi, \phi) = c \bar{\psi}_L \psi \phi f_R^* + c' \bar{\psi}_L \psi \phi f_R^* + h.c. = c(\bar{f}_L, f_L) \begin{pmatrix} \psi^* \\ -\psi^* \end{pmatrix} f_R^* + c'(\bar{f}_L, f_L) \begin{pmatrix} \psi^* \\ -\psi^* \end{pmatrix} f_R^* + h.c.
\]

(59)

where \(c\) and \(c'\) are two arbitrary constants.

After the spontaneous symmetry breaking and removing the fields \(\psi_1\), \(\psi_2\) and \(\psi_3\), we have

\[
\mathcal{L}(\psi, \phi) = \frac{i}{\sqrt{2}}(\psi \psi^* + v) \left[ c \bar{f}_L f_R^* + c' \bar{f}_L f_R^* + h.c. \right].
\]

(60)

Comparison with Eq. (55) gives

\[
m_f = -\frac{c v}{\sqrt{2}}, \quad m_f' = -\frac{c' v}{\sqrt{2}}.
\]

\[
\mathcal{L}(\psi, \phi) = -\left(1 + \frac{g v^2}{M_W^2}\right)(m_f \bar{f} f + m_f' \bar{f}' f').
\]

(61)

This relation tells us that in addition to generating the fermion masses, the physical Higgs \(\psi_2\) couples to the fermions with a) scalar coupling, b) a coupling constant which is \(\frac{m_f}{v} = \frac{g v}{M_W^2}\), where \(m\) is the fermion mass and we have used that \(M_W = g \frac{v}{2}\) (see the fig.)

\[
\begin{array}{c}
\psi_2 \\
\hline
\bar{f} \\
\end{array}
\Rightarrow \frac{g m_f}{2 M_W} \psi_2 \bar{f} f
\]
In the next section generalization to several pairs of fermions, e.g. three families of quarks and leptons, is given.

7. MORE THAN ONE FAMILY

The Standard Model accommodates several families as follows. Suppose there are \( n \) quark families (we discuss the leptons later). Then we introduce 2\( n \) pairs of fields

\[
Q = \frac{2}{3} : \quad f_{1L}, f_{2L}, \ldots, f_{nL} ; \quad f_{1R}, f_{2R}, \ldots, f_{nR} \\
Q = \frac{1}{3} : \quad f'_{1L}, f'_{2L}, \ldots, f'_{nL} ; \quad f'_{1R}, f'_{2R}, \ldots, f'_{nR} .
\]

According to the Standard Model, the left-handed fields form doublets and the right-handed ones are singlets. Without any loss of generality we may assume that \( f'_{1L} \) and \( f'_{1L} \) form isospin doublets. We introduce

\[
\psi_{1L} = \left( f_{1L}, f'_{1L} \right), \quad \psi_{iR} = f_{iR}, \quad \psi'_{iR} = f'_{iR}, \quad i = 1, 2, \ldots, n .
\]

Actually, in the Standard Model, the left- and right-handed fermions are unrelated to each other. We should have emphasized\(^2\) this fact by choosing a different notation for the right-handed fields, e.g. \( \hat{f}'_{1L} \) and \( \hat{f}'_{1R} \). However, for the discussion below this turns out to be irrelevant (see the next Section). Going back to the beginning, Eq. (2), we must generalize the terms \( \mathcal{L}(\psi, W, B) \) and \( \mathcal{L}(\psi, \varphi) \) to \( n \) families. To this end we make the following replacement

\[
\mathcal{L}(\psi, W, B) \rightarrow \sum_{j=1}^{n} \mathcal{L}(\psi_{j}, W, B),
\]

where

\[
\mathcal{L}(\psi_{j}, W, B) = \frac{i}{2} \left( \overline{\psi}_{jL} f'_{jL} \right) \gamma^{\mu} \left( \gamma_{\mu} - i\gamma_{5} \frac{\lambda_{j}}{2} \right) \overline{\psi}_{jR} W_{\mu} + i\gamma_{5} g' B_{\mu} f'_{jL} + \overline{\psi}_{jR} \gamma^{\mu} \left( \gamma_{\mu} - i\gamma_{5} g' B_{\mu} \right) f'_{jR} .
\]

This relation is the generalization of Eq. (12) to the case of \( n \) families. Similarly, the generalization of \( \mathcal{L}(\psi, \varphi) \) is evidently

\[
\mathcal{L}(\psi, \varphi) \rightarrow \sum_{j,k=1}^{n} \left[ c_{jk} \overline{\psi}_{jL} \varphi^{c} f'_{kR} + c'_{jk} \overline{\psi}_{jL} \varphi f'_{kR} + h.c. \right],
\]

where \( c_{ij} \) and \( c'_{ij} \) are \( 2n^2 \) arbitrary constants. Again after the spontaneous symmetry breaking we put

\[
\varphi \rightarrow \frac{1}{\sqrt{2}} \left( \varphi_{0} + \nu \right), \quad \varphi^{c} \rightarrow \frac{1}{\sqrt{2}} \left( \varphi_{0} - \nu \right)
\]

and get
\[ L(\psi, \varphi) = -\left(1 + \frac{Q_\psi}{\nu} \right) \sum_{j,k} \left[ \bar{f}_{jL} m_{jk} f_{kR} + \bar{f}_{jL}' m'_{jk} f'_{kR} + h.c. \right] + m_{j} \bar{u} u + m_{c} \bar{c} c + m_{t} \bar{t} t + \ldots + m_{d} \bar{d} d + m_{s} \bar{s} s + m_{b} \bar{b} b + \ldots \]

Evidently \( m \) and \( m' \) are the mass matrices for the charge 2/3 and -1/3 quarks respectively. In order to identify the physical particles one must diagonalize the mass matrices. This is always possible with the help of bi-unitary transformations

\[ U_L \theta U_R^{+} = \begin{pmatrix} m_u & m_c & 0 \\ m_t & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad U_L' \theta' U_R^{+} = \begin{pmatrix} m_d & m_s & 0 \\ m_s & m_c & 0 \\ 0 & 0 & m_c \end{pmatrix} \]

where \( U_L, U_R, U'_L \) and \( U'_R \) are four unitary \( n \times n \) matrices. Then

\[ \sum f_{jL} \theta_{jk} f_{kR} = \bar{f}_{jL} m_{jk} f_{kR} = \bar{f}_{jL} U_L^{+} U_L m U_R^{+} U_R \bar{f}_{kR} = \bar{f}_{jL} U_L^{+} U_L \left( \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \right) U_R^{+} \bar{f}_{kR}. \]

From here we conclude that the physical fermions are related to the "mathematical" quarks through

\[ \begin{pmatrix} u \\ c \\ t \end{pmatrix} = U_{L,R} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}, \quad \chi = L, R, \]

and similarly

\[ \begin{pmatrix} d \\ s \\ b \end{pmatrix} = U'_{L,R} \begin{pmatrix} f'_1 \\ f'_2 \\ f'_3 \end{pmatrix}, \quad \chi = L, R. \]

In the "physical basis" \( L(\psi, \varphi) \) becomes

\[ L(\psi, \varphi) = -\left(1 + \frac{Q_\psi}{\nu} \right) \sum \left[ m_u \bar{u} u + m_c \bar{c} c + m_t \bar{t} t + \ldots + m_d \bar{d} d + m_s \bar{s} s + m_b \bar{b} b + \ldots \right]. \]

Clearly, all the above formulae may be repeated for the leptons. The following important conclusions may be drawn

1. The very same Higgs doublet which gives masses to \( W \) and \( Z \) also gives masses to quarks and leptons.
2) The Higgs-fermion interactions are purely scalar and flavour conserving. The diagonalization of mass matrices simultaneously diagonalizes the Higgs-fermion interactions.

3) The coupling constants of a Higgs to a fermion pair is \( g m / (2M_W^\text{c}) \). Thus the Higgs preferentially decays into the heaviest fermion satisfying \( 2m_f < m_\Phi \), provided the phase space is not negligible, e.g.,

\[
\frac{\Gamma'(q_\ell \to t \bar{t})}{\Gamma'(q_\ell \to b \bar{b})} = \left( \frac{m_\ell}{m_\Phi} \right)^2 \left( \frac{1 - r_f^2}{1 - r_b^2} \right)^{3/2}, \quad r_f \equiv \frac{4m_f^2}{m_\Phi^2}, \quad m_\Phi > 2m_\ell.
\]

I shall discuss the phenomenology of the Standard Higgs particle in a subsequent article.\(^1\)

8. FLAVOUR MIXING

Having obtained the physical quarks and leptons, \( u, d, \nu_e, e, \ldots \), from diagonalization of the mass matrices in \( f(\psi, \phi) \), we should go back to the original Lagrangian of the Standard Model and everywhere replace the unphysical fermions \( f_1, f_1' \) by the physical fermions. We should also replace \( \bar{W}, \bar{B} \) by the physical gauge bosons \( \bar{W}^\pm, Z \) and \( \gamma \). We shall see now that in the physical basis there are mixing angles and phases in the charged current sector. The mixing angles/phases are reminiscents of the \( U_R \)'s (in Eq. (68)). The \( U_R \)'s leave no remnants because there are no right-handed charged currents \((\text{no } \bar{W}_R^+)\) in the Standard Model. We begin with the relation (64). Consider first the neutral currents, e.g. the term

\[
E_\Phi = \left( \bar{f}_{jL}, \bar{f}_{jL}' \right) Y^\nu \left( g' g_B \right) \left( f_{jL}, f_{jL}' \right) = \bar{f}_{jL} F f_{jL} + \bar{f}_{jL}' F f_{jL}'.
\]

(73)

Going to the physical basis (see Eq. (70) gives, for example

\[
\bar{f}_{jL} F f_{jL} = \left( \bar{u}_{L}, \bar{c}_{L}, \ldots \right) U_D F U_L^\dagger \left( \bar{c}_{L} \right) = \bar{u}_{L} F u_{L} + \bar{c}_{L} F c_{L} + \bar{t}_{L} F t_{L} + \ldots .
\]

(74)

where we have used \( U_D^\dagger U_L = 1 \). Thus in the above term there is no flavour mixing. Similarly the charge \((-1/3)\) sector is flavour diagonal and we have

\[
E_\Phi = \bar{u}_{L} F u_{L} + \bar{c}_{L} F c_{L} + \bar{t}_{L} F t_{L} + \ldots + \left( \bar{d}_{L} F d_{L} + \bar{s}_{L} F s_{L} + \bar{b}_{L} F b_{L} + \ldots \right).
\]

(75)

The term involving \( W^3 \) may be treated exactly in the same way

\[
E_{W^3} = \left( \bar{f}_{jL}, \bar{f}_{jL}' \right) Y^\nu \left( g' \frac{g_3}{2} \right) W_3 \left( f_{jL}, f_{jL}' \right) = g W_3 \left( \frac{1}{2} \left( \bar{u}_{L} Y^\nu u_{L} + \bar{c}_{L} Y^\nu c_{L} + \ldots \right) - \frac{1}{2} \left( \bar{d}_{L} Y' u_{L} + \bar{s}_{L} Y' c_{L} + \ldots \right) \right).\]

(76)
Here $\pm 1/2$ are the eigenvalues of $\sigma^3$ originating from $Q = 2/3$ and $-1/3$ sectors. We may rewrite $B^u_\mu$ and $W^3_\mu$ in terms of the $A^u_\mu$ and $Z^u_\mu$. Clearly these interactions will be flavour conserving. We have seen that the flavour conservation in neutral currents is due to the fact that $U^u_L$'s are unitary and for charge $2/3$($-1/3$) quarks we get $U^u_L U^{\dagger u}_L (U^{\dagger u}_L)^{-1}$ which is unity. This is called the GIM-mechanism\textsuperscript{21}). In the charged current sector, i.e., interactions mediated by $W^\pm$, there is a transition between the charge $2/3$ and $-1/3$ sectors. Therefore, we get $U^u_L U^{\dagger u}_L$ which is not necessarily equal to unity. From (64) we obtain

$$\mathcal{L}_{cc} = g \sum_j \left( \frac{\bar{f}_{jL}}{f_{jL}} \right)^* y^r \frac{\bar{W}_{r} W_{r}^*}{2} \left( \frac{f_{jL}}{f_{jL}} \right) =$$

$$= \frac{g}{\sqrt{2}} \left( W^{(c)}_{r} \left( \bar{u}_L, c_L, \bar{t}_L \right) \right) y^r U^u_{L} U^{\dagger u}_{L} \left( \frac{d_L}{s_L} \right) + h.c., \quad U^u_{L} U^{\dagger u}_{L} = V,$$

(77)

where $V$ is $n$ by $n$ unitary matrix and, therefore, has $n^2$ parameters. However, we are free to choose the phases of the quark fields as we wish. Since only the relative phases matter and we have 2n quark field, we may remove 2n-1 parameters of the matrix $V$ by suitable redefinition of the quark fields. Thus the number of physical parameters in $V$ is $n^2 - (2n-1) = (n-1)^2$. For two families there is only one parameter, the so-called Cabibbo angle\textsuperscript{21)},

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W^{(c)}_{r} \left( \bar{u}_L, c_L, \bar{t}_L \right) y^r \left( \frac{\cos \theta_c}{\sin \theta_c} \right) \left( \frac{d_L}{s_L} \right) + h.c.$$

(78)

The Standard Model, with only 2 families, is CP-conserving because all the coupling constants are real. However, with three families one expects\textsuperscript{22}) CP-violation in the Standard Model. The matrix $V$, which is 3 by 3, depends on $(3-1)^2 = 4$ parameters, 3 rotation angles $\theta^u_i$, $i=1,2,3$ and a phase $\delta$. The charged current Lagrangian is given by

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W^{(c)}_{r} \left( \bar{u}_L, c_L, \bar{t}_L \right) y^r \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} + h.c.$$

(79)

where $V_{ij}$ are constants, functions of the four parameters $\theta^u_i$ and $\delta$. In the Kobayashi-Maskawa parametrization\textsuperscript{23}), the $V_{ij}$ are given by

$$V_{ud} = c_1, \quad V_{cd} = -s_1 c_2, \quad V_{td} = s_1 s_2$$
$$V_{us} = s_1 c_3, \quad V_{cs} = c_1 c_2 s_3 - s_1 s_3 e^{i\delta}, \quad V_{ts} = -c_1 s_2 c_3 - c_2 s_3 e^{i\delta}$$
$$V_{ub} = s_1 s_3, \quad V_{cb} = c_1 c_2 s_3 + s_1 s_3 e^{i\delta}, \quad V_{tb} = -c_1 s_2 s_3 + c_2 s_3 e^{i\delta}$$

(80)
where
\[ C_i = \cos \theta_i, \quad S_i = \sin \theta_i, \quad i = 1, 2, 3, \]
and $\delta$ is the CP-phase.

The Kobayashi-Maskawa parametrization is reproduced\(^1\) by the following product of 3 rotations matrices and a phase matrix
\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_2 S_3 - S_2 c_1 \\
0 & S_2 c_3 + c_2 S_1 \\
0 & 0 & 1 \\
0 & -S_3 c_1 + c_2 c_3 \\
0 & c_1 c_3 + S_2 S_1 \\
\end{pmatrix}
\]
\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_2 S_3 - S_2 c_1 \\
0 & S_2 c_3 + c_2 S_1 \\
0 & 0 & 1 \\
0 & -S_3 c_1 + c_2 c_3 \\
0 & c_1 c_3 + S_2 S_1 \\
\end{pmatrix}
\]
\[ (81) \]

The coupling constants in (80) are determined from beta decay, charm production in neutrino interactions, beauty lifetime, etc. I shall not discuss the present experimental situation as it is reviewed by other authors\(^1(6)\).

Recently, I have also reviewed the subject and shall not repeat the analysis here\(^3\).

For the leptons, we may repeat the above analysis, replacing everywhere
\[
(u, c, t) \rightarrow (\nu_1, \nu_2, \nu_3)
\]
\[
(d, s, b) \rightarrow (e, \mu, \tau)
\]
\[ (82) \]
where the $\nu_i$ are the neutrino mass eigenstates. In general, we then expect three mixing angles and a phase angle $(\theta_1, \delta)$ to appear in the leptonic interactions. These quantities are, in principle, measurable but in practise it is very hard to check the identity of the neutrinos. Furthermore, if the neutrinos are massless (or by some miracle have exactly the same masses) the coupling constants $V_{ij}$ (for leptons) are unobservable because we may simply define
\[
|\nu_k^\ell\rangle = \sum_{j=1}^{3} V_{j\ell} |\nu_j^\nu\rangle, \quad \nu = e, \mu, \tau
\]
\[ (83) \]
as physical states whereby the leptonic charged current is given by
\[
\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{W}_\alpha L \left[ \bar{\nu}_L \gamma^\nu \nu_L + \bar{\nu}_L \gamma^\tau \nu_L + \bar{\nu}_L \gamma^\mu \nu_L \right] + h.c.
\]
\[ (84) \]
This relation shows that in the case of massless neutrinos there is no family mixing. If the neutrinos have different masses, the linear superpositions in (83) will not correspond to physical states, because $\nu_i$ are distinguishable. The present experimental situation on neutrino masses and mixings is reviewed by others\(^1(6)\). Processes such as neutrino oscillations and double beta decay are invaluable sources of information on leptonic mixing parameters in (83). If the present experimental evidence for a nonvanishing
neutrino mass would be confirmed one would expect family mixing in the leptonic sector. Unfortunately the Standard Model has nothing to say on the question of masses and the values of $\theta_1, \theta_2$.

9. CONCLUSIONS

In the above notes, I have given an elementary introduction to the Standard Electroweak Model of Glashow, Weinberg and Salam. I reviewed the various terms in the Standard Model Lagrangian, explaining mostly why's and how's but leaving out the experimental consequences.

It is indeed remarkable that W and Z with expected masses and branching ratios were discovered\(^1\) in 1983 at CERN. The ratio of masses $M_W/M_Z$ agrees with the expectation of the minimal model, where a Higgs doublet is responsible for the symmetry breaking. We don't understand yet what such a result means. Of course it does not prove that one or more Higgs doublets exist and are the origin of masses. If, however, the mass ratio had not agreed with the expectations we could have excluded the minimal model.

The Standard Model, in spite of being very successful, leaves a large number of questions unanswered:

- Why are there 3 families and are there any fundamental differences between them?
- Which principles fix the values of the 17-24 parameters of the Standard Model and the weak hypercharges of the fundamental fermions?
- What is the origin of parity violation? In other words why left-handed doublets but right-handed singlets?
- Who needs the quarks and leptons? We could have had a $\text{SU}(2) \times \text{U}(1)$ invariant Lagrangian with just the gauge bosons.
- Why should weak interactions be different from electromagnetic and strong interactions (as well as gravity) by having massive mediators? Short range interactions do not imply that the mediators have to be massive (compare Q.C.D.). Perhaps the true weak interactions are confined and $W, Z$ are just the mediators of some residual interactions.

There are indeed many open deep questions at the moment to which we have no answers. I hope that some of you young students, as lecturers at future CERN schools, will be able to explain the answers to the future students. Good luck!
REFERENCES


2) C. Jarlskog, Proceedings of the 1981 CERN-JINR School of Physics, CERN Yellow Report 82-04, 63.


5) O. Klein, Nature 161, 897 (1948).


13) J.R. Oppenheimer, in the 1958 Annual International Conference at CERN.

14) See, for example, the review article by A. Zichichi, CERN Yellow Report 64-26. A. Zichichi was not involved in the experiment.


16) D. Haidt, these Proceedings; J.D. Dowell, these Proceedings.

17) For a detailed discussion see, for example E.S. Abers and B.W. Lee, Phys. Reports 9C, 1 (1973); J. Iliopoulos, CERN Yellow Report 77-18, pp. 36-78.


19) C. Jarlskog, "Phenomenology of Higgs Particles", these Proceedings.

20) P.H. Frampton and C. Jarlskog, to be published.


ELEMENTARY SUPERSYMMETRY
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ABSTRACT
The supersymmetric harmonic oscillator is constructed from elementary quantum mechanics and some of its properties are discussed. Introducing supercoordinates one easily generates more general examples of N=1 and N=2 supersymmetric quantum mechanics.

1. INTRODUCTION
Each of the four fundamental interactions between elementary particles is now known to be a highly symmetric gauge theory. But still they contain too many unknown parameters and do not constitute a unified theory of all four interactions. In order to improve this situation one needs new symmetry principles.

A very promising possibility is supersymmetry which relates fermions and bosons. The essential properties of this new symmetry can be demonstrated for field theories in 1+0 dimensions, i.e. supersymmetric quantum mechanics. The very complex mathematical formalism used in higher dimensions then simplifies enormously and becomes very transparent. For beginners, it can therefore be useful to get acquainted with supersymmetric quantum mechanics before advancing to more realistic field theories.

2. THE SUPERSYMMETRIC HARMONIC OSCILLATOR
One of the simplest and probably the most important quantum mechanical system is the ordinary bosonic harmonic oscillator with Hamiltonian

\[ H_B = \frac{1}{2} p^2 + \frac{1}{2} \omega_B^2 q^2 \]  \hspace{1cm} (2.1)

and canonical commutator \([q,p] = i\). Introducing the annihilation operator

\[ a = \sqrt{\frac{1}{2 \omega_B}} (p - i \omega_B q) \]  \hspace{1cm} (2.2)

and the conjugate creation operator \(a^\dagger\), one finds the commutator

\[ [a, a^\dagger] = 1 \]  \hspace{1cm} (2.3a)

and trivially

\[ [a, a] = [a^\dagger, a^\dagger] = 0. \]  \hspace{1cm} (2.3b)

The Hamiltonian now takes the form

\[ H_B = \frac{\omega_B}{2} \{ a^\dagger, a \} \]  \hspace{1cm} (2.4)

where the anticommutator \([a, a^\dagger] = aa^\dagger + a^\dagger a\). Using (2.3) one obtains the energy eigenvalues

\[ E_B = \omega_B \left(n_B + \frac{1}{2} \right) \]  \hspace{1cm} (2.5)
where $n_B = 0, 1, 2, \ldots$ are the eigenvalues of the number operator $a^\dagger a$. One then has the well-known equally spaced energy levels of the bosonic harmonic oscillator with ground state energy $\frac{1}{2} \omega_B$.

Now we will construct a corresponding fermionic harmonic oscillator. Fermions satisfy the Pauli principle which follows from quantizing with anticommutators instead of commutators as for bosons. We introduce therefore a fermion annihilation operator $b$ and a corresponding creation operator $b^\dagger$. In order to have symmetry between bosonic and fermionic degrees of freedom, these new operators are required to satisfy quantization conditions analogous to those for bosons (2.3), but now in terms of anticommutators instead of commutators,

\begin{equation}
[b, b^\dagger] = 1 \quad \text{(2.6a)}
\end{equation}

and

\begin{equation}
[b, b] = [b^\dagger, b^\dagger] = 0. \quad \text{(2.6b)}
\end{equation}

Since now $bb = 0$, two such fermions cannot exist in the same quantum state. Again, from the requirement of maximum symmetry between bosons and fermions, we get the Hamiltonian for this new fermion from the bosonic Hamiltonian (2.4) by simply replacing the anticommutator with a commutator:

\begin{equation}
H_F = \frac{\omega_F}{2} [b^\dagger, b]. \quad \text{(2.7)}
\end{equation}

Using the anticommutator (2.6a) we get the fermionic energy eigenvalues

\begin{equation}
E_F = \omega_F (n_F - \frac{1}{2}) \quad \text{(2.8)}
\end{equation}

where now $n_F = 0, 1$ are the only allowed eigenvalues of the fermionic number operator $b^\dagger b$. The ground state energy of this oscillator is negative, $-\frac{1}{2} \omega_F$, a property which will turn out to be very important for supersymmetric theories.

We will next consider the combined system of a bosonic and a fermionic oscillator. It has energy eigenvalues which are given as the sum of (2.5) and (2.8). In the general case where the frequencies of the two oscillators are different, nothing very interesting develops. But in the particular case where $\omega_B = \omega_F = \omega$, we get the combined energy levels

\begin{equation}
\mathcal{E} = \omega (n_B + n_F) \quad \text{(2.9)}
\end{equation}

which are all doubly degenerate except for the ground state with $n_B = n_F = 0$ and hence zero energy.

Degeneracy in quantum mechanics signals the presence of symmetry in the Hamiltonian. For example, when a particle moves in a spherically symmetric potential, each energy level carrying the quantum numbers $(n, \ell)$ is $2\ell + 1$ times degenerate. The levels within one such multiplet are connected by the raising and lowering operators $L_+$ and $L_-$ which at the same time also generate the symmetric rotations of the system, $[L_3, H] = 0$.

In our case, the extra symmetry of the combined bosonic and fermionic oscillators is called supersymmetry. Since this degeneracy arises by simultaneously destroying one bosonic quantum, $n_B \to n_B - 1$, and creating one fermionic quantum, $n_F \to n_F + 1$, or vice versa, we expect the corresponding symmetry generators to behave like $ab^\dagger$ and $a^\dagger b$. In fact, defining
\[ Q = \sqrt{2\omega} a^\dagger b, \quad \bar{Q} = \sqrt{2\omega} ab^\dagger \]  
(2.10)

one finds that they both commute with the Hamiltonian
\[ H = \omega (a^\dagger a + b^\dagger b) \]  
(2.11)

of the supersymmetric harmonic oscillator:
\[ [Q, H] = [\bar{Q}, H] = 0. \]  
(2.12)

On the other hand, the Hamiltonian is also given by the fundamental anticommutator
\[ [Q, \bar{Q}] = 2\hbar \]  
(2.13)

which plays a central role in all supersymmetric theories.

While the bosonic creation and annihilation operators can be represented in terms of the canonical variables \( q \) and \( p \) as in (2.2), we can represent the corresponding fermionic operators by Pauli matrices:
\[ b = \sigma_-, \quad b^\dagger = \sigma_+ \]  
(2.14)

This representation is seen to satisfy the canonical anticommutators (2.6). The supersymmetric Hamiltonian (2.11) can now be written as
\[ H = \frac{1}{2} p^2 + \frac{1}{2} \omega q^2 + \frac{1}{2} \omega q^3 \]  
(2.15)

and it looks like a bosonic oscillator with a spin-1/2 particle coupled to a magnetic field.

Given the Hamiltonian, we can now find the Lagrangian for the supersymmetric harmonic oscillator
\[ L = \frac{1}{2} q^2 - \frac{1}{2} \omega q^2 + i\tilde{\psi}\dot{\psi} - \dot{\psi}\tilde{\psi} \]  
(2.16)

where the fermionic variable \( \psi = \psi(t) \), \( \tilde{\psi} = \psi^\dagger \) and \( \dot{\psi} \) is the time derivative of \( \psi \). The previous fermionic operators \( b \) and \( b^\dagger \) used in the Hamiltonian, are the quantized versions of \( \psi(0) \) and \( \tilde{\psi}(0) \). It should be stressed that in the Lagrangian \( \psi \) and \( \tilde{\psi} \) are not operators, but classical fermionic variables which are Grassmann numbers satisfying \( \psi\tilde{\psi} = \tilde{\psi}\psi = 0 \) and \( \tilde{\psi}\tilde{\psi} = -\tilde{\psi}\psi \). This follows from the quantum mechanical anticommutator (2.6) in the classical limit where the right-hand sides go to zero. Anticommuting variables of this kind are used in the mathematical formulation of supersymmetry as will be seen in the next section.

Before leaving the supersymmetric harmonic oscillator one should note that in contrast to the ground state energies of the bosonic and fermionic harmonic oscillators, the supersymmetric version has zero ground state or vacuum energy. This property survives in many more realistic theories in higher dimensions and can be related to the vanishing cosmological constant of the Universe.
The electric and magnetic fields \( E \) and \( B \) in 3-dimensional space is unified into an electromagnetic field tensor \( F_{\mu \nu} \) by going to 4-dimensional spacetime. Similarly, one can give a unified description of bosons and fermions by enlarging our spacetime to more abstract superspace. In this space the corresponding superfields contain both boson and fermion component fields.

For supersymmetric mechanics it is sufficient to extend the ordinary time variable \( t \) to a new supertime involving a Grassmann time variable \( \theta \) in addition to \( t \). The ordinary position variable of a particle will then become a superposition variable depending on both \( t \) and \( \theta \).

In order to discuss symmetries it is necessary to decide what should be left invariant under the symmetry transformations. The energy of a system is conserved if the Lagrangian is invariant under time translations \( t \rightarrow t + \alpha \) which leaves the differential \( dt \) invariant. Similarly, we can now define a differential line element in the \((t, \theta)\) superspace which we want to keep invariant under supersymmetry transformations:

\[
dt - i\theta d\theta = \text{invariant.} \tag{3.1}\]

The second term is essentially the only possible choice we have involving the two Grassmann variables \( \theta \) and \( d\theta \). It is multiplied by the imaginary \( i \) in order to keep the line element real since the complex conjugate of a product of two real Grassmann variables is defined by \((\alpha \beta)^* = \beta^* \alpha = -\alpha \beta \). Now we see that the line element is invariant under the combined transformation

\[
\begin{align*}
\theta &\rightarrow \theta' = \theta + \epsilon \\
t &\rightarrow t' = t + i\epsilon \theta
\end{align*}
\tag{3.2}
\]

where the Grassmann variable \( \epsilon \) parametrizes this supersymmetry coordinate transformation.

In the \((t, \theta)\) superspace we can now define a real, scalar superfield \( \Phi(t, \theta) \). It can be Taylor expanded in powers of \( \theta \) and gives only two terms since \( \theta^2 = 0 \). Writing it as

\[
\Phi(t, \theta) = q(t) + i\theta \psi(t) \tag{3.3}
\]

we see that it involves a fermionic variable \( \psi(t) \) in addition to the ordinary bosonic position variable \( q(t) \). Under the transformation (3.2) this field changes by

\[
\delta \Phi = \phi(t', \theta') - \Phi(t, \theta) \quad \text{which we can evaluate to give}

\[
\delta \Phi = i\epsilon \delta q + i\epsilon \psi. \tag{3.4}
\]

The supersymmetry transformation (3.2) in superspace induces therefore a corresponding transformation among the component fields \( q(t) \) and \( \psi(t) \):

\[
\delta q = i\epsilon \psi, \quad \delta \psi = -i \epsilon q. \tag{3.5}
\]

It mixes the fermionic field variables into the boson field and vice versa. We also notice
that the last component field in the expansion (3.3) transforms as a derivative. This property will shortly be made use of.

The transformation (3.4) can be written as

$$\epsilon \Phi = \epsilon Q \Phi$$

(3.6)

which defines the supersymmetry generator

$$Q = \frac{\partial}{\partial \theta} + i \theta \frac{\partial}{\partial t}$$

(3.7)

when

$$\frac{\partial}{\partial \theta} \theta = 0, \quad \frac{\partial}{\partial \theta} \phi = 1.$$

This new derivative with respect to $\theta$ is also defined to anticommute with other Grassmann variables. When $Q$ acts a second time on the superfield $\Phi$, we find that it gives simply the time derivative of $\Phi$. In fact, since the generator of ordinary time translations is the Hamiltonian operator $\hat{H} = i \frac{\partial}{\partial t}$, we can abstract this result to give the fundamental anticommutator

$$[Q, \Phi] = 2H$$

(3.8)

of this theory. It corresponds to (2.13) for the supersymmetric harmonic oscillator.

With $Q$ having the form (3.7) one can similarly show that the operator

$$D = \frac{\partial}{\partial \theta} - i \theta \frac{\partial}{\partial t}$$

(3.9)

satisfies $[Q, D] = 0$ in addition to being invariant under the transformation (3.2).

We now define a general superfield to transform like (3.6) under a supersymmetry transformation. The sum of two superfields $\Phi_1(t, \theta)$ and $\Phi_2(t, \theta)$ will then be a new superfield. It is also pretty straightforward to show that the product of $\Phi_1$ and $\Phi_2$ will be another superfield. Similarly, since $Q$ and the ordinary time derivative commute while $Q$ and $D$ anticommute, both $\phi$ and $D \Phi$ will be superfields.

A supersymmetric Lagrangian for a system involving fields of this kind can now be constructed. From a sum of products of different superfields one isolates the highest component field of this resulting superfield, i.e. the term multiplied with $0$ in the Taylor expansion. We have already shown that this component transforms as a time derivative and hence will constitute a supersymmetric Lagrangian since total derivatives usually do not contribute to the physics of the problem.

Formally one isolates the highest term in the Taylor expansion by a Grassmann integration defined by:

$$\int d\theta = 0, \quad \int d\theta = 1.$$  

(3.10)

It acts exactly like Grassmann derivation! As an example, we now construct a Lagrangian from the product superfield $D \Phi$:

$$L = \frac{1}{2} \int d\theta D \Phi$$

(3.11)
Taking the derivatives and integrating, we find

$$L = \frac{1}{2} \dot{\psi}^2 + \frac{1}{2} \dot{\psi}^2$$

(3.12)

As a check, we now see how this Lagrangian transforms under the supersymmetry transformation (3.5)

$$\delta L = \frac{i}{2} \epsilon (\dot{\psi} \psi + \dot{\psi} \psi) = \frac{i}{2} \epsilon \frac{d}{dt} (\psi \dot{\psi})$$

The total derivative comes out as expected and we have a supersymmetric Lagrangian (3.12).

The physics of this particular example is not very interesting since the corresponding Hamiltonian is simply $H = \frac{1}{2} p^2$ describing a free particle. However, if the real scalar superfield $\Phi(t, \theta)$ we have considered here is replaced by a 3-vector superfield $\Phi(t, \theta)$, one can show that the Lagrangian corresponding to (3.12) gives rise to the Pauli Hamiltonian for a quantized non-relativistic spin-1/2 particle. One can even couple this particle to an external magnetic field and still have supersymmetry. This supersymmetry of the Pauli equation has recently been used to determine algebraically the energy levels of an electron bound in the field of a magnetic monopole$^1$. Similarly, one can also consider a 4-vector superfield $\Phi(t, \theta)$ and one gets a relativistic and supersymmetric description of a spin-1/2 particle consistent with more conventional Dirac theory when quantized$^2$.

These different supersymmetric Lagrangians give rise to what is called $\mathbb{N}=1$ supersymmetric quantum mechanics since they are obtained from a superspace with only one $\theta$-coordinate. There is apparently nothing preventing us from investigating theories with more $\theta$-coordinates. Going to $\mathbb{N}=2$ we will recover the supersymmetric harmonic oscillator we started with.

4. **$\mathbb{N}=2$ SUPERSYMMETRIC QUANTUM MECHANICS**

Now consider the line element $dt - i \theta_1 d\theta_1 - i \theta_2 d\theta_2$ which is invariant under the transformation

$$\theta_1 \rightarrow \theta_1 + \epsilon_1, \quad \theta_2 \rightarrow \theta_2 + \epsilon_2$$

$$t \rightarrow t + i \epsilon \theta_1 + i \epsilon \theta_2.$$  

(4.1)

Introducing the complex coordinates

$$\theta = \frac{1}{\sqrt{2}} (\theta_1 - i \theta_2), \quad \bar{\theta} = \frac{1}{\sqrt{2}} (\theta_1 + i \theta_2)$$

we find the corresponding supersymmetry generators by the same line of reasoning as in the previous section:

$$\mathcal{Q} = \frac{\partial}{\partial \theta} + i \theta \frac{\partial}{\partial t}, \quad \bar{\mathcal{Q}} = \frac{\partial}{\partial \bar{\theta}} + i \bar{\theta} \frac{\partial}{\partial t}.$$  

(4.2)
The fundamental anticommutator is now
\[ \{q, \bar{q}\} = 2 \hbar \] (4.3)
which follows from letting these operators act on a superfield \( \Phi(t, \theta, \bar{\theta}) \). The invariant
derivatives
\[ D = \frac{\partial}{\partial \theta} - i \theta \frac{\partial}{\partial t}, \quad \bar{D} = \frac{\partial}{\partial \bar{\theta}} - i \bar{\theta} \frac{\partial}{\partial t} \] (4.4)
both anticommute with \( q \) and \( \bar{q} \). If the scalar superfield \( \Phi(t, \theta, \bar{\theta}) \) is assumed to be real, it
contains four real component fields:
\[ \Phi(t, \theta, \bar{\theta}) = q(t) + i \bar{\theta} \psi(t) + i \theta \bar{\psi}(t) + \theta \bar{\theta} A(t). \] (4.5)
Under (4.1) they are found to transform as
\[ \delta q = iq \psi + i \bar{\psi}, \quad \delta A = \epsilon \bar{\psi} - \bar{\epsilon} \psi \]
\[ \delta \bar{\psi} = -\epsilon (q + iA), \quad \delta \bar{\psi} = -\bar{\epsilon} (q + iA). \] (4.6)
Again we see that the highest component transforms as a pure derivative.
Making use of this property we know immediately that
\[ L = \int d\theta d\bar{\theta} \left[ \frac{1}{2} \bar{D} \Phi D\Phi - W(\Phi) \right] \] (4.7)
will be a supersymmetric Lagrangian. The potential \( W(\Phi) \) is an arbitrary function of the
superfield. Only its \( \theta \bar{\theta} \) component will contribute after the integration;
\[ W(\Phi) = \theta \bar{\theta} (AW' + \bar{\psi} \psi w) + . \]
where the derivatives are taken for \( \theta = \bar{\theta} = 0 \). Similarly expanding the first product of
derivatives in (4.7) we find the Lagrangian:
\[ L = \frac{1}{2} q^2 + \frac{1}{2} \bar{A}^2 - AW' + i \bar{\psi} \dot{\psi} - \psi \dot{\bar{\psi}}. \] (4.8)
The field \( A \) enters with no time derivatives and is therefore not a dynamical field. The
equation of motion \( \partial L / \partial A = 0 \) simply gives \( A = W' \) which substituted back into \( L \) gives the
final expression for the supersymmetric Lagrangian:
\[ L = \frac{1}{2} q^2 - \frac{1}{2} W'^2 + i \bar{\psi} \dot{\psi} - \psi \dot{\bar{\psi}}. \] (4.9)
Here \( W'(q) \) is some function of the ordinary particle coordinate \( q \). For the special choice
\( W' = w q \) we see that we have rederived the Lagrangian (2.16) for the supersymmetric
harmonic oscillator.
From the Lagrangian (4.9) one finds by canonical methods the corresponding Hamiltonian. After having quantized the fermionic variables as in Sec. 2, it can be written as
\[ H = \frac{1}{2} p^2 + \frac{1}{2} w'^2 + \frac{1}{2} \sigma_3 W'. \] (4.10)
The supersymmetry generators are now represented by the operators
\[
Q = (p + i\bar{W})\sigma_+ , \quad \bar{Q} = (p - iW)\sigma_-
\]
which are seen to satisfy (4.3). This form of the Hamiltonian for supersymmetric N=2 quantum mechanics was first introduced and discussed by Witten \(^3\). It contains a lot of interesting physics which is now being uncovered \(^4\text{-}^7\).

CONCLUSION

N=1 and N=2 supersymmetric quantum mechanics both give elementary examples of how supersymmetric theories are constructed using the powerful formalism based on superfields of anticommuting Grassmann variables. Hopefully, these illustrations will help shorten the way for those who set out to study the more realistic supersymmetric field theories of elementary particles in higher dimensions.

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\]

REFERENCES


UNIFICATION AND SUPERSYMMETRY

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ABSTRACT
A short review of the various attempts to unify all forces of Nature is presented. The concept of Grand Unification is introduced and its dynamical effects are analyzed. A particular emphasis is put on supersymmetry and its phenomenological consequences.

I. THE STANDARD MODEL AND ITS SHORTCOMINGS

The strong, electromagnetic and weak interactions are described by a gauge theory based on the group $U(1) \times SU(2) \times SU(3)$ which is spontaneously broken into $U(1)_{\text{e.m.}} \times SU(3)$. The fundamental Lagrangian contains the three following types of fields:

(i) Gauge fields: We have eight gluons associated with the strong interactions and four electroweak vector bosons. After spontaneous breaking the gluons and the photon remain massless while the three remaining ones ($W^\pm, Z^0$) acquire a mass.

(ii) Fermion matter fields: The basic unit is a "family" consisting of fifteen two-component complex spinor fields which, under $SU(2) \times SU(3)$, form the representation

$$(2, 1) + (2, 3) + (1, 1) + (1, 3)$$

The prototype is the electron family

$$\begin{align*}
&e^L, \quad \nu_e^L \\
&u^L, \quad d^L_i \\
e^R, \quad u^R_i, \quad d^R_i
\end{align*}$$

$i =$ blue, white, red

with the quarks transforming as triplets of $SU(3)$.

For the muon family $\nu_\mu \rightarrow \nu_e$, $e \rightarrow \mu$, $u_i \rightarrow c_i$ and $d_i \rightarrow s_i$ and, similarly, for the tau family $\nu_\tau \rightarrow \nu_e$, $e \rightarrow \tau$, $u_i \rightarrow t_i$, $d_i \rightarrow b_i$. A remarkable property is that the sum of the electric charges in each family vanishes. This turns out to be necessary for the cancellation of triangle anomalies in the Ward identities of axial currents and hence, for the construction of a renormalizable theory.

This family structure, which according to the latest news has been once more brilliantly verified with the discovery of top, is an as yet unexplained feature of the theory. Furthermore, the total number of families is not restricted. In fact, we know of no good reason why any, beyond the first one, should exist.

(iii) Higgs scalar fields: We would be very happy if we could live with only the first two kinds of fields, but in fact we need a third one, the scalar Higgs fields. Their non-zero

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vacuum expectation values break the gauge symmetry spontaneously, thus providing masses to \( \frac{3}{2} \), \( Z^0 \) as well as the fermions. In the standard model this is accomplished with a complex doublet of scalar fields. At the end, one neutral spin-zero boson survives as a physical particle. There are no severe restrictions on its mass. In the absence of any concrete experimental evidence, one is left to speculate on the number of physical Higgs particles as well as on their elementary or composite nature. Whichever the ultimate answer to these questions may be, we can say that the Higgs sector is at present the least understood and perhaps the most interesting sector of gauge theories.

The above uncertainties notwithstanding our confidence in this model is amply justified on the basis of its ability to accurately describe the bulk of our present day data and, especially, of its enormous success in predicting new phenomena. Nevertheless, there are several reasons to suspect that the gauge theory based on \( U(1) \times SU(2) \times SU(3) \) cannot be considered as the final theory. First, it is not a unified theory at all. We know of three fundamental interactions and we have three independent coupling strengths. Each group factor introduces its own. Second and even worse, is the presence of \( U(1) \) because an abelian gauge symmetry allows for arbitrary coupling constants. Indeed we can easily verify that we can write a consistent theory of quantum electrodynamics with two charged fields, one with charge \( e \) and the other with charge \( \pi e \). However, if we repeat the exercise with a non-abelian gauge theory, say one based on \( SU(2) \), we can show that this arbitrariness disappears; all matter fields couple to the gauge bosons with the same coupling strength, up to a possible Clebsch-Gordan coefficient depending on the representation. It follows that the standard model, because of its \( U(1) \) factor, cannot explain the observed electric charge quantization. In such a model, the observed very precise equality (up to one part in \( 10^{20} \)) of the electric charges of the positron and the proton is accidental.

II. GRAND UNIFICATION

For all these reasons, and others that I skip, several theorists tried to go beyond the standard model. The hypothesis of grand unification states that \( U(1) \times SU(2) \times SU(3) \) is the remnant of a larger, simple or semi-simple group \( G \), which is spontaneously broken at very high energies. The scheme looks like:

\[
G \rightarrow_{M} \quad U(1) \times SU(2) \times SU(3) \quad m_\text{W} \sim 10^2 \text{ GeV} \tag{1}
\]

where the breaking of \( G \) may be a multistage one and \( M \) is one (or several) characteristic mass scale(s). Two questions immediately arise concerning this idea:

(a) Is it possible? In other words are there groups which contain \( U(1) \times SU(2) \times SU(3) \) as a subgroup and which can accommodate the observed particles?

(b) Does it work? i.e. is the observed dynamics compatible with this grand-unification idea?

We shall try to answer each of these questions separately.
II.A - Candidates for G.U.T.'s

In this section we shall answer the first question by giving some explicit examples of groups \( G \) which satisfy our requirements. We first observe that \( G \) must contain electromagnetism, i.e. the photon must be one of the gauge bosons of \( G \). This is contained in the requirement that \( G \) contains the group of the standard model \( U(1) \times SU(2) \times SU(3) \). Another way to say the same thing, is to say that the electric charge operator \( Q \) must be one of the generators of the algebra of \( G \). Since \( G \) is semi-simple all its generators are represented by traceless matrices. It follows that, in any irreducible representation of \( G \), we must have:

\[
\text{Tr} (Q) = 0
\]

(2)

in other words, the sum of the electric charges of all particles in a given irreducible representation vanishes.

In order to proceed we shall make an important assumption: The fifteen two-component spinors of a family fill a representation of \( G \), i.e. we assume that there are no other, as yet unobserved, particles which sit in the same representation. Property (2) together with the above assumption have a very important consequence: As we have remarked, the fifteen members of a family satisfy (2) because the sum of their charges vanishes. This, however, is not true if we consider leptons or quarks separately. Therefore each irreducible representation of \( G \) will contain both leptons and quarks. This means that there exist gauge bosons of \( G \) which can change a lepton into a quark, or vice versa. We conclude that a grand unified theory that satisfies our assumption cannot conserve baryon and lepton numbers separately. This sounds disastrous, since it raises the spectrum of proton decay. The amplitude for such a decay is given by the exchange of the corresponding gauge boson and therefore, it is of order \( M^{-2} \), where \( M \) is the gauge boson's mass. The resulting proton life-time \( \tau_p \) will be of order:

\[
\tau_p \sim \frac{M^4}{m_p^5}
\]

(3)

Using the experimental limit of \( 10^{30} - 10^{32} \) years we can put a lower limit on \( M \):

\[
M > 10^{14} - 10^{15} \text{ GeV}
\]

(4)

Grand unification is not a low-energy phenomenon!

After these general remarks let us try to find some examples:

II.A.1. The simplest G.U.T.: \( SU(5) \): 

\( U(1) \times SU(2) \times SU(3) \) is of rank 4 (i.e. there are four generators which commute: one of \( U(1) \), one of \( SU(2) \) and two of \( SU(3) \)). Therefore, let us first look for a grand unification group of rank 4. I list all possible candidates:

\[
[SU(2)]^4, \quad [SO(5)]^2
\]

\[
[G_2]^2, \quad SO(8), \quad SO(9), \quad Sp(8), \quad F_4
\]
$[\text{SU}(3)]^2$, $\text{SU}(5)$

The first two are excluded because they have no SU(3) subgroup. The next five admit no complex representations, therefore they cannot accommodate the observed families where, as we already saw, the right- and left-handed particles do not transform the same way. (I again assume that no unobserved fermions will complete a given representation). Finally, in SU(3) x SU(3) quarks and leptons must live in separate representations because the leptons have no colour. But $\sum Q_{\text{quarks}} \neq 0$ and the same is true for leptons. This leaves us with SU(5) as the only candidate of a G.U.T. group of rank 4. It is the simplest and, in some sense, the standard model of grand unification.

The gauge bosons belong to the 24-dimensional adjoint representation. It is useful to decompose it into its SU(2) x SU(3) content. We find:

\begin{equation}
24 = \begin{array}{c}
(2, 3) + (2, \bar{3}) + (1, 8) + (3, 1) + (1, 1) \\
\text{gluons}
\end{array}
\end{equation}

where the first number denotes the SU(2) and the second the SU(3) representation. The known vector bosons can be identified as the eight gluons of Q.C.D. in the (1, 8) piece (a singlet of SU(2) and an octet of SU(3)) as well as the electroweak gauge bosons $W^\pm$, $Z^0$ and $\gamma$ in the (3, 1) + (1, 1) piece. We are left with twelve new ones, called X and Y, with electric charges 4/3 and 1/3 respectively, which transform as a doublet of SU(2) and a triplet and anti-triplet of SU(3). They must be heavy, according to the limit (4).

Let us now come to the matter-field assignment. We shall try to put the fifteen two-component spinors of a family in a representation (not necessarily irreducible) of SU(5). But before doing so, we observe that all gauge couplings, being vectorial, conserve helicity. Therefore, we cannot put right- and left-handed spinors in the same representation. We go around this problem by replacing all right-handed spinors by the corresponding left-handed charge conjugate ones. A quick glance at the representation table of SU(5) suggests to use each family in order to fill two distinct representations: the $\bar{s}$ and the 10. Their SU(2) x SU(3) content is:

\begin{equation}
\bar{s} = (1, \bar{3}) + (2, 1)
\end{equation}

It contains a singlet of SU(2) and an anti-triplet of SU(3), i.e. three anti-quarks, or equivalently, a triplet of right-handed quarks, as well as a doublet of SU(2) and a singlet of SU(3), i.e. a doublet of left-handed leptons. Since the sum of the electric charges must vanish, we see that the anti-triplet must contain the charge conjugate of the d-quarks. For the 10-dimensional representation we have:
\[ 10 = (2, 3) + (1, \bar{3}) + (1, 1) \]
\[
\begin{pmatrix}
  u^c_L \\
  d^c_L \\
  \nu^c_L \\
  e^c_L 
\end{pmatrix}
\]
\[ \sum Q = 0 \]  

and the identification is given by the same reasoning. We often write the representations as:

\[ \bar{5} = \begin{pmatrix}
  d^c_1 \\
  d^c_2 \\
  d^c_3 \\
  \nu^c \\
  e^c 
\end{pmatrix} \]
\[ 10 = \begin{pmatrix}
  0 & u^c_3 & -u^c_2 & -u^c_1 & -d^c_1 \\
  0 & u^c_1 & -u^c_2 & -d^c_2 \\
  0 & -u^c_3 & -d^c_3 \\
  0 & -e^c \\
  0 & 0 
\end{pmatrix} \]  

with the matrix of the 10 being anti-symmetric. A technical remark: It is important to notice that the sum of these two representations is anomaly-free. A second physical remark is that, unless one introduces an SU(5) singlet, there is no room for a right-handed neutrino.

Let us finally study the Higgs system. The first symmetry breaking goes through a 24-plet of scalars. It is convenient to represent the 24 as a 5 x 5 traceless matrix. The vacuum expectation value which breaks SU(5) down to U(1) x SU(2) x SU(3) is proportional to the diagonal matrix:

\[ \lambda_{24} = \begin{pmatrix}
  1 & 1 & \circ & \circ & \circ \\
  1 & -3/2 & \circ \\
  \circ & \circ & -3/2 
\end{pmatrix} \]  

Can we use the same 24-plet of Higgs in order to obtain the second breaking of the standard model? The answer is no for two reasons: First, the 24- does not contain any (2, 1) piece (see eq.(5)) which is the one needed for the U(1) x SU(2) \( \rightarrow U(1)_{\text{e.m.}} \) breaking. Second, the 24- does not have the required Yukawa couplings to the fermions. Indeed with the \( \bar{5} \) and 10 assignment the fermions can acquire masses through Yukawa couplings with scalars belonging to one of the representations in the products:

\[ \bar{5} \times 10 = 5 + 45 \]  
\[ 10 \times 10 = 5 + 45 + 50 \]  

We see that the 24- is inoperative while the 5 looks promising. If we restrict ourselves to this simplest choice we have two independent Yukawa couplings. Looking back at the assignment (8) we see that the up-quarks take their masses through (11) while the down-quarks and the leptons through (10).
This discussion answers the first question, namely it shows that there exist groups which have the required representations to be used as groups of grand unification. Before turning to the next question and extract the dynamical consequences of such a scheme, let us give a second example of a possible G.U.T. which presents some different and interesting features:

II.A.2: A rank 5 G.U.T.: SO(10):

Following the same method as before, we list all possible rank 5 candidates together with the reasons that exclude them:

\[ [SU(2)]^5 : \text{no SU}(3) \text{ subgroup}. \]

\[ SO(11), Sp(10) : \text{no complex representations}, \]
\[ \text{no 15- or 16-dimensional representations}. \]

\[ SU(6) : \] It has a 15-dimensional representation but the decomposition in \( SU(2) \times SU(3) \) representations shows that it cannot accommodate the members of a family. One finds:
\[ 15 = (2, 3) + (1, 5) + (1, 3) + (2, 1) + (1, 1) \]  
(12)

The troublesome piece is the \((1, 3)\) which is a singlet of \( SU(2) \) and a triplet of color rather than an anti-triplet.

The final candidate for a rank 5 G.U.T. is \( SO(10) \) which has a 16-dimensional representation. \( SO(10) \) contains \( SU(5) \) as a subgroup and the 16-plet decomposes into:
\[ 16 = 10 + 5 + 1 \]  
(13)
i.e. under \( SU(5) \) we find the 5 and 10 we used before and a singlet. The obvious interpretation of this last one is a right-handed neutrino (or \( \nu^C_L \)).

In the simple \( SU(5) \) scheme parity violation, which is observed at present energies, is a fundamental law of nature. Present experimental evidence notwithstanding, it is attractive to speculate that this violation is a low-energy accident and that the underlying theory is ambidextrous. This leads us to extending \( SU(2) \) of the standard model to \( SU(2)_L \times SU(2)_R \). For phenomenological purposes it is sufficient that the gauge bosons of \( SU(2)_R \) are a few times heavier than \( W^\ast \). The simplest grand-parent of this model is precisely \( SO(10) \).

The salient features of this G.U.T. are the following: It has 45 gauge bosons which, under the \( SU(5) \) subgroup, transform as:
\[ 45 = 24 + 10 + 5^2 + 1 \]  
(14)

As mentioned above, the fermions of each family, together with a corresponding right-handed neutrino, form a 16-dimensional representation. In the long journey from \( SO(10) \) down to \( U(1) \times SU(2) \times SU(3) \), nature may choose various paths. She can take the direct road (just one big break), or she may decide to go through one of the intermediate subgroups:
The Higgs system depends on the breaking pattern we choose but, in any case, it is more complex than that of SU(5). Several representations are necessary.

II.B -

II.B.1. Dynamics of G.U.T.'s : The coupling constants:

Let us now turn to the second question, namely, are the dynamical consequences of grand-unified theories compatible with experiment? This is not a trivial question and, at first sight, the answer seems to be negative. A grand-unified theory is based on a single simple group (or a direct product of simple factors with discrete symmetries which interchange them) and hence it has only one coupling constant. On the other hand, in nature we observe three distinct coupling constants $g_1$, $g_2$ and $g_3$ corresponding to the group of the standard model $U(1) \times SU(2) \times SU(3)$. They are often parametrized as $g_3$ (strong interactions), $g_2$ (weak interactions) and $sin^2 \theta_W$ given by

$$sin^2 \theta_W = \frac{g_1^2}{g_1^2 + g_2^2}$$

where the $U(1)$ generator $Y$ is related to the electric charge $Q$ and the third component of weak isospin $T_3$ by:

$$Q = T_3 - Y$$

For the embedding of $U(1) \times SU(2) \times SU(3)$ in the G.U.T. group $G$ all generators must be normalized the same way. Let us put

$$Tr(J_i J_j) = R S_{i,j}$$

where $R$ is a constant which may depend on the representation we use to compute the trace, but it is independent of $i$ and $j$. Let us now compute $Tr(T_3^2)$ using, for example, the electron family. (We assume once more that the observed members of a family completely fill a representation, possibly reducible, of $G$). We find $Tr(T_3^2) = 2$. Similarly we find $Tr(Y^2) = 10/3$. Therefore we see that for the embedding the $U(1)$ generator must be rescaled by $Y \rightarrow c Y$ with $c^2 = 5/3$. Therefore (15) gives

$$sin^2 \theta_W = \frac{g_1^2/c^2}{g_1^2/c^2 + g_2^2} = \frac{3}{8}$$

We conclude that the naive G.U.T. "prediction" is
\[
G_3 = G_2, \quad \sin^2 \theta_w = \frac{3}{8} \tag{19}
\]

These "predictions", especially the first one, are so far from the truth that the whole G.U.T. idea seems totally wrong! The claim that all interactions can be described in terms of a single coupling constant sounds incredible.

In order to answer this question we must go with some detail into the renormalization program of a spontaneously broken theory. The reader who is not interested in technicalities can skip this part and go directly to the conclusions. For pedagogical purposes let me explain the case of \( G = \text{SU}(5) \). The changes for any other group are straightforward. Furthermore, let me ignore the second breaking of \( U(1) \times \text{SU}(2) \longrightarrow U(1)_{\text{e.m.}} \). This means that we shall have all fermions massless and we shall begin only the 24-plet of Higgs scalars. We start from the Lagrangian:

\[
\mathcal{L} = -\frac{i}{4} \mathcal{T}_{\mu \nu} G_{\mu \nu} G^{\mu \nu} + i \bar{\psi} \partial \psi + \frac{i}{2} \mathcal{T}_{\mu} \left( D_{\mu} \Phi^* \Phi \right) - \frac{1}{2} \mu^2 \mathcal{T}_{\mu} \left( \Phi^2 \right) - \frac{h_1}{4} \left( \mathcal{T}_{\mu} \left( \Phi^2 \right) \right)^2 - \frac{h_2}{2} \mathcal{T}_{\mu} \left( \Phi^4 \right) \tag{20}
\]

where \( G_{\mu \nu} \) is the usual Yang-Mills field strength of SU(5) which, in terms of the 24 gauge bosons \( G_{\mu} \), is given by:

\[
G_{\mu \nu} = \partial_{\mu} G_{\nu} - \partial_{\nu} G_{\mu} - i q \left[ G_{\mu}, G_{\nu} \right] \tag{21}
\]

\( \psi \) is a column with all fermion fields and \( \Phi \) is the 24-plet of Higgs fields. \( G_{\mu \nu}, G_{\mu} \) and \( \Phi \) are written as \( 5 \times 5 \) traceless hermitian matrices. \( D \) is the SU(5) covariant derivative for the fermions and Higgs fields. The Lagrangian (20) contains four independent parameters, namely \( m^2, h_1, h_2 \) and \( g \). In order to obtain a spontaneous symmetry breaking we choose \( m^2 < 0 \) and we write

\[
\Phi \rightarrow \Phi + \frac{V}{\sqrt{2}} \lambda_{24} \tag{22}
\]

with \( \lambda_{24} \) given by eq. (9). We then find the following mass spectrum:

(i) \( M^2 = \frac{5}{3} V^2 g^2 \) for the vector gauge bosons \( G_3, \ldots G_{20} \)  \hspace{1cm} (23a)

(ii) \( M_3^2 = \frac{1}{3} V^2 h_2 \) for the scalars \( \phi_1, \ldots \phi_8 \)  \hspace{1cm} (23b)

(iii) \( M_2^2 = \frac{4}{3} V^2 h_2 \) for the scalars \( \phi_{21}, \phi_{22}, \phi_{23} \)  \hspace{1cm} (23c)

(iv) \( M_1^2 = 2V^2 (h_1 + \frac{7}{15} h_2) \) for \( \phi_{24} \)  \hspace{1cm} (23d)

In terms of the parameters of (20), \( V^2 \) is given, in the tree approximation, by:

\[
V^2 = - m^2 \left[ h_1 + \frac{7}{15} h_2 \right]^{-1} \tag{24}
\]
Notice that, as expected, the gauge bosons $G_1, ..., G_8$ (gluons) as well as $G_{21}, ..., G_{24}$ (electroweak gauge bosons) remain massless while the scalars $\Phi_1, ..., \Phi_{20}$ disappear via the Higgs mechanism. The next step is to renormalize the theory. This means that we have to assign prescribed values taken from experiment to some quantities. In our example of eq. (20) we have four independent parameters and a convenient choice is to use three masses $M_1^2$, $M_2^2$ and $M_3^2$ and the gauge coupling constant $g$. In addition we need the usual wave-function renormalization conditions for the fermion and boson fields of the theory. The mass renormalizations are done by specifying $M_1^2$, $M_2^2$ and $M_3^2$ as the poles of the corresponding propagators. The coupling constant $g$ can be chosen to be the value of the fermion-gauge boson three point function at a certain scale of the external momenta $p_i^2 = -\mu^2$.

$$\Gamma^A_{\alpha\beta}(\mu^2) = i g_A(\mu^2) \gamma_{\alpha} (T^A)_{\alpha\beta}$$ (25)

However, since SU(5) is broken, we must specify which particular gauge boson we are using in the definition (25). This is done with the index $A$ which runs from 1 to 24. The conservation of $U(1) \times SU(2) \times SU(3)$ implies that we only need to distinguish four distinct cases: $A = 1, ..., 8$; $A = 9, ..., 20$; $A = 21, 22, 23$; $A = 24$. Once the condition is imposed using any one of these, all the others become finite and calculable. The same remark applies to the wave function renormalization conditions for which again one must specify the component of the vector and scalar fields one is using.

I want to emphasize that, once these three kinds of renormalization conditions (wave functions, masses and coupling constant) have been imposed, the perturbation theory is completely defined and all Green's functions are finite and calculable as formal power series in $g_A$. Furthermore, it is not possible to impose any further conditions. Notice, in particular, that there exists only one gauge coupling constant, as one should expect from a grand unified theory. Does this mean that we could compute a Q.C.D. process, like a deep inelastic structure function, as a power series in the weak interaction coupling constant? Formally the answer is yes, but in practice this is not so. Formal perturbation theory guarantees that, if we choose, in (25), A to denote one of the SU(2) gauge bosons, all other three point functions are finite and calculable. In particular, if $\Gamma^{(3)}_3$ is the three-point function with an SU(3) external gauge boson, we can write: (I shall use the following notation: For $A = 1, ..., 8$, $g_A = g_3$; for $A = 21, ..., 23$, $g_A = g_2$; $g_A = 24 = g_1$):

$$\Gamma^{(3)}_3(p^2; -\mu^2) \equiv g_3 = g_2 + R_1 g_3^2 + R_2 g_2^3 + \cdots$$ (26)

where the $R_n$'s are finite and calculable functions of $\mu^2$ and the masses of the theory. However, it is easy to check that $R_n$ is of the form:

$$R_n \sim (\ln \frac{M^2}{\mu^2})^n$$ (27)
which, for $M/\mu \sim 10^{14}$ gives $[65]^n$. In other words, although the series (26) is well defined, it is useless for practical computations.

The remedy to this difficulty is simple. We shall renormalize the same broken SU(5) theory in three different ways where the index A in the condition (25) denotes the bosons of U(1), SU(2) or SU(3). This gives us three perturbation expansions in powers of $g_1$, $g_2$ or $g_3$, always of the same theory, but now each one is suited to particular processes. The values of the $g_i$'s will be fixed by experiment, but we must always remember that we are talking about one and the same theory, so we write the analog of eq. (26):

$$g_i = F_{ij} \left( g_j, \frac{M^2}{\mu^2}, \alpha \right)$$  \hspace{1cm} (28)

where with $\alpha$ we mean the ratios $M_1^2/M^2$ and $M_2^2/M^2$. In the limit of exact SU(5) symmetry, i.e. when $M^2/\mu^2 \rightarrow 0$, all coupling constants must be equal. This only happens at infinite energy

$$F_{ij} \left( g_j, 0, \alpha \right) = g_j$$  \hspace{1cm} (29)

By taking $\mu^2 d/d\mu^2$ on both sides of (28) we obtain

$$\beta_i \left( F_{ij}, \alpha, \alpha \right) = \left[ -\alpha \frac{\partial}{\partial \alpha} + \beta_j \left( g_j, \alpha, \alpha \right) \right] F_{ij}$$  \hspace{1cm} (30)

where

$$\beta_k \left( g_k, \alpha, \alpha \right) = \mu^2 \frac{d}{d\mu^2} g_k$$  \hspace{1cm} (31)

and $\alpha = M^2/\mu^2$.

The differential equation (30) with the boundary condition (29) is our basic equation. The $\beta$-functions are calculable at any given order of perturbation theory and they are of the form:

$$\beta_k \left( g_k, \alpha, \alpha \right) = b_k^0 \left( \alpha, \alpha \right) g_k^3 + \ldots$$  \hspace{1cm} (32)

Notice that the $b$ coefficients, unlike those of $F_{ij}$ itself, do not contain large logarithms. This can be easily understood since the $\beta$-functions, as defined by eqs. (31) and (25), possess well-defined limits both for $M^2/\mu^2 \rightarrow \infty$ (when they become the $\beta$-functions of SU(3), SU(2) or U(1)) and $M^2/\mu^2 \rightarrow 0$ when they all become equal to that of SU(5). This is a consequence of the decoupling theorem. On the contrary, $F_{ij}$ has no limit when $M^2/\mu^2 \rightarrow \infty$ with $g_j$ kept fixed.

Equation (30) can be solved by the standard methods of characteristics. The solution expresses any coupling constant in terms of any other.

$$g_i = F_{ij} = \eta \left( g_j, \alpha, \alpha \right)$$  \hspace{1cm} (33)
with \( \eta \) a given function. For example, using the one-loop \( \beta \)-functions of eq. (32) we find:

\[
\frac{1}{\alpha_i^2} = \frac{1}{\alpha_j^2} + 2 \int_0^\Lambda \frac{dx}{x} \left[ b_i^0(x, \alpha) - b_j^0(x, \alpha) \right]
\]

(34)

We now must use, as input, the experimentally measured effective strengths of strong, electromagnetic and weak interactions at moderate (say \( p^2 \sim 10\text{–}100 \text{ GeV}^2 \)) energies. Using the renormalization group for \( \mu^2 = -p^2 \), we write \( g_i = \tilde{g}_i(p^2) \) and \( \lambda = -\mu^2/p^2 \). The two independent equations given by the relations (33) (i, j = 1, 2, 3) contain three unknown parameters, namely \( \lambda \) and the Higgs masses \( M_i^2/M^2 \) and \( M_2^2/M^2 \) denoted by \( \alpha \). However, it turns out that the dependence on \( \alpha \) is very weak. If we ignore it for the moment, then the two equations can be used to determine \( \lambda \) and to predict the value of \( \sin \Theta_W \). A precise calculation must take into account the breaking of \( U(1) \times SU(2) \) as well. In fact it turns out that the value of \( \sin \Theta_W \) is quite sensitive to this last breaking. The result, including the two-loop effects is:

\[
\begin{align*}
M &= 3.1 \times 10^{14} \pm 0.3 - 0.2 (n_h - 1) + 0.1 (F - 3) \left( \frac{\Lambda_{\overline{MS}}}{0.2} \right)^{1.03} \text{ GeV} \\
\tau_p &= 10^{29} \pm 2.1 - 0.8 (n_h - 1) + 0.4 (F - 3) \left( \frac{\Lambda_{\overline{MS}}}{0.2} \right)^{0.1} \text{ years} \\
\left( \sin^2 \Theta_W \right)^{exp} \left( \mu^2 = 20 \text{ GeV}^2 \right) &= 0.216 \pm 0.0071 \\
&+ 0.004 (n_h - 1) - 0.004 (F - 3) - 0.0056 \left( \frac{\Lambda_{\overline{MS}}}{0.2} \right)
\end{align*}
\]

(35a, 35b, 35c)

where all uncertainties have been included. By far the largest one is due to the uncertainty in the Q.C.D. coupling constant, or equivalently, the parameter \( \Lambda \). This has been included in the form of the ratio of its value in the \( \overline{MS} \) scheme and 200 MeV. \( n_h \) and \( F \) are the numbers of the Higgs doublets and fermion families respectively. The remaining uncertainty reflects our ignorance of the heavy Higgs masses (the parameters \( \alpha \) in (28)), the precise value of the t-quark mass, etc. \( \tau_p \) is the predicted proton life-time in years and an estimation of the uncertainty due to the proton structure is included. We see that, unless the number of families is large, the model is already in trouble with the latest data. The calculated value of \( \sin^2 \Theta_W \) (35c) should be compared with the experimental average:

\[
\left( \sin^2 \Theta_W \right)^{exp} \left( \mu^2 = 20 \text{ GeV}^2 \right) = 0.216 \pm 0.01
\]

(36)

A new series of precision measurements are already in progress in order to reduce the experimental error on this very fundamental parameter. But we can already say that the agreement between theory and experiment is spectacular.

In Fig. 1a we show the variation of the effective coupling constants, defined by (25), with respect to the scale \( \mu^2 \). Figure 1b gives, at a larger scale, the region around \( M \). We see that for \( \mu \gtrsim 10 M \) all the three coupling constants essentially coincide and follow the variation given by the \( \beta \)-function of SU(5). Also, for \( \mu \ll 10^{-3} M \), the three coupling constants are approximately decoupled and each one varies according to the renormalization group equations of U(1), SU(2) or SU(3). This is the justification of the "step function approximation" which consists in putting, for each scale \( \mu \), all the masses larger than \( \mu \)
Fig. 1a - The variation of the effective coupling constants with the energy scale.

Fig. 1b - Expanded view of the approach to grand unification
Fig. 2a - The variation of $\sin^2 \theta_W$ with the energy scale

Fig. 2b - Expanded view near $M_W^2$
equal to infinity, and all the masses smaller than \( \mu \) equal to zero. In fact, figs. 1a and 1b give a nice illustration of the decoupling theorem: Let us consider a renormalizable theory given by a Lagrangian density \( \mathcal{L}(Q_i \bar{Q}_i, Q_i) \) in terms of a set of fields \( Q_i(x) \) \( i = 1, \ldots, n \). We are interested in the limit in which some of the fields, let us say \( Q_j(x) \) for \( j = k + 1, \ldots, n \), become very massive \( m_j \to \infty \). The decoupling theorem states that, in this limit, the resulting theory is given by a Lagrangian density \( \mathcal{L}'(Q_i \bar{Q}_i, Q_i) \) \( i = 1, \ldots, k \) which is obtained from the initial \( \mathcal{L} \) by setting all the "superheavy" fields \( Q_j \) with \( j = k + 1, \ldots, n \) equal to zero, provided the resulting \( \mathcal{L}' \) still defines a renormalizable theory. Figures 2a and 2b give the corresponding variation of \( \sin^2 \theta_W \). The nucleon decay branching ratios can be computed using any specific constituent model. The results are roughly model independent and the two-body \( \pi^0 \gamma \) decay mode is predicted to be dominant.

This analysis applies to other G.U.T. models as well. For our SO(10) example the results depend on the symmetry breaking pattern. If we choose a double stage one we must introduce two mass scales and we loose the prediction on \( \sin^2 \theta_W \) which now must be used from experiment in order to determine the value of the additional parameter of the model. On the contrary, the nucleon life-time and decay modes are not changed substantially.

II.B.2. Fermion masses

Fermion masses are generated in G.U.T.'s through the same mechanism as in the standard model, i.e. through Yukawa couplings with Higgs scalars. Therefore the detailed spectrum is model dependent. In the minimal SU(5) model with a 5-plet of Higgs scalars we saw, in eqs. (10) and (11) that we have two independent coupling constants for each family. We thus obtain the relations

\[
M_d = M_e \quad ; \quad M_s = M_\tau \quad ; \quad M_b = M_\tau
\]

(37)

These relations are valid at \( \sim 10^{14} \) GeV. We can follow them down, using the renormalization group equations and we obtain:

\[
\frac{m_b}{m_\tau} = 2.7 - 3 \quad ; \quad \frac{m_d}{m_s} = \frac{m_e}{m_\tau} = \frac{1}{200}
\]

(38)

The first one is welcome and can be qualitatively understood because the quarks, having strong interactions, become heavier. In fact one finds that:

\[
\frac{m_b}{m_\tau} = \left[ \frac{\alpha_s(Q = 2m_b)}{\alpha_s(Q = M)} \right]^{12/55-F}
\]

(39)

where \( \alpha_s \) is the strong interaction coupling constant evaluated at \( Q = 2m_b \sim m_\tau \) (the upsilon mass) and at \( M \sim 10^{14} - 10^{15} \) GeV respectively, and \( F \) is the number of families. For \( F = 3 \) we find a very good agreement with experiment \( (m_\tau \sim 1.8 \text{ GeV}, m_\gamma \sim 2m_b \sim 9.5 \text{ GeV}) \), while the agreement gets much worse with increasing \( F \).

Unfortunately, the second of the relations (38) is in very poor shape. Any estimation of \( m_d/m_s \) based on chiral dynamics gives a result which is ten times larger. We can improve the agreement by using a second Higgs multiplet, for example the 45 appearing in (10), but the number of parameters increases accordingly.
II.B.3. B-L and neutrino masses

My final remark on G.U.T.'s is devoted to baryon and lepton number violation. Let us concentrate on the minimal SU(5) model, involving a 5 and a 24 of Higgs's. In this case the Lagrangian is invariant under a group of U(1) phase transformations (global). If \( \psi_{10} \) and \( \psi_{5} \) are the fermion multiplets and \( \Phi_{24} \) and \( \Phi_{5} \) the Higgs scalars, the transformations are:

\[
\psi_{10} \rightarrow e^{i\theta} \psi_{10} \quad ; \quad \psi_{5} \rightarrow e^{-3i\theta} \psi_{5} \quad ; \quad \Phi_{5} \rightarrow e^{-2i\theta} \Phi_{5} \tag{40}
\]

with all other fields invariant. The non-zero vacuum expectation value of \( \Phi_{5} \) breaks this symmetry spontaneously. This sounds disastrous since it, normally, leads to the appearance of a truly massless Goldstone boson. However we are saved because the symmetry is not really broken, it is simply changed. We can check immediately that, even after the translation of the Higgs's, the linear combination \( f_{1} + 4Y \) remains as a global symmetry, where \( f_{1} \) is the generator of (40) and \( Y \) the U(1) part of SU(5) given by (16). The conserved charge of this symmetry is the difference B-L of baryon and lepton numbers. This conservation has some very important consequences. First it gives precise predictions for the decay properties of the proton. For example \( p \rightarrow e^{+} + n^{0} \) is allowed but \( n \not\rightarrow e^{-} + n^{+} \). This property remains true (or very nearly true) for essentially all grand unified models, so its experimental verification (assuming of course that baryon decay is observed) is of the utmost importance. A second consequence of B-L conservation is that the neutrino cannot acquire a Majorana mass. Since, on the other hand, a Dirac mass is impossible, (there is no right-hand neutrino component in the standard SU(5)) we conclude that neutrino oscillation experiments can be used to test SU(5). Notice that even if we break B-L by introducing extra Higgs scalars, the expected neutrino Majorana mass is very tiny (\( \sim 10^{-5} \) eV). Therefore, if the recent measurement of non-vanishing neutrino mass in tritium \( \beta \)-decay is confirmed, the SU(5) model, at least in its present form, should be abandoned. The only way to save it would be to introduce a \( \nu_{L}^{c} \) as singlet but then all the simplicity of the model disappears and one would be forced to go to the SO(10) model where, as we already said, a right-handed neutrino is naturally present. In fact, the main experimental prediction of SO(10), which differs substantially from those of SU(5), concerns the neutrino mass. In SO(10) B-L is a gauge generator and it is spontaneously violated when SO(10) is broken. (Since it is a gauge generator it has to be violated otherwise there would be a massless gauge boson coupled to it.) The presence of \( \nu_{R}^{c} \) allows for a neutrino Dirac mass of the form \( \nu_{R}^{c} \nu_{L} \) and the violation of B-L allows for a Majorana one. Therefore we naturally expect massive neutrinos. Let me further notice that the above-mentioned violation of B-L does not lead to measurable effects in nucleon decay because the branching ratio of forbidden to allowed nucleon decays is predicted to be very small

\[
\frac{n \rightarrow e^{-} + n^{0}}{p \rightarrow e^{+} + n^{0}} \approx \frac{M_{W}^{2}}{M_{W}^{2}} \sim 10^{-24} \tag{41}
\]

Proton decay is not sensitive to the different grand-unified models.
III. SUPERSYMMETRY

"Οὐκ ἄγω τοῦτο ἀπαλλάξομαι ἀπεκδεχόμενος...

"I am not come to destroy, but to fulfil" 

III.A -

III.A.1. The trial of scalars

The purpose of this chapter is not to destroy, but to fulfil. It is our firm belief, shared by most physicists, that gauge theories have come to stay. "Beyond" here does not mean that we propose to replace gauge theories by something else, but rather to embed them into a larger scheme with a tighter structure and higher predictive power. There are several reasons for such a search.

As we said in chapter I, gauge theories contain two and possibly three independent worlds. The world of radiation with the gauge bosons, the world of matter with the fermions and, finally, in our present understanding, the world of Higgs scalars. In the framework of gauge theories these worlds are essentially unrelated to each other. Given a group $G$ the world of radiation is completely determined, but we have no way to know a priori which and how many fermion representations should be introduced; the world of matter is, to a great extent, arbitrary.

This arbitrariness is even more disturbing if one considers the world of Higgs scalars. Not only their number and their representations are undetermined, but their mere presence introduces a large number of arbitrary parameters into the theory. Just compare Q.C.D. with massless quarks with the $U(1)$ x $SU(2)$ model. The first contains, according to our present understanding, two arbitrary parameters: a mass scale $\Lambda$ and a vacuum angle $\Theta$. $U(1)$ x $SU(2)$ on the other hand, with three families contains eighteen. Notice that this is independent of our computational ability, since these are parameters which appear in our fundamental Lagrangian. What makes things worse, is that these arbitrary parameters appear with a wild range of values. For example, in the standard model, the ratio of yukawa couplings for different fermions equals the ratio of fermion masses. But $m_t/m_e > 10^5$ and it is hard to admit that such a number is a fundamental parameter.

The situation becomes even more dramatic in grand unified theories where one may have to adjust parameters with as many as twenty-six significant figures. This is the problem of gauge hierarchy which is connected to the two enormously different mass scales at which spontaneous symmetry breaking occurs. The breaking of $G$ into $U(1)$ x $SU(2)$ x $SU(3)$ happens at $M \sim 10^{14}$ GeV. This means that a certain Higgs field $\bar{\Phi}$ acquires a non-zero vacuum expectation value $V = \langle \bar{\Phi} \rangle \sim 10^{14}$ GeV. The second breaking, that of $U(1)$ x $SU(2)$, occurs at $\sim 10^2$ GeV, i.e. we must have a second scalar field $\phi$ with $v = \langle \phi \rangle \sim 10^2$ GeV. But the combined Higgs potential will contain a term of the form $\lambda \phi^2 \bar{\Phi}^2$. Therefore, after the first breaking, the $\phi$-mass will be given by:

$$m_{\phi}^2 = \mu^2 + 2\lambda V^2$$ 

(42)
where $\mu$ is the mass appearing in the symmetric Lagrangian. On the other hand, I want to remind you that $v^2 \sim m_\rho^2$, so unless there is a very precise cancellation between $\mu^2$ and $2 \lambda v^2$, a cancellation which should extend to twenty-six decimal figures, $v^2$ will turn out to be of order $v^2$ and the two breakings will come together, in other words the theory is not able to sustain naturally a gauge hierarchy. This grand-fine tuning of parameters must be repeated order by order in perturbation theory because, unlike fermions, scalar field masses require quadratically divergent counterterms. The whole structure looks extremely unlikely. The problem is similar to that of the induced cosmological constant in any theory with spontaneous symmetry breaking. I believe that, in spite of its rather technical aspect, the problem is sufficiently important so that some new insight will be gained when it is eventually solved.

One possible remedy is to throw away the scalars as fundamental elementary particles. After all their sole purpose was to provoke the spontaneous symmetry breaking through their non-vanishing vacuum expectation values. In non-relativistic physics this phenomenon is known to occur but the role of Higgs fields is played by fermion pairs (ex. the Cooper pairs in superconductivity). Let me also remind you that the spontaneous breaking of chiral symmetry, which is supposed to be a fundamental property of Q.C.D., does show the same feature, namely the "vacuum" is formed by quark-antiquark pair condensates and the resulting Goldstone boson (the pion) is again a $q\bar{q}$ bound state. This idea of dynamical symmetry breaking has been studied extensively, especially under the name of "Technicolor". In spite of its many attractive features, it suffers, up to now, from two main difficulties. First, the available field theory technology does not allow for any precise quantitative computation of bound-state effects and everything has to be based on analogy with the chiral symmetry breaking in Q.C.D. Second, nobody has succeeded in producing a satisfactory phenomenological model. Nevertheless there is still hope that these difficulties may be overcome and, independently, the scheme has some precise predictions which can be tested experimentally in the near future.

III.A.2. The defense of scalars

The best defense of scalars is the remark that they are not the only ones to reduce the predictive power of a gauge theory. As we have already seen, going through the chain radiation-fermion matter fields- Higgs scalars we encounter an increasing degree of arbitrariness. One possibility which presents itself is to connect the three worlds with some sort of symmetry principle. Then the knowledge of the vector bosons will determine the fermions and the scalars and the absence of quadratically divergent counterterms from the fermion masses will forbid their appearance in the scalar masses. Let me point out that no such symmetry between fermions and bosons is manifest in the particle spectrum with the exception of a possible unexplained degeneracy between the photon and the neutrinos.

Is it possible to construct such a symmetry? A general form of an infinitesimal transformation acting on a set of fields $\Phi^i(x)$, $i = 1, \ldots, m$ can be written as:

$$C \Phi^i(x) = \epsilon^\alpha (T_\alpha)_j^i \Phi^j(x)$$

(43)

where $\alpha = 1, \ldots, n$, the $\epsilon$'s are infinitesimal parameters and $T_\alpha$ is the matrix of the
representation of the fields. Usually the $\epsilon$'s are taken to be c-numbers in which case the transformation (43) mixes only fields with the same spin and obeying the same statistics, fermions with fermions and bosons with bosons. It is clear that if we want to change the spin of the fields with a transformation (43), the corresponding $\epsilon$'s must transform non-trivially under rotations. If they have non-zero integer spin they can mix scalars with vectors or spin-1/2 with spin-3/2 fields. This was the case with the old attempts to construct a relativistic SU(6) theory with its well-known shortcomings. If, on the other hand, the $\epsilon$'s are anti-commuting parameters, they will mix fermions with bosons. If they have zero spin, the transformations (43) will change the statistics of the fields without changing their spin, i.e. they will turn a physical field into a ghost. This is the case with the B.R.S. transformation which is so useful in the quantization of non-abelian gauge theories. Here, however, we want to connect physical bosons with physical fermions, therefore the infinitesimal parameters must be anti-commuting spinors. We call such transformations "supersymmetry transformations" and we see that a given irreducible representation will contain both fermions and bosons. It is not a priori obvious that such supersymmetries can be implemented consistently, but in fact they can. In the following I shall give a very brief description of their properties as well as their possible applications to gauge theories.

III.B -

III.B.1 The algebra

Rather than going through mathematical preliminaries, I shall directly give the algebraic scheme which turns out to be interesting for physics. It is based on the well-known Poincaré algebra whose generators are $P_\mu$, $\mu = 0, 1, 2, 3$ for the four translations and $M_{\mu \nu}$ for rotations and Lorentz transformations. In addition we shall introduce four new generators $Q_\alpha$ forming a four-component Majorana spinor. This means that the commutator of $Q_\alpha$ with $M_{\mu \nu}$ is given by:

$$[Q_\alpha, M_{\mu \nu}] = i(\gamma^{\mu \nu})_{\alpha \beta} Q_\beta$$

(44)

with $\gamma^{\mu \nu} = \frac{1}{4} [\gamma^\mu, \gamma^\nu]$. The relation (44) only expresses the fact that $Q$ is a spinor. Furthermore we shall assume that $Q$ is translationally invariant and anti-commutes with itself:

$$[P_\mu, Q_\alpha] = 0 = [Q_\alpha, Q_\beta] = [\bar{Q}_\alpha, \bar{Q}_\beta]$$

(45)

where $\bar{Q} = Q^T \gamma^0$ and $[\ ]_+$ denotes the anti-commutator. The last relation required to close the algebra is the anti-commutator of $Q$ with $\bar{Q}$. We postulate:

$$[Q_\alpha, \bar{Q}_\beta]_+ = -2 (\gamma^\mu)_{\alpha \beta} P_\mu$$

(46)

The relations (44)-(46) form the supersymmetry algebra we are going to use. We shall not attempt to justify its particular form on mathematical grounds but we shall derive its physical consequences. Before doing so, let me write it in an alternative form, using two component complex Weyl spinors instead of four component real Majorana ones. We can rewrite (45) and (46) as:
\[ [P_r, Q] = [P_r, \bar{Q}] = [Q, \bar{Q}] = [\bar{Q}, \bar{Q}] = 0 \quad (45') \]

\[ [Q^\alpha, \bar{Q}^\beta] = 2 (\sigma^\mu)_{\alpha \beta} P^\mu \quad (46') \]

with \( \sigma^\mu = (1, \sigma^\mu) \).

An obvious generalization consists in starting from the Poincaré algebra \( x \) a compact internal symmetry \( G \) with generators \( A_i \). If the \( Q \)'s belong to a certain representation of the internal symmetry, we write \( Q^\alpha_m \) where \( \alpha \) is the spinor index and \( m \) that of the internal symmetry \( m = 1, \ldots, N \). We then have:

\[ [A_i, A_j] = i f^k_{ij} A_k \quad i, j, k = 1, \ldots, \ell \quad (47) \]

\[ [A_i, Q^\alpha_n] = i S^m_{in} Q^\alpha_n \quad n, m = 1, \ldots, N \quad (48) \]

\[ [Q^\alpha_n, Q^\beta_m] = [Q^\alpha_n, P_r] = 0 \quad (49) \]

\[ [Q^\alpha_n, \bar{Q}^\alpha_m] = 2 \delta^{mn} (\sigma^\mu)_{\alpha \beta} P^\mu \quad (50) \]

The meaning of these relations is clear: The first one defined the internal symmetry with \( f^k_{ij} \) the structure constants of the group \( G \). The second gives the \( N \)-dimensional representation of \( G \) in which the \( Q \)'s belong and \( S^m_{in} \) are the corresponding constants. Finally (49) and (50) generalize in a particular way the basic \( N = 1 \) supersymmetry relations (45) and (46) or (45') and (46').

### III.B.2. All possible supersymmetries of the S-matrix

The reader may feel uneasy with the very particular and seemingly arbitrary form of the algebra we introduced in the previous section. So this is the right moment to state, without proof, a very powerful theorem. It started in the sixties, when people tried unsuccessfully to combine Poincaré invariance with an internal symmetry (at that time it was SU(3) flavor symmetry) into a single larger group. Eventually a no-go theorem was proven showing that the only such combination which may be a symmetry of a unitary \( S \)-matrix is the trivial one given by the direct product of Poincaré and internal symmetry. The algebra (47)-(50) seems to contradict this result. In fact it does not, because one of the assumptions of the theorem was that the algebraic scheme was using only commutators among the different generators. Now we can go back to the proof of the no-go theorem and relax this assumption allowing for anti-commutators as well as commutators. The remarkable result is that the supersymmetries we considered are essentially (apart from trivial generalizations) the only admissible ones. No other scheme would lead to a unitary \( S \)-matrix.

### III.B.3. Representations in terms of one-particle states

In order to extract the possible physical consequences of supersymmetry we must construct the representations of the algebra in terms of one-particle states, i.e. the one-particle "supermultiplets". We start by observing that the spinorial charges commute with \( P_r \) and...
therefore they do not change the momentum of the one-particle state. Furthermore the operator 
\( p^2 \) commutes with all the operators of the algebra which implies that all the members of a
supermultiplet will have the same mass. We can distinguish two cases \( p^2 \neq 0 \) or \( p^2 = 0 \).

(i) Massive case. We can go to the rest frame in which the r.h.s. of (46') or of (50)
becomes a number. Let us first forget about a possible internal symmetry and consider the
case \( N = 1 \). Eq. (46') gives:

\[
\left[ Q_\alpha, \overline{Q}_\beta \right]_+ = 2M \delta_{\alpha\beta}
\]  (51)

where \( p^2 = N^2 \). Equation (51) implies that the operators \( Q/\sqrt{2N} \) and \( \overline{Q}/\sqrt{2N} \) satisfy the anti-
commutation relation for creation and annihilation operators of a system of free fermions.
Since the index \( \alpha \) can take two values, 1 and 2, and \( Q_1^2 = Q_2^2 = 0 \), starting from any one-
particle state with spin \( S \) and projection \( S_z \), we can build a four-dimensional Fock space with
states:

\[
|S, S_z; n_1, n_2\rangle = Q_2^{n_2} Q_1^{n_1} |S, S_z\rangle \quad n_1, n_2 = 0, 1
\]  (52)

Some specific examples:

\( S = 0 \) \( \Rightarrow \) one spin-\( \frac{1}{2} \) particle, two spin-zero particles

\( S = \frac{1}{2}, S_z = \pm \frac{1}{2} \) \( \Rightarrow \) one spin-zero, two spin-\( \frac{1}{2} \), one spin-one

\( S = 1, S_z = 0, \pm 1 \) \( \Rightarrow \) two spin-one, one spin-\( \frac{3}{2} \), one spin-\( \frac{1}{2} \)

etc...

The generalization to include internal symmetries is straightforward. The difference is
that now we have more creation operators and the corresponding Fock space has \( 2^{2N} \) independent
states, where \( N \) is the number of spinorial charges.

(ii) Massless case. Here we choose the frame \( P = (E, 0, 0, E) \). The relation (46')
yields:

\[
\left[ Q_\alpha, \overline{Q}_\beta \right]_+ = 2E (1-\delta_z) = 4E \delta_{\alpha 2} \delta_{\beta 2}
\]  (53)

Only \( Q_2 \) and \( \overline{Q}_2 \) can be considered as creation and annihilation operators. Starting from a one-
particle state with helicity \( \pm \chi \), we obtain the state with helicity \( \pm (\chi + \frac{1}{2}) \). Some interesting examples:

\( \chi = \frac{1}{2} \) \( \Rightarrow \) one spin-\( \frac{1}{2} \) and one spin-1 (both massless)

\( \chi = \frac{3}{2} \) \( \Rightarrow \) one spin-\( \frac{3}{2} \) and one spin-2 (both massless)

etc...

If we have more than one spinorial charge, i.e. \( N > 1 \), we obtain \( N \) creation and annihi-
lation operators. A well-established theoretical prejudice is that, if one excludes gravitation,
there exist no elementary particles with spin higher than one. This prejudice is based
on the great difficulties one encounters if one wants to write consistent field theories with high spin particles. The consequence of such a prejudice is that $N = 4$ is the largest supersymmetry which may be interesting for particle physics without gravitation. The reason is that $N = 4$ contains four creation operators and allows us to go from a helicity state $\mathcal{A} = -1$ to that of $\mathcal{A} = +1$. Any increase in the number of spinorial charges will automatically yield representations containing higher helicities. Finally, if we include gravitation, the same prejudice tells us that we must allow for elementary particles with helicities $|\mathcal{A}| \leq 2$. The previous counting argument now gives $N = 8$ as the maximum allowed supersymmetry.

A concluding remark: All representations contain equal number of bosonic and fermionic states. All states in an irreducible representation have the same mass.

III.B.4. Representations in terms of field operators

In order to use the machinery of Quantum Field Theory we must look for linear representations of supersymmetry in terms of local fields. We remind the reader that our aim was to realize supersymmetry as transformations of the general form (43). Representations of this kind were first obtained by trial and error, but now we have more powerful methods which give, at least for $N = 1$ supersymmetry, a complete classification of representations. We shall not use them in these lectures and we shall restrict ourselves to presenting some simple examples which turn out to be the most interesting in physics. Two cases should be distinguished:

The case of "global" supersymmetry corresponds to transformations (43) with infinitesimal parameters $\xi$ which are independent of the space-time point $x$. The opposite case of $x$-dependent anticommuting parameters $\xi(x)$ is called "local" supersymmetry or "supergravity". We shall examine first the case of global supersymmetry and we shall postpone the study of supergravity until the last chapter. Some interesting examples of field supermultiplets which transform linearly under global supersymmetry transformations are the following:

(i) The "chiral" multiplet. It consists of a complex Weyl spinor $\psi(x)$ and two complex scalars $A(x)$ and $F(x)$. The latter will turn out to be an auxiliary field. Under an infinitesimal supersymmetry transformation with parameter $\xi$ these fields transform as:

\[
\begin{align*}
\delta A(x) &= \xi \psi(x) \\
\delta \psi(x) &= 2i \gamma_\mu \xi \partial_\mu A(x) + 2 \xi F(x) \\
\delta F(x) &= i \partial^\mu \psi(x) \gamma_\mu \xi
\end{align*}
\]  

(54)

This multiplet will be used to describe "matter" fields i.e. quarks and leptons.

(ii) The massless vector multiplet which will be used for the gauge bosons before spontaneous symmetry breaking. It contains the free vector boson field strength $U_{\mu \nu}(x)$, a Weyl spinor $\lambda(x)$ and an auxiliary scalar field $D(x)$. Its transformation properties are:

\[
\begin{align*}
\delta U_{\mu \nu}(x) &= i \xi \gamma_\nu \partial_\mu \lambda(x) - i \xi \gamma_\mu \partial_\nu \lambda(x) + h.c. \\
\delta \lambda(x) &= \xi \gamma^\mu U_{\mu \nu}(x) + \xi D(x) \\
\delta D(x) &= -\xi \gamma^\nu \partial_\nu \lambda(x) + h.c.
\end{align*}
\]

(55)
with \( U_{\nu} = \partial_{\nu} U - \partial_{\nu} U \) and \( \sigma^{I} = \frac{1}{4} (\sigma^{I} \sigma_{\nu} - \sigma^{\nu} \sigma^{I}) \), \( \Omega_{\nu} = \left[ \left( \sigma_{\nu} \right) \right]^{*} \).

It is straightforward to verify that (54) or (55) are indeed representations, i.e. if we apply two successive supersymmetry transformations we obtain a third one.

III.C - Simple field-theoretic models

In this section we shall construct some simple field-theoretic models with interactions which are invariant under supersymmetry. These models are not realistic but they will reveal to us the mathematical properties of supersymmetry which we shall use in the phenomenological models of later sections.

III.C.1. The self-interacting chiral multiplet

We discuss here the simplest supersymmetric invariant field theory model in four dimensions, that of a self-interacting chiral multiplet. The Lagrangian density reads:

\[
\mathcal{L} = \mathcal{L}_{k} + \mathcal{L}_{m} + \mathcal{L}_{I} \tag{56}
\]

where \( \mathcal{L}_{k} \), \( \mathcal{L}_{m} \) and \( \mathcal{L}_{I} \) are the kinetic energy, the mass and the interaction term respectively. We shall not derive the form of these terms in terms of the fields of eqs. (54), but we shall directly give the result.

\[
\mathcal{L}_{k} = -\frac{i}{2} \bar{\Psi} \gamma^{I} \sigma_{\mu} \Psi - \partial_{\mu} A \gamma^{I} A^{\dagger} + F^{+} F \tag{57}
\]

\[
\mathcal{L}_{m} = m (AF - \frac{1}{2} \bar{\Psi} \Psi) + h.c. \tag{58}
\]

\[
\mathcal{L}_{I} = g \left( A^{3} F - \bar{\Psi} \gamma^{I} A \right) + h.c. \tag{59}
\]

Some remarks: (i) The Lagrangian (56) is not invariant under supersymmetry transformations. When the fields transform as in (54) we find, for the different terms in \( \mathcal{L} \), \( \delta \mathcal{L} = \partial_{\mu} R^{\mu} \), where \( R^{\mu} \) is some vector field constructed out of the basic fields \( A \), \( \Psi \), \( F \) and their derivatives. In other words the Lagrangian density is not invariant but the resulting action is. This is not surprising because supersymmetry contains space-time translations. We know that no non-trivial Lagrangian density is invariant under translations. Only actions are. (ii) As announced, the field \( F \) is auxiliary because its derivatives do not appear in the kinetic energy term. Using the equations of motion we can eliminate it and we find:

\[
F = -m A^{\dagger} - g A^{2} \tag{60}
\]

\[
\mathcal{L}_{m} = -\frac{1}{2} \left( m^{2} A^{\dagger} A + m \bar{\Psi} \Psi \right) + h.c. \tag{61}
\]

\[
\mathcal{L}_{I} = -mg A^{2} A^{\dagger} - g \bar{\Psi} \gamma^{I} A - \frac{1}{2} g^{2} (A^{\dagger} A)^{2} + h.c. \tag{62}
\]

In this form the Lagrangian describes an ordinary renormalizable theory with a Yukawa, a \( Q^{3} \) and a \( Q^{4} \) coupling. Supersymmetry manifests itself in two ways: The masses of the complex scalar field and of the two-component Weyl spinor are equal and the different coupling strengths...
are not independent but are all given in terms of a single one \( g \). Notice also that the presence of the auxiliary field \( F \) is necessary in order to ensure linear transformation properties for all fields. Indeed, if we replace \( F \) in (54) by its equation (60), we find that \( \Psi \) transforms non-linearly.

This very simple model has some remarkable renormalization properties. First, we can show that all vacuum-to-vacuum diagrams vanish, i.e. no normal ordering is required. This is a consequence of exact supersymmetry and it is valid for every supersymmetric theory. The surprising result, which could not be guessed by supersymmetry considerations alone, is that in this model, mass and coupling constant renormalizations are absent. All Green functions, to every order in perturbation theory, become finite if one introduces a single, common wave-function renormalization counterterm. In other words, in spite of the presence of scalar fields, not only we do not have any quadratically divergent mass counterterms, but we have none whatsoever. What happens is that the divergences due to boson loops are cancelled against those of fermion loops which have the opposite sign. The equality of masses and coupling constants is essential for this cancellation. We shall use this result extensively later on.

### III.C.2. The supersymmetric extension of Q.E.D.

A combination of supersymmetry with gauge invariance is clearly necessary for the application of these ideas to the real world. We shall first examine an abelian gauge theory and we shall construct the supersymmetric extension of quantum electrodynamics.

If \( \mathcal{U}_\mu \) is the photon field and \( \Phi_i \) and \( \Phi_\nu \) the real and imaginary parts of a charged field, an infinitesimal gauge transformation is given by

\[
\delta \mathcal{U}_\mu = \partial_\mu \Lambda ; \quad \delta \Phi_i = e \Lambda \Phi_\nu ; \quad \delta \Phi_\nu = -e \Phi_i \Lambda
\]

where \( \Lambda(x) \) is a scalar function. In order to extend (63) to supersymmetry we must replace \( \mathcal{U}_\mu \) by a whole vector multiplet. Let us also assume that the matter fields are given by a charged chiral multiplet. We expect, therefore, to describe the interaction of photons with charged scalars and spinors simultaneously. It is obvious that if \( \Lambda(x) \) is a scalar function, the transformation (63) is not preserved by supersymmetry. The gauge transformation must be generalized so that \( \partial_\mu \Lambda(x) \) is a member of a vector multiplet. This can be achieved if \( \Lambda(x) \) becomes an entire chiral multiplet. The construction of the Lagrangian requires some kind of tensor calculus of supermultiplets, i.e. the rules of combining supermultiplets in order to obtain new ones. The final result is:

\[
\mathcal{L} = -\frac{i}{4} \mathcal{U}^\mu \mathcal{D}_\mu \mathcal{U} + \frac{i}{2} \mathcal{D}^2 + \mathcal{D}_\mu \mathcal{D}^\mu \psi - \mathcal{D}_\mu \mathcal{D}^\nu \mathcal{A} A^\nu + \mathcal{D}_\mu \mathcal{D}^\nu \mathcal{A}^\mu + \mathcal{D}^2 + \text{mass terms} + e \mathcal{U}^\mu \left[ \frac{i}{2} \bar{\psi} \gamma_\mu \psi + \frac{i}{2} A^\mu \gamma_\nu \mathcal{A}_\nu - \frac{i}{2} A^\nu \gamma_\mu \mathcal{A}_\nu \right] + \frac{i}{4} \mathcal{D}_\mu \mathcal{U}^\nu \mathcal{A}^\mu \mathcal{A}^\nu + \frac{i}{4} \mathcal{D}^2 + \mathcal{D}^\nu \mathcal{A}^\mu \mathcal{A}^\nu + \frac{e}{2} \mathcal{D}^\nu \mathcal{A}^\mu \mathcal{A}^\nu
\]

(64)
It is easy to understand the origin of the various terms in (64). The first line contains the usual kinetic energy terms of the vector multiplet members, the photon $U_{\mu}$ and its spin-$\frac{1}{2}$ partner $\lambda$. The field $D(x)$ is again an auxiliary field. The second line is the kinetic energy of the chiral matter multiplet of eq. (57) together with a possible mass term given by eq. (58). In the third line the photon field $U_{\mu}$ is coupled to the charged current in the usual way with coupling constant $e$. The well-known "sea-gull" term is also present. The last line contains the new couplings dictated by supersymmetry. They are the transformed of those in the previous line and they describe a Yukawa-type interaction between the spinor $\lambda$ (the partner of the photon) and the scalar and spinor members of the matter multiplet. The last term, after elimination of the auxiliary field $D$, is the normal $(A^{+}A)^{2}$ term in scalar electrodynamics. Notice however that here it does not introduce any new coupling constant. Supersymmetry, once more, forces all couplings in (64) to be given in terms of the electric charge $e$.

III.C.3. Yang-Mills and supersymmetry

We can generalize the above results to non-abelian Yang-Mills theories. The gauge bosons belong to vector superfields transforming according to the adjoint representation of a group $G$. $U_{\mu}$ is the vector field, $U_{\mu\nu}$ its field-strength given by the analog of eq. (21), the Weyl spinor $\lambda$ is its spin-$\frac{1}{2}$ partner and $D$ the associate auxiliary field. Following the notation of chapter II (see eq. (21)) we write all these fields as square, traceless, hermitian matrices. The Yang-Mills Lagrangian now reads:

$$\mathcal{L} = -\frac{1}{4} \, T_{c} \, U_{\mu\nu} \, U^{\mu\nu} - \frac{i}{2} \, T_{c} \, \overline{\lambda} \, \not{D} \, \lambda + \frac{i}{2} \, T_{c} \, D^{2}$$  

(65)

where the covariant derivative $\not{D}$ acting on $\lambda$ is given by:

$$\not{D} \lambda = \partial_{\mu} \lambda + i \, g \, [U_{\mu}, \lambda]$$

(66)

The auxiliary field equation gives, in this case, $D = 0$. The Lagrangian (65) shows that a Yang-Mills interaction of a massless Weyl fermion in the adjoint representation of the gauge group is automatically supersymmetric. The corresponding spin-$3/2$ conserved current is:

$$J^{\mu} \sim T_{c} \left[ U_{\nu} \sigma^{\nu} \sigma^{\rho} \sigma^{\tau} \lambda \right]$$

(67)

where the trace is only taken in the internal symmetry indices and not the $\sigma$-matrices.

The introduction of additional matter multiplets in the form of chiral superfields belonging to any desired representation of the gauge group presents no difficulties. An interesting result is obtained if one studies the asymptotic properties of these theories. The one-loop $\beta$-function for an SU(m) Yang-Mills supersymmetric theory with $n$ chiral multiplets belonging to the adjoint representation of SU(m) is:

$$\beta(g) = \frac{m(n-3)}{16 \, n^{2}} \, g^{3}$$

(68)

which means that, for $n < 3$, the theory is asymptotically free, although it contains scalar particles.
Before closing this section I want to mention a surprising and not yet fully understood result. Until now we have been considering supersymmetric theories with only one spinorial generator. We explained earlier that the generalization to N such generators is straightforward and N = 4 is the maximum number which does not introduce spins higher than one. The astonishing result is that the N = 4 supersymmetric Yang-Mills theory based on any group SU(m) seems to have no divergences in perturbation theory. This has been verified up to, and including, three loops. What is the significance of this result? For a theory with only massive particles one can show that the absence of divergences implies either triviality (free fields) or the presence of states with negative metric. No analogous theorem is known for unbroken gauge theories.

III.D - Breaking of supersymmetry

The spectrum of elementary particles shows no sign, at any conceivable approximation, of a degeneracy between fermions and bosons. Supersymmetry, if at all relevant, must be broken. Under these circumstances an entirely explicit breaking is meaningless because the symmetry breaking part of the Hamiltonian could not be considered as a perturbation to the symmetric part. This tells us that the main source of supersymmetry breaking must be spontaneous. In this section we shall study some properties of such a theory.

The usual mechanism for spontaneous symmetry breaking is the introduction of some spin-zero field with negative square mass. This option is not available for supersymmetry because it would imply an imaginary mass for the corresponding fermion. In fact, we can make this argument more general: Taking the trace of the basic anticommutator relation (46') we find immediately that the Hamiltonian of a supersymmetric system is positive definite. If there exists a state |a> such that H |a> = 0, this must be the ground state. But (46') tells us that for such a state we have Q |a> = 0, i.e. the state |a> is supersymmetric. We have just proven an important theorem which says that there is no state with energy lower than that of a supersymmetric state. It follows that the only way to break supersymmetry spontaneously is to manage so that there is no supersymmetric state. Supersymmetry is hard to break!

Today we know of three ways to achieve spontaneous supersymmetry breaking in perturbation theory.

(i) Through a U(1) gauge group. This mechanism is not available for G.U.T.'s which are based on simple groups.

(ii) Through "special" chiral multiplets. Although technically possible, this mechanism is subject to even more severe criticism than the ordinary Higgs mechanism regarding its arbitrariness.

(iii) Through supergravity which escapes the consequences of the aforementioned theorem. We shall consider this mechanism in Chapter V.

In the rest of the section we want to present some model-independent consequences of spontaneous supersymmetry breaking.
The first one is a direct application of Goldstone's theorem. Whenever a continuous symmetry is broken spontaneously there appears a massless particle with quantum numbers given by the divergence of the corresponding current. The supersymmetry current $\mathcal{J}_\mu$ has spin-3/2, therefore its divergence has spin-1/2. It follows that the associated Goldstone particle is a spin-1/2 fermion of zero mass, the "Goldstino". Where is the Goldstino? There are three possible answers to this question.

(i) Our first reaction to the appearance of a massless spin-1/2 fermion is to rejoice because we hope to associate it with one of the neutrinos. Alas, appearances are deceptive! There is a low-energy theorem, satisfied by any Goldstone particle. The field $\chi(x)$ of the Goldstino is proportional to the divergence of the supersymmetry current

$$\Psi(x) \sim \partial^\mu \mathcal{J}_\mu(x)$$

Eq. (69) seems strange because the r.h.s. vanishes for a conserved current. Strictly speaking we should introduce a small explicit breaking, in which case the constant of proportionality contains a factor $m^{-1}$, where $m$ is the Goldstino mass. In the limit when the explicit breaking goes to zero, the ratio of the matrix elements of $\partial^\mu \mathcal{J}_\mu$ divided by $m$ is well defined.

Eq. (69) implies that the amplitude $M(a \to b + \chi)$ of the emission (or absorption) of a Goldstino with momentum $k_\mu$ satisfies the low energy theorem:

$$\lim_{k_\mu \to 0} M(k) = 0$$

(70)

This is a very powerful prediction and can be checked by studying the end-point spectrum in nuclear $\beta$-decay. Unfortunately, it is a wrong one! Experiments show no such suppression which means that the electron neutrino cannot be the Goldstino.

(ii) Can the Goldstino be a new, as yet unobserved, massless, spin-1/2 fermion? The answer is yes and I know of at least two ways to implement this idea. One suggestion is to identify it with the right-hand component of the physical neutrino. The low-energy theorem would thus explain why it is not coupled. A second one is to make the Goldstino hard to detect by endowing it with a new, conserved quantum number. This possibility is the only one which has given some semi-realistic models based on low-energy global supersymmetry. Their predictions have not been ruled out by experiment and can be tested with available technology. We shall study them in the next chapter.

(iii) The third answer to the Goldstino puzzle is a "super-Higgs" mechanism. In the normal Higgs phenomenon we have:

$$m = 0, \text{spin} = 1 \text{ or } m \neq 0, \text{spin} = 0$$

In a "super-Higgs" mechanism we get:

$$m = 0, \text{spin} = 3/2 \text{ or } m \neq 0, \text{spin} = 1/2$$

i.e. we need to start with a gauge spin-3/2 field which will absorb the massless Goldstino to give a massive spin-3/2 particle. This mechanism can be applied in the framework of supergravity theories.

The second model-independent consequence of spontaneous breaking of global supersymmetry concerns the particle spectrum. In exact supersymmetry fermions and bosons are degenerate.
After the breaking the masses are split but a certain pattern remains. The masses squared of the boson fields are equally spaced above and below those of the fermions. More precisely we obtain the mass formula

\[ \sum J (-)^{2J} (2J+1) m_J^2 = 0 \]  

(70)

where \( m_J \) is the mass of the particle of spin \( J \). This formula plays an important role in model building.

IV. PHENOMENOLOGY OF GLOBAL SUPERSYMMETRY

Let us now try to apply these ideas to the real world. We want to build a supersymmetric model which describes the low-energy phenomenology. There may be several answers to this question but, to my knowledge, there is only one class of models which come close to being realistic. They assume a superalgebra with only one spinorial generator, consequently all particles of a given supermultiplet must belong to the same representation of the gauge group. In the following we shall try to keep the discussion as general as possible, so that our conclusions will be valid in essentially all models.

IV.A - BUILDING BLOCKS OF SUPERSYMMETRIC MODELS

All models based on global supersymmetry use three types of multiplets:

(i) Chiral multiplets. As we said already they contain one Weyl (or Majorana) fermion and two scalars. Chiral multiplets are used to represent the matter (leptons and quarks) fields as well as the Higgs fields of the standard model.

(ii) Massless vector multiplets. They contain one vector and one Weyl (or Majorana) fermion, both in the adjoint representation of the gauge group. They are the obvious candidates to generalize the gauge bosons.

(iii) Massive vector multiplets. They are the result of ordinary Higgs mechanism in the presence of supersymmetry. A massive vector multiplet is formed by a vector field, a Dirac spinor and a scalar. These degrees of freedom are the combination of those of a massless vector multiplet and of a chiral multiplet.

IV.B - Supersymmetric extension of the standard model

IV.B.1. The particle content

In the standard model we have:

- Number of bosonic degrees of freedom = 28
- Number of fermionic degrees of freedom = 90

It follows that a supersymmetric extension of the standard model will necessarily introduce new particles. Let us go one step further: In \( N = 1 \) supersymmetry all the particles of a given supermultiplet must belong to the same representation of the gauge group. For the various particles of the standard model this yields:

(i) The gauge bosons are one color octet (gluons), one SU(2) triplet and one singlet (\( W^\pm, Z^0, \gamma \)). No known fermions have these quantum numbers.
The Higgs scalars transform as SU(2) doublets but they receive a non-zero vacuum expectation value, consequently they cannot be the partners of leptons or quarks otherwise we would have induced a spontaneous violation of lepton or baryon number. Furthermore, we must enlarge the Higgs sector by introducing two complex chiral supermultiplets. This is necessary for several technical reasons which are related to the fact that, in supersymmetry, the Higgs scalars must have their own spin-\(\frac{1}{2}\) partners. This in turn creates new problems like, for example, new triangle anomalies which must be cancelled. Furthermore, now the operation of complex conjugation on the scalars induces a helicity change of the corresponding spinors. Therefore we cannot use the same Higgs doublet to give masses to both up and down quarks. Finally, with just one Higgs supermultiplet, we cannot give masses to the charged partners of the W's. The net result of this operation is a much richer spectrum of physical "Higgs" scalars. Since we start with eight scalars (rather than four) we end up having five physical ones (rather than one). They are the scalar partners of the massive vector bosons \(W^\pm\), \(Z^0\) and two separate neutral ones.

The conclusion is that, in the standard model, supersymmetry associates known bosons with unknown fermions and known fermions with unknown bosons. We are far from obtaining a connection between the three independent worlds. For this reason this model should not be considered as a fundamental theory but as an intermediate step towards the higher N supergravity theories which we shall present in the last chapter. Nevertheless, the phenomenological conclusions we shall derive are sufficiently general to be valid, unless otherwise stated, in every theory based on supersymmetry.

We close this section with a table of the particle content in the supersymmetric standard model. Although the spectrum of these particles, as we shall see shortly, is model dependent, their very existence is a crucial test of the whole supersymmetry idea. We shall argue in the following chapters that its experimental verification is within the reach of present machines.

### Table 1

The particle content of the supersymmetric standard model

<table>
<thead>
<tr>
<th>SPIN-1</th>
<th>SPIN-(\frac{1}{2})</th>
<th>SPIN-0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluons</td>
<td>Gluinos</td>
<td></td>
</tr>
<tr>
<td>Photon</td>
<td>Photino</td>
<td></td>
</tr>
<tr>
<td>(W^\pm)</td>
<td>2 Dirac Winos</td>
<td>(W^\pm)</td>
</tr>
<tr>
<td>(Z^0)</td>
<td>2 Major. Zinos</td>
<td>(z)</td>
</tr>
<tr>
<td>1 Major. Higgsino</td>
<td>standard (\phi^0) pseudosc. (\phi^0^*)</td>
<td></td>
</tr>
<tr>
<td>Leptons</td>
<td>Spin-0 leptons</td>
<td></td>
</tr>
<tr>
<td>Quarks</td>
<td>Spin-0 quarks</td>
<td></td>
</tr>
</tbody>
</table>
IV.B.2. R-parity

The motivation for introducing a new quantum number for the supersymmetric partners of known particles was the problem of the Goldstino. However, such a quantum number appears naturally in the framework of supersymmetric theories and it is present even in models in which the above motivation is absent. This number is called "R-parity" and one possible definition is:

\[ (-)^R = (-)^{2S} (-)^{3(B-L)} \]

(71)

where \( S \) is the spin of the particle and \( B \) and \( L \) the baryon and lepton numbers, respectively. It is easy to check that eq. (71) gives \( R = 0 \) for all known particles, fermions as well as bosons, while it gives \( R = \pm 1 \) for their supersymmetric partners. Since \( R \) is conserved, the R-particles are produced in pairs and the lightest one is stable. In a spontaneously broken global supersymmetry this is the Goldstino, which is massless, but in the supergravity theories the latter is absorbed by the super-Higgs mechanism. The general consensus is that, in this case, the lightest, stable R-particle is the partner of the photon, the "photino".

IV.B.3. The Goldstino

Looking back at Table 1 we see that an important question - before going into the details of any model - is the identity of the Goldstino; in particular, can it be identical, say to the photino? As we said earlier, the mechanism of spontaneous symmetry breaking, which is at the origin of the existence of the Goldstino, allows us to find some properties of the latter, independently of the details of a particular model. In a spontaneously broken theory the spin-3/2 conserved current is given by:

\[ J^R = d \gamma^R \gamma_5 \psi_3 + \hat{J}^R \]

(72)

where \( d \) is a parameter with dimensions \((\text{mass})^2\), \( \psi_3 \) is the Goldstino field and \( \hat{J}^R \) is the usual part of the current which is at least bilinear in the fields. In other words, the field of a Goldstone particle can be identified with the linear piece in the current. The conservation of \( J^R \) gives:

\[ d \gamma_5 \not\! \psi_3 = \not\! \partial_\mu \hat{J}^R \]

(73)

This is the equations of motion of the Goldstino. In the absence of spontaneous breaking \( d = 0 \) and \( \not\! \partial_\mu \hat{J}^R = 0 \). In fact, to lowest order, the contribution of a given multiplet to \( \not\! \partial_\mu \hat{J}^R \) is proportional to the square mass-splitting \( \Delta m^2 \). Thus, the coupling constant of the Goldstino to a spin-0-spin-1/2 pair is given by

\[ f_3 = \pm \frac{\Delta m^2}{d} \]

(74)

where the sign depends on the chirality of the fermion. It follows that if the Goldstino were the photino, \( f_3 \sim e \) and the \((\text{mass})^2\)-splittings would have been proportional to the electric charge. For example, if \( S_e \) and \( t_e \) were the charged spin-zero partners of the electron, we would have:

\[ m^2(S_e) + m^2(t_e) = 2m^2(e) \]

(75)
Eq. (75), which is clearly unacceptable, is a particular example of the mass-formula (70) and here we see how tight the latter is. The conclusion is that the photon cannot be the bosonic partner of the Goldstino. With a similar argument we prove that the same is true for the Z boson, the Higgs's or any linear combination of them. This is a model-independent result. Not only can we not identify the Goldstino with the neutrino but also we cannot pair it with any of the known neutral particles. Therefore, strictly speaking, there is no acceptable supersymmetric extension of the standard model. The one that comes closest to it assumes an enlargement of the gauge group to $U(1) \times U(1) \times SU(2) \times SU(3)$ thus involving a new neutral gauge boson. We shall not study it in detail here but we shall rather extract those features which are model independent and are likely to be present in any supersymmetric theory.

IV.B.4. Particle spectrum and decay modes

As we earlier said, supersymmetry predicts a rich spectroscopy of new particles whose existence is an important test of the theory. Such a test, however, is only meaningful if the masses of the new particles are also predicted, at least to an order of magnitude. Let me remind you the situation when the charmed particles were predicted. The motivation was the need to suppress unwanted processes like strangeness changing neutral current transitions. Such a suppression was effective only if the charmed particles were not too heavy. No precise value could be given but the prediction was powerful enough to be testable. We have a similar situation with supersymmetry. The need for introducing supersymmetry was to control the bad behaviour of elementary scalar fields and this can be achieved only if supersymmetry is not too badly broken which, in practice, means that the masses of the partners of the known particles cannot exceed the mass of the W too much. The most convincing argument comes from grand-unified theories and it will be presented in the next chapter. The conclusion is that supersymmetry can be tested with present machines, existing or under construction (CERN and FNAL pp colliders and LEP).

Let us now briefly discuss some results on masses and decay properties. In the absence of any concrete experimental evidence I can only quote limits and expected signatures. The mass spectrum is very model dependent but some general features can be extracted.

(i) Scalar partners of quarks and leptons (squarks, sleptons). The best limits on the masses of the charged ones come from PETRA. With small variations they are of the order of 20 GeV. Theoretical arguments almost always predict squarks heavier than sleptons. The reason is that in most models the masses are set equal at the grand unification scale and the differences are due to the strong interactions of the squarks (see eq. (39)). For the same reason the masses of sneutrinos are predicted to be of the same order as those of the corresponding charged sleptons. The recent collider results, which at the moment I am writing these lines still need confirmation, suggest that squarks are at least as heavy as 40 GeV.

(ii) Gluinos. From beam dump results we can extract a model-dependent limit of the order of 2 GeV. From the same theoretical arguments, the gluinos are always heavier than photinos. The same UA1 results indicate again a limit of the order of 40 GeV.

(iii) Charged gauginos, the partners of $W^\pm$. We have the same PETRA bounds.

(iv) Neutral gauginos and Higgsinos. They mix among themselves, so they should be treated simultaneously. The lightest among them, which for the purposes of this discussion we shall
call the photino, is likely to be the lightest particle with R ≠ 0 and therefore stable. I remind you that the Goldstino, which, if it exists, is massless, is absent from theories derived from supergravity (see last chapter). No precise prediction for the photino mass exists but we shall assume M_\tilde{\gamma} \sim 1-10 \text{ GeV}. Supergravity-type models have a tendency to give masses close to the upper value while cosmological arguments exclude values much lighter than 1 GeV.

The picture that emerges is that supersymmetric particles may be spread all over from 10 to 100 GeV with squarks and gluinos heavier than sleptons and photino.

Let us now come to possible decay modes and signatures. They obviously depend on the detailed mass spectrum but two considerations must be kept in mind: Supersymmetric particles do not introduce new coupling constants. Thus squarks and gluinos are coupled with \alpha_s and sleptons and gauginos with \alpha. Secondly, we must remember that R is conserved, therefore all new particles will eventually end up giving photinos whose interactions are comparable to those of the neutrinos and they leave undetected. Hence the great importance of a precise determination of missing transverse momentum as a handle in the search of supersymmetric particles.

After these preliminaries we list some specific examples:

Squarks are produced in hadron collisions either in pairs or in association with gluinos (R must be conserved). Their decay modes are

\[ \tilde{q} \to q + \tilde{\gamma} \]  \hspace{1cm} \text{(quark + photino)} \hspace{1cm} \text{(76)}

or, if phase space permits,

\[ \tilde{q} \to q + \tilde{g} \]  \hspace{1cm} \text{(quark + gluino)} \hspace{1cm} \text{(77)}

The gluino in turn decays either as

\[ \tilde{g} \to q + \tilde{q} + \tilde{\gamma} \]  \hspace{1cm} \text{(78)}

or as

\[ \tilde{g} \to g + \tilde{\gamma} \]  \hspace{1cm} \text{(gluon + photino)} \hspace{1cm} \text{(79)}

In most models (78) dominates over (79). We see that the signature for squarks or gluinos is missing \text{P}_T + \text{jets}. It was from the study of the small sample of the UA1 monojet events that the limits of 40 GeV for squarks and gluinos were obtained.

Sleptons behave similarly and give

\[ \tilde{\ell} \to \ell + \tilde{\gamma} \]  \hspace{1cm} \text{(lepton + photino)} \hspace{1cm} \text{(80)}

Thus the signal at PETRA or LEP will be again missing energy and acoplanar and acollinear events. The jump in R (the total hadronic e^+e^- cross section divided by the \mu^+\mu^- one) due to a single charged scalar particle is too small to be detectable but, if LEP is above several thresholds it will be seen.

An important source of information is provided by the W and Z decays. A precise measurement of the Z° width, like R in e^+e^- collisions, counts the number of particle species that
are produced. In the most favorable case, i.e. several decay channels available, the total $Z^0$
width could be increased by a factor of two. If the charged gauginos are substantially lighter
than the $W$, one could have, for example:

$$ W \rightarrow \text{wino} + \text{photino} \quad \text{(81)} $$

which could be detected by a precise measurement of the electron spectrum. Another possible
decay mode of the $W$, if $m_W > m_e + m_{\nu_e}$, is:

$$ W \rightarrow \text{selectron} + \text{sneutrino} \quad \text{(82)} $$

The sneutrino, if it is relatively light, decays into neutrino + photino. If it is heavier
than the other gauginos it can decay into them. In all cases (82) will result into a final
state which, like that of (81), looks like an ordinary one and can be identified only through
detailed measurement of the electron spectrum.

A final interesting possibility for LEP is the study of quarkonium decay. This has been
already considered for $\Psi$ in order to put limits on the photino mass. The process was

$$ \Psi \rightarrow \text{Goldstino} + \text{photino}. \text{At present our prejudices exclude such a decay but the toponium}
\text{is still interesting. If } m_t > m_{\chi^0} + m_{\nu_e} \text{ one could have } T \rightarrow \bar{t} + \bar{t} + \gamma \text{. If } m_t < m_{\chi^0} + m_{\nu_e}
\text{but } m_{\chi^0} < m_t \text{ we could have } T \rightarrow \bar{t} + W^\pm \text{. Depending on the gluino mass we could have } T \rightarrow g + g + \gamma
\text{ or } T \rightarrow g + g + \gamma \text{ or even } T \rightarrow \bar{g} + \tilde{g}. $$

I believe that looking for supersymmetric particles will dominate experimental research
in the second half of this decade. I hope that it is going to be both exciting and rewarding
and, in any case, by the early nineties we shall know for sure whether supersymmetry is a
fundamental symmetry of particle forces.

IV.C - Supersymmetry and grand-unified theories

Let me remind you that one of the reasons why we decided to study supersymmetry in
connection with gauge theories was the gauge hierarchy problem which plagues all known G.U.T.'s.
It is now time to study this question. The problem has two aspects: The first one, the
physical aspect, is to find a natural way to create these too largely separated mass scales.
Supersymmetry offers no new insight to this very fundamental question. The second is the
technical aspect. In the notation of section III.A.1. the 24-plet $\Phi$ takes a vacuum expectation value $V$. As before, in order for the model to be able to sustain a gauge hierarchy, we
must impose a very precise relation among the parameters of the potential of the form $m \sim \phi V$.
It is this relation which is destroyed by renormalization effects and has to be enforced arti-
factually order by order in perturbation theory. This is the technical aspect. Supersymmetry
can eliminate this part of the problem. The key is the non-renormalization theorems we men-
tioned in section III.C.1. If supersymmetry is exact, the parameters of the potential do not
get renormalized. What happens is that the infinities coming from fermion loops cancel against
those coming from boson loops. When supersymmetry is spontaneously broken the cancellation is
not exact, but the corrections are finite and calculable. They are of the order $\sim \Delta m^2$
where $\Delta m$ is the mass-splitting in the supermultiplet. Here comes the estimation we used in the previous section. For the gauge hierarchy to remain, $\Delta m^2$ should not be much larger than the small mass scale, namely $m_\text{W}^2$. A badly broken supersymmetry is not effective in protecting the small mass scale. Let me make the logic of this argument clear: Supersymmetry does not solve the gauge hierarchy problem in the sense of providing an explanation for the existence of two very different mass scales. It only provides a framework to stabilize the hierarchy, once it is imposed. For this last function the breaking cannot be arbitrarily large. Hence the upper limit on the masses of supersymmetric particles.

After these remarks on the gauge hierarchy problem, one can proceed in supersymmetrizing one's favorite G.U.T. model. The construction parallels that of the low-energy standard model with similar conclusions. Again, no known particle can be the superpartner of another known particle. Furthermore, assuming a spontaneous symmetry breaking, we can repeat the analysis which led us to conclude that $U(1) \times SU(2) \times SU(3)$ was too small. The corresponding conclusion here will be that $SU(5)$ is too small, since $SU(5)$ does not contain anything larger than the group of the standard model.

Finally, we can repeat the renormalization group estimation of the grand-unification scale and the proton life-time. We had found in Chapter II that at low energies the effective coupling constants evolve following, approximately, the renormalization group equations of $U(1)$, $SU(2)$ or $SU(3)$. The same remains true in a supersymmetric theory, but now the values of the $\beta$-functions are different. The number of Yang-Mills gauge bosons is the same as before. They are the ones which give rise to negative $\beta$-functions. On the other hand supersymmetric theories have a larger number of "matter" fields, spinors and scalars, which give positive contributions. The net result is a smaller, in absolute value, $\beta$-function and, therefore, a slower variation of the asymptotically free coupling constants. We expect larger values of the grand-unification scale $M$ and indeed, we find $M \sim 10^{16} - 10^{17}$ GeV. If nothing else contributes to proton decay, it will be invisible! Fortunately, there are other contributions, which although of higher order, turn out to be dominant. An example is shown in figure 3.

![Diagram](image)

**Fig. 3**

The important point is that the superheavy particle is now a fermion and the amplitude is of order $M^{-1}$ rather than $M^{-2}$. The resulting proton life-time is $\tau_p \sim 10^{30}$ years with large uncertainties. The absolute value has not changed significantly with the introduction of supersymmetry but the dominant decay modes are now different. Since the intermediate particle is a Higgsino, heavy flavors are favored. We expect to find
\[ P \rightarrow \overline{\gamma}_\mu + K^+ \]
\[ \rightarrow \overline{\nu}_e + K^+ \]  \hspace{1cm} (83)

which considerably complicates the task of experimental detection. One cannot rely on a two-body back-to-back decay mode like the \( e^+ \eta^+ \) for background rejection. Maybe the last word has not been said in baryon decay.

V. SUPERGRAVITY

Supergravity is the theory of local supersymmetry, i.e. supersymmetry transformations whose infinitesimal parameters - which are anticommuting spinors - are also functions of the space-time point \( x \). There are several reasons to go from global to local supersymmetry:

(i) We have learned in the last years that all fundamental symmetries in nature are local (or gauge) symmetries.

(ii) The supersymmetry algebra contains the translations and we know that invariance under local translations leads to general relativity which, at least at the classical level, gives a perfect description of the gravitational interactions.

(iii) As we already noticed, local supersymmetry provided the most attractive explanation for the absence of a physical Goldstino.

(iv) In the last chapter we saw that in a supersymmetric grand-unified theory the unification scale approaches the Planck mass \( (10^{19} \text{ GeV}) \) at which gravitational interactions can no more be neglected.

The gauge fields of local supersymmetry can be easily deduced. Let us introduce an anticommuting spinor \( \epsilon \) for every spinorial charge \( \xi \) and write the basic relation (50) as a commutator:

\[
[\epsilon^m \xi^m, \bar{\xi}^n \xi^n] = 2 \delta^{m n} \epsilon^m \Sigma^\mu \bar{\epsilon}^n P^\mu \quad ; \quad \eta_{\mu} = 1, \ldots, N \quad (84)
\]

where no summation over \( m \) and \( n \) is implied. In a local supersymmetry transformation \( \epsilon \) becomes a function \( \epsilon(x) \). Eq. (84) implies that the product of two supersymmetry transformations with parameters \( \epsilon_1(x) \) and \( \epsilon_2(x) \) is a local translation with parameter

\[
\alpha_\mu(x) = \epsilon_1(x) \Sigma^\mu \bar{\epsilon}_2(x) \quad (85)
\]

On the other hand we know that going from a global symmetry with parameter \( \Theta \) to the corresponding local one with parameter \( \Theta(x) \), results in the introduction of a set of gauge fields which have the quantum numbers of \( \Sigma^\mu \Theta(x) \). If \( \Theta(x) \) is a scalar function, which is the case for internal symmetries, \( \Sigma^\mu \Theta(x) \) is a vector and so are the corresponding gauge fields (ex. gluons, \( W^+, Z^-, \gamma \)). If the parameter is itself a vector, like \( \alpha_\mu(x) \) of translations, \( \Sigma^\mu \alpha_\mu(x) \) is a two-index tensor and the associated gauge field has spin two. In supersymmetry the parameters \( \epsilon^m(x) \) have spin one-half so the gauge fields will have spin three-half. We conclude that the gauge fields of local supersymmetry, otherwise called supergravity, are one spin-two field and \( N \) spin-three-half ones. To those we have to add the
ordinary vector gauge fields of whichever internal symmetry we are considering.

**V.A - N = 1 Supergravity**

"Οὐκ ἐσείς τοῦ φῶς, ἀλλ’ ὤνα μάρτυριην περί τοῦ φωτός"

"He was not that Light, but was sent to bear witness of that Light"  
*John A B*

This is the simplest supergravity theory. As I shall explain in the next section, I do not consider it as the fundamental theory of particle physics, but I believe that it provides for a good basis for a phenomenological analysis. The gauge fields are the metric tensor $g_{\mu\nu}(x)$ which represents the graviton and a spin-three-half Majorana "gravitino" $\Psi_\mu(x)$. We can start by writing the Lagrangian of "pure" supergravity, i.e. without any matter fields. The Lagrangian of general relativity can be written as:

$$\mathcal{L}_G = -\frac{1}{2k^2} \sqrt{-g} \ R = \frac{i}{2k^2} \epsilon \ R$$  
(86)

where $g_{\mu\nu}$ is the metric tensor and $g = \det g_{\mu\nu}$. $R$ is the curvature constructed out of $g_{\mu\nu}$ and its derivatives. We have also introduced the vierbein field $e^m_\mu$ in terms of which $g_{\mu\nu}$ is given as $g_{\mu\nu} = e^m_\mu e^n_\nu \eta_{mn}$ with $\eta_{mn}$ the Minkowski space metric. It is well known that if one wants to study spinor fields in general relativity the vierbein, or tetrad, formalism is more convenient. $\epsilon$ equals $\sqrt{-g}$; $k^2$ is the gravitational coupling constant. 

Eq. (86) is the Lagrangian of the gravitational field in empty space. We add to it the Rarita-Schwinger Lagrangian of a spin-three-half massless field in interaction with gravitation:

$$\mathcal{L}_{RS} = -\frac{1}{2} \varepsilon^{\nu\rho\sigma} \Psi_\mu \gamma_\nu \gamma_\rho \partial_\sigma \Psi_\sigma$$  
(87)

where $\varepsilon^{\nu\rho\sigma}$ is the completely antisymmetric tensor which equals one (minus one) if its four indices form an even (odd) permutation of $1 2 3 4$ and zero otherwise. $\partial_\sigma$ is the covariant derivative

$$\partial_\sigma = \partial_\sigma + \frac{i}{2} \omega^m_{\rho \sigma} \gamma_m$$  
(88)

and $\omega^m_{\rho \sigma}$ is the spin connection. Although $\omega^m_{\rho \sigma}$ can be treated as an independent field, its equation of motion expresses it in terms of the vierbein and its derivatives.

The remarkable result is that the sum of (86) and (87)

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{RS}$$  
(89)

gives a theory invariant under local supersymmetry transformations with parameter $\epsilon(x)$

$$\delta e^m_\mu = \frac{k}{2} \epsilon(x) \gamma^m \Psi_\mu$$  
(90a)

$$\delta \omega^m_{\rho \sigma} = 0$$  
(90b)

$$\delta \Psi_\mu = \frac{i}{k} \gamma^\sigma \epsilon(x) = \frac{i}{k} (\partial_\mu \epsilon + \frac{i}{2} \omega^m_{\rho \sigma} \gamma_m \epsilon)$$  
(90c)
Two remarks are in order here: First the invariance of \((89)\) reminds us of the similar result obtained in global supersymmetry, where we found that the sum of a Yang-Mills Lagrangian and that of a set of Majorana spinors belonging to the adjoint representation, was automatically supersymmetric. Second, we must point out that the transformations \((90)\) close an algebra only if one uses the equations of motion derived from \((89)\). We can avoid this inconvenience by introducing a set of auxiliary fields. In fact, we have partly done so, because the spin connection is already an auxiliary field.

The next step is to couple the \(N = 1\) supergravity fields with matter in the form of chiral or vector multiplets. The resulting Lagrangian is quite complicated and will not be given explicitly here. Let me only mention that, in the most general case, it involves two arbitrary functions. If I call \(z\) the set of complex scalar fields, the two functions are:

\[
G(z, z^*) \quad \text{a real function, invariant under whichever gauge group we have used.}
\]

\[
f_{ij}(z) \quad \text{an analytic function which transforms as a symmetric product of two adjoint representations of the gauge group.}
\]

One may wonder why we have obtained arbitrary functions of the fields, but we must remember that, in the absence of gravity, we impose to our theories the requirement of renormalizability which restricts the possible terms in a Lagrangian to monomials of low degree. In the presence of gravity, however, renormalizability is anyway lost, so no such restriction exists. In view of this, it is quite remarkable that only the two aforementioned functions occur.

As in ordinary gauge theories, the spontaneous breaking of local supersymmetry results in a super-Higgs mechanism. The gravitino, which is the massless gauge field of local supersymmetry, absorbs the massless Goldstino and becomes a massive spin-three-half field. At ordinary energies we can take the limit of the Planck mass going to infinity. In this case gravitational interactions decouple and the spontaneously broken supergravity behaves like an explicitly but softly broken global supersymmetry. The details of the final theory, like particle spectra, depend on the initial choice of the functions \(G\) and \(f_{ij}\), but the general phenomenology is the same as the one presented in the previous chapter.

Before closing this section let me mention a famous unsolved problem, for which supergravity offers a new line of approach. The Einstein Lagrangian \((86)\) is not the most general one. We could add a constant \(\Lambda\) with dimensions \(\text{mass}^4\) and write:

\[
\mathcal{L}_G = -\frac{1}{2k^2} \sqrt{-g} \left( R + \Lambda \right)
\]  

\((91)\)

\(\Lambda\) is called "the cosmological constant" and represents the energy density of empty space, but in the presence of the gravitational field this is no more an unphysical quantity which one can set equal to zero. In fact, any matter field gives an infinite contribution to \(\Lambda\). Experimentally, \(\Lambda\) is very small, \(\Lambda < 10^{-48} \text{[GeV]}^4\). If we have exact supersymmetry \(\Lambda\) vanishes identically because the infinite vacuum energy of the bosons cancels that of the fermions. However, in a spontaneously broken global supersymmetry the vacuum energy is always positive, as we explained in section III.D and this yields a positive cosmological constant.
In a spontaneously broken supergravity this is no more true and one can arrange to have $E_{\text{vac}} = 0$ and hence $\Lambda = 0$. In a realistic theory this must be the consequence of a certain symmetry and, indeed, such models have been constructed and are under study. I believe that ultimately this problem will be connected to the way one obtains $N = 1$ supergravity as an intermediate step between low-energy phenomenology and the fundamental theory to which I shall now turn.

V.B - $N = 8$ Supergravity

"... ένα τρεξελώμενος είσ 'έν..."

"... that they may be made perfect in one ;..."  

Let me remind you that one of the arguments to introduce supersymmetry was the desire to obtain a connection among the three independent worlds of gauge theories, the worlds of radiation matter and Higgs fields. None of the models presented so far achieved this goal. They all enlarged each world separately into a whole supermultiplet, but they did not put them together. $N = 8$ supergravity is the only one which attempts such a unification. It is the largest supersymmetry we can consider if we do not want to introduce states with spin higher than two. Following the method of section III.B.3. we construct the irreducible representation of one-particle states which contains:

1 spin-2 graviton
8 spin-$\frac{3}{2}$ Majorana gravitini
28 spin-1 vector bosons
56 spin-$\frac{1}{2}$ Majorana fermions
70 spin-0 scalars

(92)

We shall not write down the Lagrangian which involves all these fields and is invariant under eight local supersymmetry transformations but we shall mention some of its properties. Contrary to the $N = 1$ case, there is no known system of auxiliary fields. Since we have 28 vector bosons we expect the natural gauge symmetry to be $SO(8)$. This is bad news because $SO(8)$ does not contain $U(1) \times SU(2) \times SU(3)$ as subgroup. The remarkable property of the theory, which raised $N = 8$ to the status of a candidate for a truly fundamental theory, is the fact that the final Lagrangian has unexpected symmetries: (i) A global non-compact $E_7$ symmetry and (ii) a gauge $SU(8)$ symmetry whose gauge bosons are not elementary fields. They are composites made out of the 70 scalars. $SU(8)$ is large enough to contain the symmetries of the standard model, but this implies that all known gauge fields ($\text{gluons, } W^\pm, Z^0, \gamma$) are in fact composite states. The elementary fields are only the members of the fundamental multiplet (92). We believe that none of the particles we know is among them, they should all be obtained as bound states.

$N = 8$ supergravity promises to give us a truly unified theory of all interactions, including gravitation and a description of the world in terms of a single fundamental multiplet. We still have many problems to solve before reaching this final step and I shall list some of them: (i) Is it a consistent field theory? The Lagrangian is certainly not renormalizable, but the large number of supersymmetries and the good convergence properties that often accompany
them may make the problem tractable. (ii) Can we solve the bound state problem and obtain the known particles out of the fields of the fundamental supermultiplet? (iii) It seems that only $N = 1$ supersymmetry may be relevant for low-energy phenomenology. Is it possible to break $N = 8$ into $N = 1$ spontaneously? (iv) Can $N = 8$ accommodate chiral fermions? The problems are formidable but the expectations are great!
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