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NATURAL CP BREAKING IN LEFT-RIGHT SYMMETRIC THEORIES

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A B S T R A C T

It is shown that in the minimal left-right symmetric gauge theory based on $SU(2)_L \times SU(2)_R \times U(1)$, a CP violating vacuum cannot be obtained in a natural way, independently of the CP transformation properties of the Higgs fields. We point out that the simplest modification in the Higgs sector which can lead to spontaneous CP breaking involves the introduction of a pseudoscalar singlet. Other Higgs structures are analyzed, emphasizing the interplay between CP transformation properties and the possibility of generating spontaneous CP violation.

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In spite of its great successes, the standard $U(1) \times SU(2) \times SU(3)$ gauge theory leaves open many fundamental questions. One of these questions is the origin of P and CP breaking. A possible route to a deeper understanding of the breaking of these symmetries is to require that they be symmetries of the Lagrangian, only broken by the vacuum. The simplest realization of this idea is within $SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric (LRS) gauge theories¹⁾ where P and CP can indeed be broken spontaneously. There is another reason why LRS theories are an interesting framework to study CP violation: they provide a possible understanding²⁾ of the smallness of the recently measured³⁾ ϵ'/ϵ ratio [$\epsilon'/\epsilon = (-4.6 \pm 5.3 \pm 2.4) \times 10^{-3}$]. In this letter, we investigate how a naturally CP breaking vacuum can arise in LRS gauge theories. It will be shown that actually in the so-called minimal LRS theory the tree approximation vacuum does not break CP, provided no fine-tuning of parameters is done. This result holds true for any CP transformation properties chosen for the scalar fields. The crucial point is that the value of the potentially CP violating phase is controlled by quartic terms in the Higgs potential, all with the same phase dependence, which leads to a CP conserving vacuum.

In view of the above result, we look for the minimal Higgs system necessary for obtaining a CP violating vacuum, without having to resort to fine-tuning of parameters. It turns out that the simplest solution involves the introduction of a pseudoscalar singlet. We find this result interesting since it shows that if one requires spontaneous CP violation, the minimal Higgs content of a LRS theory allows for the possibility of separating the scales of P and $SU(2)_R$ breaking⁴⁾. However, it will be shown that, at least in the context of a minimal model, these two scales cannot be vastly different, since otherwise the model becomes unrealistic. Recently, it has been emphasized⁵⁾ that the introduction of non-trivial CP transformations for the scalar fields may be of physical interest since their extension to the Yukawa interactions can lead to calculability of mixing angles in terms of quark masses. We will analyze the interplay between non-standard CP transformation properties and the possibility of generating spontaneous CP violation.

THE MINIMAL MODEL

In the minimal left-right symmetric model one introduces the following Higgs fields:

$$\begin{aligned} \phi(2, 2, 0) \equiv \phi_1 \quad ; \quad \tilde{\phi} = \tau_2 \phi^* \tau_2 \equiv \phi_2 \\ \Delta_L(3, 1, 2) \quad ; \quad \Delta_R(1, 3, 2) \end{aligned} \quad (1)$$

Under P these fields transform as

$$P: \quad \phi_i \longrightarrow \phi_i^\dagger \quad ; \quad \Delta_L \longrightarrow \Delta_R \quad ; \quad \Delta_R \longrightarrow \Delta_L \quad (2)$$

Since our arguments crucially depend on the scalar potential, we give next its most general gauge invariant form, consistent with renormalizability and P invariance:

$$\begin{aligned} V &= V_\phi + V_\Delta + V_{\Delta\phi} \\ V_\phi &= m_{ij} \text{tr} \phi_i^\dagger \phi_j + c_{ijkl} \text{tr} (\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l) + d_{ijkl} \text{tr} (\phi_i^\dagger \phi_j) \text{tr} (\phi_k^\dagger \phi_l) \\ V_{\Delta\phi} &= \alpha_{ij} [\text{tr} (\phi_i^\dagger \phi_j) \text{tr} (\Delta_L^\dagger \Delta_L) + \text{tr} (\phi_j^\dagger \phi_i) \text{tr} (\Delta_R^\dagger \Delta_R)] + \beta_{ij} \text{tr} (\Delta_L^\dagger \Delta_L \phi_i^\dagger \phi_j + \Delta_R^\dagger \Delta_R \phi_i^\dagger \phi_j) \\ &+ \beta'_{ij} \text{tr} (\Delta_L \Delta_L^\dagger \phi_i^\dagger \phi_j + \Delta_R \Delta_R^\dagger \phi_i^\dagger \phi_j) + \gamma_{ij} [\text{tr} \Delta_L^\dagger \phi_i \Delta_R \phi_j^\dagger + h.c.] \end{aligned} \quad (3)$$

where V_Δ stands for the part of the potential depending only on Δ_L, Δ_R . Hermiticity together with P-invariance imply some relations among the coupling constants:

$$\begin{aligned} m_{ij} = m_{ji} \quad ; \quad c_{ijkl} = c_{lijk} = c_{klji} = c_{jkl i} \\ d_{ijkl} = d_{jilk} \quad ; \quad \alpha_{ij} = \alpha_{ji}^* \quad ; \quad \beta_{ij} = \beta_{ji}^* \quad ; \quad \beta'_{ij} = \beta'_{ji}^* \end{aligned} \quad (4)$$

Furthermore, P invariance and Hermiticity constrain all coupling constants to be real, except α_{ij} , β_{ij} and β'_{ij} ($i \neq j$). So far, we have not imposed CP invariance on the Lagrangian. It will be shown that, independently of the CP transformation properties of the Higgs fields, it is not possible to obtain in a natural way spontaneous CP violation. We will start with the usual choice of CP transformation, where CP invariance is manifest and then consider the case of non-manifest CP invariance.

MANIFEST CP INVARIANCE

The scalar fields transform in the following way

$$\text{CP: } \phi_i \rightarrow \phi_i^* ; \Delta_L \rightarrow \Delta_L^* ; \Delta_R \rightarrow \Delta_R^* \quad (5)$$

In this case, CP invariance constrains all coupling constants to be real, but it does not introduce any further constraints beyond those of Eq. (4). Without loss of generality, the charge conserving vacuum can be written as:

$$\langle \phi \rangle = e^{i\alpha/2} \text{diag.}(|k_1|, |k_2|); \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ u_L e^{i\delta} & 0 \end{pmatrix}; \Delta_R = \begin{pmatrix} 0 & 0 \\ u_R & 0 \end{pmatrix} \quad (6)$$

where $u_{L,R}$ are real. In order to analyze the CP properties of the vacuum, it is convenient to introduce the following parametrization:

$$u_L = U \sin r ; u_R = U \cos r ; |k_1| = K \cos s ; |k_2| = K \sin s \quad (7)$$

The part of the potential depending on the angles r, s, α, δ can then be written

$$\begin{aligned} V_f &= V_1 + V_2 \\ V_1 &= A_1 k^4 \sin^2 2s + A_2 U^4 \sin^2 2r + A_3 U^2 K^2 \cos 2s \\ V_2 &= B_1 K^2 \sin 2s \cos \alpha + B_2 K^4 \sin^2 2s \cos 2\alpha + B_3 K^4 \sin 2s \cos \alpha + \\ &+ B_4 U^2 K^2 \sin 2s \cos \alpha + U^2 K^2 \sin 2r [B_5 \sin 2s \cos \delta + B_6 \sin^2 s \cos(\delta - \alpha) + B_7 \cos^2 s \cos(\delta + \alpha)] \end{aligned} \quad (8)$$

where A_1, B_1 are linear combinations of the coupling constants appearing in (3).

The stationarity conditions for r and δ lead to:

$$\sin 2r = \frac{1}{2} \frac{K^2}{A_2 U^2} [B_5 \sin 2s \cos \delta + B_6 \sin^2 s \cos(\delta - \alpha) + B_7 \cos^2 s \cos(\delta + \alpha)] \quad (9)$$

$$\tan \delta = \left[\frac{B_6 \sin^2 s - B_7 \cos^2 s}{B_5 \sin 2s + B_6 \sin^2 s \cos \alpha + B_7 \cos^2 s \cos \alpha} \right] \sin \alpha \quad (10)$$

From (8), (9) and (10) it is clear that, provided no unnatural fine-tuning of parameters is done, the value of α at the minimum will be controlled by the term $B_4 U^2 K^2 \sin 2s \cos \alpha$ and will be at $\alpha = 0$ or $\alpha = \pi$ depending on whether B_4 is negative or positive. From (10) one concludes that $\delta = n\pi$ at the minimum and therefore the tree approximation vacuum is CP invariant. A theorem⁶⁾ due to Georgi and Pais guarantees that CP will be conserved to all orders in perturbation expansion.

At this point, it is worthwhile to examine the question of fine-tuning in the present context. For a CP breaking solution to exist, it is necessary to choose a very small B_4 ($B_4 \approx K^2/U^2$). Noting now that $B_4 = 2(\alpha_{12} + \beta_{12} + \beta'_{12})$ it is clear that choosing B_4 small is not natural in the technical sense, since the Lagrangian does not acquire any symmetry in the limit of vanishing B_4 .

So far, we have considered the case where the $\phi(2,2)$ have the standard CP transformations. We will see next that the previous result holds for any CP transformation property of the Higgs fields.

NON-MANIFEST CP INVARIANCE

Although the CP transformation introduced in (5) is usually adopted, it is certainly not unique. Of special interest are those non-trivial CP transformations whose extension to the Yukawa interactions can lead to calculability of mixing angles in terms of quark masses. In the present case, since one has only one $(2,2,0)$ multiplet, there are only two possible CP transformations:

$$\begin{aligned} (i) \text{ CP: } \phi_1 &\rightarrow e^{i\psi} \phi_1^* & ; & \phi_2 \rightarrow e^{-i\psi} \phi_2^* \\ (ii) \text{ CP: } \phi_1 &\rightarrow e^{i\psi} \phi_2^* & ; & \phi_2 \rightarrow e^{-i\psi} \phi_1^* \end{aligned} \quad (11)$$

It can be shown that the choice (ii) leads, at least in the case of three generations, to unrealistic quark masses and therefore will not be considered

further. The implications of CP transformation (i) have been recently analyzed⁵⁾. A particularly attractive choice is

$$CP: \quad \phi_1 \rightarrow i\phi_1^* \quad ; \quad \phi_2 \rightarrow -i\phi_2^* \quad (12)$$

since in this case, CP invariance alone, when extended to Yukawa couplings leads to phenomenologically interesting relations for mixing angles and quark masses. However, a close look at the potential shows that also in this case it is not possible to obtain a CP violating vacuum in a natural way. The value of the phase of $\langle \phi_1 \rangle$ will again be determined by terms in the Higgs potential of order $U_2 K_2$, with non-trivial phase dependence, which have coefficients α_{12} , β_{12} , β'_{12} in expression (3). Noticing that for $\phi = \pi/2$ in (i), CP invariance constrains these three coefficients to be pure imaginary, one finds that the relevant part of the potential is now of the form $k^2 U^2 \sin \alpha \sin 2S$, leading to:

$$\langle \phi \rangle = e^{i\pi/4} \text{diag.} (|k_1|, |k_2|) \quad (13)$$

In view of (12), it is clear that this vacuum is CP invariant. It is trivial to verify that for any other value of ϕ in (11), one cannot obtain a CP violating vacuum, with the minimal Higgs system. At this point, we would like to point out that the phase appearing in (13) is of the geometrical type⁷⁾, that is, its value is determined entirely by the symmetry of the Lagrangian and it is therefore independent of the specific values of the arbitrary parameters of the scalar potential. Some time ago, it was shown that, apart from some notable exceptions⁷⁾, vacua with geometrical phases are CP conserving.

In view of the above result, we analyze next what is the minimal modification in the Higgs sector, in order to obtain a CP violating vacuum.

MINIMAL MODEL WITH NATURAL CP BREAKING

We introduce an additional singlet η which transforms under P, CP in the following way:

$$P: \quad \eta \rightarrow -\eta \quad ; \quad CP: \quad \eta \rightarrow -\eta \quad (14)$$

Furthermore, we assume that ϕ , $\Delta_{L,R}$ transform in the standard way indicated in

Eq. (5). The new terms in the scalar potential which are relevant for our analysis are:

$$V_3 = i C_1 V \eta \operatorname{tr}(\phi_2^\dagger \phi_1 - \phi_1^\dagger \phi_2) + C_2 \eta^2 \operatorname{tr}(\phi_2^\dagger \phi_1 + \phi_1^\dagger \phi_2) \quad (15)$$

In order to find physically acceptable CP violating solutions we evaluate α , s at the stationary point:

$$\tan \alpha = \frac{C_1 V^2}{B_4 U^2 + C_2 V^2} \quad ; \quad \tan 2s = \frac{(B_4 \cos \alpha) U + (C_1 \sin \alpha + C_2 \cos \alpha) V^2}{A_3 U^2} \quad (16)$$

From (16) it follows that in general the vacuum will violate CP and a realistic value of s can also be obtained. At this stage, we should note that in this minimal model with natural CP breaking there are in principle three different mass scales, associated with the breaking of $SU(2)_L$, $SU(2)_R$ and parity, which can be identified with the vacuum expectation values K , U and V . Due to the predominantly V-A character of the observed charged weak interactions, one necessarily has $U^2 \gg K^2$. The two other scales U and V may in principle coincide and in that case the breakings of $SU(2)_R$ and parity are at the same level. However, from (16) it follows that, at least within the context of the present minimal model, one cannot make U and V vastly different without rendering the model physically unrealistic. For example, in the limit where V^2/U^2 becomes infinite, $s = \pi/4$ which corresponds to $K_1 = K_2$ and therefore to vanishing Cabibbo mixing. On the other hand, in the limit of vanishing V^2/U^2 the vacuum becomes CP invariant. At this point, it is worthwhile emphasizing that the pseudoscalar character of η is crucial in order to obtain a CP breaking vacuum. For η even under parity, the first term in (15) is not present and one obtains $\alpha = 0, \pi$.

We have presented what we consider to be the minimal model with spontaneous CP violation in the framework of left-right symmetric theories. The phase appearing in $\langle \phi \rangle$ will in general lead to complex mixing matrices in the left- and right-handed sectors. However, since we have adopted the standard CP transformations of the various fields and no extra symmetries were introduced, the various mixing angles will be arbitrary. We consider next the case of non-standard CP transformations.

MINIMAL MODEL WITH NATURAL CP BREAKING AND CALCULABILITY OF MIXING ANGLES

The most interesting non-standard CP transformation is that of Eq. (12), which leads to calculable weak mixing angles in agreement with all present experimental data. However, we have previously shown that the model, as initially suggested, cannot lead to a naturally CP breaking vacuum. The minimal modification in the Higgs potential will again involve the introduction of a singlet η under the gauge group. Due to the new CP transformation properties of Φ given by Eq. (12), it is easy to convince oneself that in order to generate spontaneous CP breaking, a term of the form $\eta \text{tr}(\phi_2^\dagger \phi_1 + \phi_1^\dagger \phi_2)$ has to be introduced in the scalar potential, which in turn uniquely fixes the P and CP transformation properties of η :

$$P: \eta \rightarrow \eta \quad ; \quad CP: \eta \rightarrow -\eta \quad (17)$$

The relevant part of the scalar potential has now the following phase dependence:

$$V'_3 = A_3 U^2 K^2 \cos 2s + B'_4 U^2 K^2 \sin 2s \sin \alpha + C'_1 V^2 K^2 \sin 2s \cos \alpha \quad (18)$$

which leads to:

$$\tan \alpha = \frac{B'_4 U^2}{C'_1 V^2} ; \quad \tan 2s = \frac{B'_4 U^2 \sin \alpha + C'_1 V^2 \cos \alpha}{A_3 U^2} \quad (19)$$

It is clear from (18) that again the two mass scales U and V cannot be vastly different, since in the limit where V^2/U^2 becomes infinite, $s = \pi/4$ (no Cabibbo mixing) while in the limit of vanishing V^2/U^2 , $\alpha = \pi/2$, which in view of Eq. (12) means no CP violation. As previously mentioned, in this model the non-standard CP transformations of Eq. (12) constrain the Yukawa couplings in such a way that they lead to calculable weak mixing angles. However, it follows from (19) that the Kobayashi-Maskawa phase will be arbitrary.

OTHER HIGGS STRUCTURES

We consider next alternative Higgs structures which can lead to natural CP breaking in L.R.S. gauge theories. So far, we have considered only one $\phi(2,2,0)$ multiplet. It can be easily seen that if two such multiplets ϕ and ϕ' are introduced, then a naturally CP breaking vacuum can be obtained, even without the introduction of an additional singlet. In this case there will be three physically meaningful phases appearing in $\langle\phi\rangle$, $\langle\phi'\rangle$, which can be parametrized in the following way:

$$\begin{aligned}\langle\phi\rangle &= \text{diag.} (k_1, k_2) \equiv \text{diag.} (|k_1| e^{i\alpha/2}, |k_2| e^{i\alpha/2}) \\ \langle\phi'\rangle &= \text{diag.} (k_3, k_4) \equiv \text{diag.} (|k_3| e^{i\beta/2}, |k_4| e^{i\delta/2})\end{aligned}\quad (20)$$

We assume that under P the scalar fields transform as in Eq. (2). The dominant part of the scalar potential with non-trivial phase dependence is then constrained by gauge and P invariance to be of the form:

$$\begin{aligned}V_3'' &= b_{12} (|u_L|^2 k_1 k_2 + |u_R|^2 k_1^* k_2^*) + b_{13} (|u_L|^2 k_1 k_3^* + |u_R|^2 k_1^* k_3) + \\ &+ b_{14} (|u_L|^2 k_1 k_4 + |u_R|^2 k_1^* k_4^*) + b_{23} (|u_L|^2 k_2 k_3 + |u_R|^2 k_2^* k_3^*) + \\ &+ b_{24} (|u_L|^2 k_2 k_4^* + |u_R|^2 k_2^* k_4) + b_{34} (|u_L|^2 k_3 k_4 + |u_R|^2 k_3^* k_4^*) + \text{h.c.}\end{aligned}\quad (21)$$

If we adopt the standard CP transformations of Eq. (5), then CP invariance constrains all b_{ij} to be real. The stationarity conditions for α, β, γ are rather involved, but it is clear that for an appropriate range of the parameters b_{ij} , the vacuum will be CP violating.

At this point, one may ask whether this will still hold true when one tries to implement calculability⁸⁾ of weak mixing angles. A complete answer would require an exhaustive study of all models with two Φ multiplets, which is clearly beyond the scope of this paper. However, we would like to emphasize that the introduction of an additional symmetry in order to achieve calculability, can often make impossible the generation of spontaneous CP violation. As an example of this, we mention models⁵⁾ where, in order to achieve calculability, an additional symmetry is introduced, under which $\phi \rightarrow i\phi$, $\phi' \rightarrow i\phi'$. With this additional symmetry, all b_{ij} in (21) vanish, except b_{13} and b_{24} . It can then easily be seen from (21) that one cannot obtain a CP breaking vacuum.

Finally, we would like to mention another possibility, which consists of introducing two sets of $\Delta_{L,R}$. This Higgs structure has been previously considered⁹⁾ as an attempt to solve the strong CP problem and it has been shown that one may have spontaneous CP violation, but the phase of $\langle\phi\rangle$ is suppressed by V_L/V_R and therefore the dominant contribution to CP violation in the kaon sector comes from Higgs exchange.

In conclusion, we have investigated under what conditions a naturally CP breaking vacuum can be obtained in L.R.S. theories. Essentially, one may classify the various alternatives in the following way.

- (i) The simplest solution adopts the standard CP transformations for the various fields and involves the introduction of a pseudoscalar singlet. This naturally leads to a separation of the breaking scales for $SU(2)_R$ and parity.
- (ii) Another interesting solution introduces non-standard CP transformations, which constrain the Yukawa couplings in such a way that the weak mixing angles become calculable and in good agreement with their experimental value. A CP violating phase can be generated from the vacuum, provided a singlet even under P and odd under CP, is introduced. However, this CP violating phase is not calculable¹⁰⁾, depending on the arbitrary parameters of the Higgs potential.
- (iii) A third alternative consists of introducing two Φ multiplets. Its distinct feature relies on the fact that in this case there are only two physically relevant mass scales: M_{WL} , M_{WR} .
- (iv) The last possibility involves the introduction of two sets of $\Delta_{L,R}$. It has the special feature that both the left-handed and the right-handed mixing matrices are essentially real and therefore CP violation arises only from flavour-changing Higgs exchange.

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