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THE FOURTH GENERATION IN SUPERGRAVITY

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ABSTRACT

We consider model-independent constraints on the fourth-generation fermion masses and the magnitude of the D-term contribution to the scalar masses. We find that the ratio of vacuum expectation values is limited to the range $1/5 \lesssim |v_2/v_1| \lesssim 4.5$. We compute the mass spectrum in the class of models where the only source of soft supersymmetry breaking is the gaugino mass $m_{\frac{1}{2}}$. The lightest charged scalar turns out to be the fourth-generation slepton, which may be light enough to be pair-produced at LEP. The present PETRA bound on new charged particles implies that $m_{\frac{1}{2}} \gtrsim 150$ GeV. A general feature of the four-generation models is thus a heavy spectrum of sparticles.

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At present, the only known phenomenologically viable supersymmetric models are based on $N = 1$ supergravity¹⁾. In models with three generations and minimal Higgs content, the breakdown of $SU(2)_L \times U(1)_Y$ is determined by the evolution of the mass parameters in the scalar potential, according to the corresponding renormalization group equations²⁾. In particular, in the so-called "dimensional transmutation" models³⁾ the electroweak breaking occurs radiatively, and its scale is determined to be exponentially small with respect to the scale (usually the GUT scale M_X) where the boundary conditions are imposed on the soft supersymmetry breaking part of the Lagrangian: $m_W \sim e^{-O(1)/\alpha_t \cdot M_X}$ where $\alpha_t \equiv h_t^2/4\pi$ and h_t is the top quark Yukawa coupling. A further step is taken in the "no-scale" models⁴⁾, where also the scale of supersymmetry breaking is determined dynamically. It is remarkable that in all these models the many free parameters of supersymmetry breaking are considerably reduced; consequently, as many studies indicate⁵⁾, the low-energy spectrum and other relevant properties are relatively well constrained.

Since already the CERN $p\bar{p}$ collider may possibly be used to obtain non-trivial limits on the fourth generation particles⁶⁾, it is interesting to ask what the predictions of $N = 1$ supergravity models with four generations are⁷⁾⁻⁹⁾. In addition, theoretical motivations for considering a fourth generation are found in the currently popular superstring theories¹⁰⁾: although three-generation models also exist, the effective low-energy theory of the heterotic superstring¹¹⁾ could easily be a four-generation¹²⁾, $N = 1$ no-scale supergravity model^{4),13)}. In the present paper we will consider some model-independent features of the fourth-generation fermion masses and the mass spectrum of sparticles in the phenomenologically interesting class of models which have the gaugino masses as the only SUSY breaking parameters. Such a situation is encountered in the no-scale models which have a curved Kähler manifold⁴⁾ and which may well be the effective low-energy limit¹³⁾⁻¹⁵⁾ of the heterotic string¹¹⁾. Here our approach differs from the recent work^{7),8)} done on the four-generation models in supergravity which assumes a flat Kähler manifold.

We will assume that, at scales $m_W < \mu < M_X$, the effective Lagrangian is the sum of a supersymmetric part, determined by the superpotential

$$\mathcal{J} = h_{ab}^U H_2 Q_a U_b^c + h_{ab}^D H_1 Q_a D_b^c + h_{ab}^E H_1 L_a E_b^c + \mu H_1 H_2 \quad (1)$$

and a soft supersymmetry-breaking part, given by

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + m_{Q_a}^2 |Q_a|^2 + m_{U_a}^2 |U_a^c|^2 + m_{D_a}^2 |D_a^c|^2 \\ & + m_{L_a}^2 |L_a|^2 + m_{E_a}^2 |E_a^c|^2 + (B\mu H_1 H_2 + \text{h.c.}) + [(A_{ab}^U h_{ab}^U H_2 Q_a U_b^c + \\ & + A_{ab}^D h_{ab}^D H_1 Q_a U_b^c + A_{ab}^E h_{ab}^E H_1 L_a E_b^c) + \text{h.c.}] + \frac{1}{2} M_i (\lambda_i \lambda_i + \text{h.c.}) \end{aligned} \quad (2)$$

where obvious contractions of the group indices are understood, $a, b = 1, \dots, n_G$ are generation indices and λ_i ($i = 1, 2, 3$) are Majorana gaugino fields. Correspondingly, the quadratic part of the scalar potential involving the neutral Higgs fields is:

$$V_{\text{Higgs}}^{(2)} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + \text{h.c.}) \quad (3)$$

where

$$m_1^2 \equiv \mu_1^2 + \mu^2, \quad m_2^2 \equiv \mu_2^2 + \mu^2, \quad m_3^2 \equiv -B\mu.$$

As boundary conditions at $\mu = M_X$, we shall impose:

$$\mu_1^2 = \mu_2^2 = m_{Q_a}^2 = m_{U_a}^2 = m_{D_a}^2 = m_{L_a}^2 = m_{E_a}^2 = m_0^2, \quad (4)$$

$$A_{ab}^U = A_{ab}^D = A_{ab}^E = A_0, \quad (5)$$

$$M_1 = M_2 = M_3 = m_{1/2}, \quad (6)$$

$$\mu = \mu_0, \quad B = B_0. \quad (7)$$

The above boundary conditions, with equal soft scalar masses m_0 and equal gaugino masses $m_{1/2}$, are not the most general ones. However, Eqs. (4) and (5) are true if the Kähler potential of supergravity is $U(N)$ invariant (where N is the number of chiral scalars), whereas Eq. (6) may be true [but need not be true¹⁶⁾] if there is grand unification of interactions.

The parameters appearing in the effective Lagrangian are subject to renormalization group scaling. In the following, we shall work in the one-loop approximation, neglecting the Yukawa couplings of the first two generations and intergenerational mixing. The evolution of the gauge couplings and the gaugino masses is trivial: introducing the variable $t \equiv \ln M_X^2/\mu^2$, and defining $\tilde{\alpha}_i \equiv g_i^2/(4\pi)^2$ ($i = 1, 2, 3$), one has

$$\tilde{\alpha}_i(t) = \frac{\tilde{\alpha}_U}{1 + \tilde{\alpha}_U b_i t}, \quad M_i(t) = \frac{\tilde{\alpha}_i(t)}{\tilde{\alpha}_U} m_{1/2}, \quad (8)$$

where $b_1 = 2n_G + 3/5$, $b_2 = 2n_G - 5$, $b_3 = 2n_G - 9$ and $\tilde{\alpha}_U$ is the common value of the gauge couplings $\tilde{\alpha}_i(t)$ at the scale M_X where they come together (this need not imply, however, that there is conventional grand unification). Note that for $n_G > 4$ strong interactions are no longer asymptotically free; in the following, we shall assume $n_G = 4$. Taking $m_W = 82$ GeV, $\alpha(m_W) = 1/128$, $\alpha_3(m_W) = 1/8$, one finds

$$M_X = 2.9 \times 10^{16} \text{ GeV}, \quad \sin^2 \theta_w(m_W) = 0.229, \quad \tilde{\alpha}_U = \frac{3}{160\pi}. \quad (9)$$

Passing now to the evolution of the Yukawa couplings and of the mass parameters in the effective Lagrangian, the relevant renormalization group equations (RGE) for the case of four generations can easily be derived from Ref. 17). We do not reproduce them here.

Let us first consider the RGE for the Yukawa couplings, defining also, for further use, the quantity:

$$\omega \equiv \frac{v_2}{v_1}, \quad (10)$$

which measures the difference of the two VEVs $\langle H_1^0 \rangle \equiv v_1$, $\langle H_2^0 \rangle \equiv v_2$. In three- $\frac{2}{3}$ -5 generation models with a light top quark ($m_t \sim 40$ GeV), one finds that $\omega \approx 1$.

In our analysis of the Yukawa couplings, we shall assume the following boundary conditions at the scale M_X (corresponding to $t = 0$):

$$Y_b(0) = Y_\tau(0), \quad Y'_b(0) = Y'_\tau(0),$$

where $Y_b \equiv (h_{33}^D/4\pi)^2$, $Y_\tau \equiv (h_{33}^E/4\pi)^2$, $Y'_b \equiv (h_{44}^D/4\pi)^2$, etc... . The first relation is responsible for the successful prediction of the ratio m_b/m_τ in grand

unification, while the second one is a natural assumption, since we expect that the heavier the fermions, the better this kind of relations is justified. Although other possibilities exist⁸⁾, we do not find them particularly appealing.

Note that the mass parameters appearing in the effective Lagrangian do not affect the evolution of the Yukawa couplings; this will allow us to derive constraints on the fourth-generation masses and on the quantity ω independently of the soft supersymmetry breaking parameters and of the details of electroweak symmetry breaking. It seems reasonable to demand that the following conditions be satisfied:

- (i) A perturbative treatment of the Yukawa couplings should be possible in the whole energy range between m_W and M_X ; following the last of Ref. 18), we require that

$$Y(t) < 0.08 \quad (0 \leq t \leq 67) \quad (11)$$

- (ii) The observed spectrum of the fermions of the third generation should be reproduced; taking into account our approximations, we impose the requirements

$$\begin{aligned} 30 \text{ GeV} &\leq m_t \leq 50 \text{ GeV} \\ 3 \text{ GeV} &\leq m_b \leq 6 \text{ GeV} \\ 1.7 \text{ GeV} &\leq m_\tau \leq 1.85 \text{ GeV} \end{aligned} \quad (12)$$

where all the masses are evaluated at the scale $\mu = m_W$.

- (iii) The masses of the fermions of the fourth generation should satisfy the limits

$$m_{b'}, m_{t'} \geq 40 \text{ GeV}, \quad m_{\tau'} \geq 22 \text{ GeV}, \quad (13)$$

where the limit on $m_{b'}$ and $m_{t'}$ reflects the fact that they should weigh more than the top quark. The latter limit may soon be improved by the $p\bar{p}$ collider⁶⁾.

Of course, the exact relations between the Yukawa couplings and the corresponding fermion masses depend on the value of the parameter ω . The system of

RGEs for the Yukawa couplings cannot be solved analytically. We have then integrated the RGE numerically for different values of the parameter ω .

It is obvious that because of the constraints (11)-(13), ω cannot be arbitrarily large or small, and it is interesting that with these mild requirements one finds

$$0.2 \lesssim |\omega| \lesssim 4.5 \quad (14)$$

Therefore the difference between the top and the bottom masses cannot be explained by the hierarchy of the VEVs¹⁹⁾, but their Yukawas must be intrinsically different. We have verified that this statement remains true also in the case of three generations, where the analysis of the Yukawa couplings alone is not sufficient to prove this. However, we have checked that in supergravity models with a light top quark, such a situation would be incompatible with the known mechanisms of the electroweak symmetry breaking.

We find also that the lightest quark must be lighter than 150 GeV, while the heaviest must be lighter than 210 GeV and the charged lepton must be lighter than 105 GeV. For fixed ω , one can obtain more severe constraints on the masses. These results are displayed in Fig. 1 as allowed regions in the (m'_b, m'_t) plane corresponding to selected values of ω . It is interesting to note that, for $|\omega| \sim 1$, both quarks of the fourth generation should be lighter than 150 GeV.

Other interesting results are those concerning the mass ratios m_b/m_τ and m'_b/m'_τ . If the effects of the Yukawa couplings could be neglected, one would have $m_b/m_\tau|_{m_W} = m'_b/m'_\tau|_{m_W} = 3.48$ (to be compared with 2.42 in the case of three generations). However, the effect of the Yukawa couplings is to diminish those ratios: in general (without specifying the values of m'_t, m'_b and ω), one has $2.2 \lesssim m_b/m_\tau \lesssim 3.48$ and $1.8 \lesssim m'_b/m'_\tau \lesssim 3.34$; higher masses of the fourth generation fermions correspond to smaller values of the mass ratios, and vice versa. As a result, if one neglects the case of exceedingly high values for Y'_t , the experimental limit $m'_\tau > 22$ GeV corresponds to a region where the ratio $m'_b/m'_\tau \sim 3$ and thus induces the corresponding limit $m'_b \gtrsim 70$ GeV. As has been noticed in Refs. 8) and 9), in the limit where the Yukawa couplings of the fourth generation are large ($Y'_t, Y'_b \gtrsim 0.005$), they have infra-red fixed points, which dictate $m'_b \approx m'_t \approx 140$ GeV and $m'_\tau \approx 60$ -70 GeV in the case $\omega^2 \approx 1$. However, for smaller values of the Yukawa couplings [but still compatible with the limits in (13)], this behaviour is lost, as is displayed in Fig. 2.

The previous results were independent of any boundary conditions imposed on the soft supersymmetry breaking part of the effective Lagrangian. Now we consider the more model-dependent issue of the evolution of the mass parameters in the scalar potential, determining the electroweak breaking and the spectrum of sparticle masses. To do this, we need to specify the boundary conditions (4÷7), which we choose to be the simplest imaginable:

$$m_0 = A_0 = B_0 = 0, \quad m_{1/2}, \mu_0 \neq 0 \quad (15)$$

Therefore, the only source of supersymmetry breaking is given by gaugino masses, the scalar potential at M_X is globally supersymmetric and the only free parameters (in addition to the unknown masses of the fourth generation fermions) are $m_{1/2}$ and μ_0 . The situation summarized in (15) occurs in no-scale models⁴⁾, which may be the effective low-energy framework resulting from superstring theories after compactification¹³⁾⁻¹⁵⁾.

The necessary requirements for the radiatively-induced breakdown of $SU(2) \times U(1)$ have been extensively discussed in Refs. 3) and 5). Briefly, one must make sure that the potential is bounded from below at all scales; that there is no $SU(2) \times U(1)$ breaking already at scales $O(M_X)$; and that there are no minima which break the electric charge and/or colour. We implement the last condition by demanding that

$$\begin{aligned} A_{t_i}^2 &\leq 3(m_{Q_i}^2 + m_{U_i}^2 + m_2^2) \\ A_{b_i}^2 &\leq 3(m_{Q_i}^2 + m_{D_i}^2 + m_1^2) \\ A_{\tau_i}^2 &\leq 3(m_{L_i}^2 + m_{E_i}^2 + m_1^2) \end{aligned} \quad (i=3,4) \quad (16)$$

It may be²⁰⁾ that the conditions (16) are not exhaustive; there may be other conditions that should be imposed on some part of the parameter space. However, we believe that (16) is representative for a large class of possible models.

For a given set of the top quark and the fourth generation fermion masses, the known scale of $SU(2) \times U(1)$ breaking will fix one degree of freedom. Thus, for a given gaugino mass $m_{1/2}$ the parameter μ_0 is fixed. We have imposed a further requirement $m_{1/2}, \mu_0 < 1$ TeV in order not to destroy the hierarchy of scales protected by supersymmetry. Of course, any such limit can be taken as representative only.

We have integrated numerically the RGEs imposing the constraints discussed above. The gaugino masses are trivially found to be

$$M_1 = 0.23 m_{1/2}; \quad M_2 = 0.46 m_{1/2}; \quad m_{\tilde{g}} = 1.67 m_{1/2}. \quad (17)$$

The squark and slepton masses can be found by diagonalizing the mass matrix

$$\mathcal{M}_a^2 = \begin{bmatrix} m_a^2(LL) & m_a^2(LR) \\ m_a^2(LR) & m_a^2(RR) \end{bmatrix} \quad (18)$$

where a runs over squarks and sleptons. Here

$$\begin{aligned} m_a^2(LL) &= (C_a^L + H_a^L) m_{1/2}^2 + m_{f_a}^2 + \frac{\omega^2 - 1}{\omega^2 + 1} m_Z^2 D_{aL} \\ m_a^2(RR) &= (C_a^R + H_a^R) m_{1/2}^2 + m_{f_a}^2 + \frac{\omega^2 - 1}{\omega^2 + 1} m_Z^2 D_{aR} \\ m_a^2(LR) &= m_{f_a} [A_{f_a} + K\mu] \end{aligned} \quad (19)$$

where m_{f_a} is the mass of the fermionic partner (quark or lepton) of the scalar in question; $K = 1/\omega$ for u-squarks, and $K = \omega$ for d-squarks and charged sleptons. C_a^L and C_a^R are integrals depending on gauge couplings; their form can be found in Ref. 3). H_a^L and H_a^R are integrals depending on Yukawa couplings and are naturally important for the third and fourth generations only. The values of these integrals are shown in the Table (in the case of H_a^L and H_a^R for some examples of fourth-generation quark masses). Finally, D_{aL} and D_{aR} are the D-term contributions to the scalar masses, which are important only if $\omega^2 \neq 1$. These are also shown in the Table, for completeness.

Before going into details, let us discuss some general features of the sparticle spectrum in the case of four generations. The most striking feature is that invariably it is $\tilde{\tau}'$ which turns out to be the lightest scalar sparticle. The present PETRA bound of 22 GeV can then be used to put a limit on $m_{1/2}$. In all cases we have investigated $m_{1/2} \gtrsim 150$ GeV. This fact has two obvious but important

consequences. First, apart from $\tilde{\tau}'$, the whole sparticle spectrum, with the possible exception of right-handed sleptons, is so heavy that it is even beyond the reach of LEP. Second, because the scale of soft SUSY breaking is so large, D-term contributions, and in most cases even the supersymmetric contributions to the scalar masses, are almost negligible. Therefore, all squarks of different generations and all sleptons of different generations, with the exception of the right-handed sleptons, are roughly degenerate in mass. The order of magnitude of the parameter μ_0 is $m_{\frac{1}{2}}$.

A typical mass spectrum in the case $\omega^2 \approx 1$ is shown in Fig. 3, where $m'_t = 64$ GeV, $m'_b = 78$ GeV and $m'_\tau = 24$ GeV. Other quark mass values result in similar spectra with the mass of $\tilde{\tau}'_1$ (the lightest $\tilde{\tau}'$ mass eigenstate) becoming smaller as the quark mass scale becomes larger. For instance, because of this, the case $m'_t = 100$ GeV, $m'_b = 112$ GeV and $m'_\tau = 40$ GeV would have led us to conclude that $m_{\frac{1}{2}} > 350$ GeV. In general, the approximate lower limit is $m_{\frac{1}{2}} > 150$ GeV, giving the bound $m_g \gtrsim 250$ GeV. Except for the right-handed slepton, the masses of the first three generation sparticles are approximately equal to those of the fourth generation. Typically, one finds that $m_{e_R} \gtrsim 0(50)$ GeV, not light enough to be pair-produced at LEP 1.

In contrast to the three-generation models where the smallness of the top quark mass drives³⁾ the ratio of VEVs towards $|\omega| \approx 1$ to a high accuracy, the four-generation models can easily accommodate $|\omega| \neq 1$. However, the requirement that electroweak symmetry breaking takes place at the correct energy scale puts bounds on ω that are more severe than those obtained from our analysis of the Yukawa couplings only. We find that for $m_{\frac{1}{2}}$ and $\mu_0 \lesssim 1$ TeV

$$0.5 \lesssim |\omega| \lesssim 2 \quad (20)$$

In Fig. 4 we display examples of the mass spectrum in the cases $\omega^2 = 2$ and $\omega^2 = 0.5$. The spectra are qualitatively similar to the $\omega^2 \approx 1$ case, with $\tilde{\tau}'$ being always the lightest SUSY sparticle and $m_{\frac{1}{2}} > 150$ GeV. If $m_{\frac{1}{2}} > 0(\frac{1}{2})$ TeV, the D-term contributions are guaranteed to be negligible compared to the overall scale by virtue of the bound (20) and the upper limits on the fourth-generation quark masses discussed previously.

Some further comments on the mass spectrum are in order. The charginos are usually lighter than squarks, whereas the neutralinos are always heavier than about 45 GeV, the lightest one being predominantly the photino. Both the charged

and neutral Higgs scalars are very heavy except in the case $\omega^2 \rightarrow 1$, when one of the neutrals becomes a pseudo-Goldstone boson. In that case, its mass will be generated radiatively. However, because $m_{\frac{1}{2}} \gtrsim 150$ GeV, it can probably not be seen in $T \rightarrow H\gamma$ decays²¹⁾.

In conclusion, we have considered some model-independent features of the four-generation supergravity models with radiatively-induced breakdown of $SU(2) \times U(1)$. Useful limits can be obtained from the structure of the renormalization group only which dictate that the fourth-generation quarks should lie within reach of near-future experiments. We found also that the D-term contributions to the sparticle masses cannot be arbitrarily large, but must be confined to a rather narrow range. We considered also the spectrum of sparticles in the theoretically interesting case where the only soft supersymmetry breaking parameter is $m_{\frac{1}{2}}$, the primordial gaugino mass. The unambiguous prediction of such models is that the scalar sparticle masses are very heavy, with the possible exception of the fourth-generation (mostly right-handed) slepton. When applicable, our results are in general agreement with previous work⁷⁾⁻⁹⁾. It is interesting to note that unlike minimal-type supergravity models, which have been shown by Goldberg⁷⁾ to be capable of accommodating a gluino with mass less than 50 GeV, the four-generation models with $m_{3/2} = A_0 = B_0 = 0$ cannot give rise to spectacular physics at the CERN collider.

In many of the cases that we have considered, $\tilde{\tau}_1'$ was the lightest supersymmetric particle. Such a situation is not acceptable on cosmological grounds²²⁾, and it may well be that the most stringent constraints on the four-generation supergravity models come²³⁾ from cosmology.

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Table

The relevant parameters involved in the mass matrices for squarks and sleptons, according to the notation of formula (19) and assuming $m_t = 40$ GeV. The parameters H_a^L and H_a^R , depending on the Yukawa couplings, are tabulated (when they are not negligible) for the four illustrative cases A, B, C and D:

	ω^2	$m_t, [\text{GeV}]$	$m_b, [\text{GeV}]$	$m_\tau, [\text{GeV}]$
A	1	64	78	24
B	1	100	112	40
C	0.5	55	150	54
D	2	138	74	25

$C_Q = 5.14$	$D_{UL} = -\frac{1}{2} + 2/3 \sin^2\theta_W$
$C_U = 4.80$	$D_{UR} = -2/3 \sin^2\theta_W$
$C_D = 4.75$	$D_{DL} = \frac{1}{2} - 1/3 \sin^2\theta_W$
$C_L = 0.43$	$D_{DR} = 1/3 \sin^2\theta_W$
$C_E = 0.13$	$D_{EL} = \frac{1}{2} - \sin^2\theta_W$
	$D_{ER} = \sin^2\theta_W$
	$D_{NL} = -\frac{1}{2}$

	A	B	C	D
H_{3Q}	-0.20	-0.19	-0.30	-0.15
H_{3U}	-0.40	-0.37	-0.60	-0.29
H_{4Q}	-1.01	-1.82	-1.59	-1.78
H_{4U}	-0.82	-1.65	-0.83	-2.13
H_{4D}	-1.20	-1.99	-2.34	-1.43

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FIGURE CAPTIONS

Fig. 1 : The allowed regions in the plane (m'_b, m'_t) of the fourth-generation quark masses for different $\omega = v_2/v_1$.

Fig. 2 : The evolution of the fourth-generation Yukawa couplings as a function of $t \equiv \ln(M_X^2/\mu^2)$. $Y'_t(t)$ and $Y'_b(t)$ are shown in Figs. 2a and 2b respectively for the following boundary conditions:

- (a) $Y'_t(0) = Y'_b(0) = 0.08$;
- (b) $Y'_t(0) = Y'_b(0) = 8 \times 10^{-3}$;
- (c) $Y'_t(0) = 3 \times 10^{-5}$, $Y'_b(0) = 9 \times 10^{-5}$;
- (d) $Y'_t(0) = 0.08$, $Y'_b(0) = 1 \times 10^{-4}$.

On the left-hand side the values of $m'_t(m_W)$ and $m'_b(m_W)$ are plotted, corresponding to the case $|\omega| = 1$.

Fig. 3 : The scalar mass spectrum of the fourth generation in the case $\omega^2 \approx 1$, with $m_t = 40$ GeV, $m'_t = 64$ GeV, $m'_b = 78$ GeV and $m'_\tau = 24$ GeV (solid line). The dotted line shows the masses of the first three generations (averaged in the case of squarks). The splitting inside the third generation is about 100 GeV for stop at $m_{\frac{1}{2}} = 150$ GeV, which diminishes to about 20 GeV at $m_{\frac{1}{2}} = 500$ GeV. Sbottom splittings are less than 15 GeV.

Fig. 4 : The scalar mass spectrum of the fourth-generation in the case $\omega^2 = 0.5$, with $m_t = 40$ GeV, $m'_t = 55$ GeV, $m'_b = 150$ GeV and $m'_\tau = 54$ GeV (solid line). Also shown is the spectrum in the case $\omega^2 = 2$ with $m_t = 40$ GeV, $m'_t = 138$ GeV, $m'_b = 74$ GeV and $m'_\tau = 25$ GeV (dotted line). The masses of $\tilde{t}_1$, $\tilde{b}_1$, $\tilde{b}_2$ and $\tilde{\tau}_2$ are virtually identical to the $\omega^2 = 0.5$ case and are not shown.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4