ON THE LOW-ENERGY LIMIT OF SUPERSTRING THEORIESJ.P. Derendinger^{*)}, L.E. Ibáñez^{+)~} and H.P. Nilles^{‡)}

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ABSTRACT

We study some aspects of the low-energy supersymmetric $d = 4$ theory obtained as a limit of superstring theories. We first consider a general $O(6)$ invariant truncation of the $d = 10$ supergravity + Yang-Mills theory, including bosonic as well as fermionic bilinear terms. As a particular case, one may recover the results of Witten's truncation. The Kähler potential is easily obtained from the fermionic sector of the theory. We next study the process of supersymmetry breaking and argue that only in the case where the hidden E_8 symmetry is badly broken on compactification may one obtain a relatively small gravitino mass. The problem of SUSY breaking in the "observable" sector is also discussed, as well as the role of the "dilaton" and "axions" of the theory in connection with the strong-CP and cosmological constant problems. Finally, we classify possible low-energy phenomenological models and show how the residual gauge symmetries may be broken radiatively in a natural way. Some problems of the whole approach are pointed out.

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1. INTRODUCTION

Superstring theories [1] are a further step in an attempt to make progress in physics by imposing theoretically-motivated restrictions. From a theoretical point of view string theories are supposed to have less ultraviolet divergences, since in usual field theory many of the divergences can be attributed to the fact that one is dealing with pointlike particles. In that sense string theories are a better candidate for a unified field theory that is completely finite in perturbation theory. Since string theories necessarily contain gravitational interactions, finiteness might be the only way to arrive at a consistent quantum theory unifying all known interactions.

The construction of string theories is much more restrictive than that of pointlike particle field theories. Quantization of these theories is only consistent in certain space-time dimensions. The presence of tachyons in bosonic string theories seems to require supersymmetry as the only possible cure. The resulting superstring theories predict the dimensionality of space-time to be $d = 10$. In principle, it is very welcome that the theory makes such predictions, but since here the prediction is $d = 10$ and not 4 one still has to find out whether they have anything to do with the real world.

A closer examination of the consistency of these theories has brought more surprises. The choice of the gauge group is severely limited to $O(32)$ [2] or $E_8 \times E_8$ [2,3], as has been deduced both from the examination of anomalies [2] and finiteness [4]. Such superstring theories provide us with a possible finite theory of all interactions including gravity which predicts the dimension of space-time and almost uniquely the choice of the gauge group. Having obtained such a powerful and unique theory from theoretical considerations, it is inevitable to ask whether this theory has something to do with physics in the four-dimensional world as we know it. One of the first questions to ask in this direction is the question of the chirality of the fermion representations as observed in the $SU(3) \times SU(2) \times U(1)$ standard model. Contrary to many attempts

to explain the $d = 4$ theory from high dimensions, the superstring theory has this property at least in ten dimensions. Whether this property still survives in $d = 4$ is a question that concerns the compactification of the six extra dimensions. The theory tells us that $d = 10$ and the only way it can be relevant for physics is that it admits a compactification to $d = 4$.

We are at the moment not in a situation to answer the question whether the dynamics of the theory induces this compactification, but one can investigate the question of whether such a compactification could really lead to realistic four-dimensional theories [5-8]. The requirement of the chirality of fermion representations in $d = 4$ already restricts the possible compactification schemes. Gauge symmetries have to be broken during compactification very much unlike the situation in the usual attempts involving high-dimensional theories. The final answer to these questions can only be obtained once we understand string theories and know how to compute the relevant quantities. These remarks are valid for the arguments used throughout this paper and should be kept in mind.

To investigate these problems one would like first to have an approximation to string theories. Such an approximation could be the field theory in the zero-slope limit, i.e. the field theory of the ground states of strings disregarding the excited states. In the case at hand, a first step would be to consider $N = 1$ supergravity in $d = 10$ coupled to pure supersymmetric Yang-Mills gauge theory [9-11] with group $O(32)$ or $E_8 \times E_8$. However, this cannot be the whole story since the theory has gauge and gravitational anomalies, although it contains all the massless modes of the string theory. These anomalies can, however, be cancelled by the addition of interaction terms [2] exactly for the gauge groups mentioned above. The presence of these terms can be understood from the anomaly-free string theory where excited states appear in internal lines of graphs that still give a contribution in the zero-slope limit. These terms cancel anomalies that turn up once one computes the effective action at the one-loop level. Terms that cancel gauge anomalies have, therefore, to be gauge variant, a rather strange

situation if one wants to construct the theory as a pointlike particle field theory in its own right. The motivation, however, comes from the full superstring theories. Nonetheless, it is non-trivial that the field theory can be made anomaly-free with just a finite number of counter terms, and in that sense it might still be useful to consider it as an approximation. There might, of course, be additional terms that have to be added to get a meaningful approximation. The naïve zero-slope limit theory has gravitational anomalies and we therefore expect it also to possess a supersymmetry anomaly. It is not clear whether the terms added cancel this anomaly as well [12]. In any case, the field theory at the tree graph level will look non-supersymmetric since it has to contain terms that cancel an anomaly appearing at the one-loop level. This will make the discussion of a possible survival of supersymmetry in $d = 4$ very delicate as we will see later in several places.

Taking this improved $d = 10$ field theory as a starting point for our discussion, we can investigate the question of compactification. This discussion is very short because of the existence of a no-go theorem [13]: the classical equations of motion do not allow a non-trivial compactification to $d = 4$. This clearly shows a limitation of the approximation. The argument cannot be used to prove a similar statement for the string theory, since the inclusion of terms proportional to the square of the Riemann tensor [7] (motivated from string calculations) evades the no-go theorem.

Let us now discuss the question of the mass scales in more detail, and first repeat the discussion in type I-superstring models [1]. In ten dimensions trilinear closed string interactions are characterized by a coupling constant κ_{10} of dimension M^{-4} and open-string interactions by g_{10} of dimension $(\text{mass})^{-3}$ with the relation

$$M_s^2 = \kappa_{10} / g_{10}^2 \quad (1.1)$$

which defines the string mass scale. Assume now that six dimensions are compactified on a manifold with volume M_c^{-6} . In the four-dimensional theory we will then have the coupling constants

$$g_4 = g_{10} M_c^3, \quad \kappa_4 = \kappa_{10} M_c^3 \quad (1.2)$$

and if this describes the real world we would expect $\kappa_4^{-1} \sim M_{\text{Planck}}$ and $g_4 = O(1)$, leading to

$$M_s^2 \simeq \frac{M_c^3}{M_p} \quad (1.3)$$

Thus the situation $M_c \ll M_s$ is only possible if both scales are large compared to the Planck scale [14]. In fact, a situation with $M_c \ll M_s$ is not desirable, since for the string theory to have a sensible perturbative expansion one would require [15,16]

$$g_{10} \lesssim M_s^{-3}, \quad \kappa_{10} \lesssim M_s^{-4} \quad (1.4)$$

which would lead to unreasonable gauge coupling constants in four dimensions.

The corresponding relation in the heterotic [17] string theory is

$$M_s = g_{10} / \kappa_{10} \quad (1.5)$$

which together with expressions (1.2) leads to

$$M_s = g_4 M_p \quad (1.6)$$

giving an approximate equality of M_s and M_p independent of the scale of compactification. The relation (5) in the case of the heterotic string reflects the fact that the only interaction in this theory is the three-closed-string vertex. Since one coupling constant depends on the vacuum expectation value of the scalar ϕ in the supergravity multiplet, perturbation theory can be shown [12,15] to be an expansion in $\langle\phi\rangle$. In the case of type I superstrings such a simple behaviour cannot be expected since g and κ arise from different string vertices, giving an ambiguity in the loop expansions. Some graphs that are loop graphs if one considers open-string interactions are in fact tree-level contributions with exchange of a closed string.

The theoretical argument that M_c cannot be small compared to M_s can be backed up by 'phenomenological' arguments [15,16]. Since the string theories are only consistent for certain gauge theories, one expects the breakdown of these gauge theories at a scale M_{GUT} not larger than M_c . The gauge groups under consideration are very large, and viewed as grand unified interactions they would give problems with proton stability if M_c is too small. Given the arguments on chirality of fermion representations, as discussed previously, one would identify M_{GUT} and M_c . We would then expect all these scales M_s , M_c , and M_{GUT} to be of the order of the Planck scale. One should stress here that a sufficient size of M_{GUT} is a necessary condition for the absence of proton decay at an observable level. Models in four dimensions 'derived' from the superstring still have severe problems with proton stability due to non-gauge interactions, as we will discuss later.

Given our previous discussion it should be clear that a discussion of the improved ten-dimensional field theory as an approximation to superstring theories has severe limitations. Nevertheless, it contains all the massless modes of the string theory and has therefore to be discussed. This, of course, is true in the case where string theory has a well-defined perturbative expansion such that the full theory is not too different from its classical counter-

part. The fact that certain aspects of superstring theory can be simulated in field theory with a finite number of counterterms makes us confident that a study of the field theory is a sensible starting point to investigate superstring models. These results could then perhaps be improved once we understand more about string theories.

The second section of this paper contains the formulation of the $d = 10$ theory as a four-dimensional theory with six 'internal dimensions' and $SO(6)$ symmetry, which can serve as the starting point to simulate compactification schemes such as, for example, those on Calabi-Yau manifolds that preserve one supersymmetry in four dimensions [7]. The formulae given will be useful to read off the defining functions (such as the Kähler potential) of the four-dimensional supersymmetric theory. Our discussion includes that of Witten [18] for the bosonic terms as the simplest case. We argue, however, that this case, although very interesting as a starting point, might not be a very good simulation of compactification on Calabi-Yau manifolds.

We try to isolate those properties of the $d = 4$ theories that only weakly depend on the special compactification scheme used. These include a discussion of the gauge kinetic terms in the four-dimensional theory and the presence of axion-like objects originating from the antisymmetric tensor field B_{MN} that is crucial for the cancellation of the anomalies.

The next logical step is a discussion of the breakdown of $N = 1$ supergravity in four dimensions. The superstring models contain a natural hidden sector that breaks supersymmetry through gaugino condensates [19,20]. With such a mechanism a wide separation between the scales of grand unification and supersymmetry breakdown is likely to occur. The supersymmetry breakdown scale is determined through the renormalization group-invariant scale of the hidden sector gauge group, and we compute this scale for the hidden sector E_8' in the $E_8 \times E_8'$ model including various patterns of breakdown during compactification. It turns out that in order to obtain a scale of 10^{13} GeV (leading to a mass of

the order of 100 GeV for the gravitino), E'_8 has to be broken down to a small non-Abelian subgroup.

With supersymmetry broken in the hidden sector coupled only gravitationally to the observable sector, we then have to discuss the feeddown of supersymmetry breakdown to the latter. In general, one expects the scalar masses m and gaugino masses M to be related to the gravitino mass $m_{3/2}$, and the trilinear coupling A of order unity. In a special limiting case obtained through naïve reduction and truncation to $N = 1$ supergravity with some additional assumptions, one obtains a model in which the observable sector stays completely supersymmetric at the tree-graph level although supersymmetry is broken in the hidden sector at a large scale. Here one would then have to argue that radiative corrections (similarly to gravitational ones) still transmit supersymmetry breakdown to the observable sector [21,22]. In this case, however, one would have to discuss the influence of these radiative corrections from the beginning and to see whether this limiting case remains a sensible possibility with their inclusion -- a question which at the moment cannot be completely answered.

Nonetheless, this limiting case can serve as a toy model to discuss a possible relation between dilatons and axions in view of a simultaneous solution of the strong CP problem [23,24] and the problem of the cosmological constant. Axions appear through the zero modes of the antisymmetric tensor field, and their existence and properties are very much model independent. Their scalar partners are dilatons coupling to the field strength through the non-minimal gauge kinetic terms. In general, one obtains two axion candidates corresponding to the two gauge groups. We will discuss the solution of the strong CP problem and the conjectured relation to the problem of the cosmological constant. This discussion includes an investigation of the terms added to cancel the anomalies in the ten-dimensional theory, which are especially important for the coupling of the second axion.

In the following section we investigate the question of realistic grand unified model building [25,26], starting with a model of minimal particle con-

tent and later generalizing to non-minimal models. While several properties of the models are very promising, such as chirality of fermion representations and sources for symmetry breakdown, there remain nonetheless severe problems. These include primarily the absence of sufficient suppression of proton decay through non-gauge interactions and the problem of neutrino masses. These problems arise in a rather model-independent way and seem to be hard to overcome unless some appropriate discrete symmetries [26] exist in the compact manifold.

In the last section we will give our conclusions.

2. TEN-DIMENSIONAL SUPERGRAVITY AS A FOUR-DIMENSIONAL MODEL

The ten-dimensional field theory obtained in the zero-slope limit of superstrings is a version of chiral $N = 1$ supergravity coupled to a Yang-Mills multiplet. The supergravity multiplet possesses 128 on-shell degrees of freedom. Its boson fields are a vielbein e_M^A , an antisymmetric tensor $B_{MN} = -B_{NM}$, and a real scalar ϕ . The fermions are a gravitino ψ_M and a spinor λ . In the Yang-Mills sector a gaugino spinor χ^α corresponds to each gauge field A_M^α . All fermion fields are Majorana-Weyl spinors. A supersymmetry algebra as well as a Lagrangian has been obtained for these multiplets [9-11]. However, this theory does not exactly reproduce the zero-slope limit of superstrings. For instance, Green and Schwarz [2] have shown that the cancellation of anomalies in superstrings with gauge group $SO(32)$ or $E_8 \times E_8$ requires the addition of higher derivative bosonic interaction terms to the supergravity Lagrangian. The fully modified supergravity theory obtained in the low-energy limit of superstrings is not known. It may only exist if all the massive modes of the strings are included, or alternatively it may close using only the massless modes. The same problem arises for the supersymmetry transformation rules, which should probably be different from the standard supergravity algebra.

At the present stage, compactification is discussed using the standard supergravity Lagrangian^{*}) [11]:

$$\begin{aligned}
 E^{-1} \mathcal{L} = & -\frac{1}{2} R - \frac{i}{2} \bar{\Psi}_M \Gamma^{MNP} D_N(\omega) \Psi_P + \\
 & + \frac{3}{4} \varphi^{-3/2} H_{MNP} H^{MNP} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M(\omega) \lambda + \\
 & + \frac{9}{16} (\partial_M \varphi / \varphi)^2 + \frac{3}{8} \sqrt{2} \bar{\Psi}_M (\not{\partial} \varphi / \varphi) \Gamma^M \lambda - \\
 & - \frac{\sqrt{2}}{16} \varphi^{-3/4} H_{MNP} (i \bar{\Psi}_Q \Gamma^{QMNP R} \Psi_R + 6i \bar{\Psi}^M \Gamma^N \Psi_P \\
 & + \sqrt{2} \bar{\Psi}_Q \Gamma^{MNP} \Gamma^Q \lambda - i \bar{\chi}^\alpha \Gamma^{MNP} \chi^\alpha) - \\
 & - \frac{1}{4} \varphi^{-3/4} F_{MN}^\alpha F^{\alpha MN} + \frac{i}{2} \bar{\chi}^\alpha \Gamma^M (D_M(\omega) \chi)^\alpha - \\
 & - \frac{i}{4} \kappa \varphi^{-3/8} (\bar{\chi}^\alpha \Gamma^M \Gamma^{NP} F_{NP}^\alpha) (\Psi_M + i \frac{\sqrt{2}}{12} \Gamma_M \lambda) \\
 & + \text{four fermion interactions} .
 \end{aligned} \tag{2.1}$$

Here E is the ten-dimensional vielbein determinant. The Γ 's are $SO(1,9)$ gamma matrices and $M, N, P, \dots = 0, \dots, 9$. The $SO(1,9)$ metric is $(+ - \dots -)$. The only parameter is in fact the gravitational constant κ since the gauge coupling g can be reabsorbed into φ . The covariant derivatives are

$$D_N \Psi_P = (\partial_N - \frac{1}{2} \omega_{N[RS]} \Gamma^{RS}) \Psi_P - 2 \omega_{NP}^Q \Psi_Q$$

^{*}) As shown in Ref. 20, a four-fermion coupling is missing in the Lagrangian of Ref. 11.

$$D_N \lambda = \left(\partial_N - \frac{1}{2} \omega_{N[RS]} \Gamma^{RS} \right) \lambda$$

$$(D_N \chi)^\alpha = \left(\partial_N - \frac{1}{2} \omega_{N[RS]} \Gamma^{RS} \right) \chi^\alpha - f^{\alpha\beta\gamma} A_N^\beta \chi^\gamma$$

The Yang-Mills field strengths are as usual

$$F_{MN}^\alpha = \frac{1}{2} \partial_{[M} A_{N]}^\alpha + f^{\alpha\beta\gamma} A_M^\beta A_N^\gamma \quad (2.2)$$

where $f^{\alpha\beta\gamma}$ are structure constants and α, β, γ are indices in the adjoint representation of the gauge group. From the fermion bilinear couplings it is apparent that ψ_M and χ^α have the same 'handedness' (we choose left), while λ will have the opposite (right) chirality. H_{MNP} is the gauge-invariant field strength for the antisymmetric tensor B_{MN} . It is known [2] to have different form in the superstring case from that in the standard supergravity:

$$H_{MNP} = \partial_{[M} B_{NP]} - \frac{\kappa}{\sqrt{2}} \left(\omega_{MNP}^{(Y.M.)} - \omega_{MNP}^{(L)} \right) \quad (2.3)$$

The Yang-Mills Chern-Simons form is

$$\omega_{MNP}^{(Y.M.)} = \text{Tr} \left(A_{[M} F_{NP]} - \frac{2}{3} A_{[M} A_N A_{P]} \right) \quad (2.4)$$

where the trace runs over the adjoint representation of the gauge group. The Lorentz Chern-Simons from $\omega_{MNP}^{(L)}$, present only in the limit of superstrings, reads

$$\omega_{MNP}^{(L)} = \text{tr} \left(\omega_{[M} R_{NP]} - \frac{2}{3} \omega_{[M} \omega_N \omega_{P]} \right) \quad (2.5)$$

in terms of connections w and of the curvature tensor. The trace runs over (omitted) $SO(1,9)$ vector indices.

The Lagrangian (2.1) without the Chern-Simons Lorentz form (2.5) is invariant under the supersymmetry transformation rules given in Ref. 11. The modification of H_{MNP} indicates that some terms have to be added to restore, if possible, a supersymmetry invariance. It has also been argued [7] that the field theory limit of superstrings contains an R^2 term. The Lagrangian (2.1) with the definition (2.2) to (2.5) will be our basic tool to discuss the effective four-dimensional theory resulting from superstrings. This was also the approach of Ref. 7.

The vacuum of relevant compactifications is of the form $M_4 \times K_6$. The dynamics of the ten-dimensional theory selects the six-dimensional compact manifold K_6 and should also impose that M_4 is Minkowski space-time. The tangent space symmetry group $SO(1,9)$ splits into $SO(1,3) \times SO(6)$, $SO(6)$, being then in general broken due to the geometry of K_6 . We now consider, as a first step of the compactification procedure, the theory with the tangent space group $SO(1,3) \times SO(6)$. Ten-dimensional indices $M, N, P, \dots$ split into four-dimensional indices $\mu, \nu, \rho, \dots = 0, 1, 2, 3$ and 'internal' $SO(6)$ indices $m, n, p, \dots = 1, \dots, 6$. The breaking of the tangent space group is achieved through a gauge choice for the vielbein, $E_m^\alpha = 0$, ($\alpha = 0, 1, 2, 3$; $m = 4, \dots, 9$). The Lagrangian (2.1) rewritten with this procedure exhibits the possible spin content of the compactified theory. All fields, however, depend also on the internal coordinates x_m . The fully compactified four-dimensional theory is obtained when this dependence is integrated in the action, using an expansion of the fields around their background value defining K_6 . This process will provide the mass spectrum and the interactions of the compactified theory. The simplest case is to assume no dependence on x_m , i.e. $\partial_m \phi = 0$ for any field ϕ . This choice corresponds to simple dimensional reduction from ten to four dimensions for which K_6 is a six-torus. In that case the resulting theory obtained from Eq. (2.1) is an $N = 4$ super-

gravity theory coupled to six Abelian vector multiplets [9], in addition to the $N = 4$ Yang-Mills sector with the same gauge group as the ten-dimensional theory. Notice that, for this simple case, the additional term in H_{MNP} due to strings does not contribute after reduction. More complicated compactifications lead, in general, to a smaller number of supersymmetries in the four-dimensional theory.

We will further on concentrate our discussion on compactifications which preserve $N = 1$ supergravity in four dimensions. It has been argued [7] that in this case K_6 is a 'Calabi-Yau manifold', i.e. K_6 is a Ricci-flat Kähler manifold with $SU(3)$ holonomy group^{*}). Then the compactified theory will be defined in terms of two functions $G(z^i, z_i^*)$ and $f_{\alpha\beta}(z^i)$ of the four-dimensional scalars z^i , according to the general $N = 1$ supergravity Lagrangian in four dimensions [28]. The compactification procedure uniquely fixes the field content and the two functions of the effective theory. Because of supersymmetry, these two functions can be obtained from different sectors of the theory. The real part of $f_{\alpha\beta}$ is the metric of the gauge field and gaugino kinetic terms. G and its derivatives appear in the scalar sector which is, however, subject to complicated field redefinitions to define properly scalars of the four-dimensional theory. Alternatively, G can be obtained from the part of the Lagrangian bilinear in the fermion fields. This is probably the most convenient method to obtain G , since redefinitions of fermion fields are, in general, only rescaling by simple functions of scalar components. Also, the gravitino and the gauginos are gauge fields in four dimensions and the gauge algebra helps to define these fields correctly.

As a first aspect of the compactification process, we now reformulate the Lagrangian (2.1) in '4 + 6 dimensions', with a tangent space group $SO(1,3) \times SO(6)$. We only consider explicitly the bilinear fermionic terms relevant in order to

^{*}) A different possibility may occur when fermion condensates are allowed to form at the compactification scale [27].

obtain the form of G and $f_{\alpha\beta}$ in the compactified theory, when supersymmetric. A Majorana-Weyl spinor in ten dimensions corresponds to four Weyl spinors in four dimensions. Moreover, ψ_M gives rise to spin 3/2 gravitinos (for $M = \mu$) and spin 1/2 Weyl spinors (for $M = m$) in four dimensions. We need an explicit form of $SO(1,9)$ Γ -matrices in terms of $SO(1,3)$ and $SO(6)$ matrices. The conventions we use are listed in Appendix A.

The gravitino kinetic term in Eq. (2.1) will give rise to spin 3/2 and 1/2 kinetic terms as well as kinetic terms mixing both spin states. A redefinition of the four-dimensional gravitino will be necessary to obtain diagonal kinetic terms. We use the following form of the ten-dimensional Majorana-Weyl gravitino ψ_M :

$$\psi_M = \begin{pmatrix} \psi_{iML} \\ \tilde{\psi}_{mR}^i \end{pmatrix} \quad (2.6)$$

where $i = 1, \dots, 4$ is an $SO(6)$ spinor index. The ψ_{iML} are left-handed four-dimensional spinors transforming as a $\underline{4}$ of $SO(6)$ and $\tilde{\psi}_{mR}^i$ are the charge conjugate spinors. The rescaling of the gravitino is of the form

$$\psi_{\mu iL} + \frac{i}{2} \gamma_\mu \gamma^5 (\Gamma^m)_{ij} \tilde{\psi}_{mR}^j \longrightarrow \psi_{\mu iL} \quad (2.7)$$

After this redefinition, the gravitino kinetic term in Lagrangian (2.1) reads

$$\begin{aligned} & -iE \bar{\psi}_{mR}^i \gamma^{\mu\nu\rho} \partial_\nu \psi_{\rho iL} + \\ & + \frac{i}{2} E \bar{\psi}_{mR}^i \left[\delta_i^j g^{mn} + \frac{1}{2} (\Gamma^m)_{ik} (\Gamma^n)^{kj} \right] \gamma^\mu \partial_\mu \psi_{njL} \end{aligned} \quad (2.8)$$

and there is no mixing. The relation between E and the $SO(1,3)$ vierbein determinant e is of the form

$$E = \exp(-3\sigma) e \quad (2.9)$$

(see Appendix A), where σ is a real $SO(6)$ -singlet scalar field. This field also gives the convenient rescaling of the metric tensor $g_{\mu\nu}$ [9]. Writing explicitly these factors for the kinetic terms in Eq. (2.8) indicates that a factor

$$\exp\left(\frac{3}{4}\sigma\right)$$

has to be absorbed within ψ_{MIL} by a rescaling in order to get canonical spin 3/2 kinetic terms in four dimensions. Using the definition (2.7) for the four-dimensional spin 3/2 fields, one can then rewrite the spin 3/2 'mass terms' appearing in Lagrangian (2.1), i.e. all terms bilinear in ψ_M without derivatives. This leads to the following expression, after rescaling of the spin 3/2 fields:

$$-e \bar{\psi}_{\mu i L} \gamma^{\mu\nu} \gamma^5 M^{ij} \psi_{\nu j L} \quad (2.10)$$

The 'mass' matrix is

$$M^{ij} = \exp(-2\sigma) \left[\partial_m \delta^i_k + \frac{1}{2} \omega_{m[np]} (\sigma^{np})^i_k \right] (\Gamma^m)^{kj} - \exp(-3\sigma) \left[\frac{\sqrt{2}}{8} \varphi^{-3/4} (\Gamma^{mnp})^{ij} H_{mnp} \right] \quad (2.11)$$

The first term is the covariant derivative on the internal manifold K_6 , while the second involves the modified field strength of B_{MN} in the internal directions. For compactifications with $N = 1$ supersymmetry in four dimensions, the corresponding interactions for the massless gravitino ψ_μ are of the form [28]

$$-e \exp(G/2) \bar{\psi}_{\mu L} \gamma^{\mu\nu} \gamma^5 \psi_{\nu L} \quad (2.12)$$

so that M_{ij} in Eq. (2.11) directly gives the Kähler function G when scalar fields are properly defined in four dimensions.

The next terms of interest contain non-derivative interactions linear in both spin 3/2 and spin 1/2 states. In '4 + 6 formulation' this part of the Lagrangian reads

$$\frac{1}{2} E \Psi_{\mu R}^i \gamma^\mu \alpha_{iL} + \frac{1}{2} E \bar{\Psi}_{\mu iL} \gamma^\mu \tilde{\alpha}_R^i \quad (2.13)$$

where the left-handed spinor α_{iL} is

$$\begin{aligned} \alpha_{iL} = & i (\Gamma^{mn})_i^j \left[\partial_m \Psi_{njL} + 2 \omega_{mn}^p \Psi_{pjL} + \omega_{m[rs]} (\Gamma^{rs})_j^k \Psi_{nkL} \right] - \\ & - \frac{\sqrt{2}}{4} \varphi^{-3/4} H_{mnp} \left[i (\Gamma^{mnpq})_i^j \Psi_{qjL} + \frac{i}{\sqrt{2}} \gamma^5 (\Gamma^{mnp})_{ij} \tilde{\chi}_L^j \right] + \\ & + \frac{i}{2} \varphi^{-3/8} (\Gamma^{mn})_i^j F_{mn}^\alpha \chi_{jL}^\alpha + \\ & + \frac{3\sqrt{2}}{4} i \gamma^5 (\Gamma^m)_{ij} (\partial_m \varphi / \varphi) \tilde{\chi}_L^j \end{aligned} \quad (2.14)$$

The last two equations hold prior to the redefinition of $g_{\mu\nu}$ and E , and the rescaling of the spinor fields by powers of $\exp \sigma$. Redefinitions of the spin 1/2 fields are more difficult to obtain. Their kinetic terms are not in general canonical and the complete compactified Lagrangian and/or supersymmetry transformation rules are necessary to fix their normalization.

The expression (2.13) corresponds, for compactifications preserving supersymmetry, to [28]

$$\frac{g}{2} G^i T_i^\alpha z_j (\bar{\Psi}_L \cdot \gamma) \lambda_R^\alpha - i \exp(G/2) G^i (\bar{\Psi}_R \cdot \gamma) \chi_{iL} \quad (2.15)$$

in terms of the Kähler function G and its derivative $G_i = \partial G / \partial z^i$. It will be the source of a possible Goldstino state when vacuum expectations values of some z^i scalars break supersymmetry.

The gravitino mass term, Eq. (2.10), and its goldstino coupling Eq. (2.13), are, in principle, sufficient to obtain the Kähler function G in terms of correctly normalized fields z^i . The second function $f_{\alpha\beta}(z^i)$ has then to be extracted from gauge fields or gaugino kinetic terms for its real part, while the imaginary part only appears in the $\epsilon^{\mu\nu\rho\delta} F_{\mu\nu}^\alpha F_{\rho\delta}^\beta$ coupling.

The bosonic sector of ten-dimensional supergravity can easily be reformulated in the $SO(1,3) \times SO(6)$ case. Contrary to gauge or fermion fields, the scalars are subject to non-polynomial redefinitions which can strongly differ depending on the choice of K_6 . We will not explicitly discuss the bosonic sector further here, except for some particular aspects which will be relevant in the following sections.

The ten-dimensional Lagrangian has a characteristic dilatational invariance due to the presence of the scalar φ . This invariance will not in general be broken by the compactification process. Consequently, φ will generate in four dimensions a real massless scalar field. The same will hold for σ when K_6 is Ricci-flat. This last property is probably necessary if the compactified theory has to arise from an underlying consistent, finite superstring theory [29]. The size of the internal manifold remains a free parameter. For compactification -- preserving supersymmetry, these two scalar fields will have pseudoscalar partners, arising from B_{MN} . In particular, $B_{\mu\nu}$ will in general correspond to a massless pseudoscalar.

The tensor $B_{\mu\nu}$ appears in the Lagrangian (2.1) only through the field strengths $H_{\mu\nu\rho}$ or $H_{\mu\nu m}$, as given by Eq. (2.3). For simple reductions, i.e. $\partial_m B_{\mu\nu} = 0$, the Lagrangian for $B_{\mu\nu}$ is

$$\left(\frac{3}{4} \psi^{-3/2} \exp(\sigma\sigma) H_{\mu\nu\rho} + O_{\mu\nu\rho} \right) H^{\mu\nu\rho} \quad (2.16)$$

where $O_{\mu\nu\rho}$ contains only fermion bilinears. $H_{\mu\nu\rho}$ satisfies the constraint

$$\partial_{[\lambda} H_{\mu\nu\rho]} + \frac{\kappa}{2\sqrt{2}} \text{Tr} (F_{[\lambda\mu} F_{\nu\rho]}) = 0 \quad (2.17)$$

which can be implemented in the Lagrangian by an additional term

$$- \frac{\theta}{2\sqrt{2}} \varepsilon^{\mu\nu\rho\lambda} \left(\partial_{[\mu} H_{\nu\rho\lambda]} + \frac{\kappa}{2\sqrt{2}} \text{Tr} (F_{[\mu\nu} F_{\rho\lambda]}) \right) \quad (2.18)$$

where θ is a real field acting like a Lagrange multiplier. The expression (2.18) can be rewritten in the form

$$\begin{aligned} \frac{1}{2\sqrt{2}} \varepsilon^{\mu\nu\rho\lambda} H_{\nu\rho\lambda} (\partial_\mu \theta) - \frac{\kappa}{8} \theta \varepsilon^{\mu\nu\rho\lambda} \text{Tr} (F_{[\mu\nu} F_{\rho\lambda]}) \\ + \text{total derivative,} \end{aligned} \quad (2.19)$$

which is manifestly gauge invariant. We can now integrate $H_{\mu\nu\rho}$ to get finally, instead of (2.16), the Lagrangian

$$\begin{aligned} \frac{1}{4} \psi^{3/2} \exp(-\sigma\sigma) (\partial_\lambda \theta) (\partial^\lambda \theta) + \\ + \frac{1}{3\sqrt{2}} \psi^{3/2} \exp(-\sigma\sigma) \varepsilon^{\mu\nu\rho\lambda} O_{\mu\nu\rho} \partial_\lambda \theta \\ + \frac{1}{3} \psi^{3/2} \exp(-\sigma\sigma) O^{\mu\nu\rho} O_{\mu\nu\rho} \\ - \frac{\kappa}{8} \theta \varepsilon^{\mu\nu\rho\lambda} \text{Tr} (F_{[\mu\nu} F_{\rho\lambda]}) \end{aligned} \quad (2.20)$$

This illustrates how the duality transformation from $B_{\mu\nu}$ to θ in the case of simple reduction generates new terms in the Lagrangian, containing the $F\tilde{F}$ coupling and four-fermion interactions. This mechanism will be important for the

derivation of $\text{Im } f_{\alpha\beta}(z^i)$ when compactification preserves supersymmetry. One can also check that, due to the Lorentz Chern-Simons form in $H_{\mu\nu\rho}$, a term of the form $\Theta R\tilde{R}$ does also appear.

3. TRUNCATION TO FOUR DIMENSIONS

Understanding compactification on a Calabi-Yau manifold is a difficult task owing to the complicated structure of these manifolds whose metric is unknown. In order to make a guess at how the four-dimensional theory might look after compactification, Witten [18] has recently suggested a truncation which presumably has some of the general features of Calabi-Yau compactification. The truncation goes as follows. One first assumes that the light fields are invariant under translations on x^m , $m = 5-10$. This would give rise to an $N = 4$, $d = 4$ supergravity theory, as we discussed in the previous section. The four SUSY generators sit on a $\underline{4}$ of $SU(4) \sim SO(6)$. To get rid of the three unwanted supersymmetries one splits $\underline{4} = \underline{3} + \underline{1}$ and requires invariance under this $SU(3)$ group for the light fields. This applies to the singlet fields. For the fields with gauge quantum numbers, owing to the identification of vacuum expectation values (v.e.v.'s) of gauge fields and manifold connections, one requires invariance under diagonal $SU(3) + SU(3)_G$, where $SU(3)_G$ is the subgroup in E_8 which commutes with E_6 . Under these conditions, one trivially gets that the light fields are: a) $E_6 \times E_8'$ gauge fields and gauginos; b) one E_6 family (27) of fermions from χ^α and scalar partners from A_m^α ; c) the two usual dilation fields φ and σ with we have already discussed in the previous section; d) the two parity partners of the above, i.e. a pseudoscalar from the ansatz $B_{mn} = \eta \epsilon_{mn}$ and the pseudoscalar θ that is obtained from $H_{\mu\nu\rho}$ after a duality transformation, as we discussed in the previous section; e) the fermionic partners of the above scalars which come from $SU(3)$ singlet components of the fermion λ and the gravitino field ψ_m ; f) graviton and gravitino.

With these rules it is straightforward to obtain a truncated version of the Chapline-Manton Lagrangian. For example, using the truncated ansatz

$$g_{\mu\nu}^{10} = e^{-3\sigma} g_{\mu\nu} ; \quad g_{mn}^{10} = e^{\sigma} \delta_{mn} ; \quad g_{m\mu}^{10} = g_{\mu m}^{10} = 0 \quad (3.1)$$

one obtains the following kinetic bosonic terms:

$$\begin{aligned} e^{-1} \mathcal{L}_{KB} = & -\frac{1}{2} R + \frac{9}{16} \left(\frac{\partial_{\mu} \psi \partial^{\mu} \psi}{\psi^2} \right) + 3 \partial_{\mu} \sigma \partial^{\mu} \sigma - \\ & - \frac{1}{4} \psi^{-3/4} e^{3\sigma} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} \theta \text{Tr} (F_{\mu\nu} \tilde{F}^{\mu\nu}) \\ & + \frac{3}{2} e^{-2\sigma} \psi^{-3/2} \left(\partial_{\mu} \eta - \frac{i}{\sqrt{2}} C_x^* \overleftrightarrow{D}_{\mu} C^x \right)^2 + \frac{1}{4} \psi^{3/2} e^{-6\sigma} (\partial_{\mu} \theta)^2 \\ & + 3 e^{-\sigma} \psi^{-3/4} D_{\mu} C_x^* D^{\mu} C^x \end{aligned} \quad (3.2)$$

where θ and η are the pseudoscalars obtained from $H_{\mu\nu\rho}$ and B_{mn} , respectively, and C^* denote the scalars transforming as a $\underline{27}$. The scalings under compactification (e^{σ} factors) are easily obtained from the assumed ansatz for the metric Eq. (3.1). The structure of the kinetic terms of the scalars looks quite complicated and it seems non-trivial to guess to what $N = 1$ Kähler potential it corresponds. However, one immediately recognizes the existence of a non-minimal kinetic gauge function:

$$f_{ab} = \delta_{ab} (\psi^{-3/4} e^{3\sigma} + i\theta) \quad (3.3)$$

In fact it is very easy to obtain the form of the Kähler potential from the general fermionic terms obtained in the previous section, Eqs. (2.10) and (2.11). In the general case one can write

$$G = 2 \log (M) \quad (3.4)$$

where M is the quantity in Eq. (2.11). In the case of the truncation considered here, internal coordinate derivatives on light fields vanish, so that the first term in Eq. (2.11) disappears and one gets from that formula

$$G = \log \left(\frac{1}{16} e^{-6\sigma} \varphi^{-3/2} \right) + \log \left| \frac{1}{\sqrt{2}} \langle \Gamma^{mnp} \rangle H_{mnp} \right|^2 \quad (3.5)$$

Furthermore, only the Chern-Simons form $\omega_{mnp}^{(YM)}$ in Eq. (2.3) survives after truncation. As first pointed out in Ref. 19, in this limit ω_{mnp} is related to the superpotential W

$$\langle \Gamma^{mnp} \rangle \omega_{mnp}^{(Y.M.)} \sim \sum^{ijk} \omega_{ijk}^{(Y.M.)} \sim d_{abc} (C^a C^b C^c) \quad (3.6)$$

where i, j, k are $SU(3)$ holonomy indices, d_{abc} is the E_6 symmetric invariant tensor and the C 's are the scalars in the $\underline{27}$. In the second step in Eq. (3.6) an identification of holonomy $SU(3)$ indices and $SU(3)_G \in E_6$ indices has been done, in the spirit of the identification of gauge and manifold connections in Calabi-Yau compactifications. From Eq. (3.5) one finally writes

$$G = \log (e^{-6\sigma} \varphi^{-3/2}) + \log |W|^2 \quad (3.7)$$

This is certainly the easiest way to calculate the Kähler potential. In fact, the dependence of G on σ and φ can easily be traced back to an overall factor $e^{-3\sigma}$ from the ten-dimensional vielbein times the $\varphi^{-3/4}$ factor already present in the $\psi_M \Gamma^{MNPQR} \psi_R H_{NPQ}$ coupling. The other rescalings of fields compensate^{*)} after normalization of the gravitino kinetic term. Still Eq. (3.7) does not give us the form of the Kähler potential as a function of the two complex

*) One must remember here that contractions with Γ matrices scale like the vierbein and sechsbein.

fields (let us call them S and T) which we know remain light in this truncation. However, Eq. (3.3) tells us that one of the complex fields is

$$S = \varphi^{-3/4} e^{3\sigma} + i\theta \equiv s + i\theta \quad (3.8)$$

There must be another complex scalar field T , which includes the other scalar degree of freedom and the axion η . We see, however, that the Kähler potential [Eq. (3.7)] seems to depend only on the real fields σ and φ . We thus expect in this situation (let us forget for the moment the C^X sector):

$$G = \log(e^{-6\sigma} \varphi^{-3/2}) = -\log(s^m t^n) \quad (3.9)$$

It is easy to see that one must have $\hat{n} \equiv n/m = 3$ in order to reproduce the σ and φ kinetic terms. Thus, for example, imposing that no crossed $(\partial\sigma)(\partial\varphi)$ terms exist implies

$$(\partial\sigma)(\partial\varphi) \left(-\frac{q}{2} + \frac{\hat{n}}{2} \frac{27}{\hat{n}^2} \right) = 0 \longrightarrow \hat{n} = 3 \quad (3.10)$$

From Eq. (3.9) we then find that

$$T = e^{\sigma} \varphi^{3/4} + i\eta \equiv t + i\eta \quad (3.11)$$

as the other light complex scalar in the truncation. Thus from Eq. (3.9) we can finally write the Kähler potential as

$$G = -\log(S+S^*) - 3\log(T+T^* - 2|C^X|^2) + \log|W|^2 \quad (3.12)$$

where for convenience we have changed the normalization of the superpotential.

We have also changed the definition of the T field $\text{Re } T = e^{\sigma} \varphi^{3/4} + |C^X|^2$ to

include a term giving rise to kinetic terms for the observable fields C_x . Thus from now on we define

$$T \equiv e^{\sigma} \varphi^{3/4} + |C^x|^2 + i\eta \equiv t + i\eta \quad (3.13)$$

How well does this truncation mimic the compactification on a Calabi-Yau manifold? The light states fit nicely into supersymmetric multiplets but this spectrum is certainly an oversimplification. For example, there are no $\overline{27}$ multiplets in this scheme, whereas we know that in any Calabi-Yau compactification there must be at least one light $\overline{27}$ from the existence of the Kähler metric [7]. More obviously, in this truncation one only obtains one family of quarks and leptons (instead of 3 or 4). Thus it is not clear that the correct Calabi-Yau compactification, including all the necessary light states implied by topological arguments, will have qualitatively the same properties as this truncation ansatz. This is particularly the case for properties such as the Yukawa and generation structure. The above truncation also does not give rise to the extra light singlets associated with the deformations of the shape of the manifold. These appear [7] as extra zero modes of the internal Calabi-Yau metric g_{mn} ($m, n = 5-10$). For example, for some manifolds with $b_{1,1} = 1$ (such as the CP^N surfaces of Ref. 7) there is one of these singlets for each light $\overline{27}$ generation. This sort of 'generation structure' in the metric zero modes of this type of manifold is intriguing. In fact, one expects that these singlet fields associated with the shape of the manifold will play a role in the specific form of the kinetic terms of, for example, the observable chiral fields since there are terms in the action

$$e^{-\sigma} \varphi^{-3/4} g_{(0)}^{mn} (D_\mu A_m^a) (D^\mu A_n^a) \quad (3.14)$$

This, in turn, will affect the sizes of the four-dimensional observable sector couplings after the appropriate rescalings to get canonical kinetic terms. This could be related to the observed hierarchy of fermion masses. This is true in general; one of the changes one expects when going from this truncation to a more general compactification is the appearance of interactions with the metric zero modes [which, by the way, are not $SU(3)$ singlets] owing to a more general ansatz $g_{mn} = e^{\sigma} g_{mn}(0)$ instead of Eq. (3.1).

However, one expects that several properties will remain true in a proper Calabi-Yau compactification. Thus, for example, the ϕ and σ (S and T) dependence should be quite universal since it just depends on the definition of a general dilaton mode $g_{mn} = e^{\sigma} g_{mn}(0)$.

From the truncated Kähler potential one easily obtains the scalar potential (see Appendix B for general useful formulae):

$$V = \frac{s^{-1} t_c^{-3}}{16} \left(|W|^2 + \frac{t_c}{3} |W_c|^2 \right) + \frac{q}{2} g^2 t_c^{-2} s^{-1} (C_x^* \lambda^i C_x)^2, \quad (3.15)$$

where $s = \text{Re } S$ and $t_c = \text{Re } T - |C_x|^2$. The superpotential in the truncation can be obtained to be

$$W = 8\sqrt{2} g d_{abc} C^a C^b C^c \quad (3.16)$$

This potential is positive definite owing to the structure of the Kähler potential in Eq. (3.12). The usual $-3e^G$ term in the scalar potential is exactly cancelled due to the structure of the second term in Eq. (3.12) [the factor 3 obtained in Eq. (3.10) is crucial in this cancellation]. This is what happens in the so-called 'no-scale' models [30] to which the T -dependent term in the Kähler potential has a close similarity. On the other hand, this positiveness of V is to be expected since in this truncation the F^2 and H^2 terms in extra dimensions

are also positive definite. The first term in Eq. (3.12) comes from $H_{mnp}H^{mnp}$ and the second and third from $F_{mn}F^{mn}$.

Notice that, as it stands, the potential is minimized for $C_x = 0$ and/or $t_c, s \rightarrow \infty$ (free, non-compactified theory) in a supersymmetric minimum. For $C_x = 0$ there is a classical degeneracy with respect to any value of S and T . Eventually one expects that this degeneracy must be lifted (see next section) and one has to normalize all the kinetic terms to put them in the canonical form. After the redefinitions

$$C'_x, \chi' = \sqrt{3} e^{-\sigma/2} \psi^{-3/8} C_x, \chi \quad (3.17a)$$

$$A'_\mu = \psi^{-3/8} e^{3/2 \sigma} A_\mu \quad (3.17b)$$

$$F'_{\mu\nu} = \psi^{-3/8} e^{3/2 \sigma} F_{\mu\nu} \quad (3.17c)$$

$$g' = \psi^{3/8} e^{-3/2 \sigma} g \quad (3.17d)$$

which lead to canonical terms, one gets the shape of the scalar potential for the C'_x fields:

$$V_r = |W_N + m_{3/2}|^2 + \sum_x \left| \frac{\partial W_N}{\partial C'_x} \right|^2 + \frac{g'^2}{2} (C'_x \lambda^i C_x)^2 \quad (3.18)$$

where we have defined

$$W_N = \frac{1}{3} \sqrt{\frac{8}{3}} g' d_{xyz} C'_x C'_y C'_z \quad (3.19)$$

$$m_{3/2} = e^{\langle G \rangle / 2} = \frac{\langle W \rangle}{4 s^{1/2} t_c^{3/2}} \quad (3.20)$$

$\langle W \rangle$ being any possible v.e.v. the superpotential may have. Notice that, as will be further discussed in the next section, the 'A' parameter of low-energy supergravity models is $A = 1$ in this particular case. Of course, the rescalings in

Eq. (3.17) also properly normalize the fermionic sector, as can easily be checked using the formulae of Appendix B. In particular, one obtains in the above simple example gaugino masses [19]

$$M = F_5 f'^5 = e^{\langle G \rangle / 2} = m_{3/2} \quad (3.21)$$

As we will discuss in detail in the next section, the situation changes if the supersymmetry breaking in this simple truncated theory occurs through gaugino condensation.

Before closing this section, let us again remark that proper compactification on a Calabi-Yau manifold may change many of the general properties obtained in the trivial truncation discussed above. This will especially be the case if it finally turns out that a 'string-like' compactification is needed but will still be the case even if ten-dimensional field theory is a good approximation.

4. SUPERSYMMETRY BREAKDOWN

In the previous sections we considered the interesting possibility of retaining one unbroken supersymmetry in the process of compactification. However, one has eventually to break that remnant supersymmetry. It has been observed [19,20] that superstring models already inherently contain a mechanism for the breakdown of this last supersymmetry. This is especially transparent in the $E_8 \times E_8$ model, where the second E_8 serves as the hidden sector of supergravity models breaking supersymmetry through gaugino condensates. The same mechanism can also take place in $O(32)$ models if the group is broken into a product group upon compactification. Such a dynamical breakdown of supersymmetry has the nice property that the scale of supersymmetry breakdown can be naturally small

compared to the Planck and grand unification (compactification) scale. We will, therefore, mainly concentrate on this mechanism.

The proposal for a 'hidden sector' has been at the origin of the low-energy supergravity models and the fact that gaugino condensation might break local supersymmetry has been discussed in the literature [31]. It was shown that a crucial ingredient for supersymmetry breaking was the presence of non-trivial gauge kinetic terms (f_{ab} function) and this is, in general, the case in the reduced $d = 4$ models obtained from the superstring. The supersymmetry transformations of the fields λ and ψ_m [31]

$$\delta \lambda = \frac{\sqrt{2}}{384} \Gamma^{mnp} \text{Tr} (\chi \Gamma_{mnp} \chi) + \dots \quad (4.1)$$

$$\delta \psi_m = \frac{1}{256} \Gamma_{mnpq} \text{Tr} (\chi \Gamma^{npq} \chi) + \dots \quad (4.2)$$

show that a four-dimensional condensate $\langle \chi \chi \rangle = \Lambda^3$ breaks supersymmetry and the goldstino will be a combination of zero modes from λ and ψ_m . The scale of supersymmetry breaking will be [31] $M_{ss}^2 \sim \kappa \Lambda^3 = \Lambda^3 / M_P$ and the gravitino will receive a mass through the super-Higgs effect from terms such as

$$\kappa^2 \text{Tr} (\chi \Gamma_{mnp} \chi) \psi_\mu \Gamma^\mu \Gamma^{mnp} \lambda \quad (4.3)$$

giving rise to values

$$m_{3/2} \sim M_{ss}^2 / M_P \sim \frac{\Lambda^3}{M_P^2} \quad (4.4)$$

A value of Λ in the 10^{13} - 10^{14} GeV range would lead to gravitino masses in the TeV range. E_6 may be broken in the compactification process down to some subgroup through the same ('Hosotani') mechanism [33,7] which is at work with E_6

breaking. In fact it could even be desirable that this breaking takes place [34]. E_8 has some nice subgroups such as $SU(5) \times SU(5)$, $SO(16)$, $SU(9)$, $E_7 \times SU(2)$, and $E_6 \times SU(3)$. However, all of these subgroups (and of course E_8) lead to large condensation scales Λ , close to the Planck mass because of their large quadratic Casimir. This would lead to very large gravitino masses and also would surely affect the process of compactification. Thus to obtain a relatively light gravitino in general one needs E'_8 to break down according to

$$E'_8 \rightarrow SU(N, N \leq 5) \otimes (\text{groups with } \leq \text{Casimirs}) \quad (4.5)$$

or to other subgroups with similar Casimir invariants.

To illustrate the above statements let us consider the example of a model with three $\underline{27}$ families which are light down to the weak scale with a gauge symmetry in the observable sector $SU(3) \times SU(2)_L \times U(1)^3$. In this case $\alpha_G = \alpha_3(M_W) \simeq 0.12$ and $M_G = M_P / \sqrt{8\pi} \simeq 10^{18}$ GeV. Then one finds, for example, the following condensation scales and gravitino masses:

$$\begin{aligned} SU(5) : \quad \Lambda &\sim 3 \times 10^{16} \text{ GeV} , \quad m_{3/2} \sim 3 \times 10^{13} \text{ GeV} \\ SU(3) : \quad \Lambda &\sim 3 \times 10^{15} \text{ GeV} , \quad m_{3/2} \sim 10^{10} \text{ GeV} \\ SU(2) : \quad \Lambda &\sim 10^{14} \text{ GeV} , \quad m_{3/2} \sim 4 \times 10^6 \text{ GeV} \end{aligned} \quad (4.6)$$

So that it seems that only if one breaks strongly the E_8 symmetry down to, for example,

$$E'_8 \rightarrow SU(2)^n \times U(1)^m \quad (n \geq 1) \quad (4.7)$$

we will be able to generate a relatively light gravitino. Of course, this depends on the assumed value of α_G as well as on the value of M_{GUT} . An absolute lower bound on α_G is given by its value in the minimal SUSY model, which gives $\alpha_G \simeq 0.04$. Taking this value for $\alpha_G(E'_8)$ one finds that E'_8 must be broken down to a subgroup whose Casimir must be smaller than that of $SU(5)$, as stated in Eq. (4.5). Notice, however, that this case is over-optimistic since after compactification one always has at least three $\underline{27}$ families and the extra $SU(5)$ ($\underline{5} + \bar{\underline{5}}$) multiplets in them will tend to increase the value of α_G . Thus the most likely situation would require E'_8 to be broken down to $SU(2)$'s or at most $SU(3)$ [or $SO(5)$] subgroups. Models with four families all lead to large gravitino masses since α_G is very large to start with. Thus the overall conclusion is that only if E'_8 is badly broken may one obtain a hierarchically small gravitino mass.

It may be argued that M_G could be lower than M_P (leading to lower Λ 's) but this is not desirable in the present scheme since it would imply a strongly interacting superstring [15,16,35] which would (presumably) ruin the $d = 10$ field theory approximation which is assumed in the present context. Another possibility is that although $m_{3/2}$ may be quite large, this does not necessarily mean that the soft terms in the observable sector are $\sim m_{3/2}$; they could be smaller. We discuss this possibility later on in this section. A third logical possibility is that it could well happen that the E'_8 gauge coupling constant were different from the E_6 one. We consider this possibility in the next section.

Breaking E'_8 down to a subgroup which contains only non-Abelian components $SU(2)$, $SU(3)$ [or $SO(5)$], etc., is not that easy in the Hosotani mechanism. To show this, let us consider (in analogy with Witten's breaking of E_6 [26]) the $SU(5) \times SU(5)$ subgroup of E_8 in order to classify possible non-trivial gauge configurations. Under this subgroup one has

$$248 = (1, 24) + (24, 1) + (5, \bar{10}) + (10, 5) + (\bar{5}, 10) + (\bar{10}, \bar{5}) \quad (4.8)$$

Consider as an example a case with a $Z^5 \times Z^5$ discrete symmetry in the manifold. Then the following ansatz for ' U_g ' breaks the symmetry down to $SU(5) \times SU(5)$

$$U_g = \begin{pmatrix} \alpha & & & & \\ & \alpha & & & \\ & & \alpha & & \\ & & & \alpha & \\ & & & & \alpha \end{pmatrix} \times \begin{pmatrix} \beta & & & & \\ & \beta & & & \\ & & \beta & & \\ & & & \beta & \\ & & & & \beta \end{pmatrix} \quad \alpha, \beta \in Z^5 \quad (4.9)$$

where $\beta \neq \alpha^3$; otherwise E'_8 remains unbroken. One may break the symmetry down to a group of the form $SU(3) \times$ (something with $\leq$ Casimir) through, for example, an ansatz:

$$U_g = \begin{pmatrix} \alpha & & & & \\ & \alpha & & & \\ & & \alpha & & \\ & & & \alpha^2 & \\ & & & & 1 \end{pmatrix} \times \begin{pmatrix} \alpha^2 & & & & \\ & \alpha^2 & & & \\ & & \alpha^2 & & \\ & & & \alpha^4 & \\ & & & & 1 \end{pmatrix}, \quad \alpha \in Z^5 \quad (4.10)$$

One can check that no extra gauge bosons from inside $(\underline{5}, \overline{\underline{10}}) + (\underline{10}, \underline{5}) + \text{h.c.}$ remain light in this example. This is not always the case for many ansätze. Very often ansätze which apparently lead to $SU(3) \times \dots$ lead in fact to bigger unbroken symmetries because there are extra gauge bosons included in $(\underline{5}, \overline{\underline{10}}) + (\underline{10}, \underline{5}) + \text{h.c.}$ which remain light. Thus breaking E'_8 down to a group the form $SU(2)^m \times U(1)^n$ is not so easy. The following ansatz with discrete symmetry Z^{16} does it:

$$U_g = \begin{pmatrix} \alpha & & & & \\ & \alpha & & & \\ & & \alpha^3 & & \\ & & & \alpha^3 & \\ & & & & \alpha^{-8} \end{pmatrix} \times \begin{pmatrix} \alpha & & & & \\ & \alpha & & & \\ & & \alpha^3 & & \\ & & & \alpha^3 & \\ & & & & \alpha^{-8} \end{pmatrix} \quad (4.11)$$

Simpler examples surely exist but certainly the E'_8 gauge fields have to cleverly wrap around the manifold to get such a small Casimir gauge group.

It seems, therefore, not so easy to obtain gravitino masses $m_{3/2}$ as small as 100 GeV through hidden sector gaugino condensation if the gauge couplings of hidden and observable sectors coincide. The question, however, as to whether such a low mass for the gravitino is desired or not can only be decided after we have examined the supersymmetry-breaking parameters in the low-energy theory of the observable sector.

Gaugino condensation is not the only way in which supersymmetry breaking may occur. As noticed in Ref. 19, a v.e.v. for the antisymmetric tensor H_{mnp} in extra dimensions does also break supersymmetry, as is easy to observe from the term in the Lagrangian, Eq. (2.1):

$$-i \frac{\sqrt{2}}{16} \varphi^{-3/4} H_{MNP} (\bar{\Psi}_a \Gamma^{QMPR} \Psi_R) \quad (4.12)$$

This can also be seen from the supersymmetry transformations

$$\delta \lambda = \frac{1}{8} \varphi^{-3/4} \Gamma^{MNP} H_{MNP} + \dots \quad (4.13)$$

$$\delta \Psi_M = \frac{\sqrt{2}}{32} \varphi^{-3/4} (\Gamma_M^{NPA} - 9 \delta_M^N \Gamma^{PA}) H_{NPA} \quad (4.14)$$

In this case one obtains a gravitino mass

$$m_{3/2} \sim \frac{\kappa^2}{4g^2} \langle H \cdot \Gamma \rangle \sim \frac{\langle H \cdot \Gamma \rangle}{M_P^2} \quad (4.15)$$

If such a v.e.v. were produced upon compactification we would expect [19,20,32]

$$H_{ijk} = c \sum_{i,j,k} M_P^3 \quad (4.16)$$

where $i, j, k = 1, 2, 3$ are SU(3) holonomy indices. It has been argued [20] that such a vacuum expectation value could originate from the $\partial_i B_{jk}$ piece inside H_{ijk} .

However, it could also come from a small miscancellation of Yang-Mills and Lorentz Chern-Simons symbols

$$H_{ijk} \sim \omega_{ijk}^{(Y.M.)} - \omega_{ijk}^{(L)} \sim c \sum_{ijk} M_P^3 \quad (4.17)$$

or else from just a v.e.v. of the low-energy superpotential. In any case, we do not know of any mechanism that produces a v.e.v. for $\langle H_{mnp} \Gamma^{mnp} \rangle$ that is small compared to the Planck scale and, as such, $H \cdot \Gamma$ would be a free input parameter.

Notice that the above breaking of supersymmetry through $\langle H_{mnp} \rangle$ causes soft terms [19] to appear in the observable sector such as scalar trilinear terms

$$\frac{3}{4} \psi^{-3/2} H_{mnp} H^{mnp} \longrightarrow \sim c \psi^{-3/2} W \quad (4.18)$$

and gaugino masses

$$i \frac{\sqrt{2}}{16} \psi^{-3/4} (\bar{\chi} H \cdot \Gamma \chi) \longrightarrow \sim c \psi^{-3/4} (\bar{\chi} \chi) \quad (4.19)$$

In fact, as we discussed at the end of the last section, upon trivial truncation and rescaling of fields one gets for these soft terms

$$A = 1, \quad M = m_{3/2} \quad (4.20)$$

In a more general compactification, masses for the observable scalars C^X may appear from terms such as $H^{mnp} A_{[m} F_{np]}$ upon proper integration (and not just truncation) of the heavy fields. In a naively truncated Lagrangian (as in the previous section) no masses for the scalars C^X are obtained at the tree level. This is a dangerous situation if $\langle H_{ijk} \rangle$ is the only source of SUSY breaking, since [according to Eq. (3.18)] minimization would require $W_N \sim m_{3/2} M_P^2$. This would give unwanted large v.e.v.'s to observable fields C_X . Apart from this problem one gets a large cosmological constant:

$$V_0 = \frac{|c|^2}{16 s t_c^3} = m_{3/2}^2 M_P^2 \quad (4.21)$$

Supersymmetry breaking through a gaugino condensate, as discussed at the beginning of this section, seems to be a more attractive possibility than the one induced by a H_{ijk} v.e.v. However, both mechanisms induce a large cosmological constant.

The situation changes if one considers simultaneously both mechanisms of supersymmetry breaking. It has been pointed out [20] that H_{mnp} and the relevant gaugino condensate appear in the Lagrangian in the form of a perfect square:

$$\frac{3}{4} \Psi^{-3/2} \left[H_{mnp} + i \frac{\sqrt{2}}{24} \Psi^{3/4} \text{Tr} (\chi \Gamma_{mnp} \chi) \right]^2 \quad (4.22)$$

This allows for the possibility of cancelling both contributions (H and condensate) to the cosmological constant against each other. One can check that still $\delta\lambda, \delta\psi_m \neq 0$ and supersymmetry is broken. However, in this case also the contributions of H_{mnp} and the condensate $\text{tr} (\chi \Gamma_{mnp} \chi)$ to the soft terms A and M cancel:

$$A = M = 0 \quad (4.23)$$

There are various ways in which this cancellation could happen. One possibility is that the gauge-coupling constant and grand unifications scale (or $\langle\phi\rangle$ and $\langle\sigma\rangle$) were determined in the process of compactification [19]. Then the values of $\langle\chi\chi\rangle$ would be determined by the value of $g(M_P)$ and the renormalization group equations. Then, if there were a light zero mode associated with H_{ijk} , it could adjust itself to cancel the bracket in Eq. (4.15). Unfortunately, that light zero mode is not expected and the cancellation in Eq. (4.22) should be fine-tuned.

There exists the opposite possibility [20]: a given fixed value $H_{ijk} \sim c \epsilon_{ijk} M_P^3$ is assumed in Eq. (4.22) and φ and σ are running fields. This may easily be realized in the truncated Lagrangian of the previous section. With the 'running condensate' one associates an S -dependent superpotential [20]

$$W = W_0 + h \exp(-3S/2b_0) \equiv W_0 + W(S) \quad (4.24)$$

where b_0 is the one-loop β -function coefficient of the unbroken subgroup of E_8 and $\langle W_0 \rangle = c$, including the $H_{ijk} \sim c M_P^3 \epsilon_{ijk}$ v.e.v. Then one obtains [20] the scalar potential^{*}) (see Appendix B for notation):

$$V = \frac{s^{-1} t_c^{-3}}{16} \left| W_0 + h \frac{3s}{b_0} \exp\left(\frac{-3S}{2b_0}\right) \right|^2 + \frac{s^{-1} t_c^{-2}}{48} \left| \frac{\partial W_0}{\partial C^x} \right|^2 + D\text{-term} \quad (4.25)$$

The complex field S will adjust itself to cancel the first term in Eq. (4.25). It and its partner $\tilde{S}$ will acquire a mass $\sim m_{3/2}$ [see Eq. (B.23) in Appendix B] and the value of g is determined in the process. The goldstino in this particular case is $\tilde{T}$ since

$$F_T = -e^{G/2} (G_T G_T^{-1} + G_C G_T^{-1}) = \frac{(c + W(s))}{2 e^{2\sigma}} = \frac{-3s W(s)}{2b_0 e^{2\sigma}} \quad (4.26)$$

whereas $F_S = 0$ since it is proportional to the first bracket in Eq. (4.25). Since gaugino masses are proportional to $f_S F^S = 0$, there are no gaugino masses in this simple case. Still supersymmetry is broken with a vanishing tree-level cosmological constant and a gravitino mass

$$m_{3/2} \sim \frac{c}{s^{1/2} t_c^{3/2}}, \quad (4.27)$$

^{*}) One should include the effects of the effective potential for the bound states, but the qualitative result remains the same [36].

which is undetermined since t_c has not yet been fixed. One assumes here that t [and hence the compactification scale $M_X \sim (st)^{1/2}$] will be determined by radiative corrections. Thus the compactification scale is undetermined at the tree-level but the gravitino mass $m_{3/2}$ is really given by the free parameter c since we know from other arguments (see the comments in the Introduction) that $t, s \sim 1$.

Notice that, independently of how the bracket in Eq. (4.22) cancels [H runs to cancel the condensate or vice versa] we have a supersymmetric observable sector at the tree-level. One expects, however, that radiative corrections will induce [21,22,37], in general, both scalar and gaugino masses. One expects that the SUSY breaking in the hidden sector will feed down to the observable sector through gravitational couplings [see the terms in Eqs. (B.25) in Appendix B]. Recently, the quadratically divergent contributions to the effective scalar potential in the truncated model have been computed [22]. It seems that there is no contribution to the masses of the observable fields C^X . However, there is no reason to believe that this situation will persist in higher corrections so that one expects that $m_c \sim m_{3/2} \times (\text{some small factor})$ will appear at some level. Furthermore, one will also expect contributions [22] to the vacuum energy $\sim -m_{3/2}^2 M_P^2$ so that, since t is still undetermined, one will get $m_{3/2} \sim M_P$ upon minimization. All this may be an indication that the truncation discussed in the previous section may be too naive: we are led to a situation in which $m_{3/2} \sim M_P$, but we have to protect the scalars from getting large masses although supersymmetry has been broken. In this situation, perhaps more attractive is the possibility discussed above in which g is assumed to be fixed by the dynamics of compactification. Then one may obtain a hierarchically small gravitino mass as given by the renormalization group estimate and soft terms will appear in the observable sector from radiative corrections. Still the tree-level cosmological

constant may be cancelled (if no cleverer solution is found) by fine-tuning H_{mnp} in Eq. (4.22).

5. DILATONS AND AXIONS

In the previous sections we have seen that there are (at least) two candidates for dilatons as the real scalar components of chiral superfields. With them come two pseudoscalars as their supersymmetric partners originating from the antisymmetric tensor field B_{MN} :

$$\begin{aligned} S &= \varphi^{-3/4} e^{3\sigma} + i\theta \\ T &= \varphi^{3/4} e^{\sigma} + i\eta \end{aligned} \quad (5.1)$$

The field S corresponds directly to the function f representing the non-triviality of the gauge field kinetic term. With a certain normalization the gauge coupling constant is therefore given by the vacuum expectation value of the real part $\text{Re}(S)$:

$$\frac{1}{g^2} = \langle \varphi^{-3/4} e^{3\sigma} \rangle \quad (5.2)$$

At the classical level the vacuum expectation values of $\text{Re}(S)$ and $\text{Re}(T)$ are undetermined and we shall call these fields dilaton fields. The field φ corresponds to the usual dilaton mode in string theories and the field σ corresponds to a fluctuation of the overall size of the internal compact manifold. Under these circumstances all the parameters are as yet undetermined and the values of the gauge coupling constant and the compactification scale have to be determined through quantum corrections.

The pseudoscalar mode θ was obtained from $B_{\mu\nu}$ using the duality transformation described previously. It couples to $F\tilde{F}$ and is called an axion [5,24]. This duality transformation is straightforward as all terms in the action depend only on $B_{\mu\nu}$ through the field strength $H_{\mu\nu\rho}$ which was the case under consideration in the previous sections.

At first sight this situation changes once we add the terms [2] to $N = 1$ supergravity in $d = 10$ that cancel the anomalies. These terms contain B_{MN} explicitly, e.g.

$$S_g \sim B \wedge (T_r(F \wedge F) T_r(F \wedge F) + T_r(F' \wedge F') T_r(F' \wedge F') - T_r(F \wedge F) T_r(F' \wedge F')) \quad (5.3)$$

and once reduced to four dimensions one might question the validity of the duality transformation. A closer inspection of these terms, however, shows that via an integration by parts they can be written as a sum of terms that only contain H_{MNP} and terms that are total derivatives. The latter can probably be ignored in $d = 10$ since $\pi_g(E_g) = 0$. With this redefinition the theory reduced to $d = 4$ has no problems attached to the duality transformation. It will just produce some new interaction terms. This is relevant for the terms that contain $\partial_\mu B_{\mu\nu}$ in four dimensions.

We have now also considered the inclusion of the anomaly terms and we have to point out that especially in the context of supersymmetry their discussion is delicate. These terms are not supersymmetric although the full string theory is supersymmetric and the improved field theory seems to be a better approximation of superstring theory than the naïve field theory. There remain some open questions. Do we have to add new terms to the theory to discuss supersymmetry appropriately in the low-energy approximation, and what would these terms look like? Or is it just enough to have these terms to cancel a supersymmetry anomaly in the naïve field theory as happens for the gauge and gravitational anomalies? These questions are far from being solved at the moment. Nonetheless, we have to discuss the influence of these terms on the $d = 4$ theory since they are so crucial for the consistency of the theory. One has, however, to keep in mind that our discussion has to be preliminary as long as the above-mentioned questions are not answered.

The influence of these new terms on the four-dimensional theory seem to be particularly important [23] for the imaginary part of T , $\text{Im}(T) = \eta$, given by $\epsilon^{mn} B_{mn}$. The term in Eq. (5.3) and

$$S_{\text{mix}} \sim B \wedge \text{Tr}(R \wedge R) \wedge (\text{Tr}(F \wedge F) + \text{Tr}(F' \wedge F')) \quad (5.4a)$$

and also

$$S_{\text{grav}} \sim B \wedge ((\text{Tr} R \wedge R)(\text{Tr} R \wedge R) + 4 \text{Tr}(R \wedge R \wedge R \wedge R)) \quad (5.4b)$$

couple this field to F and R . In general these are highly complicated interaction terms but we have to remember that in the course of compactification both $\text{Tr}(F^2)$ and $\text{Tr}(R^2)$ receive a v.e.v. With this expression (5.4) gives rise to the couplings

$$\mathcal{L}_{E_6} \sim 5 \eta F \tilde{F} \quad ; \quad \mathcal{L}_{E_8} \sim -\eta F' \tilde{F}' \quad ; \quad \mathcal{L}_R \sim 15 \eta R \tilde{R} \quad (5.5)$$

that is, similarly to θ , η also couples like an axion. In the presence of only one gauge group a linear combination of θ and η would be the real axion. But here we discuss the model in four dimensions where the gauge symmetry is already broken; and in the $E_6 \times E_8'$ model a product gauge group is present from the beginning. While θ couples with the same coefficient to the field strengths of E_6 and E_8' an inspection of the anomaly cancellation terms shows that this is not true for η . Instead, the relative coefficient of the η -coupling to E_6 and E_8' is given by -5 . Thus we can diagonalize the axion couplings with an axion θ_6 for the observable sector and θ_8 for the hidden sector.

Unfortunately, with the addition of just the anomaly cancellation terms, the theory is no longer supersymmetric as discussed before. While the axions are

diagonalized, the two dilatons couple still in the same way to observable and hidden sectors and the theory looks inherently non supersymmetric even just considering the bosonic sector. Since the string theory is supersymmetric this is thought to be a disease of our approximation and requires the addition of new terms to the low-energy field theory.

There are two extreme possibilities in this situation. Either (a) this extra axion coupling is a fake of our approximation and it is cancelled by some other couplings from the superstring or (b) extra terms appear which supersymmetrize the axion coupling by adding the appropriate $N = 1$ supersymmetric counterparts.

In the first case only θ would be a true axion and its v.e.v. would be determined by the hidden sector E'_8 interactions in order to cancel the bracket in Eq. (4.20). Thus it can no longer be used to solve the strong CP problem. In the second case η also acts as an axion and thus can be used to solve the strong CP problems of the observable world. Admittedly, it is a very strong assumption to suppose that a whole bunch of low-energy terms may be generated in order to supersymmetrize the axion-like coupling of η . This would, in general, imply a dependence of the gauge functions f_{ab} both on S and T . Let us, however, speculate about this possibility and explore its possible consequences. The axions θ_6 and θ_8 would then be the pseudoscalar partners of the scalar dilatons d_6 and d_8 that couple to the field strength of E_6 and E'_8 , respectively,

$$\begin{aligned} Z_6 &= d_6 + i\theta_6 \\ Z_8 &= d_8 + i\theta_8 \end{aligned} \tag{5.6}$$

where d_6 and d_8 are given by the appropriate linear combinations of φ and σ in exactly the same way as θ_6 and θ_8 were given by θ and η . With such a formulation we then have two axions that could be responsible for the solution of the

'strong CP problem' of the hidden E'_8 and that of the observable E_6 , where in a realistic model this problem concerns just $SU(3)_{\text{colour}}$. The coupling constants of E_6 and E'_8 would be related to the v.e.v.'s of d_6 and d_8 , respectively, in the same way as expressed in Eq. (5.2) before the addition of the new terms.

Let us now suppose that the v.e.v.'s of d_6 and d_8 are undetermined at the classical level. Radiative corrections will thus play a crucial role in the determination of the corresponding gauge coupling constants. While we do not know the influence of gravitational radiative corrections we can discuss the corrections due to gauge interactions. We shall also discuss here some aspects of the question of the vacuum energy. As we commented in the previous section, it has been observed that there is a particular relation [Eq. (4.15)] between fermionic bilinears and H_{MNP} terms in the ten-dimensional action. In the approximation discussed in four dimensions this will correspond to a part of the semipositive definite potential. Let us now suppose that in the extra dimensions H_{mnp} requires a v.e.v. Here the coupling constant is undetermined and the v.e.v. of d_8 will arrange itself to cancel the vacuum energy induced by $\langle H_{\text{mnp}} \rangle$. g_8^2 is given through $\langle d_8 \rangle$ in a formula similar to Eq. (5.2) now including also the new terms through the diagonalization of the fields. Hidden E'_8 radiative corrections thus determine the vacuum expectation value of d_8 . In the process given above also d_6 and θ_8 will no longer remain massless but receive a mass of the order of the gravitino mass $m_{3/2}$ as discussed in the previous section [for example through Eq. (B.23)].

The mechanism of the v.e.v. of d_8 sliding to cancel the vacuum energy might look very promising for a solution of the problem of the cosmological constant [20]. It is in a certain way similar to the solution of the strong CP problem through an axion: a certain quantity that is a free parameter in the theory is replaced by a dynamical degree of freedom and the dynamics of the theory deter-

mines the formerly free parameter and can, for example, explain why this parameter vanishes.

In the supersymmetric case it is actually natural to expect a relation between an axion and a dilaton since they are members of the lowest component of the same chiral superfield. This has already been observed at the level of models based on global supersymmetry. To incorporate an axion one usually tries to impose an R-symmetry (which in general will be anomalous). This is done most easily by imposing classical scale invariance such that the superpotential only consists of trilinear terms exhibiting such an R-invariance in addition and the potential Goldstone bosons of these two symmetries form the lowest component of a chiral superfield in the same way as considered here. Both symmetries are in general anomalous and the dilaton might play the same role with respect to the vacuum energy as the axion plays with respect to the θ -angle of the strong gauge interactions. The special relation of certain terms in the action, such as the terms displayed in Eq. (4.15), actually supports this speculation. The v.e.v. of d_g slides to cancel the vacuum energy (a phase of H will be cancelled by the v.e.v. of θ_g). But this cancellation is likely to be incomplete. The field d_g receives a mass of order $m_{3/2}$ and it is still questionable whether it remains flexible enough to cancel the cosmological constant at the required level of 10^{-12} GeV. Standard arguments about decoupling of heavy fields would lead us to expect this cancellation to work only with sufficient accuracy if the mass of d_g is 10^{-12} GeV or even smaller, so that it can readjust for other possible contributions to the vacuum energies at low energies. One such contribution, for example, will be given by QCD condensates and it is unlikely that the v.e.v. of d_g will cancel this contribution, especially since d_g is the field that couples to the hidden sector.

But there is the other dilaton-axion combination (d_g, θ_g) that couples to the observable sector. θ_g is the obvious candidate to cancel the QCD-vacuum

angle and dynamically solve the strong CP problem. The question of the cosmological constant is more difficult to discuss. Some authors [38] claim that the axion η is responsible for the absence of the cosmological constant needs not necessarily be true. The more natural candidate is the dilaton d_6 . Its vacuum expectation value determines the observable sector gauge coupling and hence also α_{QCD} and the strength of the QCD condensate. Could the mechanism that worked in the hidden sector be repeated in the observable sector? The question is hard to answer at the moment since we do not yet have a low-energy theory including the chiral quarks. In the hidden sector we could work directly with the E'_8 (or subgroup thereof) gauginos and use the form of the ten-dimensional action. For the QCD condensates we need already to know the special form of compactification, the way in which zero modes arise and how they interact amongst themselves. One could, however, imagine that the mechanism of the hidden sector also works here, especially because of the axion-dilaton analogy and the sliding of the observable sector coupling constant. Because of the small scale of the QCD condensates compared to the E'_8 scale we would then also expect the dilaton mass to be small compared to 10^{-12} GeV and this mechanism could work with the required precision.

But even with all these ingredients present there are still some obstacles. Supersymmetry is broken in the hidden sector and d_6 should be protected and not receive a mass of order $m_{3/2}$ in this process. One might construct examples where the mass of d_6 is protected at the tree-graph (and even one-loop) level, but such a mechanism is not yet understood. This question would also usually require the inclusion of the full gravitational radiative corrections. Their presence might prevent the gauge couplings from sliding. The only hope would be that in the full string theory some symmetries exist that sufficiently protect the dilaton from acquiring a mass.

This problem of mass protection of the dilaton is actually closely related to a phenomenological problem such a light dilaton would pose. This extremely light dilaton gives contributions to long-range gravitational interactions which are inconsistent with present observation as long as the couplings of d_6 to observable matter are not sufficiently suppressed. The necessary suppression, however, would make sure that the radiative contributions to the dilaton mass are sufficiently small. The mechanism seems exciting enough to deserve further study.

6. ATTEMPTS AT REALISTIC MODEL-BUILDING

In the previous chapters we have studied the problems of compactification and supersymmetry breaking in the low-energy limit of superstrings. Upon compactification one obtains [7] an E_6 SUSY-GUT [39] coupled to $N = 1$ supergravity. To obtain a realistic theory one has to (i) break the E_6 symmetry [25,26] down to the standard model and (ii) give masses to the unwanted states in the fundamental $\underline{27}$'s of E_6 . The first task can be achieved if the gauge fields in extra dimensions get trapped in the non-simply connected structure of the Calabi-Yau manifold (Wilson loop or 'Hosotani' mechanism [33,7]). This needs in general to be supplemented by further symmetry breaking induced by the light non-singlet chiral multiplets of the theory. The second task is more complicated and, as we will show, leads to phenomenological problems in superstring theories.

The chiral non-singlet zero modes of superstring theories [7] consist of

$$n_g(\underline{27}) + b_{1,1}(\underline{27}_H + \overline{\underline{27}}_H) \quad (6.1)$$

where (hopefully) $n_g = 3$ or 4 and $b_{1,1}$ (which is always ≥ 1) is the corresponding Betti-Hodge number [7,26]. To fix the notation let us show the $SU(5)$ content as well as a tentative quark-lepton assignment of the states inside a $\underline{27}$:

$$27_i = (10 + \bar{5} + 1)_i + (5 + \bar{5})_i + 1_i \quad (6.2)$$

$\swarrow \qquad \searrow \qquad \searrow \qquad \searrow \qquad \searrow$
 $(U_R, Q_L, E_R)_i \quad (L, d_R)_i \quad \nu_{R_i} \quad (D, \bar{D}; H, \bar{H})_i \quad N_i$

where $i = 1-3(4)$ families. To indicate that a certain particle is associated with the light objects from $27 + \overline{27}$ we will add an H subindex.

The usual E_6 generations are protected by index theorems and cannot get any mass term upon compactification. However, the states in $(27 + \overline{27})$ are not protected and may gain masses in that process [26]. Notice that after E_6 breaking through the 'Hosotani' mechanism the light states protected through index theorems will not correspond to complete 27 's of E_6 but will be linear combinations of the original states in the 27 's. The states in the $(27 + \overline{27})$'s which will be light will depend on the way in which E_6 is broken. Only states which are invariant under the gauge transformation U_g induced by a corresponding discrete transformation on the manifold will be light [26]. These states will come in $b(1,1)$ copies. One can see [26,40] that one can have E_6 -breaking gauge configurations U_g such that no dangerous colour triplets which could mediate fast proton decay are light. However, this does not ensure that we are free from dangerous proton decay. Each light family of a 27 contains colour triplets $(D_i, \bar{D}_i)$ which do mediate fast proton decay. Although in some models one can give a large mass to these triplets (see models B-I and B-II later on), in others one has to assume that there is some (discrete?) symmetry [25,26] forbidding B-violating couplings such as

$$Q_L Q_L D \quad , \quad U_R \bar{D} d_R \quad (6.3)$$

This is the first phenomenological problem of superstring models. Another (related?) problem poses itself again because we obtain light families of 27 states instead of the 15 observed. This is the existence of a light right-handed neutrino ν_R (sometimes it is N), which in general combines with the left-handed one ν_L to form a (phenomenologically unacceptable) Dirac neutrino mass. The traditional solution [41] to this problem is to give a large Majorana mass to ν_R through the v.e.v. of a high dimensionality [126 of $SO(10)$, 351 of E_6] representation. However, this is not possible in superstring models since such a representation is not available, there are only 27's which do not have any component with the quantum numbers of a ν_R Majorana mass^{*)}. It cannot be argued that a large ν_R Majorana mass will be developed radiatively because supersymmetry will only allow for a minute ν_R radiative mass [42]. The only other alternative^{**)} seems to forbid the coupling of the ν_R 's to the ν_L 's through some symmetry [25,26] as we did in the case of proton decay. However, discrete symmetries cannot be arbitrarily imposed in superstring models but must come from the symmetry properties of the compact manifold [26]. It would be nice to find a scheme in which both Dirac ν -masses and B-violating couplings were forbidden by the same symmetry. This seems, however, hard to obtain.

Alternatively, one may try to give masses to ν_R 's through some more elaborate mechanism. Three possible alternatives come to mind:

a) There could be some non-renormalizable terms in the superpotential of the form

*) Excited string states with these quantum numbers do exist in the heterotic string. However it would be inconsistent to keep them, in the spirit of Ref. [7].

**) If supersymmetric compactifications on manifolds of $O(6)$ holonomy exist [26] the problem still remains since the unbroken group is $SO(10)$ which still possesses ν_R 's. Especially interesting could be compactifications leading to an $SU(5)$ gauge group [8] but then one would expect the breaking of low-energy supersymmetry.

$$O(1/M_P) \underline{27} \times \underline{27} \times \overline{\underline{27}}_H \times \overline{\underline{27}}_H \longrightarrow \nu_R \times \nu_R \times \frac{\langle \bar{N}_R \rangle^2}{M_P} \quad (6.4)$$

where $\overline{\underline{27}}_H$ is one of the multiplets implied by $b_{1,1} \geq 1$. One of the fields in $\overline{\underline{27}}_H$ ($\bar{N}_R$) has the quantum numbers of a ν_R and a $\langle \bar{N}_R \rangle \sim v$ would give ν_R masses

$$m_{\nu_R} \sim v^2 / M_P \quad (6.5)$$

If $v \geq 10^{11}$ GeV one gets masses $m_{\nu_R} \geq 10^4$ GeV which are large enough to avoid phenomenological problems. This v.e.v. v may appear without breaking supersymmetry since the D^2 term in the potential can be made to vanish through the conjugate v.e.v. $\langle N_R \rangle \sim v$. Terms such as that in Eq. (6.4) are not obtained from the $d = 10$ Lagrangian upon trivial truncation. However, this type of term is expected to arise from the effect of higher Kaluza-klein and string modes, especially if (as it seems the case) the compactification, Planck, and string scales are close to each other.

b) In many Calaby-Yau compactifications there are usually a number of chiral singlet superfields which are light [7] (apart from the 'dilaton' S and T). In fact, for some models with $b_{1,1} = 1$ one has one of these singlets per light $\underline{27}$ generation, as we remarked above. Then the possibility could exist of obtaining a low-energy superpotential term of the form

$$\underline{27} \times \overline{\underline{27}}_H \times \underline{1} \longrightarrow \nu_R \times \underline{1} \langle \bar{N}_R \rangle \quad (6.6)$$

where $\underline{1}$ stands for the above-mentioned singlets. A v.e.v. $\langle \bar{N}_R \rangle \sim v$ will give a mass of the order of v to a ν_R in the $\underline{27}$ by combining it with the singlet. Three singlets of this type are needed to give masses to the three v -generations. As in possibility (a), a term of this type is not obtained upon trivial truncation

from the $d = 10$ Lagrangian since, to start with, these singlets are missed in that truncation. It may happen that a proper truncation could induce couplings similar to that in Eq. (6.6), although Eq. (3.6) suggests that this is not the case.

c) This third possibility is only a partial solution to the problem. If a scalar neutrino $\tilde{\nu}_R$ gets a v.e.v. $\langle \tilde{\nu}_R \rangle \sim v$, it will, in general, break an originally unbroken $U(1)_{B-L}$ which is usually left over after E_6 breaking (unless it has already been broken by some non-vanishing $\langle \tilde{27}_H \rangle$). In such a case the ν_R is swallowed by the corresponding gaugino and disappears from the low-energy world [43]. The mass of this ν_R cannot be too large since then a large D-term would be induced. Also R-parity is broken so that some bizarre processes can take place. However, it seems one can manage to get rid of the ν_R in a manner consistent with known phenomenology. Still, one is left with the problem of the other two ν_R neutrinos which remain light. One should then forbid Dirac masses for these two residual ν_R 's.

None of the above possibilities is especially compelling but at least they show some possible ways in which the ν -mass problem of superstring models could be avoided.

The two problems we have discussed above, i.e. too-fast proton decay and ν -mass problems, are generic of any superstring model. A third problem, as we already remarked in previous sections, is the complete understanding of the induced supersymmetry breaking in the 'observable' sector of the theory. In the rest of this section we are going to assume (unless otherwise stated) that one has the usual general soft terms of $N = 1$ supergravity models [44], i.e. scalar masses (m), gaugino masses (M), and trilinear scalar couplings (parametrized by the parameter A). We will thus only concentrate on the symmetry breaking and viability of low-energy superstring-motivated models.

An important point to recall is that E_6 can at most be broken down to a group of rank 5 through the Hosotani mechanism [26]. Thus unless further breaking through chiral multiplets takes place one expects new gauge interactions (at least new Z^0 's) close to the TeV region. This turns out to be the case in most of the models studied.

We are going to concentrate on three $\underline{27}$ -families models. This is because four-family models lead in general [25] to bad predictions for $\sin^2 \theta_W$ and M_X owing to the large quantity of chiral multiplets involved (even if an intermediate mass scale is introduced). Also the gauge coupling constant at the GUT scale in four-family models is very large and it would be doubtful to expect in this case a perturbative regime. Although no three-family Calabi-Yau manifold has been reported in the literature up to now, most probably they exist and lead to better phenomenology than four-family models. Also because of phenomenological considerations (values of $\sin^2 \theta_W$ and M_X) we are going to consider only low-energy gauge groups of the type

$$SU(3)_C \times SU(2)_L \times U(1) \times T \quad (6.7)$$

where T is any of the following groups

$$U(1) \quad (6.8a)$$

$$U(1) \times U(1) \quad (6.8b)$$

$$SU(2) \times U(1) \quad (6.8c)$$

and $SU(2)$ may or may not be identified with an $SU(2)_R$ gauge group.

According to the low-energy chiral superfield content the following classification of models is relevant:

- A) Minimal model with just three $\underline{27}$'s and no extra Higgses.
- B-I) Model with three $\underline{27}$'s and light Weinberg-Salam (W-S) doublets from $\underline{27} + \overline{\underline{27}}$.
- B-II) Model with three $\underline{27}$'s and just light $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ singlets $N_H + \bar{N}_H$ from $\underline{27} + \overline{\underline{27}}$.
- B-III) Model with three $\underline{27}$'s and light ν_R -like objects $N_R + \bar{N}_R$ from $\underline{27} + \overline{\underline{27}}$ [or complete $SU(2)_R$ doublets].

These (and small variations) are the models of possible phenomenological interest. Of course in B-type models one can have $b_{1,1}$ copies of each representation coming from $\underline{27} + \overline{\underline{27}}$. One cannot have models such as, for example, one with both $N_R + \bar{N}_R$ and light W-S doublets since then the unbroken subgroup from E_6 would contain an $SU(3)_L$ symmetry. This is a consequence of the important fact that the low-energy states from $\underline{27} + \overline{\underline{27}}$ one may have are strongly correlated with the unbroken gauge group one has, as we will see in specific examples. Let us now discuss the different models in turn.

A) Minimal model

As we noted above, this contains three families of $\underline{27}$'s and no extra non-singlet matter around. Then one must use the fields in Eq. (6.2) to break the symmetry to $SU(3)_C \times U(1)_{e.m.}$. Notice that in this minimal model all the symmetry breaking must occur close to the weak scale since otherwise the v.e.v.'s of the fields would give non-vanishing contributions to the D^2 terms in the potential and would break supersymmetry earlier than desired. In fact there are some combinations of v.e.v.'s for which the D-terms vanish: $\langle \tilde{\nu}_R \rangle = \langle \tilde{\nu}_L \rangle = \langle H_2^0 \rangle = v$ or $\langle H_1^0 \rangle = \langle H_2^0 \rangle = \langle \tilde{N} \rangle = v$. However (apart of other problems) in these directions the extra Z^0 bosons would get masses of order M_Z , which is ruled out experimentally. In this model we will have to use, for example, singlets N to break the $SO(10)$ invariant $U(1)_6$ which is contained in E_6 , the $\tilde{\nu}_R$'s to break B-L symmetry [and maybe $SU(2)_R$] and the Higgses $H, \bar{H}$ to break $SU(2) \times U(1)$. The precise symmetry one has to break will depend on which of the gauge symmetries T one conserves unbroken after E_6 breaking. However, the general analysis of symmetry breaking is similar for any of the T symmetries in Eq. (6.8). The low-energy

superpotential of the minimal model will be [up to $O(1/M_P$ or $1/M_S)$ terms] of the following general form:

$$\begin{aligned}
 W = \sum_{\text{families}} & h_L H L E_R + h_d H Q d_R + h_u \bar{H} Q u_R + \\
 & + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} + \\
 & + h_\nu \bar{H} L \nu_R + (\text{B-violating terms})
 \end{aligned} \tag{6.9}$$

where each coupling constant is a three-index quantity in family space. The possible B-violating terms must be forbidden by some symmetry in order to forbid too-fast proton decay. Also, as we will comment later on, some (or all) of the neutrino-Yukawa couplings $h_{\nu}^{ijk} \bar{H}_i L_j \nu_{R_k}$ ($i, j, k = 1, 2, 3$ families) must also be forbidden. Notice that if there is an $SU(2)_R$ symmetry one has

$$h_L^{ijk} = h_\nu^{ijk} \tag{6.10a}$$

$$h_d^{ijk} = h_u^{ijk} \tag{6.10b}$$

so that one has to be careful when forbidding Dirac neutrino-masses not to forbid also lepton masses! This can be avoided if the Higgses H_i and $\bar{H}_j$ acquiring v.e.v.'s are not $SU(2)_R$ partners.

One has, in general, to do a triple breaking^{*)}:

$$\begin{aligned}
 \langle N_i \rangle & \neq 0 \\
 \langle \tilde{\nu}_{R_i} \rangle & \neq 0 \\
 \langle H_i \rangle, \langle \bar{H}_i \rangle & \neq 0
 \end{aligned} \tag{6.11}$$

Interestingly enough this symmetry breaking can be induced in a natural manner if one considers radiative corrections as in the models of low-energy supergravity [45,46]. Assuming a universal soft mass m for all scalars at the GUT (compactification) scale and soft trilinear scalar terms:

^{*)} Double breaking in the case of a non-Abelian discrete manifold symmetry [26].

$$\begin{aligned}
& m \sum_{\text{families}} A_L h_L H \tilde{L} \tilde{E}_R + A_d h_d H \tilde{Q} \tilde{d}_R + A_u h_u H \tilde{Q} \tilde{u}_R + \\
& + A_2 \lambda_2 \tilde{N} H \bar{H} + A_3 \lambda_3 \tilde{N} D \bar{D} + A_\nu h_\nu \bar{H} \tilde{L} \tilde{\nu}_R + \text{h.c.} \quad (6.12)
\end{aligned}$$

the mass of each scalar evolves differently at lower energies. Thus one obtains for the singlets $\tilde{N}$ the evolution equation^{*)}:

$$\begin{aligned}
\frac{d m_{\tilde{N}}^2}{dt} = & \sum_a \frac{\alpha_a}{2\pi} C_a^N M_a^2 - 3 \left(\frac{\lambda_3}{4\pi} \right)^2 (m_{\tilde{D}}^2 + m_{\tilde{D}}^2 + m_{\tilde{N}}^2 + A_3^2 m^2) \\
& - 2 \left(\frac{\lambda_2}{4\pi} \right)^2 (m_{\tilde{H}}^2 + m_{\tilde{H}}^2 + m_{\tilde{N}}^2 + A_2^2 m^2) \quad (6.13)
\end{aligned}$$

where $t = 2 \log (M_X/Q)$ and C_a^N is the quadratic Casimir of the gauge groups on the representation in which $\tilde{N}$ is written. Since $\tilde{N}$ are $SU(3) \times SU(2)_L \times U(1)_Y$ singlets, the Yukawa couplings (especially λ_3 , which involves coloured objects) can easily overcome the gauge interactions and $m_{\tilde{N}}^2$ can become negative at a certain scale Λ_N at which a minimum with some (or all) $\langle \tilde{N}_i \rangle \neq 0$ may develop. One expects v.e.v.'s $\langle \tilde{N}_i \rangle^2 \sim -m_{\tilde{N}_i}^2/g_a^2$. A dimensional transmutation scheme [46] for this breaking does not seem possible since these v.e.v.'s will induce non-vanishing $1/2 D^a{}^2$ terms in the potential and hence the evolution equations have to be cut off at a scale $Q \sim m$. This also implies that one should expect large squarks and sleptons (large m) in this 'minimal' model.

Once these singlets $\tilde{N}_i$ acquire a v.e.v. the $D_i, \bar{D}_i$ fields and the W-S doublets $H_i, \bar{H}_i$ will get masses through the term in the superpotential $\lambda_2 N H \bar{H} + \lambda_3 N D \bar{D}$. This gives an extra reason why the $\langle \tilde{N}_i \rangle$ v.e.v.'s cannot be too large.

The evolution of the W-S doublets is given by the following equation:

*) Here and in the rest of the renormalization group equations we have omitted family indices for simplicity.

$$\frac{dm_{\tilde{H}}^2}{dt} = \sum_a \frac{\alpha_a}{2\pi} C_a^H M_a^2 - 3 \left(\frac{h_t}{4\pi} \right)^2 (m_{\tilde{Q}}^2 + m_{\tilde{U}}^2 + m_{\tilde{H}}^2 + A_t^2 m^2) - \left(\frac{\lambda_2}{4\pi} \right)^2 (m_{\tilde{H}}^2 + m_{\tilde{H}}^2 + m_{\tilde{N}}^2 + A_2^2 m^2) \quad (6.14a)$$

$$\frac{dm_{\tilde{H}}^2}{dt} = \sum_a \frac{\alpha_a}{2\pi} C_a^H M_a^2 - \left(\frac{\lambda_2}{4\pi} \right)^2 (m_{\tilde{H}}^2 + m_{\tilde{H}}^2 + m_{\tilde{N}}^2 + A_2^2 m^2) \quad (6.14b)$$

where we have neglected all Yukawa couplings except λ_2, λ_3 , and that of the top quark h_t . Which symmetry breaking ($\langle \tilde{N} \rangle \neq 0$ or $\langle H, \bar{H} \rangle \neq 0$) takes place first depends on the detailed values of $\lambda_2, \lambda_3, h_t$, as well as the gauge quantum numbers (under the group T) of the $\tilde{N}$'s. Notice, however, that if one had the boundary condition at the compactification scale

$$h_t \sim \lambda_3 \sim \lambda_2 \quad (6.15)$$

then one would obtain naturally the hierarchy of masses,

$$m_{\tilde{N}}^2 \ll m_{\tilde{H}}^2 \ll m_{\tilde{H}}^2 \quad (6.16)$$

due to the form of the renormalization group equations (6.13) and (6.14). Thus one expects the breaking of the gauge symmetry T to occur before the W-S breaking. The relevant scalar potential for the breaking of the $SU(2)_L \times U(1)_Y$ symmetry has the form:

$$V = \sum_{a \in T} \frac{D_a^2}{2} + \frac{1}{8} (g^2 + g'^2) (|H|^2 - |\bar{H}|^2)^2 + \lambda_2^2 |H \bar{H}|^2 + \lambda_2^2 |\tilde{N}|^2 (|H|^2 + |\bar{H}|^2) + m_{\tilde{N}}^2 |\tilde{N}|^2 + m_{\tilde{H}}^2 |H|^2 + m_{\tilde{H}}^2 |\bar{H}|^2 + m (h_2 A_2 \tilde{N} H \bar{H} + h.c.) \quad (6.17)$$

where $D^a{}^2$ stands for the gauge contribution of the extra interactions in T. Once the first breaking, giving $\langle \tilde{N}_i \rangle \neq 0$, has taken place, one is left with a potential for H and $\bar{H}$ which is of the usual form in $N = 1$ supergravity models with a mixed Higgs mass term [45,46]:

$$\mu_{3ij}^2 = (h_2 A_2)_{ij\kappa} m \langle \tilde{N}_\kappa \rangle \quad (6.18)$$

where the indices run in 'generation space'. In this situation the quantities μ_{3ij}^2 may be large, $\mu_j^2 \sim m$, so that no large top-quark mass is required to induce the $SU(2) \times U(1)$ breaking. In particular, it is not necessary to have a negative $\frac{m^2}{H}$; the $SU(2) \times U(1)$ breaking is induced through a large $\mu_j^2 \sim m^2$ once the $\tilde{N}_i$'s get v.e.v.'s.

Notice that we have here a set of three $H + \bar{H}$ states, one for each family. In order to avoid tree-level flavour changing neutral currents (FCNC) only one set of $H + \bar{H}$ Higgses should remain light enough to give rise to $SU(2) \times U(1)$ breaking and induce quark and lepton masses. We assume here that the v.e.v.'s $\langle \tilde{N}_i \rangle$ are such that only some linear combinations,

$$H \equiv \sum_{i=1-3} a_i H_i, \quad \bar{H} \equiv \sum_{i=1-3} b_i \bar{H}_i \quad (6.19)$$

remain light and get non-vanishing v.e.v.'s $\langle H \rangle$ and $\langle \bar{H} \rangle$.

If there is an $U(1)_{B-L}$ [or $SU(2)_R \times U(1)_{B-L}$] symmetry the right-handed sneutrinos can break it through a non-vanishing v.e.v. The renormalization group equation for the $\tilde{\nu}_R$'s has the form

$$\frac{d m_{\tilde{\nu}_R}^2}{d t} = \sum_a \frac{\alpha_a}{2\pi} C_a^\nu M_a^2 - 2 \left(\frac{h_\nu}{4\pi} \right)^2 (m_{\tilde{L}}^2 + m_{\tilde{H}}^2 + m_{\tilde{\nu}}^2 + A_\nu^2 m^2) \quad (6.20)$$

Again, for a large enough h_v coupling a negative $m_{\tilde{\nu}_R}^2$ can develop and a v.e.v. $\langle \tilde{\nu}_R \rangle$ may be induced, breaking (if present) the $U(1)_{B-L}$ [or $SU(2)_R \times U(1)_{B-L}$] symmetry. Notice that, as stated above, one has to forbid in this case some of the h_v^{ijk} ($i, j, k = 1, 2, 3$) couplings to avoid Dirac ν -masses. However, presumably one must not set all $h_v = 0$ because then no possible terms driving negative $(\text{mass})^2$ would be present in Eq. (6.20) and the origin of symmetry breaking (if necessary) would be unclear. A v.e.v. for some $\tilde{\nu}_R$ breaks R-parity so that one expects the appearance of some peculiar decay modes of SUSY particles. These will occur because of the mixing of gauginos of the T gauge sector (e.g. $\tilde{Z}_R^0, \tilde{W}_R^0, \dots$, etc.) with usual charged and neutral leptons. The Higgsino-gaugino mass matrices in these models are quite complicated since they involve: three Higgsinos $\tilde{H} + 3\tilde{H}$; three right-handed neutrinos ν_R ; three singlinos N_i ; the usual winos and zeenos plus the new gauginos of the T sector. The diagonalization of such mass matrices is thus a hard task. It seems, however, that there is enough freedom to prevent unwanted flavour changing effects by giving large enough masses (up to ~ 1 TeV) to some of the particles. If some the singlinos N or right-hand neutrinos ν_R remain massless after diagonalization they may get a radiative mass through the diagrams in Fig. 1 where the Higgses in the loops are supposed to be orthogonal to H and $\tilde{H}$. The fact that all these very weakly interacting neutral fermions get a non-vanishing mass may be important in order to avoid that all these particles populate the present universe in densities incompatible with observations. Notice that these particles do not annihilate very efficiently since their interactions always imply particles which are relatively heavy.

Let us finally comment [25] on the predictions of this model for $\sin^2 \theta_w$ and M_X . If the extra gauge group is just $U(1)$ or $U(1)^2$ one gets the same result as in the minimal supersymmetric $SU(5)$ model (with no Higgses):

$$\sin^2 \theta_w \approx 0.20, \quad M_x \approx 10^{18} \text{ GeV} \quad (6.21)$$

This is a nice result, especially since in superstring models the GUT scale essentially coincides with the compactification scale and one expects both to be $\sim M_P/\sqrt{8\pi} \sim 10^{18}$ GeV. If there is an unbroken $SU(2)_R$ symmetry it is known from results on $SO(10)$ SUSY-GUT's that the predictions get worse; one gets $\sin^2 \theta_w \approx 0.27$ and $M_x \sim 10^{24}$ GeV. Thus minimally modified gauge groups are in better agreement with experiment.

B-I model: three $\underline{27}$'s + light W-S doublets.

In this case [26] we have some light W-S doublets from the $(\underline{27} + \overline{\underline{27}})$'s. Let us assume, for the moment, that $b_{1,1} = 1$, so that only one set of these is available. It is easy to check that within the kind of gauge groups we are considering here^{*}), only an ansatz such as, for example,

$$U_g = \mathbb{1}_c \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix}_L \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix}_R \quad (6.22)$$

leads to light W-S doublets which couple to quarks and leptons as required. Here α is assumed to be a phase corresponding to some Z^n symmetry of the compact manifold. This ansatz implies that the unbroken gauge group is of the type

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_6$$

and the light fields are

^{*}) For simplicity we assume here the usual embedding of the fermions in the $\underline{27}$ of E_6 .

$$(1, 2, 2)_{0,-2} + (1, 1, 1)_{0,4} + (\text{same from } \overline{27})^* \quad (6.23)$$

where the subindices refer to the $U(1)$ charges and the singlet has the quantum numbers of an N state. We still call them N_H and $\bar{N}_H$. Notice that Witten splitting mechanism for obtaining light doublets implies an unbroken $SU(2)_R$. This is not surprising since it is equivalent to the assumption of a v.e.v. for an $SO(10)$ adjoint field $\varphi_{\alpha\beta}$ with vanishing entries in the $SO(4)$ directions. A coupling $H^\alpha \varphi_{\alpha\beta} H^\beta$ to $\underline{10}$'s of $SO(10)$ will not give mass to doublets. Another point to remark is that we have two $(1, 2, 2)$'s, one from $\underline{27}$ and one from $\overline{27}$. However, the doublets from $\overline{27}$ are, in principle useless since they do not couple to quarks and leptons. Thus the naïve impression that the Kähler nature ($b_{1,1} \geq 1$) of the Calabi-Yau manifolds implies the existence of Higgses $H + \bar{H}$ from $\underline{27} + \overline{27}$ is unfortunately incorrect. $H + \bar{H}$ are contained in $\underline{27}$'s if we want them to couple to quarks and leptons. Not only this, the existence of these 'mirror doublets' tends to spoil the predictions for $\sin^2 \theta_W$ and M_X .

The superpotential in this model will be as in the minimal one but letting the family indices in H_i , $\bar{H}_i$, and N_i run also over the Higgses $H, \bar{H}$ and singlet N_H from $\underline{27}_H^*$). There will also be, in general terms

$$W_H = \lambda N_H H \bar{H} \quad ; \quad W'_H = \lambda' \bar{N}_H H' \bar{H}' \quad (6.24)$$

where H' and $\bar{H}'$ are the 'mirror doublets' of the $\overline{27}_H$.

A first stage of symmetry breaking may occur as in the minimal case through the v.e.v.'s $\langle N_H \rangle = \langle \bar{N}_H \rangle$, which will break the $U(1)_6$ symmetry. A renormalization

*) There could also be terms of the form $\underline{27} \times \underline{27}_H \times \underline{27}_H$ as we will comment later on.

group equation like in Eq. (6.13) will drive a negative (mass)² for $\tilde{N}_H$ and a v.e.v. for $\tilde{N}_H$, will be induced. The equation for $\tilde{N}_H$ is simply

$$\frac{d m_{\tilde{N}_H}^2}{d t} = \frac{2}{3} \frac{\alpha_6}{2\pi} M_6^2 - 4 \left(\frac{\lambda'}{4\pi} \right)^2 (m_{H_1}^2 + m_{\bar{H}_1}^2 + m_{\tilde{N}_H}^2 + A'^2 m^2) \quad (6.25)$$

where A' is the soft term of the λ' coupling and M_6 is the mass of the $U(1)_6$ gaugino. Thus it seems more complicated for $\tilde{N}_H$ to obtain a negative (mass)² unless λ' is very large. However, $\tilde{N}_H$ will get a v.e.v. $\langle \tilde{N}_H \rangle \simeq \langle \tilde{N}_H \rangle$ even with $\lambda' = 0$ because the cancellation of the $U(1)_6$ D^2 term will force $\tilde{N}_H$ to get such a v.e.v. upon minimization of the potential. Unlike the case of the minimal model, the breaking of the symmetry may, in general, appear through dimensional transmutation^{*}), since the D^2 term will cancel along the minimum. This has the important consequence that the scale of $U(1)_6$ symmetry breaking can be very large since it is essentially given by the scale Q at which $m_{\tilde{N}_H}^2(Q) \simeq 0$, which may be hierarchically larger than the soft SUSY-breaking scale m . This may be useful [25,21] to get rid of the dangerous colour triplets D_i and $\bar{D}_i$ as well as of the extra W-S doublets $H_i, \bar{H}_i$. If one has, for example, $\langle \tilde{N}_H \rangle \gtrsim 10^{11}$ GeV these states will get very large masses ($\gtrsim 10^{11}$ GeV) through the Yukawas in Eq. (6.12) so that B-number violating couplings will no longer be dangerous and no discrete symmetries would be required to forbid proton decay. On the other hand, we do not want to get rid of all W-S doublets so that one must assume, for example, that the coupling $\lambda \simeq 0$ in Eq. (6.24) to obtain a couple of light Higgses $H, \bar{H}$. Concerning the mirror Higgses, one may or may not assume that $\lambda' \simeq 0$. In the former case they would get large masses and decouple from the low-energy world. As stated above, the symmetry breaking scale of $U(1)_6$ may be quite large. Thus from Eq. (6.13) one can estimate (neglecting gaugino mass effects):

^{*}) It is not true, even though often stated in the literature, that the scalar potential becomes unbounded below. It is stabilized by the field dependence of $m^2(\tilde{N})$.

$$m_{\tilde{N}_H}^2 \simeq m^2 - 20 \left(\frac{\lambda_{2,3}}{4\pi} \right)^2 m^2 \text{Log} (M_P^2/Q^2) \quad (6.26)$$

If, for example, $\lambda_{2,3} \sim g_{\text{GUT}}$ one would get $m_{\tilde{N}_H}^2 \sim 0$ for $Q \sim 10^{14}$ GeV so that large scales may naturally appear and proton decay mediated by D_i and $\bar{D}_i$ would be sufficiently suppressed.

After this symmetry breaking, the low-energy states consist of three families of quarks + leptons + ν_R 's, three N singlets, one other (unswallowed) singlet N'' from $N_H + \bar{N}_H$, and the W-S doublet $H + \bar{H}$ (there could also be mirror Higgses $H' + \bar{H}'$). The effective superpotential now has the form^{*})

$$W = \sum_{\text{families}} h_L H L E_R + h_d H Q d_R + h_u \bar{H} Q u_R + h_\nu \bar{H} L \nu_R + \lambda'' N'' H \bar{H} \quad (6.27)$$

in which the singlets N_i do not appear. This could be cosmologically dangerous since 'singlinos' N_i would be massless and practically non-interacting. However, one may also have an additional term in the superpotential

$$\Delta W = \lambda_\Delta N H \bar{H} \quad (6.28)$$

for the different N_i , coming from E_6 invariant couplings of the type

$$27 \times 27_H \times 27_H.$$

The breaking of the $SU(2)_R \times U(1)_{B-L}$ and $SU(2)_L \times U(1)_Y$ symmetries will then proceed in a manner analogous to the minimal case, close to the M_W region. There will only be minor differences due to the fact that in the present model

^{*}) In fact since there is an $SU(2)_R$ symmetry it is $h_L = h_u$ and $h_d = h_\nu$ and Eq. (6.27) could be written in a manifestly $SU(2)_L \times SU(2)_R$ symmetric way.

the low-energy chiral content of the theory is different. There is a severe problem though with neutrino masses. Since, as in the minimal model, no mechanism is available to give a Majorana mass to the ν_R 's, one has to forbid the coupling giving Dirac neutrino masses, $h_\nu = 0$. However, in the present case we have an unbroken $SU(2)_R$ symmetry and hence $h_\nu = h_L$: thus if we forbid ν -masses we also forbid charged lepton masses! There does not seem to be a way out of this problem in models with the particle content considered^{*)}. However, this could be avoided in models with $b_{1,1} > 1$, i.e. more than one set of Higgses from $(27_H + \overline{27}_H)$'s. If two $(2, 2)$ $SU(2)_L \times SU(2)_R$ multiplets coupling to quark and leptons exist at low energies and if the H and $\bar{H}$ fields with non-vanishing v.e.v.'s are not $SU(2)_R$ partners then $h_\nu = 0$ does not imply vanishing charged lepton masses. However, one must admit that in this case the model becomes somewhat contrived.

This model similarly to the minimal model, has new gauge interactions $[SU(2)_R \times U(1)_{B-L}]$ close to the weak scale. Also R-parity has to be broken since the right-handed neutrinos get v.e.v.'s, as also happened in the minimal model. The predictions for $\sin^2 \theta_w$ and M_x are not very good [25] owing to the existence of the $SU(2)_R \times U(1)_{B-L}$ symmetry. Neglecting the effect of the 'mirror Higgses' H' and $\bar{H}'$, which are assumed to get a very large mass, one gets

$$\sin^2 \theta_w \approx 0.29, \quad M_x \sim 10^{24} \text{ GeV} \quad (6.29)$$

B-II model: three 27 's + light $(N_H + \bar{N}_H)$ from $(27 + \overline{27})$.

This is similar to the 'minimal' model A but with extra singlets $N_H + \bar{N}_H$ which could be useful in getting rid of the dangerous triplets D_i and $\bar{D}_i$. Furthermore, no $SU(2)_R$ is necessary in this case at low energies so that one can

*) One can show that if one tries other fermion assignments [e.g. $SU(2) \neq SU(2)_R$ with ν_R or e_R as singlets instead of N] the problems remain: if one forbids ν -masses one also forbids lepton masses or if one forbids too-fast proton decay one also forbids some quark mass.

keep reasonable values for $\sin^2 \theta_W$ and M_X . This model can be obtained from a gauge ansatz

$$U_g = \mathbb{1}_C \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix}_L \times \begin{pmatrix} \beta & & \\ & \gamma & \\ & & \alpha^{-2} \end{pmatrix}_R \quad (6.30)$$

so that one gets a gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R \times U(1)_6$. Now the W-S doublets must be contained in the three sets of $H_i + \bar{H}_i$ inside the $\underline{27}$'s. The $U(1)_6$ symmetry will be radiatively broken by dimensional transmutation as in model B-I, through the v.e.v.'s $\langle \tilde{N}_H \rangle = \langle \tilde{\bar{N}}_H \rangle$. The v.e.v. of $\tilde{N}_H$ will, in general, give mass to the dangerous $D_i + \bar{D}_i$ but also to all Higgses $H_i + \bar{H}_i$ so that one has to arrange couplings in order that there is at least one couple of light W-S doublets, $H + \bar{H}$, to break $SU(2) \times U(1)$. The corresponding $D + \bar{D}$ triplets could then also be light, in which case one would have again to invoke some discrete symmetry to bar proton decay. Thus there does not seem to be much improvement concerning the p-decay problem in this model compared to the minimal one. However, the existence of $N_H + \bar{N}_H$ allows for a dimensional transmutation breaking of $U(1)_6$ which could be a useful modification of the minimal model. The situation with ν -masses is as in the minimal model. The values for $\sin^2 \theta_W$ and M_X are also as in model A, $\sin^2 \theta_W \simeq 0.20$, $M_X \simeq 10^{18}$ GeV, which is an improvement over model B-I.

B-III model: three $\underline{27}$'s and light ν_R -like objects $N_R + \bar{N}_R$ from $(\underline{27} + \bar{\underline{27}})$.

This could be obtained from an ansatz of the type

$$U_g = \mathbb{1}_C \times \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix}_L \times \begin{pmatrix} \alpha^{-2} & & \\ & \beta & \\ & & \gamma \end{pmatrix}_R \quad (6.31)$$

and the unbroken gauge symmetry may be any of the T groups considered in Eq. (6.8) [e.g. there is an unbroken $SU(2)_R$ for $\beta = \alpha^{-2}$]. The possible advantage

of this scheme is that one may use the $N_R + \bar{N}_R$ states to break $U(1)_{B-L}$ [and $SU(2)_R$, if present] and give a large mass to the right-handed neutrinos through one of the first two mechanisms at the beginning of this section. As happened in the minimal model with the sneutrinos $\tilde{\nu}_R$, $\tilde{N}_R$ and $\tilde{\bar{N}}_R$ may get a v.e.v. which in this case can be very large since a dimensional transmutation [46] mechanism is available (cancellation of D^2). If $\langle \tilde{N}_R \rangle \gtrsim 10^{10}$ GeV, which is feasible, we would obtain a large enough Majorana mass for the ν_R 's. Then, the rest of the gauge symmetry breaking would proceed as in the 'minimal' model A. In particular, we would have to forbid some couplings to avoid too-fast proton decay, but at least, in principle, the ν -mass problem would be solved.

There are, however, some problems also in this case. For $\tilde{N}_R$ to get a v.e.v. through dimensional transmutation^{*}, $\tilde{N}_R$ should have some Yukawa coupling in order to get a negative contribution to its (mass)². However, it cannot couple to itself owing to its quantum numbers. In terms of the $SO(10)$ and $SU(5)$ contents, the $N_R(16, 1)$ will couple to $(16, \bar{5})$ and $(10, 5)$. Hence these objects inside the 27 's will get a mass once $\tilde{N}_R$ gets a v.e.v., or at least those corresponding to one family. Then for that family the remnant states will transform as

$$(10, \bar{5}) + (16, 10) + (1, 1) \quad (6.32)$$

The other two families will conserve the complete 27 states except for their ν_R 's $[(16, 1)]$'s. The usual W-S doublets will be contained in these states and will (in the most favourable case) transform as $(10, 5)_H + (10, \bar{5})_H$. Then Dirac ν -masses, at least for the family in Eq. (6.32), will appear:

$$(10, \bar{5}) \times (1, 1) \times \langle (10, 5) \rangle_H \quad (6.33)$$

^{*}) Of course if N_R does not couple at all to the matter 27 's (except for non-renormalizable terms) this problem is not present and one would have a chance to solve the ν -mass problem. But then an alternative way to generate the v.e.v.'s for N_R and $\bar{N}_R$ would need to be found.

Thus, even in this case, which gives large masses to the ν_R 's one still has a Dirac neutrino mass problem because the other SU(5) singlet in the $\underline{27}$'s may behave as right-handed neutrinos. Thus this B-III model does not seem to solve the problem for which it was created!

The overall conclusion is that one may build models consistent with phenomenology. However, one is forced to forbid certain combinations of coupling constants to avoid too-fast proton decay and/or large Dirac ν -masses. Model B-I may avoid fast proton decay by giving large masses to all the dangerous triplets. However, one cannot forbid Dirac ν -masses (unless $b_{1,1} > 1$) owing to the unbroken $SU(2)_R$ symmetry. It seems as if model B-III could solve the ν -mass problem (if non-renormalizable terms exist in the superpotential) but it turns out that it cannot, and also one has to impose B-conservations by hand. One might think of hybrid models of the B-type such as, for example, a model with both light doublets and light $N_R + \bar{N}_R$ from $(\underline{27} + \bar{\underline{27}})$. However, that would imply an ansatz U_g explicitly $SU(3)_L \times SU(3)_R$ invariant with massless Higgses transforming similarly to $(\underline{1}, \underline{3}, \bar{\underline{3}}) + \text{h.c.}$ It is easy to see that even though one may induce radiatively a v.e.v. for $(\underline{1}, \underline{3}, \bar{\underline{3}})$ producing further symmetry breaking, the objects transforming as $N_R + \bar{N}_R$ are swallowed in the process of symmetry breaking^{*}) and are no longer available at low energies. Thus in this case one reobtains model B-I. Something similar happens if we want to obtain both $N_R + \bar{N}_R$ and $N_H + \bar{N}_H$ light. An unbroken $SU(2)$ symmetry between them exists in such a way that, when it is spontaneously broken, some of the states are swallowed and are no longer available at lower energies.

The mass of supersymmetric particles depends on the size of the soft terms m , M , and A , as well as on the couplings of the singlets. One would anyway expect a heavy spectrum of squarks and sleptons, since at least some of the

^{*}) Of course some states remain at lower energies if $b_{1,1} > 1$. But this usually implies worse predictions for $\sin^2 \theta_W$ and M_X if there are W-S doublets in the game.

symmetry breakings cannot take place through dimensional transmutation. Without knowing in more detail the way in which supersymmetry breaking takes place in the observable sector one cannot say much more. If SUSY-breaking is induced from gravitational corrections one would expect $m \sim m_{3/2}$ and $M \ll m$, $A \ll 1$. In this case one could have light gauginos, particularly light charginos. This occurs because in most of the examples discussed above the Higgsino bare mass μ is

$$\mu \sim M_{Z^0} \quad (6.34)$$

where M_{Z^0} is the mass of the extra neutral gauge interaction(s) which always remain close to the weak scale. Now if phenomenology requires $M_{Z^0}' \gg M_{Z^0}$ we expect a large μ but a small Majorana wino mass M_2 and under these conditions it is well known that there is always a chargino, which is lighter than M_W . In fact, one has to be careful that M_{Z^0}' (or μ) is not too large in order to avoid the experimental bound $M_{\tilde{W}} \gtrsim 20$ GeV. The close interplay between extra-low-energy gauge interactions and SUSY-breaking parameters may make important restrictions on superstring models.

7. CONCLUSIONS

In the present paper we have studied some aspects of the low-energy limit of superstring theories including the truncation of the theory down to four dimensions, supersymmetry breaking, dilaton and axion couplings as well as the low-energy phenomenological models which can appear. There are a number of attractive features of the whole approach. To start with, superstrings offer a framework in which all known interactions may be unified into a possibly finite theory. One can obtain an $N = 1$ supergravity theory in four dimensions coupled to phenomenologically interesting gauge groups (subgroups of E_6). There is also a 'hidden' sector which can induce supersymmetry breaking in a natural way, similarly to what happens in the low energy supergravity models studied in the

last few years [44-46]. The residual gauge symmetry [e.g. $SU(3) \times SU(2)_L \times U(1)^3$] may be naturally broken down to the standard model through radiative corrections and the unbroken supersymmetry will help to solve the 'gauge hierarchy' problem. Another interesting feature of these models is the model-independent existence of dilatons and axions which could shed some light on the strong-CP and cosmological constant problems.

Although all the above points are very encouraging, there remain many theoretical and phenomenological problems. Leaving aside the correct understanding of the superstring complexities, one still would like to understand how (and why) the compactification to four dimensions takes place. It is also not clear at all that there is an 'intermediate' stage with $d = 10$ field theory being approximately valid. There are, in fact, indications of the contrary, as we commented in the Introduction. A further unsolved problem is the cancellation of the cosmological constant to all orders. This problem is even more crucial in superstring models, because in general one does not even have the possibility of fine-tuning the cosmological constant to zero as happened in $d = 4$, $N = 1$ supergravity. Finally, no compactification is known up to now in which one can appropriately suppress Higgs-mediated too fast proton decay and/or large neutrino masses. It may well be that a unique manifold exists in which all phenomenological requirements are met. Or else one could be forced to compactify directly on the superstring, with no intermediate field theory step. Still one may hope that some of the conclusions extracted from the field theory Lagrangian may remain correct in a 'stringy' compactification.

Let us finally comment that some information may come from phenomenology. If extra gauge interactions along with a generally complex Higgsino-gaugino spectrum are found close to the weak scale, one may deduce (although the inference is certainly not univocal) that one is perhaps dealing with the effective theory of a superstring.

APPENDIX A

NOTATIONS AND FORMULAE

In this appendix are given some formulae and conventions which are used in the formulation of the ten-dimensional Lagrangian with tangent space group $SO(1,3) \times SO(6)$ and in dimensional reduction.

We use the $SO(1,9)$ metric

$$\eta_{MN} = \text{diag} (1, -1, -1, \dots, -1) ; \quad M, N = 0, 1, \dots, 9 \quad (\text{A.1})$$

The corresponding (32×32) gamma matrices Γ^M satisfy

$$\{ \Gamma^M, \Gamma^N \} = 2 \eta_{MN} \mathbb{1}_{32} \quad (\text{A.2a})$$

and

$$\Gamma^{11} = \Gamma^0 \Gamma^1 \dots \Gamma^9 = \Gamma^{11*} \quad (\text{A.2b})$$

Antisymmetric products are denoted by

$$\begin{aligned} \Gamma^{M_1 \dots M_n} &= \Gamma^{[M_1} \Gamma^{M_2} \dots \Gamma^{M_n]} \\ &= \frac{1}{n!} (\Gamma^{M_1} \Gamma^{M_2} \dots \Gamma^{M_n} \pm \dots) \end{aligned} \quad (\text{A.3})$$

i.e. brackets indicate antisymmetrization with weight 1.

Weyl spinors are eigenstates of Γ^{11} with eigenvalue +1 or -1. A Majorana condition can, moreover, be imposed on a Weyl spinor ψ , reducing the number of independent components to eight.

To reduce $SO(1,9)$ down to $SO(1,3) \times SO(6)$, we use the following form for the Γ^μ

$$\Gamma^\mu = \gamma^\mu \otimes \mathbb{1}_8, \quad \mu = 0, 1, 2, 3 \quad (\text{A.4})$$

$$\Gamma^{m+3} = i \gamma^5 \otimes \Gamma^m, \quad m = 1, 2, \dots, 6$$

where γ^μ are usual Dirac matrices ($\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$) and Γ^m are (Hermitian) $SO(6)$ gamma matrices satisfying

$$\{\Gamma^m, \Gamma^n\} = 2 \delta^{mn} \mathbb{1}_8 \quad (\text{A.5})$$

Then, the eigenvalue of

$$\Gamma^7 = i \Gamma^1 \Gamma^2 \dots \Gamma^6 \quad (\text{A.6})$$

is +1 for an $SO(6)$ spinor ($\underline{4}$) or -1 for an antispinor ($\bar{\underline{4}}$).

We can choose

$$\Gamma^7 = \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix} \quad (\text{A.7})$$

A Weyl $SO(1,9)$ spinor $\underline{16}$ then contains

$$\underline{16} = (\underline{4}, \underline{2}_L) + (\bar{\underline{4}}, \underline{2}_R) \quad (\text{A.8})$$

where $\underline{2}_L$ ($\underline{2}_R$) is a left-(right) handed $SO(1,3)$ spinor, i.e. $\gamma^5 = +1$ (-1). We choose a representation for which the $SO(1,9)$ charge-conjugation matrix C_{10} reads

$$C_{10} = C \otimes \begin{pmatrix} 0 & \mathbb{1}_4 \\ \mathbb{1}_4 & 0 \end{pmatrix} \quad (\text{A.9})$$

where C is the $SO(1,3)$ charge conjugation matrix. A Majorana-Weyl $SO(1,9)$ spinor can then be written

$$\Psi = \begin{pmatrix} \Psi_{iL} \\ \tilde{\Psi}_R^i \end{pmatrix} \quad (\text{A.10})$$

where $i=1,2,3,4$ is an $SO(6)$ spinor index. $\tilde{\Psi}_R^i$ is the charge conjugate of Ψ_{iL} :

$$\tilde{\Psi}^i = C \overline{\Psi}_i^T \quad (\text{A.11})$$

We use an explicit form of Γ^m which allows a simple $SO(6)$ -covariant rewriting of $SO(1,9)$ -invariant spinor couplings:

$$\Gamma^m = \begin{pmatrix} 0 & (\Gamma^m)_{ij} \\ (\Gamma^m)^{kl} & 0 \end{pmatrix} \quad (\text{A.12})$$

with

$$(\Gamma^m)^{ij} = (\Gamma^m)_{ji}^* \quad (\text{A.13})$$

since $\Gamma^{m\dagger} = \Gamma^m$.

With respect to $SO(1,3) \times SO(6)$, the metric tensor g_{MN} contains an invariant part which can be used to rescale the $SO(1,3)$ metric $g_{\mu\nu}$ in order to get the canonical Einstein Lagrangian in four space-time dimensions. We write g_{MN} in the following form

$$g_{MN} = \begin{pmatrix} \exp(-3\sigma) g_{\mu\nu} & 0 \\ 0 & \exp(\sigma) g_{mn} \end{pmatrix} \quad (\text{A.14})$$

omitting vector fields in the off-diagonal entries. σ is a real scalar field and g_{mn} is the internal metric tensor with unit determinant. We then get

$$E = \exp(-3\sigma) e \tag{A.15}$$

where e is the four-dimensional vierbein determinant.

APPENDIX B

KAHLER POTENTIAL AND DERIVATIVES IN THE TRUNCATED LAGRANGIAN

The Kähler potential upon truncation has the form

$$G \equiv K + \log |W|^2 = -\log(S+S^*) - 3\log(T+T^*-2|C|^2) + \log |W|^2 \quad (B.1)$$

From now on we shall define

$$s \equiv \text{Re } S, \quad t \equiv \text{Re } T, \quad t_c \equiv t - |C|^2 \quad (B.2)$$

so that one has

$$K = -\log(2s) - 3\log(2t_c) \quad (B.3)$$

Notice that the following useful identities hold

$$t_c = e^{\sigma} \varphi^{3/4} = g_4^2 e^{4\sigma} \quad (B.4)$$

$$s = e^{3\sigma} \varphi^{-3/4} = g_4^{-2} \quad (B.5)$$

$$\varphi = t_c s^{-1/3} \quad (B.6)$$

$$e^{4\sigma} = s t_c \quad (B.7)$$

$$e^K = \frac{1}{16} e^{-6\sigma} \varphi^{-3/2} = \frac{1}{16} s^{-1} t_c^{-3} \quad (B.8)$$

The derivatives of the Kähler potential with respect to the scalar fields T , S , and C are useful in order to obtain the explicit form of the $N = 1$ supergravity Lagrangian. For the convenience of the reader we present many of these derivatives in this appendix. Denoting, for example,

$$G'^S \equiv \frac{\partial G}{\partial S} \quad , \quad G'_S = \frac{\partial G}{\partial S^*} \quad (\text{B.9})$$

one gets the following results (we assume for the moment that W does not depend on S or T):

$$G'_S = G'^S = \frac{-1}{2S} \quad ; \quad G''_S = G''^S = G''_{SS} = \frac{1}{4S^2} \quad ; \quad G'''_S = G'''^S = G'''_{SS} = \dots = \frac{-1}{4S^3} \quad (\text{B.10})$$

There are no mixed derivatives of S and the other scalars T and C . The fields T and C do have mixed derivatives. One gets

$$G'^T = G'_T = \frac{-3}{2t_c} \quad , \quad G''^{TT} = \frac{-3}{4t_c^2} \quad (\text{B.11})$$

$$G'^C = \frac{3C^*}{t_c} + \frac{W'_C}{W} \quad (\text{B.12})$$

The double derivatives give the Kähler metric:

$$G_{i;\bar{j}} = \frac{3}{4t_c^2} \begin{pmatrix} 1 & -2C^* \\ -2C & 4t \end{pmatrix} \quad (\text{B.13})$$

where $i = 1$ corresponds to T and $i = 2$ to C . The inverse metric is also needed very often

$$(G^{-1})_{i;\bar{j}} = \frac{t_c}{6} \begin{pmatrix} 4t & 2C^* \\ 2C & 1 \end{pmatrix} \quad (\text{B.14})$$

Other mixed derivatives which are useful for fermionic couplings are

$$G_T^{'''CT} = \frac{3C^*}{2t_c^3} \quad ; \quad G_c^{'''CT} = \frac{-3t_c - 6|c|^2}{2t_c^3} \quad (B.15)$$

$$G_{cc}^{'''CT} = \frac{-6C}{t_c^3} \quad ; \quad G_{CT}^{'''CT} = \frac{3}{2t_c^3} \quad (B.16)$$

Derivatives involving the C fields which are necessary to obtain the usual Yukawa couplings are the following:

$$G^{cc} = \frac{3C^*C^*}{t_c^2} + \frac{W_{cc}''}{W} - \frac{W_c' W_c'}{W^2} \quad (B.17)$$

$$G_c^{cc} = \frac{6C^*t}{t_c^3} \quad , \quad G_T^{cc} = \frac{-3C^*C^*}{t_c^3} \quad (B.18)$$

If the superpotential W depends on S there are some changes. In particular, if there is an effective superpotential of the form

$$W = W_0 + W(s) \\ W(s) = h \exp(-3S/2b_0) \quad (B.19)$$

one has the following modifications for the derivatives with respect to S (no changes for the other derivatives):

$$G^{1S} = \frac{-1}{2s} - \frac{3W(S)}{2b_0 W} ; G_s^{1S} = \frac{1}{4s^2} \quad (B.20)$$

$$G^{1SS} = \frac{1}{4s^2} + \frac{9}{4b_0^2} \frac{W(S)(W_0)}{W^2} ; G_s^{1SS} = \frac{-1}{4s^3} \quad (B.21)$$

The non-derivative chiral fermion bilinears have the general form

$$e^{G/2} \left[G^{1ij} + G^{il} G^{lj} - G^{il} G_l^{1k} G_k^{1ij} \right] \bar{\chi}_{li} \chi_{lj} \quad (B.22)$$

where $i, j = S, T, C$. The terms in brackets evaluated for the fermionic partners of $(\tilde{S}, \tilde{T}, \chi_c)$ give the following results:

- i) For $(\tilde{S}\tilde{S})$ one gets $[] = 0$. If there is an effective non-perturbative superpotential, as in Eq. (A.19), one gets

$$[] = \frac{9}{4b_0^2} \frac{W(S)}{W} \quad (B.23)$$

- ii) For $(\tilde{T}\tilde{T})$ one obtains

$$[] = \frac{3}{2t_c^2} \quad (B.24)$$

- iii) For $(\tilde{\chi}_c \tilde{T})$ one gets

$$[] = - \frac{G^{1c}}{t_c} \quad (B.25)$$

- iv) For the non-singlet chiral fermions χ_c the result is

$$e^{G/2} [] = e^{K/2} \left(6 \frac{C^* C^*}{t_c^2} W + 4 \frac{C^* W_c'}{t_c} + W_{cc}'' \right) \quad (B.26)$$

After the rescaling:

$$\chi' = \sqrt{3} e^{-\sigma/2 - 3/8} \chi \quad (\text{B.27})$$

$$W^N = \frac{1}{3} \sqrt{\frac{8}{3}} g' (d_{abc} C^a C^b C^c) \quad (\text{B.28})$$

The last term in Eq. (B.26) gives the usual Yukawa coupling $W_{cc}^{N\alpha} \tilde{\chi}' \tilde{\chi}'$. The χ' fields have properly normalized kinetic terms. The scalar potential which is given by

$$V = e^G (G'_K G'^{K-1}_L G'^L - 3) \quad (\text{B.29})$$

yields the result

$$V = \frac{s^{-1} t_c^{-3}}{16} \left(|W|^2 + \frac{t_c}{3} |W'_c|^2 \right) + D^2\text{-term} \quad (\text{B.30})$$

in the case where W does not depend on S or T . If there is a $W(S)$ term the result is

$$V = \frac{s^{-1} t_c^{-3}}{16} \left(\left| W + \frac{3s}{b_0} h \exp(-3S/2b_0) \right|^2 + \frac{t_c}{3} |W'_c|^2 \right) + D^2 \quad (\text{B.31})$$

In all these formulae we have not yet redefined the fields in order to get properly normalized kinetic terms in the vacuum. Let us finally show the gravitino-chiral fermion bilinear couplings which are relevant for the super-Higgs effect. They are

$$e^{G/2} \left[\left(\frac{3W(S)}{2b_0 W} - \frac{1}{2s} \right) (\bar{\Psi}_R \cdot \gamma \tilde{S}_L) - \frac{3}{2t_c} (\bar{\Psi}_R \cdot \gamma \tilde{T}_L) + \left(\frac{3C^*}{t_c} + \frac{W'_c}{W} \right) (\bar{\Psi}_R \cdot \gamma \chi_c) \right] \quad (\text{B.32})$$

Notice that the $\tilde{S}_L$ term cancels upon minimization of Eq. (B.31) in the case of the existence of a superpotential $W(S)$. Thus the goldstino will be $\tilde{T}$. In a more general situation it will be a linear combination of $\tilde{S}$ and $\tilde{T}$.

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FIGURE CAPTION

Radiative contributions to a) "singlinos" and b) right-handed neutrino masses