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EXPERIMENTAL PREDICTIONS FROM THE SUPERSTRING

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A B S T R A C T

We present phenomenological constraints on, and experimental predictions from, the minimal low energy model extracted from the superstring which appears able to generate the hierarchy $m_W \ll m_P$. (A) Quarks and charged leptons have specified couplings to a new neutral boson Z_E which must be heavier than the Z^0 . (B) Squark, slepton and gaugino masses are in the ratios

$$m_{\tilde{Q}} : m_{\tilde{L}} : m_{\tilde{U}_R} : m_{\tilde{D}_R} : m_{\tilde{E}} = 1.95:0.74:0.48:1:1/7$$

(C) Estimates are given of the Yukawa couplings which lead to $m_t \sim 55$ GeV and $m_W = m_P \exp(-O(1)/\alpha_t) \sim 80$ GeV. (D) The absolute magnitudes of sparticle masses are estimated approximately.

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Many theorists take the superstring^{1),2)} very seriously as a candidate Theory of Everything (TOE), but just how the theory makes contact with everyday, low energy, four-dimensional experiments is not yet completely clear. It is believed that the original 10-dimensional theory with its $E_8 \times E_8$ gauge group reduces³⁾ to a no-scale supergravity model in four dimensions^{4),5)}, with E_6 as a grand unification gauge group⁶⁾. The low energy gauge group must⁶⁾ be larger than $SU(3)_C \times SU(2)_L \times U(1)_Y$, containing at least one extra $U(1)$ gauge factor and possibly more low energy gauge bosons, depending on unknown details of the compactification from 10 to 4 dimensions. The number of fermion generations is likewise unclear⁷⁾, though it is known⁶⁾ that each one contains additional fields besides the conventional quarks and leptons. The masses of supersymmetric particles are determined by the scale of global supersymmetry breaking in the observable sector which should be $O(m_W)$. Although mechanisms exist^{8),9)} for generating supersymmetry breaking from the hidden sector, feeding it through to the observable sector¹⁰⁾ and determining its size dynamically through some variant of a no-scale model, the precise magnitudes of global supersymmetry breaking and of sparticle masses are not yet known. Thus superstring models offer a potential bonanza to experimentalists - more gauge bosons, possibly more generations, additional particles besides quarks and leptons, as well as conventional sparticles - the problems are to decide how and where to prospect for them. Although it may seem foolhardy and premature to attack these problems, we tackle them here in the minimal low energy model¹⁰⁾ extracted from the superstring which appears able to generate the hierarchy $m_W \ll m_P$. Thus we assume a low energy $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ gauge group with three quark and lepton generations and emphasize the rôle of an $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet field N coupled to the supersymmetric standard model Higgses $H, \bar{H}$. Our results are arranged in the order of increasing model-dependence.

(A) We first show that conventional quarks and charged leptons have well-defined vector and axial couplings to the new Z_E gauge boson. Low energy neutrino neutral current data^{11),12)} force the Z_E to be significantly heavier than the Z^0 , while $\bar{p}p$ collider data¹³⁾ allow it to be almost degenerate with the Z^0 : $90 \text{ GeV} < m_{Z_E} < 100 \text{ GeV}$ (which seems rather unlikely), or else weigh above 110 GeV.

(B) The ratios of soft supersymmetry breaking squark, slepton and gaugino masses are calculated from renormalization effects¹⁴⁾:

$$m_{\tilde{q}} : m_{\tilde{e}_L} : m_{\tilde{e}_R} : m_{\tilde{g}} : m_{\tilde{\gamma}} = 1.95 : 0.70 : 0.38 : 1 : 1/7 \quad (1.a)$$

(C) Including the more model-dependent effects of D term contributions, which are not negligible in this class of models, the results (1.a) are modified in one plausible variant to :

$$m_{\tilde{q}} : m_{\tilde{e}_L} : m_{\tilde{e}_R} : m_{\tilde{g}} : m_{\tilde{\gamma}} = 1.95 : 0.74 : 0.48 : 1 : 1/7 \quad (1.b)$$

Therefore, using $m_{\tilde{e}_R} \gtrsim 23$ GeV from fruitless e^+e^- searches¹⁵⁾ and/or $m_{\tilde{g}} \gtrsim 40$ GeV from fruitless $\bar{p}p$ searches¹⁶⁾, we infer

$$m_{\tilde{q}} \gtrsim 93 \text{ GeV}, m_{\tilde{e}_L} \gtrsim 35 \text{ GeV}, m_{\tilde{g}} \gtrsim 48 \text{ GeV}, m_{\tilde{\gamma}} \gtrsim 6.9 \text{ GeV} \quad (2)$$

Renormalization calculations also demonstrate that weak gauge symmetry breaking is possible at a scale $v, \bar{v}, x \equiv \langle 0 | H, \bar{H}, N | 0 \rangle = O(200)$ GeV : $m_W \sim 80$ GeV if the t quark and HHN Yukawa couplings are chosen :

$$\lambda^2(m_W) = 0.078, \quad \lambda_t^2(m_W) = 0.12 \quad (3)$$

The Z_E boson is too light in this minimal model, but not all important effects have yet been included, and we regard these calculations only as an illustrative existence proof.

(D) Uncertainties in the minimization of the effective potential do not yet permit a precise determination of the global supersymmetry breaking scale, but we estimate it as

$$m_{1/2} \simeq O(2) m_W \quad (4)$$

with other sparticle masses scaled according to equation (1).

To discuss the additional neutral currents expected at low energies, it is convenient to consider the maximal $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of E_6 . As pointed out in Ref. 6), if one wishes to keep $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the low energy gauge group, then since $Y = \frac{1}{2}(Y_L + Y_R)$: $Y_L \equiv \text{diag}(1/3, 1/3, -2/3)_L$ and $Y_R = \text{diag}(4/3, -2/3, -2/3)_R$, one cannot avoid keeping both Y_L and Y_R unbroken down to low energies. The low energy group may be larger, but $SU(3)_C \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R}$ is the minimum possible extension of the standard model at low

energies in the superstring. It is convenient to work with Y and the orthogonal combination

$$Y_E = \sqrt{\frac{3}{5}} \left(Y_L - \frac{1}{4} Y_R \right) \quad (5)$$

where we have introduced a normalization factor ensuring that the gauge coupling constant g_E associated with Y_E is equal to g_2 and g_3 in the E_6 symmetry limit. A simple renormalization group calculation gives

$$\alpha_E \equiv g_E^2 / 4\pi = 0.0164 \quad (6.a)$$

at scales $\lesssim m_W$. Using the definition (5) we find the following Y_E assignments for conventional quarks and leptons (q, q^c, ℓ, ℓ^c), the Higgses ($\bar{H}, H$), the additional charge $-1/3$ quarks (g, g^c) and the two $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet and neutral fields (N, N') appearing in each 27 of E_6 :

$$Y_E = \sqrt{\frac{3}{5}} \times \left(\begin{array}{cccccccccccc} (u,d)_L & u_L^c & d_L^c & (\nu, e^-)_L & e_L^c & \bar{H} & H & g & g^c & N & N' \end{array} \right) \quad (6.b)$$

$$Y_E = \sqrt{\frac{3}{5}} \times \left(\frac{1}{3}, \frac{1}{3}, -\frac{1}{6}, -\frac{1}{6}, \frac{1}{3}, -\frac{1}{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{6}, \frac{5}{6}, \frac{5}{6} \right)$$

Equations (6.a) and (6.b) together specify completely the new neutral current couplings of the gauge boson Z_E coupled to Y_E . Hence we are now in a position to calculate the effects of Z_E exchange on low energy neutral current processes.

The most precise pieces of information about low energy neutral currents come from neutrino interactions, parity violation in ep scattering and in atoms¹¹⁾, and the forward-backward asymmetries in e^+e^- collisions¹²⁾. Table 1 shows Z_E effects on neutrino neutral current parameters¹¹⁾ and on the $e^+e^- \rightarrow \ell^+\ell^-$ forward-backward asymmetries¹²⁾. Also shown are the experimental values¹¹⁾ of these quantities: note that the e^+e^- constraints¹²⁾ are less important: although not exhibited, bounds from parity violation are similar. Taking $\sin^2\theta_W = 0.227 \pm 0.010$ from m_W measured at the $\bar{p}p$ collider¹³⁾ (as discussed later, the minimal superstring spectrum is consistent with $\sin^2\theta_W = 0.21$ in leading order), we deduce from the low energy neutral current data that

$$y \equiv (g_E^2/g_2^2) (m_W^2/m_{Z_E}^2) = -0.12 \pm 0.24 \quad (7)$$

and hence, allowing 1 or 2σ and using the value (6.a) for g_E^2 :

$$m_{Z_E} > (1.6, 0.9) m_W \approx (130, 70) \text{ GeV} \quad (8)$$

Next we turn to $\bar{p}p$ collider production of the Z^0 and Z_E . It is easy to check using Equations (6.a) and (6.b) that for equal masses

$$\sigma_{Z_E}/\sigma_{Z^0} \approx 0.22 \pm 0.02 \quad (9)$$

The luminosity for $\bar{q}q \rightarrow Z_E$ collisions at the present $\bar{p}p$ collider can be described¹⁷⁾ to better than 10% accuracy in the mass range $70 \text{ GeV} < m_Z < 110 \text{ GeV}$ by the formula

$$\exp(-0.068 \Delta m) \times L_{Z^0} \quad (10)$$

where $\Delta m = m_{Z_E} - m_{Z^0}$. If the Z_E can decay freely into all three generations of conventional quarks and leptons and also into three generations of N scalars (neons) and their $\tilde{N}$ fermion partners (ninos), then

$$B_E(Z_E \rightarrow e^+e^-) \approx 1/108 \quad (11)$$

to be compared with the conventional $B_0(Z^0 \rightarrow e^+e^-) \approx 3\%$. Formula (11) may underestimate B_E , but using it as a lower bound we combine it with (9) and (10) to find

$$\sigma_E B_E / \sigma_0 B_0 \gtrsim 0.02 \quad (12)$$

for $70 \text{ GeV} < m_{Z_E} < 110 \text{ GeV}$. All high-mass l^+l^- pairs seen by the UA1 and UA2 collaborations¹³⁾ are compatible with the conventional Z^0 . Requiring less than one event in each of the channels $Z_E \rightarrow e^+e^-$, $\mu^+\mu^-$ (UA1) and $Z_E \rightarrow e^+e^-$ (UA2), we find that

$$m_{Z_E} > 110 \text{ GeV} \quad (13)$$

is allowed. There is one caveat to this analysis : if $m_{Z_E} \sim m_{Z^0}$ then a few $Z_E \rightarrow \lambda^+ \lambda^-$ events could be hiding under the observed Z^0 peaks if

$$90 \text{ GeV} < m_{Z_E} < 100 \text{ GeV} \quad (14)$$

Equations (8), (13) and (14) summarize the allowed ranges for m_{Z_E} .

In the above analyses we have ignored Z_E - Z^0 mixing. We now examine this assumption a posteriori, and show that if the diagonal entries in the (Z^0, Z_E) mass matrix were equal, the off-diagonal entries would push them far apart for representative values of the Higgs vacuum expectation values, so that the approximate degeneracy (14) is a highly unlikely scenario. The (Z^0, Z_E) mass squared matrix is

$$M_Z^2 \begin{bmatrix} 1 & \sin \theta_W \frac{4v^2 - \bar{v}^2}{6(v^2 + \bar{v}^2)} \\ \sin \theta_W \frac{4v^2 - \bar{v}^2}{6(v^2 + \bar{v}^2)} & \sin^2 \theta_W \frac{4v^2 + \bar{v}^2 + 25x^2}{9(v^2 + \bar{v}^2)} \end{bmatrix} \quad (15)$$

Using the relations (6) we find that in the limit of small off-diagonal elements relative to the difference in the diagonal elements (small mixing), the physical Z^0 boson is shifted :

$$M_Z^2 \left[1 + \frac{\frac{1}{36} \sin^2 \theta_W \left(\frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} \right)^2}{1 - \frac{1}{9} \sin^2 \theta_W \frac{25x^2 + 4v^2 + \bar{v}^2}{v^2 + \bar{v}^2}} \right] \quad (16)$$

as is the physical Z_E boson :

$$m_{Z_E}^2 = m_Z^2 \left[\frac{1}{9} \sin^2 \theta_W \frac{25x^2 + 4v^2 + \bar{v}^2}{v^2 + \bar{v}^2} - \frac{\frac{1}{36} \sin^2 \theta_W \left(\frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} \right)^2}{1 - \frac{1}{9} \sin^2 \theta_W \frac{25x^2 + 4v^2 + \bar{v}^2}{v^2 + \bar{v}^2}} \right] \quad (17)$$

On the other hand, if the diagonal entries in (15) are equal (maximal mixing), the splitting between the two mass eigenstates is

$$\Delta m_Z \approx \frac{1}{6} m_Z \sin \theta_W \frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} \quad (18)$$

For typical values of v and $\bar{v}$ to be discussed later : $v/\bar{v} \sim 5$, the splitting (18) is impossibly large. We cannot exclude the possibility that in a different version of the model

$$\frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} < 0.7 \quad (19)$$

so that $\Delta m_Z < 5$ GeV, in which case the possibility (already mentioned in our analysis of the $\bar{p}p$ collider data) of approximate degeneracy between the Z^0 and the Z_E cannot be ruled out. It will be interesting to see if future, more detailed, experiments find a split Z^0 , although we regard the possibility as fortuitous and highly unlikely.

We now analyze the renormalization group equations of the low energy theory and derive the results (1) for the supersymmetry breaking mass parameters. It is easy to check that with the low energy particle content of three generations (6.b), in one-loop order

$$\alpha_3 = \text{const.}, \quad \alpha_2 = \frac{\alpha_3}{1 + \frac{3}{2\pi} \alpha_3 t}, \quad \alpha_1 = \alpha_E = \frac{\alpha_3}{1 + \frac{9}{2\pi} \alpha_3 t} \quad (20)$$

where $t = \ln(m_X/\mu)$. Therefore the GUT scale m_X is given by

$$t_X = \ln \frac{m_X}{m_W} = \frac{2\pi}{3\alpha_3} \left[\frac{\alpha_3 \sin^2 \theta_W}{\alpha_{em}} - 1 \right] \quad (21)$$

and with $\alpha_{em}^{-1} = 128$ and $\alpha_3 = 0.11$, this gives $\sin^2 \theta_W = 0.207$ and $t_X = 36.2$ or $m_X = 4.4 \times 10^{17}$ GeV, which is acceptable given the uncertainties of thresholds, two-loop effects, etc. The gaugino masses are in the ratios of the α_i (20) as usual :

$$m_{\tilde{g}} : m_{\tilde{W}} : m_{\tilde{B}} : m_{\tilde{Z}} : m_{1/2} = 1 : 0.34 : 0.15 : 0.15 : 1 \quad (22)$$

where $m_{1/2}$ is the primordial supersymmetry breaking gaugino mass, which is⁽¹⁰⁾ the primary source of supersymmetry breaking. Assuming that third generation Yukawa couplings are the largest, the most important parts of the low energy scalar potential are derived from the superpotential

$$W = \lambda N H \bar{H} + \lambda_t U Q H \quad (23)$$

with the soft supersymmetry breaking terms

$$V_{soft} = 2\lambda A \text{Re} N H \bar{H} + 2\lambda_t A_t \text{Re} U Q H + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 \quad (24)$$

which obey the renormalization group equations

$$8\pi^2 \frac{dA}{dt} = 4\lambda^2 A + 3\lambda_t^2 A_t + (3g_2^4 + 2g_1^4) m_{1/2} g_3^{-2} \quad (25.a)$$

$$8\pi^2 \frac{dA_t}{dt} = \lambda^2 A + 6\lambda_t^2 A_t + \frac{16}{3} g_3^2 m_{1/2} + (3g_2^4 + \frac{5}{3} g_1^4) m_{1/2} g_3^{-2} \quad (25.b)$$

$$-8\pi^2 \frac{dm_N^2}{dt} = \frac{5}{3} g_1^6 m_{1/2}^2 g_3^{-4} - 2\lambda^2 (m_N^2 + m_H^2 + m_{\bar{H}}^2 + A^2) \quad (25.c)$$

$$-8\pi^2 \frac{dm_H^2}{dt} = (3g_2^6 + \frac{2}{3} g_1^6) m_{1/2}^2 g_3^{-4} - \lambda^2 (m_N^2 + m_H^2 + m_{\bar{H}}^2 + A^2) \quad (25.d)$$

$$-8\pi^2 \frac{dm_H^2}{dt} = -8\pi^2 \frac{dm_{\frac{1}{2}}^2}{dt} + g_1^6 m_{\frac{1}{2}}^2 g_3^{-4} - 3\lambda_t^2 (m_H^2 + m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \quad (25.e)$$

$$8\pi^2 \frac{d\lambda}{dt} = \left(2\lambda^2 + \frac{3}{2}\lambda_t^2 - \frac{3}{2}g_2^2 - g_1^2 \right) \lambda \quad (25.f)$$

$$8\pi^2 \frac{d\lambda_t}{dt} = \left(\frac{1}{2}\lambda^2 + 3\lambda_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{5}{6}g_1^2 \right) \lambda_t \quad (25.g)$$

The squark and slepton masses are given by

$$-8\pi^2 \frac{dm_{U_3}^2}{dt} = \left(\frac{16}{3}g_3^6 + \frac{4}{3}g_1^6 \right) m_{\frac{1}{2}}^2 g_3^{-4} - 2\lambda_t^2 (m_H^2 + m_{U_3}^2 + m_{Q_3}^2 + A_t^2) \quad (26.a)$$

$$-8\pi^2 \frac{dm_{D_3}^2}{dt} = \left(\frac{16}{3}g_3^6 + \frac{1}{3}g_1^6 \right) m_{\frac{1}{2}}^2 g_3^{-4} \quad (26.b)$$

$$-8\pi^2 \frac{dm_{Q_3}^2}{dt} = \left(\frac{16}{3}g_3^6 + 3g_2^6 + \frac{1}{3}g_1^6 \right) m_{\frac{1}{2}}^2 g_3^{-4} - \lambda_t^2 (m_H^2 + m_{U_3}^2 + m_{Q_3}^2 + A_t^2) \quad (26.c)$$

$$-8\pi^2 \frac{dm_L^2}{dt} = \left(3g_2^6 + \frac{2}{3}g_1^6 \right) m_{\frac{1}{2}}^2 g_3^{-4} \quad (26.d)$$

$$-8\pi^2 \frac{dm_E^2}{dt} = \frac{8}{3} g_1^6 m_{\frac{1}{2}}^2 g_3^{-4} \quad (26.e)$$

where $t = \ln(Q/\mu)$, with obvious modifications for the first two generations of squarks. The renormalization group equations (25) and (26) can be solved numerically with boundary conditions¹⁰⁾ $m_0 = 0$, $A = A_\lambda = 0$, $m_{\tilde{V}} = m_{\frac{1}{2}}$ at $\mu = m_X \approx m_p$, to give the results in Table 2. These results were obtained using the

representative values $\lambda_t^2(m_X) = 0.0011$ and $\lambda^2(m_X) = 0.02$ which correspond to $\lambda_t^2(m_W) = 0.12$, $\lambda^2(m_W) = 0.078$ and $m_t \sim 55$ GeV, and also lead to weak gauge symmetry breaking at a realistic scale $m_W = O(\mu_0) = O(100)$ GeV, as will be discussed shortly. The numerical predictions (1.a) are obtained directly from equation (22) and Table 2. In this class of models, D-term contributions to scalar masses are not negligible. We have evaluated these using the representative field vacuum expectation values (37), obtaining the results (1.b).

Experimentally, we know

$$m_{LR} \gtrsim 23 \text{ GeV}, \quad m_{\tilde{g}} \gtrsim 40 \text{ GeV}, \quad m_{\tilde{f}} \gtrsim 40 \text{ GeV} \quad (27)$$

of which the first two are the most important constraints for the spectrum of equation (1.b). Inserting this into equation (1.b) we get the lower bounds on sparticle masses listed in equation (2). A posteriori, the lower bounds on $(m_{\tilde{e}_R}, m_{\tilde{\gamma}})$ are at best marginally compatible with the MAC lower limits from a search for $e^+e^- \rightarrow \gamma + \text{missing energy-momentum}$ ¹⁸⁾. We infer from equation (2) that e^+e^- experiments could well find single photons and possibly right-handed sleptons in the near future. The latter might also be detectable in Z^0 decays at the $\bar{p}p$ collider, though left-handed sleptons might well not be accessible in either $W^\pm$ or Z^0 decay. The $\bar{p}p$ collider may be able to see gluinos, but would be unlikely to see squarks according to equation (2).

Finally, we turn to the minimization of the effective potential, weak gauge symmetry breaking and the dynamical determination of the supersymmetry breaking scale $m_{\frac{1}{2}}$. The tree-level low energy potential for $H, \bar{H}$ and N takes the following form for the real field values $v, \bar{v}, x$:

$$V_{eff} = m_H^2 v^2 + m_{\bar{H}}^2 \bar{v}^2 + m_N^2 x^2 + 2\lambda A v \bar{v} x + \lambda^2 (v^2 \bar{v}^2 + v^2 x^2 + \bar{v}^2 x^2) + \frac{5}{24} g_E^2 \left(x^2 - \frac{4}{5} v^2 - \frac{1}{5} \bar{v}^2 \right) + \frac{1}{8} (g_2^2 + g_{12}^2) (v^2 - \bar{v}^2)^2 \quad (28)$$

As we bring the renormalization scale μ down from m_X , the first squared mass parameter to go negative is m_H^2 , which leads to a non-zero vacuum expectation value for v :

$$V^2 = |m_H^2| / \left[\frac{4}{15} g_E^2 + \frac{1}{4} (g_2^2 + g_1^2) \right] \quad (29)$$

However, since¹⁰⁾

$$m_{1/2} = - \frac{b g^2}{4} \left(\frac{1}{16 s_R T_R^3} \right)^{1/2} \lambda \left(\frac{V X}{\bar{V}} + \frac{\bar{V} X}{V} + \frac{V \bar{V}}{X} \right) \quad (30)$$

we need all three of $v, \bar{v}, x \neq 0$ to get global supersymmetry breaking, and the renormalization group equations keep on evolving until the line $\bar{v} = x = 0$ becomes unstable. The trilinear term in (28) ensures that if one of $\bar{v}, x \neq 0$, then so also is the other. Instability sets in when

$$\begin{aligned} & \left[m_N^2 + \left(\lambda^2 - \frac{1}{3} g_E^2 \right) V^2 \right] \left[m_H^2 + \left(\lambda^2 + \frac{1}{15} g_E^2 - \frac{1}{4} (g_2^2 + g_1^2) \right) V^2 \right] \\ & - \lambda^2 A^2 V^2 < 0 \end{aligned} \quad (31)$$

at a renormalization scale $\mu_0 = m_p \exp(-O(1)/\lambda_t^2, \lambda^2) : t_0 \equiv \ln(m_X/\mu_0)$. In a no-scale model⁵⁾, the global supersymmetry breaking scale $m_{1/2}$ (30) can be determined dynamically to be $O(\mu_0)$, as we now indicate. The full low energy effective potential is

$$V = V_{\text{eff}} + \delta V : \delta V = \frac{1}{64\pi^2} \text{Str } m^4 \left(\ln m^2/\mu^2 - c \right) \quad (32)$$

where c is a renormalization-scheme-dependent constant^{*)}. Following Ref. 5), we choose $\mu : \delta V = 0$ which corresponds to

$$\mu = O(1) m_{1/2} \quad (33)$$

With this choice of μ ,

*) Note that the naive cut-off-dependent $\text{Str } m^4 \ln m^2/\Lambda^2$ is replaced¹⁹⁾ by $\text{Str } m^4 \ln m^2/\mu^2$ when a consistent renormalization prescription is adopted. Note also that the quadratically divergent $\text{Str } m^2 \Lambda^2$ term can be absorbed in a renormalization of the parameters in the initial Kähler potential^{10),20)}.

$$V = V_{\text{eff}}(\mu) = e^{-4t} P(t) \quad (34)$$

where $P(t)$ is a polynomial of t in the neighbourhood of the contour in $v, \bar{v}, x$ space $V_{\text{eff}}(\mu_0) = 0$. The potential V is therefore minimized with respect to t when

$$P(t) = \frac{1}{4} P'(t) : P' = O\left(\frac{\alpha}{\pi}\right) \quad (35)$$

Combining equations (33) and (35), we see that

$$t = t_0 + O(1) \quad (36)$$

We therefore stop the renormalization group clock at a scale $\mu : t \equiv \ln(m_X/\mu)$ is given by the expression (36), and find for $\lambda_t^2(m_W) = 0.12$, $\lambda^2(m_W) = 0.078$

$$(v^2, \bar{v}^2, x^2) = (0.81, 0.04, 0.24) \quad (37)$$

Using $m_W^{2+} = \frac{1}{2} g_2^2 (v^2 + \bar{v}^2)$ we can now calculate $m_{1/2}$:

$$m_{1/2} = \frac{\sqrt{2}}{g_2} m_W \left(\frac{m_{1/2}}{\sqrt{v^2 + \bar{v}^2}} \right) \simeq 190 \text{ GeV} \quad (38)$$

and hence the absolute values of the sparticle masses :

$$m_{\tilde{q}} = 360 \text{ GeV}, m_{\tilde{e}_L} = 140 \text{ GeV}, m_{\tilde{e}_R} = 90 \text{ GeV}, m_{\tilde{\nu}} = 110 \text{ GeV} \quad (39)$$

which are above the previous lower limits (2). Needless to say, the calculation of these absolute values involves many extra dynamical assumptions^{*)} about the determination of field values in no-scale models which go beyond those used to obtain the ratio (1), and are sensitive to the model-dependent D-terms in particular. There could also be important contributions to the low energy effective potential beyond those discussed so far, for example from an extra

*) As emphasized in Ref. 10), the form (30) of $m_{1/2}$ implies $T_R = O(1)$ at the minimum. We have assumed that $SU(2) \times U(1)$ gauge breaking fixes T_R , though it is also possible²¹⁾ that Planck mass dynamics fixes $T_R = O(1)$ [see also Binétruy and Gaillard, Ref. 20)] in which case the minimization of the potential could give somewhat different results, as we will discuss in a future publication.

27+27 in addition to three generations like (6.b), or from additional low energy gauge interactions. Moreover, the third generation N' field in (6.b) may be important, as could be the first two generations of N fields. We actually need some modification to the above analysis, since the vacuum expectation values (37) would give $m_{Z_E} \approx 40$ GeV when substituted into equation (17), which conflicts with the allowed range (13), (14). Many of the above effects would tend to increase m_{Z_E} , but the effect on the sparticle mass scale is unclear. However, given our dynamical assumptions the values (39) of the sparticle masses and of the Z_E mass could be correct to within a factor $O(2)$.

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TABLE 1 : Neutral Current Parameters

[the notation is that of Ref. 11)]

Parameter	Value with $U(1)_E$	Value with $x \equiv \sin^2 \theta_W = 0.227 \pm 0.010$	Experiment	Inferred value of $y = (g_E^2/g_Z^2)(m_W^2/m_Z^2)$
$\epsilon_L(u)$	$\frac{1}{2} - \frac{2}{3}x_W - \frac{1}{15}y$	$(0.349 \pm 0.007) - \frac{1}{15}y$	0.340 ± 0.033	-0.12 ± 0.24
$\epsilon_R(u)$	$-\frac{2}{3}x_W + \frac{1}{15}y$	$(-0.151 \pm 0.007) + \frac{1}{15}y$	-0.179 ± 0.019	
$\epsilon_L(d)$	$-\frac{1}{2} + \frac{1}{3}x_W - \frac{1}{15}y$	$(-0.424 \pm 0.003) - \frac{1}{15}y$	-0.424 ± 0.026	
$\epsilon_R(d)$	$\frac{1}{3}x_W - \frac{1}{30}y$	$(0.076 \pm 0.003) - \frac{1}{30}y$	-0.017 ± 0.058	
$g_A^{e\mu}$	$\frac{1}{4} + y/60$	$\frac{1}{4} + y/60$	0.297 ± 0.031	0.57 ± 0.89
$g_A^{e\tau}$	$\frac{1}{4} + y/60$	$\frac{1}{4} + y/60$	0.214 ± 0.040	

TABLE 2 : Supersymmetry Breaking Parameters

The parameters A , A_t have the dimensions of mass. They and all the other mass parameters are expressed in units of $m_{\frac{1}{2}}$. These values were obtained with the representative inputs $\lambda_t^2(m_W) = 0.12$, $\lambda^2(m_W) = 0.078$

A	-0.67
A_t	-3.96
m_N^2	0.04
m_H^2	0.46
m_H^2	-0.15
$m_{U_{3R}}^2$	2.90
$m_{U_{3L}}^2$	3.55
$m_{D_{3L}}^2$	3.83
$m_{U_{1,2R}}^2$	3.55
$m_{U_{1,2L}}^2$	3.94
$m_{D_{1,2L}}^2$	4.12
$m_{D_{1,2,3R}}^2$	3.61
$m_{L_L}^2$	0.54
$m_{L_L^0}^2$	0.36
$m_{L_R}^2$	0.23

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