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GAUGE SYMMETRIES OF THE FREE SUPERSYMMETRIC STRING FIELD THEORIES

A. Neveu and P.C. West^{*)}

CERN - Geneva

A B S T R A C T

The gauge covariant local formulations of the free supersymmetric strings that contained a finite number of supplementary fields are extended so as to place all the generators of the Ramond-Neveu-Schwarz algebra on a more equal footing.

^{*)} Permanent address: Kings College, Mathematics Department, London WC2R 2LS, U.K.

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Despite the long history of string theory, there has not existed a systematic second quantized theory except in a given gauge^{1),2)}. Recently, however, it was shown³⁾ how one can find a gauge covariant local second quantized field theory of strings by the systematic introduction of supplementary fields. This work was extended in Ref. 4) to give gauge covariant free actions for all known string theories in terms of a finite set of supplementary fields. In Ref. 5), these formulations of the bosonic string were extended to include an infinite set of supplementary fields with a corresponding increase in the number of gauge invariances. An advantage of this latter formulation is that the generators of the Virasoro algebra appeared on a more equal footing. In this letter we find similar results for the open superstring^{6),7)} and the heterotic string⁸⁾. Results related to those in Ref. 5) can be found in Ref. 9). [Recent attempts at the problem of finding gauge invariant, not always local, field theories of strings have been made using other methods¹⁰⁾.]

Without further ado and using the conventions of Ref. 4), we begin with the open superstring. The action for the Neveu-Schwarz sector is most easily constructed by reintroducing the L_{-n} generators and it contains the string fields

$$\psi, \phi^{(2)}, \chi^{(n)}, \zeta^{(r,s)}, \zeta^{(r,n)}, \zeta^{(n,r)}, \zeta^{(n,m)};$$

$$s, r = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots; \quad m, n = 1, 2, 3, \dots \quad (1)$$

The action is given by

$$\begin{aligned} S = & \frac{1}{2}(\psi, (L_0 - \frac{1}{2})\psi) + (G_2\psi, \phi^{(2)}) + (L_n\psi, \chi^{(n)}) + (\phi^{(2)}, \phi^{(2)}) \\ & + (\phi^{(2)}, G_{-2}\zeta^{(2,2)}) + (L_{-n}\zeta^{(n,r)}, \phi^{(2)}) + (s + \frac{3}{2}n)(\phi^{(n,s)}, \zeta^{(n,s)}) \\ & + n(\chi^{(n)}, \chi^{(n)}) + (G_s\chi^{(s)}, \zeta^{(s,n)}) + (L_m\chi^{(m)}, \zeta^{(m,n)}) \\ & + (2m+n)(\chi^{(m+n)}, \zeta^{(m,n)}) + 2(\phi^{(n+s)}, \zeta^{(s,n)}) \\ & - \frac{1}{2}(3n+s)(\zeta^{(r,s)}, \chi^{(r+s)}) - \frac{1}{2}(\zeta^{(r,s)}, (L_0 + r + s - \frac{1}{2})\zeta^{(s,r)})_{(2)} \\ & - (\zeta^{(n,s)}, (L_0 + n + s - \frac{1}{2})\zeta^{(s,n)}) - \frac{1}{2}(\zeta^{(m,n)}, (L_0 + m + n - \frac{1}{2})\zeta^{(n,m)}) \end{aligned}$$

where the sums over $m, n, \dots, r, s, \dots$ are to be understood in this and the other actions given in this paper. It is invariant only for $D = 10$ under the transformations

$$\begin{aligned}
 \delta \psi &= \sum_{n=-\frac{1}{2}}^{\infty} G_n \Lambda_n + \sum_{n=1}^{\infty} L_{-n} \Omega_n, \\
 \delta \phi^{(n)} &= -(6+n-\frac{1}{2}) \Lambda_n, \quad \delta \chi^{(n)} = -(6+n-\frac{1}{2}) \Omega_n, \\
 \delta \xi^{(n,s)} &= G_s \Lambda_n - \frac{1}{2} (3s+2) \Omega_{n+s}, \\
 \delta \xi^{(n,m)} &= -L_m \Lambda_n - (n+\frac{3}{2}m) \Lambda_{n+m}, \\
 \delta \zeta^{(n,r)} &= -G_r \Omega_n - 2 \Lambda_{n+r}, \\
 \delta \xi^{(n,m)} &= -L_m \Omega_n - (2m+n) \Omega_{n+m}.
 \end{aligned} \tag{3}$$

The action for the Ramond sector involves the fields

$$\psi, \phi^{(n)}, \chi^{(n)}, \xi^{(n,m)}, \mu^{(n,m)}, \lambda^{(n,m)}, \tau^{(n,m)}$$

and takes the form

$$\begin{aligned}
 S &= \frac{1}{2} (\psi, F_0 \psi) + (F_n \psi, \phi^{(n)}) + (L_n \psi, \chi^{(n)}) \\
 &+ (\xi^{(n,m)}, F_n \phi^{(m)} - \frac{1}{2} (m+3n) \chi^{(n+m)}) \\
 &+ (\mu^{(n,m)}, F_n \chi^{(m)} + 2 \phi^{(n+m)}) \\
 &+ (\lambda^{(n,m)}, L_n \phi^{(m)} + (2n+m) \phi^{(n+m)}) \\
 &+ (\tau^{(n,m)}, L_n \chi^{(m)} + (2n+m) \chi^{(n+m)}) \\
 &- (\phi^{(n)}, F_0 \phi^{(n)}) + \frac{5}{2} n (\phi^{(n)}, \chi^{(n)}) + 2n (\chi^{(n)}, \phi^{(n)}) \\
 &- 2 (\mu^{(n,m)}, \tau^{(m,n)}) + 2 (\xi^{(m,n)}, \mu^{(n,m)}) \\
 &- \frac{1}{2} m (\xi^{(n,m)}, \lambda^{(m,n)}) - 2 (\xi^{(n,m)}, \mu^{(m,n)}) \\
 &- \frac{1}{2} m (\lambda^{(n,m)}, \tau^{(m,n)}) + \frac{1}{2} (\xi^{(m,n)}, F_0 \xi^{(n,m)}) \\
 &+ (\mu^{(n,m)}, F_0 \lambda^{(m,n)}) - (\tau^{(n,m)}, F_0 \tau^{(m,n)})
 \end{aligned} \tag{4}$$

In ten dimensions it is invariant under the Abelian gauge transformations

$$\begin{aligned}
 \delta \psi &= \sum_{n=1}^{\infty} (F_{-n} \Lambda_n + L_{-n} \Omega_n), \\
 \delta \phi^{(n)} &= F_0 \Lambda_n - \frac{1}{2} n \Omega_n, \quad \delta \chi^{(n)} = -F_0 \Omega_n - 2 \Lambda_n \\
 \delta \xi^{(n,m)} &= F_m \Lambda_n - \frac{1}{2} (n+3m) \Omega_{n+m} \\
 \delta \mu^{(n,m)} &= -L_m \Lambda_n - 2 \Lambda_{n+m}
 \end{aligned} \tag{5}$$

$$\delta \lambda^{(n,m)} = F_m \Omega_n - 2 \Lambda_{n+m}, \quad (5)$$

$$\delta \tau^{(n,m)} = L_m \Omega_n - (2m+n) \Omega_{n+m}. \quad \text{Contd.}$$

We now consider the heterotic string which is built from two sectors. The Ramond sector contains the fields

$$\begin{aligned} & \psi, \phi^{(n)}, \chi^{(n)}, \xi^{(n)}, \xi^{(n,m)}, \mu^{(n,m)}, \\ & \lambda^{(n,m)}, \tau^{(n,m)}, \bar{\lambda}^{(m,n)}, \bar{\tau}^{(m,n)} \\ & \alpha^{(m,n)}, \beta^{(m,n)} \text{ and } \gamma^{(m,n)} \end{aligned} \quad (6)$$

The fields $\phi^{(n)}$, $\chi^{(n)}$, $\xi^{(n,m)}$, $\mu^{(n,m)}$, $\lambda^{(n,m)}$ and $\tau^{(n,m)}$ are similar to those that appear in the Ramond sector of the open superstring while $\xi^{(n)}$ and $\gamma^{(m,n)}$ play an equivalent role to the fields of the D = 26 open bosonic string. The action is obtained from that of the Ramond sector of the open superstring provided one adds the terms

$$\begin{aligned} & (\xi^{(n)}, \bar{L}_n \psi) + (\bar{\lambda}^{(m,n)}, \bar{L}_m \phi^{(n)}) + (\bar{\tau}^{(m,n)}, \bar{L}_m \chi_n) \\ & + 2n (\xi^{(n)}, F_0 \xi^{(n)}) + (\alpha^{(m,n)}, F_m \xi^{(n)}) + (\beta^{(m,n)}, L_m \xi_n) \\ & + (\gamma^{(m,n)}, \bar{L}_m \xi^{(n)}) + (2m+p) (\xi^{(m+p)}, \gamma^{(m,p)}) \\ & + (\lambda^{(m,n)}, \bar{L}_m \phi^{(n)}) + (\bar{\lambda}^{(m,n)}, F_0 \alpha^{(n,m)}) \\ & - \frac{1}{2} n (\bar{\lambda}^{(m,n)}, \beta^{(n,m)}) - (\bar{\tau}^{(m,n)}, F_0 \beta^{(n,m)}) \\ & - 2 (\bar{\tau}^{(m,n)}, \alpha^{(n,m)}) + (\alpha^{(m,n)}, F_0 \bar{\lambda}^{(n,m)}) \\ & - (\gamma^{(m,n)}, F_0 \gamma^{(n,m)}). \end{aligned}$$

(7)

This action is, in ten dimensions, invariant under the following field variations

$$\begin{aligned} \delta \psi &= \sum_{n=-1}^{\infty} (F_n \Lambda_n + L_n \Omega_n + \bar{L}_n \bar{\Omega}_n), \\ \delta \xi^{(n)} &= -F_0 \bar{\Omega}_n, \quad \delta \bar{\lambda}^{(m,n)} = -F_n \bar{\Omega}_m \end{aligned} \quad (8)$$

$$\begin{aligned}
 \delta \bar{\epsilon}^{(m,n)} &= -L_n \bar{\Omega}_m, \quad \delta \alpha^{(m,n)} = -\bar{L}_n \Lambda_m \\
 \delta \beta^{(m,n)} &= -\bar{L}_n \Omega_m \\
 \delta \gamma^{(m,n)} &= -\bar{L}_n \bar{\Omega}_m - (2n+m) \bar{\Omega}_{m+n}
 \end{aligned} \tag{8}$$

Contd.

while those of $\phi^{(n)}$, $\chi^{(n)}$, $\zeta^{(n,m)}$, $\mu^{(n,m)}$, $\lambda^{(n,m)}$ and $\tau^{(n,m)}$ are as for the open Ramond model of Eq. (5).

The other sector of the heterotic string is a Neveu-Schwarz-Veneziano type sector which now contains the field ϕ as well as

$$\phi^{(2)}, \chi^{(n)}, \zeta^{(2,0)}, \zeta^{(n,0)}, \zeta^{(0,n)}, \zeta^{(m,n)},$$

and

$$\bar{\chi}^{(n)}, \bar{\lambda}^{(m,n)}$$

and also

$$\bar{\zeta}^{(n,2)}, \bar{\zeta}^{(n,m)}, \mu^{(2,n)}, \lambda^{(m,n)} \tag{9}$$

The first set of fields are those associated with the Ramond sector while $\bar{\chi}^{(n)}$ and $\bar{\lambda}^{(m,n)}$ are associated with the Veneziano sector. The action is given by taking the actions from these two sectors, i.e., Eq. (2) and Eq. (12) of Ref. 5), and rescaling terms involving two fields with only one index by $\frac{1}{2}$ and those involving fields with two indices by 2 as well as adding the terms

$$\begin{aligned}
 &(\bar{\zeta}^{(n,2)}, \bar{L}_n \phi^{(2)}) + (\bar{\zeta}^{(n,m)}, \bar{L}_n \chi_m) + (\mu^{(2,n)}, G_n \bar{\chi}_n) \\
 &+ (\lambda^{(m,n)}, L_m \bar{\chi}^{(n)}) - 2(\bar{\zeta}^{(n,2)}, (L_0 + 2 - \frac{1}{2}) \mu^{(2,n)}) \\
 &- 2(\bar{\zeta}^{(n,m)}, (L_0 + m - \frac{1}{2}) \lambda^{(m,n)})
 \end{aligned} \tag{10}$$

It is invariant under the transformations

$$\begin{aligned}
 \delta \psi &= \sum_{r=\frac{1}{2}}^{\infty} G_{-r} \Lambda_r + \sum_{n=1}^{\infty} (L_{-n} \Omega_n + \bar{L}_{-n} \bar{\Omega}_n) \\
 \delta \bar{\zeta}^{(n,2)} &= -G_n \bar{\Omega}_n, \quad \delta \bar{\zeta}^{(m,n)} = -L_n \bar{\Omega}_m \\
 \delta \mu^{(2,n)} &= -\bar{L}_n \Lambda_n, \quad \delta \lambda^{(m,n)} = -\bar{L}_n \Omega_m
 \end{aligned} \tag{11}$$

while the variations of the remaining fields are as for their appropriate sectors with the rescaling mentioned above:

$$\phi^{(n)} \rightarrow \frac{1}{2} \phi^{(n)}, \quad \chi^{(n)} \rightarrow \frac{1}{2} \chi^{(n)}, \quad \bar{\chi}^{(n)} \rightarrow \frac{1}{2} \bar{\chi}^{(n)} \quad (12)$$

One can obtain similar actions and gauge transformation laws for closed superstrings; this is left as an exercise for the reader. One can also easily verify, following the method given in Ref. 4), that it is possible to choose a gauge where all supplementary fields vanish and ψ satisfies its usual equations of motion and ghost eliminating conditions; for the Neveu-Schwarz-Ramond model, these read:

$$\begin{aligned} (L_0 - \frac{1}{2}) \psi_{NS} &= 0, \\ G_2 \psi_{NS} = L_n \psi_{NS} &= 0, \\ F_0 \psi_R &= 0, \\ F_m \psi_R = L_m \psi_R &= 0. \end{aligned} \quad (13)$$

Hence, no new physical degrees of freedom have been introduced, and we are indeed dealing with the same dynamical systems.

In a forthcoming publication¹¹⁾, we shall derive the supersymmetry transformation laws of the gauge invariant NSR model. We shall see that these are most easily expressed in the formulation of the present paper, rather than in the formulation of Ref. 4). This is because the commutation laws of the supersymmetry generator, built out of the fermion emission vertex, with the superconformal generators F_n and G_r involve infinite series in these generators¹²⁾.

In the functional integral which defines the quantum theory, gauge fixing introduces Faddeev-Popov ghosts. Insertion of naive gauge fixing conditions such as, for example, $\delta(L_1\psi)\delta(L_2\psi)\delta(L_3\psi)\dots$ in the functional integral leads to a set of Faddeev-Popov ghosts and antighosts which does not give a correct counting of the number of physical degrees of freedom as they appear in the light cone gauge. This is to be expected since the identity

$$L_3 = [L_2, L_1] \quad (14)$$

demonstrates that the gauge fixing constraints are not independent. This feature occurs at the third level for the formulation of Ref. 5), and occurs for the first time at the fifth level for the formulation of Ref. 4), because of the identity¹³⁾

$$[L_1, [L_1, [L_1, L_2]]] = -6 [L_2, [L_1, L_2]] \quad (15)$$

The correct procedure involves the addition of Nielsen-Kallosh ghosts¹⁴⁾; the systematic insertion of these ghosts goes beyond the scope of this paper, and will be dealt with in a separate publication.

Finally, the results of this paper can be put in a functional form in the same fashion as in Ref. 5).

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