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Phenomenology of High Colour Fermions

D. Lüst *

Sektion Physik, Ludwig-Maximilians-Universität
Munich, F.R.G.

E. Papantonopoulos +

CERN - Geneva

K.H. Streng

Sektion Physik, Ludwig-Maximilians-Universität
Munich, F.R.G.

and

G. Zoupanos +**

Max-Planck-Institut für Physik und Astrophysik
- Werner-Heisenberg-Institut für Physik -
Föhringer Ring 6, 8000 München 40
Federal Republic of Germany

Abstract:

We present the phenomenological consequences of a dynamical scenario for electroweak symmetry breaking and generation of fermion masses, involving the presence of fermions which transform under high colour representations. Particular emphasis is given to the predictions for rare processes and to the possible signals in present and future machines.

* Address after October 1, 1985: California Institute of Technology, Pasadena, U.S.A.

+ On leave from Physics Department, National Technical University, Athens, Greece

** Alexander von Humboldt fellow

1. Introduction

The discovery of the W and Z vector bosons has established the standard electroweak model⁽¹⁾ $SU(2)_L \times U(1)_Y$ as a fundamental ingredient of our understanding of elementary particle interactions. However, there remains always the challenge of understanding the mechanism which is responsible for the spontaneous symmetry breakdown of the theory. A number of important open problems of the standard model, such as lack of predictivity of fermion masses and mixing angles, large disparity between fermion and gauge boson masses, uncontrollable behaviour of scalar masses, if it is unified with strong interactions in any GUT scheme, suggest that the Higgs scalar sector cannot be a fundamental part of the theory but it rather serves as a parametrization of some as yet unknown, dynamical mechanism. It is natural then to speculate on alternative schemes that could overcome these problems. A popular suggestion was to substitute the elementary Higgs fields by fermion-antifermion composite fields, that could lead to dynamical generation of gauge boson and fermion masses. This suggestion led to the development of the technicolour (TC) scenario⁽²⁾, where the gauge $SU(2)_L \times U(1)_Y$ symmetry is broken dynamically by the condensates of the technifermions (new fermions that feel the technicolour force)

$$\langle \bar{T} T \rangle \sim O(\Lambda_{TC}^3) \quad (1.1)$$

with Λ_{TC} the scale of the extra strong technicolour interactions. The technipion decay constant sets the W and Z mass scales, while isospin conservation guarantees the successful relation $M_W = M_Z \cos \theta_W$. The generation of quarks and lepton masses necessitated the enlargement of the technicolour gauge group to the extended technicolour (ETC). Breaking subsequently G_{ETC} to G_{TC} the broken generators of G_{ETC}/G_{TC} generate radiative masses to the ordinary quarks and leptons. However, once the ETC gauge boson masses are fixed by fitting the observed fermion spectrum, large FCNC are induced in tree order⁽³⁾.

We shall follow another suggestion for dynamical mass generation⁽⁴⁻¹¹⁾, which is much more attractive than the technicolour scenario, since the dynamical breaking of the electroweak symmetry requires only ordinary Q.C.D.

with additional fermions in higher than triplet representations of the $SU(3)_c$ gauge group. Chiral symmetry breaking in the exotic quark sector may occur at much larger mass scales than the ordinary triplet chiral symmetry breaking and therefore could produce the right values of the W and Z boson masses. Such a dynamical scheme is supported by recent lattice Monte Carlo calculations.⁽¹²⁾

To generate ordinary fermion masses, one has to embed $SU(3)_c$ -colour group into a larger gauge group G_s and subsequently to break it down to $SU(3)_c$. Then the massive $G_s/SU(3)_c$ gauge bosons mediate transitions between ordinary fermions and fermions with high colour (echo-fermions) if they all belong to a common irreducible representation of G_s .^(5,6)

The models that can be constructed within the above framework can be divided into two classes, with the main characteristic feature whether quarks and leptons belong to separate or common representations of G_s . Models of the first class have no lepton-quark unification and have been introduced in ref. (5) assuming $G_s = SU(3) \times SU(3)$. Models of the second class, which we shall call Pati-Salam type-models, have been introduced in ref. (6) assuming $G_s = SU(4)$. In ref. (7) the possibility to construct a realistic model was investigated extensively in both classes and was concluded that the first class models are more promising.

As we shall discuss later it is very appealing that these models, contrary to E.T.C., do not suffer from flavour changing neutral currents due to tree order heavy gauge boson exchanges. The reason is that the G_s gauge bosons have only flavour diagonal couplings to the fermions exactly as the usual gluons of Q.C.D. Flavour changing neutral current processes involving the heavy gauge bosons are induced by high order diagrams but nevertheless they are safe. However the real difficulties for this scenario arise from the existence of massless Goldstone bosons in these models, a defect that they share with the technicolour type models.

In this paper we mainly want to discuss the phenomenological consequences of the introduction of high colour fermions in the context of the enlarged strong gauge groups. It is worth mentioning that high colour

fermions appear naturally in composite models, namely in models in which quark and leptons have coloured constituents.⁽¹³⁾ However many of the phenomenological consequences of the composite high colour fermions, for example radiative decays, are quite different from those of the elementary ones. Therefore our discussion in this paper does not refer to composite high colour fermions which in any case have been discussed before.⁽¹⁴⁾

The paper is organized as follows: In section 2 we recall the basic mechanism of dynamical symmetry breaking in Q.C.D. with high colour fermions. In chapter 3, we present the general mechanisms of dynamical fermion mass generation and family mixing. Then in the 4th chapter, we discuss the properties of the new hadronic bound states composed out of high colour fermions. In chapter 5, we investigate the low energy phenomenology of high colour fermions. Finally in section 6, we discuss the production cross-sections of the high colour fermions and their bound states, as well as their decay signatures which could be observed in present and future accelerator experiments.

2. Dynamical Symmetry Breaking from Fermions in High $SU(3)_C$ -Representations

Let us recall the basis of the conjecture that ordinary Q.C.D. with new quarks Q_r in representations $r > 3$ can produce the dynamical symmetry breakdown of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$. For N_r massless fermions in some representation $r > 3$, the Q.C.D. Lagrangian processes in each fermion sector r a separate global chiral symmetry (neglecting the effects of the axial anomaly):

$$G_r = U(N_r)_L \times U(N_r)_R \quad (2.1)$$

At some value of the effective strength of the colour forces, it is assumed that condensates of the high colour fermions are formed:

$$\langle \bar{Q}_r Q_r \rangle = \mu_r^3 \quad (2.2)$$

These condensates break the global symmetry G_r to the vector symmetry $U(N_r)_{L+R}$. Therefore N_r^2 Goldstone bosons, which are high colour fermion boundstates

arise with decay constant F_{π_r} . In analogy to the decay constant of the ordinary pion we set:

$$\mu_r \approx 2.5 F_{\pi_r} \quad (2.3)$$

The high colour fermions obtain at the same time a dynamical mass of order μ_r which reflects itself in a momentum dependent self-energy of the high colour fermion propagator⁽¹⁵⁾:

$$D_r(p) = \frac{1}{p - \Sigma_r(p)} \quad ; \quad \Sigma_r(p) \approx \frac{3 C_2(r) \mu_r^3}{p^2} \quad (p^2 \gg \mu_r^2) \quad (2.4)$$

where $C_2(r)$ is the quadratic Casimir invariant of the representation r .

Switching on the $SU(2)_L \times U(1)_Y$ gauge symmetry the $W^\pm$ and Z bosons acquire masses absorbing three of the Goldstone bosons

$$\begin{aligned} M_{W^\pm} &= \frac{1}{2} g_2 \sum_i F_{\pi_{r_i}^\pm} \\ M_{Z^0} &= \frac{1}{4} (g_1^2 + g_2^2) \sum_i F_{\pi_{r_i}^0} \end{aligned} \quad (2.5)$$

where g_2, g_1 are the coupling constants of $SU(2)$ and $U(1)$. Since isospin is conserved the $F_{\pi_{r_i}^\pm}$ for every r are the same. Therefore the successful mass relation $M_W = M_Z \cos \theta_W$ is preserved. To reproduce the known values of the weak gauge boson masses one has to require

$$\sum_i F_{\pi_{r_i}} = 250 \text{ GeV} \iff \sum_i \mu_{r_i} \approx 625 \text{ GeV} \quad (2.6)$$

In order to obtain such large scales Marciano⁽⁴⁾ assumed, that chiral symmetry breaking depends entirely on the strength of the effective quark-antiquark binding potential, based on the one gluon approximation. Effective potential calculations⁽¹⁶⁾ of chiral symmetry breaking yield that chiral symmetry breaking is induced for

$$C_2(r) \alpha_s(\mu_r) \gtrsim \text{const.} \approx \frac{\pi}{3} \quad (2.7)$$

Also non-perturbative analytic methods support the validity of eq. (2.7).⁽¹⁷⁾

Next one can integrate the renormalization group equation and finds, neglecting the contribution of the high colour fermions to the β -function, the so-called exponential "Casimir scaling":

$$\frac{\mu_r}{\mu_3} \approx \exp \left[\frac{2\pi}{7\alpha_s(\mu_3)} \left(\frac{C_2(r)}{C_2(3)} - 1 \right) \right] \quad (2.8)$$

Then, it can be easily seen, that with $r = 6$ scales $\mu_r \approx 0$ (100 GeV) can be reached. However one also realizes that the predicted scales become very large and totally unrealistic if the simple Casimir scaling really works (see table I of ref. (7)). Therefore we have assumed⁽⁷⁾ that the Casimir scaling does not take place with full strength, but there exists some screening of the Casimir scaling. We have parametrized this effect associating with every representation r a screening factor $f(r)$. Then eq. (2.7) is replaced by:

$$f^2(r) \cdot C_2(r) \cdot \alpha_s(\mu_r) \geq \text{const.} \quad (2.9)$$

These screening factors are estimated by fitting the fermion mass spectrum and mixing angles. They are in a way the predictions of our models which could, in principle, be tested in lattice calculations. The introduction of the screening factor is somehow justified by considering other possible sources of chiral symmetry breaking. For example instantons as source of chiral symmetry breaking lead instead of eq. (2.7) to the other extreme⁽¹⁸⁾

$$\alpha_s(\mu_r) \geq \text{const.} \quad (2.10)$$

This means that the condensation for every fermionic representation takes place at an universal scale μ . Therefore one can regard eq. (2.9) as a compromise between eqs. (2.7) and (2.10). It is very interesting that a gradually accumulating evidence from Monte Carlo studies of the chiral symmetry breaking in SU(2) and SU(3) lattice Q.C.D. supports the results of Casimir scaling.⁽¹²⁾ In the lattice calculations one finds evidence even for the screening we have speculated about.

We close this chapter with a remark on the asymptotic freedom problem in Q.C.D. with high colour representations. For simplicity, let us consider an example with one fermion Q_r in high representation r whose contribution to the β -function spoils asymptotic freedom. Let us assume that the fermion condenses at a scale μ_r and obtains a dynamical mass of the same order. Then the energy, needed for pair production is $2\mu_r$ and this is also the threshold where r -fermions start to contribute to the β -function. Therefore asymptotic freedom is lost at energies larger than $2\mu_r$, a fact that strictly speaking makes the theory inconsistent. The reason is that then α_s is monotonically increasing with μ and it is not justified to use eq. (2.7) to determine the right $\alpha_s(\mu_r)$ for condensation to take place. Remember in addition that in the derivation of eq. (2.8) we have neglected the contribution of high colour fermions in the β -function. Thus, in a consistent model one needs a cut-off Λ ($\Lambda > 2\mu_r$) where the theory changes and $SU(3)_c$ becomes again asymptotically free. Moreover, in order to obtain hierarchical scales $\mu_r \gg \mu_3$ according to eq. (2.8) in this simple picture, one must ensure that $\alpha_s(\Lambda) < \alpha_s(\mu_r)$ (see also ref. (19)). The behaviour of $\alpha_s(\mu)$ is summarized in figure 1. In a concrete scenario, one could interpret Λ as the scale where $SU(3)_c$ is embedded into G_s or where quarks, leptons and echo-fermions become composite out of preons. We conclude that self-consistency of our picture requires

$$2\mu_{r_i} \geq M_{r_{max}} \quad (2.11)$$

for every r_i that destroys asymptotic freedom.

3. Dynamical Fermion Mass Generation

The basic mechanism for fermion mass generation, as we have already briefly outlined, consists of the breaking of a strong gauge group G_s in one or more steps to a $SU(3)_c$ subgroup under which irreducible representations of G_s decompose to colour-singlets, colour-triplets and high-colour representations of $SU(3)_c$. Then, the massive gauge bosons V_μ of $G_s/SU(3)_c$ give rise to radiative diagrams like the ones shown in Fig. 2. Using the propagators of the high colour fermions Q_r (eq. (2.4)) we can calculate these diagrams^(6,7) and obtain

$$m_f \approx \left(\frac{3\alpha_s}{4\pi} \right)^{l(2)} \frac{3 C_2(\tau) \mu_r^3}{M_V^2} \ln \left(\frac{M_V^2}{(3 C_2(\tau))^{2/3} \mu_r^2} \right) \quad (3.1)$$

where $l(2)$ refers to the order of the loop and M_V is the heavy gauge boson mass. We shall discuss with some detail the fermion mass generation mechanism in the Pati-Salam type-models and in the non-unification $SU(3)^N$ -type models.

3.1 $[SU(3)]^N$ -class models

One can realize quite easily models of the class 1 by enlarging $SU(3)_C$ to $G_S = [SU(3)]^N$. Then G_S can be broken down to $SU(3)_C$ dynamically by the introduction of a new hypercolour force as it was described in ref. (9). Another possibility for the breaking can arise from Higgs fields which have geometrical origin in a dimensional reduction scheme.⁽⁸⁾ These breaking mechanisms reduce the gauge symmetry of G_S to an unbroken diagonal $SU(3)$ gauge group which we identify with the colour group. The rest $n-1$ octets of vector bosons V_{8i} ($i = 1 \dots n-1$) receive masses which are ordered $M_{V8i} > M_{V8i+1}$. They connect ordinary quarks and leptons with the corresponding echos, but they never couple identical representations to each other. Semirealistic models for two quark and lepton families based on $G_S = SU(3) \times SU(3)^{(7)}$ and $G_S = SU(3) \times SU(3) \times SU(3)^{(9)}$ have been proposed by three of the authors.

To avoid tree order FCNC, each quark or lepton family must be assigned to different G_S -representations, otherwise horizontal-type couplings will induce tree order flavour changing processes. Taking into account this requirement the general structure of transformation properties of different quark and lepton families under $SU(3)_C \times SU(2)_L \times U(1)_Y$ looks as follows (H denotes the G_S representations which decompose into several $SU(3)_C$ -representations):

$$\begin{aligned} \begin{pmatrix} H^u \\ H^d \end{pmatrix}_L &= \left[(3 + \tau_1^{u,d}), 2, \frac{1}{3} \right] \\ H_R^u &= \left[(3 + \tau_1^u), 1, \frac{4}{3} \right] \\ H_R^d &= \left[(3 + \tau_1^d), 1, -\frac{2}{3} \right] \end{aligned} \quad (3.2)$$

$$\begin{pmatrix} H^c \\ H^s \end{pmatrix}_L = \left[(3 + \tau_1^{c,s} + \tau_2^{c,s}), 2, \frac{1}{3} \right]$$

$$H_R^c = \left[(3 + \tau_1^c + \tau_2^c), 1, \frac{4}{3} \right]$$

$$H_R^s = \left[(3 + \tau_1^s), 1, -\frac{2}{3} \right]$$

$$\begin{pmatrix} H^t \\ H^b \end{pmatrix}_L = \left[(3 + \tau_1^{t,b} + \tau_2^{t,b} + \tau_3^{t,b}), 2, \frac{1}{3} \right]$$

$$H_R^t = \left[(3 + \tau_1^t + \tau_2^t + \tau_3^t), 1, \frac{4}{3} \right]$$

$$H_R^b = \left[(3 + \tau_1^b + \tau_2^b), 1, -\frac{2}{3} \right]$$

(3.2)

$$\begin{pmatrix} H^{\nu_e} \\ H^e \end{pmatrix}_L = \left[(1 + \tilde{\tau}_1^{\nu_e, e}), 2, 1 \right]$$

$$H_R^e = \left[(1 + \tilde{\tau}_1^e), 1, -2 \right]$$

$$\begin{pmatrix} H^{\nu_\mu} \\ H^\mu \end{pmatrix}_L = \left[(1 + \tilde{\tau}_1^{\nu_\mu, \mu} + \tilde{\tau}_2^{\nu_\mu, \mu}), 2, 1 \right]$$

$$H_R^\mu = \left[(1 + \tilde{\tau}_1^\mu + \tilde{\tau}_2^\mu), 1, -2 \right]$$

$$\begin{pmatrix} H^{\nu_\tau} \\ H^\tau \end{pmatrix}_L = \left[(1 + \tilde{\tau}_1^{\nu_\tau, \tau} + \tilde{\tau}_2^{\nu_\tau, \tau} + \tilde{\tau}_3^{\nu_\tau, \tau}), 2, 1 \right]$$

$$H_R^\tau = \left[(1 + \tilde{\tau}_1^\tau + \tilde{\tau}_2^\tau + \tilde{\tau}_3^\tau), 1, -2 \right]$$

where r_i denote complex and $\tilde{r}_i$ real $SU(3)_c$ -representations. In addition, one has to add the following multiplets with only high colour representations which are necessary in order to cancel the strong anomalies (for a detailed discussion see ref. (9)) and to conserve weak isospin:

$$H_{1R} = \left(\tau_2, 1, -\frac{2}{3} \right), \quad H_{2R} = \left(\tau_3, 1, -\frac{2}{3} \right) \quad (3.3)$$

To obtain the correct W^+ and Z^0 masses all μ_r should lie in the following range

$$50 \text{ GeV} \leq \mu_{r_i} \leq 200 \text{ GeV}$$

More specifically one can choose

$$\mu_{r_1} \sim \mu_{\tilde{r}} \sim 70 \text{ GeV}, \quad \mu_{r_2} \sim 100 \text{ GeV}, \quad \mu_{r_3} \sim 140 \text{ GeV} \quad (3.4)$$

in agreement with eqs. (2.6) and (2.11).

The mass splitting between different families, respectively between quarks and leptons of the same family or between different flavours of the same family can originate from two possible sources: (1) different high colour condensates are responsible for the radiative mass of different fermions. Looking for example to eq. (3.2) one sees that the $\begin{pmatrix} t \\ b \end{pmatrix}$ -system obtains the dominant part of its mass from r_3 , while the $\begin{pmatrix} c \\ s \end{pmatrix}$ -system obtains its mass mainly from r_2 . The t, b -splitting arises, because the right handed t -quark is accompanied by different high colour representations than the b -quark. (2) Different octets of heavy vector bosons with independent masses are involved in the diagrams which provide masses to different fermions.

The mixing between different quark families arises, because some echo-quarks of one family condense with equal strength with echo-quarks of the same family but also with the echo-quark of another family. Therefore there exist condensates which conserve flavour numbers, such as $\langle \bar{r}_1^{u,d} r_1^{u,d} \rangle$, $\langle \bar{r}_2^{c,s} r_2^{c,s} \rangle$ etc. as well as condensates which break it spontaneously such as $\langle \bar{r}_1^d r_1^s \rangle$, $\langle \bar{r}_2^s r_2^b \rangle$. Thus, via the radiative mass generation mechanism, the condensates produce diagonal and also off diagonal elements in the

fermion mass matrix which lead to flavour mixing. It is interesting to note that in the leptonic sector such a mixing does not occur. The reason is that leptons are accompanied by echo-leptons which belong to real representations of $SU(3)_C$ and therefore can only condense flavour diagonally because of Pauli statistics. This fact will become important later, when we shall discuss flavour changing processes involving leptons.

According to the above discussion the quark mass matrices have the following form:

$$M_u \simeq \frac{g \alpha_s}{4 \pi} \begin{pmatrix} \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_3}^3}{M_{V_8}^2} \\ \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_2) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_2) \mu_{\tau_2}^3}{M_{V_8}^2} \\ \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_2) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_3) \mu_{\tau_3}^3}{M_{V_8}^2} \end{pmatrix} \quad (3.5a)$$

$$M_d \simeq \frac{g \alpha_s}{4 \pi} \begin{pmatrix} \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_3}^3}{M_{V_8}^2} \\ \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_1) \mu_{\tau_3}^3}{M_{V_8}^2} \\ \frac{C_2(\tau_1) \mu_{\tau_1}^3}{M_{V_8}^2} & \frac{C_2(\tau_2) \mu_{\tau_2}^3}{M_{V_8}^2} & \frac{C_2(\tau_2) \mu_{\tau_2}^3}{M_{V_8}^2} \end{pmatrix} \quad (3.5b)$$

(The M_{V_8} 's must not be necessarily identical.)

It is worth noting that by diagonalizing the ordinary quark mass matrix it is in general not guaranteed that also the mass matrix of echo-quarks

is diagonalized. Since ordinary and echo-quarks are in common G_S representations a rotation of the ordinary quarks also rotates the corresponding echo-quarks simultaneously. However, since various echo-quarks contribute to the ordinary quark mass matrix the diagonalization of the latter in general does not diagonalize the various r-echo-quark mass matrices.

To have a specific example for the subsequent phenomenological discussion we introduce the following model which describes two quark and lepton families. It is based on the gauge group $G_S = SU(3) \times SU(3) \times SU(3)$ with the following multiplet assignment under $G_S \times SU(2)_L \times U(1)_Y$ and $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned}
 \begin{pmatrix} H^u \\ H^d \end{pmatrix}_L &= [(\bar{3}, \bar{3}, 1), 2, \frac{1}{3}] = [(3 + \bar{6}), 2, \frac{1}{3}] \\
 H^u_R &= [(\bar{3}, \bar{3}, 1), 1, \frac{4}{3}] = [(3 + \bar{6}), 1, \frac{4}{3}] \\
 H^d_R &= [(\bar{3}, \bar{3}, 1), 1, -\frac{2}{3}] = [(3 + \bar{6}), 1, -\frac{2}{3}] \\
 \\
 \begin{pmatrix} H^c \\ H^s \end{pmatrix}_L &= [(1, 8, 3), 2, \frac{1}{3}] = [(3 + \bar{6} + 15), 2, \frac{1}{3}] \\
 H^c_R &= [(1, 8, 3), 1, \frac{4}{3}] = [(3 + \bar{6} + 15), 1, \frac{4}{3}] \\
 H^s_R &= [(1, \bar{3}, \bar{3}), 1, -\frac{2}{3}] = [(3 + \bar{6}), 1, -\frac{2}{3}] \\
 \\
 \begin{pmatrix} H^{\nu e} \\ H^e \end{pmatrix}_L &= [(3, \bar{3}, 1), 2, 1] = [(1 + 8), 2, 1] \\
 H^e_R &= [(3, \bar{3}, 1), 1, -2] = [(1 + 8), 1, -2]
 \end{aligned} \tag{3.6}$$

$$\begin{pmatrix} H^{\nu_r} \\ H^r \end{pmatrix}_L = [(\bar{3}, 1, 3), 2, 1] = [(1+8), 2, 1]$$

$$H^r_R = [(\bar{3}, 1, 3), 1, -2] = [(1+8), 1, -2]$$

According to the previous discussion the above assignement leads to the following quark mass matrices and lepton masses:

$$M_u = \begin{pmatrix} K_1(6) & K_2(6) \\ K_1(6) & K_2(15) \end{pmatrix}, \quad M_d = \begin{pmatrix} K_1(6) & K_2(6) \\ K_1(6) & K_2(6) \end{pmatrix}$$

$$m_e = K_1(8), \quad m_\mu = K_2(8), \quad m_{\nu_e} = m_{\nu_\mu} = 0$$

$$K_{1(2)}(\tau) = \frac{g \alpha_s C_2(\tau) \mu_\tau^2}{4 \pi M_{\nu_{8,1(2)}}^2} \ln \left(\frac{M_{\nu_{8,1(2)}}^2}{(3 C_2(\tau))^{2/3} \mu_\tau^2} \right) \quad (3.7)$$

with

$$\begin{aligned} \alpha_s &= 0.1, \quad \mu_6 = 77 \text{ GeV}, \quad \mu_{15} = 150 \text{ GeV}, \\ \mu_8 &= 66 \text{ GeV}, \quad M_{\nu_{8,1}} = 37 \text{ TeV}, \quad M_{\nu_{8,2}} = 1.7 \text{ TeV} \end{aligned} \quad (3.8)$$

one obtains

$$m_u \sim m_d \sim 1 \text{ MeV}, \quad m_c \sim 1.3 \text{ GeV}, \quad m_s \sim 175 \text{ MeV},$$

$$\tan \theta_c \sim \frac{m_d}{m_s}, \quad m_e \sim 0.5 \text{ MeV}, \quad m_\mu \sim 105 \text{ MeV} \quad (3.9)$$

3.2 Pati-Salam type models

In these models quarks and leptons are unified in one G_S -representation. Their structure has been discussed in refs. (6,7). In general the family hierarchy and flavour mixing arises by the same mechanism explained above. The only difference between those two schemes lies in the fact that the same heavy gauge boson that connects quarks with echo-quarks now connects also leptons and quarks. Therefore the leptons obtain masses from the same high colour representations as the quarks, but their radiative diagrams always involve the exchange of one more heavy gauge boson which then leads to the relation

$$m_\ell \sim \frac{\alpha_s}{4\pi} m_q \quad (3.10)$$

The simplest of these models is based on the enlarged strong gauge group $G_S = SU(4)$ with a 10-plet of fermions. The $SU(4)_S$ is assumed to break spontaneously to $SU(3)_C$ and the 10-plet decomposes as $10 = 1 + 3 + 6$. The 15 gauge bosons on $SU(4)_S$ decompose under $SU(3)_C$ as $15 = 1 + 3 + \bar{3} + 8$. The 8-plet has to be identified with the usual gluons, while the heavy triplet V_3 mediates transitions between the singlet fermions (i.e. leptons) and the triplet quark and also between the triplet and the echo-sextets. Singlets and sextets do not couple directly.

4. High Colour Bound States

In this chapter we shall discuss some properties of the $SU(3)_C$ -singlet bound states involving high colour quarks (see also ref. (20)).

4.1 High Colour Baryons/Mesons

In the following we consider bound states which contain different numbers of echo-quarks and echo antiquarks. We want to restrict our discussion to the lightest of them, which contain only one high colour fermion Q_T and several ordinary light quarks:

$$B_{\tau} \sim Q_{\tau} (q \dots q) (\bar{q} \dots \bar{q}) \quad (4.1)$$

Depending on the number of the light quarks, which build up a colour singlet, the resulting hadrons can be fermions or bosons. For example $6 \times \bar{3} \times \bar{3}$ form a singlet which is a fermion (baryon), while $15 \times 3 \times \bar{3} \times \bar{3}$ form a singlet, which is a boson (meson). We estimate the masses of the high colour baryons/mesons in analogy to the mass of ordinary baryons i.e. by adding the constituent masses of the echo and ordinary quarks that build them up, which is roughly speaking μ_{τ} . In fact just scaling the relation between the proton mass, constituent quark masses and μ , (as it is obtained from Q.C.D.) one finds, using eq. (3.4), the following typical masses for the lightest echo-baryons/mesons:

$$\begin{aligned} M_{B_{\tau_1}} &\approx 100 \text{ GeV}, \quad M_{B_{\tau_2}} \approx 150 \text{ GeV}, \\ M_{B_{\tau_3}} &\approx 200 \text{ GeV}, \quad M_{B_{\tau_4}} \approx 100 \text{ GeV} \end{aligned} \quad (4.2)$$

Of course the high colour baryons/mesons built up from the same echo but different ordinary quarks are almost degenerate in mass. The high colour baryons/mesons containing the heavy echo-quarks Q_{τ_2} and Q_{τ_3} will decay into B_{τ_1} via the exchange of a heavy gauge boson V_8 with the following rate:

$$\Gamma (Q_{\tau_{2,3}} \rightarrow Q_{\tau_1} q q) = \frac{\alpha_s^2 M_{Q_{\tau_{2,3}}}^5}{348 \pi M_{V_8}^4} \quad (4.3)$$

Now the question is if B_{τ_1} , B_{τ_4} are stable or they can decay finally into ordinary quark matter. The answer depends crucially on the structure of the considered model. In the first option where quarks and leptons belong to different representations of $G_S = [SU(3)]^n$ the heavy vector bosons V_8 couple only ordinary quarks to echo-quarks and respectively ordinary leptons to echo-leptons but they never couple two ordinary fermions. In addition there does not exist a trilinear self coupling between these vector bosons. Therefore in this scheme a high colour fermion can never decay into a light quark or lepton which means that the baryons/mesons are absolutely stable. However the situation is totally different in the Pati-Salam type models, where the heavy gauge boson V , couples also quarks and leptons. In this

scheme the echo-quarks can decay into two quarks and one antilepton (see fig. 3). This leads to the following baryon/meson decay rate:

$$\Gamma(Q_{r,2,3} \rightarrow qq\bar{\ell}) \approx \frac{\alpha_s^2 M_{Q_{r,2,3}}^5}{348\pi M_{V_3}^4} \approx (0.1, 0.8, 4) \times 10^{-4} \text{ eV} \quad (4.4)$$

with $m_{V_3} \approx 50 \text{ TeV}$.

It is worth to comment, that in composite models these considerations do not longer hold because there, the echo-quarks can decay rapidly into ordinary fermions via magnetic transitions by emitting one gluon.

Finally we would like to mention that in our type of models and also in the sub-quark models there cannot exist high colour baryons/mesons with fractional charge. The reason is the following. Triality tells us that there is no fractionally charged colour singlet bound states if the $SU(3)$ representation which belongs to the triality class zero have electric charge $Q = 1 + n$ (n -integer). Also representations, which belongs to triality class 1, must have charge $Q = \frac{2}{3} + n$ and every representation which belongs to the triality class 2 must have charge $Q = \frac{1}{3} + n$. This is of course satisfied by the ordinary quarks and leptons. But it is also true for the models based on the strong gauge group $G_s = [SU(3)]^n$. Here every G_s -multiplet consists particles with definite electric charge which belong to $SU(3)_c$ -representations of the same triality class. Therefore, the high colour fermions fall automatically into the same triality classes as their corresponding ordinary quark or lepton partners. In the Pati-Salam type models the situation is a little bit more subtle. Here, the electric charge is a linear combination of the 15th generator of $SU(4)$ and the generator of $U(1)_Y$, (see ref. (7)). The coefficients have to be chosen in such a way that quark and lepton charge take their correct values. But doing so the charges of the accompanying echo-quarks are also fixed, in a way that the charges of the echo-quarks fall into the correct triality class. Therefore in both schemes there are no fractionally charged bound states.

4.2 High Colour $\mathcal{P}$ -Mesons

The next kind of high colour bound states are the vector mesons (high

colour ρ -mesons) which contain an echo-quark-antiquark pair: $\rho_r \sim Q_r \bar{Q}_r$. Again we estimate its mass in analogy to the mass of the ordinary ρ -meson:

$$m_{\rho_r} \simeq \frac{m_\rho}{f_\pi} \cdot F_{\pi_r} \simeq 3.3 \mu_r \quad (4.5)$$

Using eq. (3.4) their masses are about

$$m_{\rho_{r_1}} \simeq 230 \text{ GeV}, \quad m_{\rho_{r_2}} \simeq 330 \text{ GeV}, \quad m_{\rho_{r_3}} \simeq 460 \text{ GeV} \quad (4.6)$$

The high colour ρ -mesons decay mainly into the light high colour (pseudo) Goldstone bosons which we shall discuss in the next chapter. From comparison with the strong decay of the ordinary ρ 's one obtains for the decay rate of the high colour ρ :

$$\begin{aligned} \Gamma(\rho_r \rightarrow \pi_r \pi_r) &\simeq \Gamma(\rho \rightarrow \pi \pi) \left(\frac{m_{\rho_r}}{m_\rho} \right) \left(1 - 4 \frac{m_{\pi_r}^2}{m_{\rho_r}^2} \right)^{-3/2} \\ &\simeq (14, 20, 28) \text{ GeV} \end{aligned} \quad (4.7)$$

4.3 High Colour (Pseudo) Goldstone Bosons-Pions

In our discussion on high colour boundstates we finally come to the most important particles for the low energy phenomenology namely the high colour pions. Their importance lies on the fact that their mass could be already in the energy range of present accelerators. As we shall see, some of them can be even totally massless, a fact which has unpleasant consequences for rare decays.

Let us first look to the mass spectrum of the high colour pions. To do so it is necessary to determine the global symmetries of the model. To be specific we take $G_g = [SU(3)]^n$ with the multiplet structure given in eq. (3.2). The global symmetry G of Q.C.D. in the echo-quark sector (neglecting the effect of the $SU(3)_c$ anomaly) is given by:

$$G = U(6)_L^{\tau_1^{u,d,c,s,t,b}} \times U(6)_R^{\tau_1^{u,d,c,s,t,b}} \times U(4)_L^{\tau_2^{c,s,t,b}} \times U(4)_R^{\tau_2^{c,s,t,b}} \times U(2)_L^{\tau_3^{t,b}} \times U(2)_R^{\tau_3^{t,b}} \quad (4.8)$$

The condensates $\langle \bar{r}_1 r_1 \rangle$, $\langle \bar{r}_2 r_2 \rangle$ and $\langle \bar{r}_3 r_3 \rangle$ break G spontaneously to H :

$$H = U(6)_{L+R}^{\tau_1^{u,d,c,s,t,b}} \times U(4)_{L+R}^{\tau_2^{c,s,t,b}} \times U(2)_{L+R}^{\tau_3^{t,b}} \quad (4.9)$$

This leads to 56 massless Goldstone bosons π_r composed out of an echo-quark-antiquark pair. Three of them (mainly π_{r_3}) are eaten by W 's and Z bosons. Of course not all of the remaining stay massless.

Part of G is explicitly broken by the massive $G_S/SU(3)_C$ generators. A better way to understand this explicit breaking of G is that some of the echo-quarks receive radiative masses from the condensations of higher colour echo-quarks which are their partners in the same representation of G_S . Looking to the specific multiplet structure of eq. (3.2) one sees that the following equo-quarks become massive: r_1^c with a current quark mass of the order of charm quark mass ($m(r_1^c) \sim 1$ GeV), r_1^t , r_2^t with a current mass of the order of the top quark mass ($m(r_1^t, r_2^t) \sim 40$ GeV) and r_1^b with a mass of the order of the b-quark mass ($m(r_1^b) \sim 5$ GeV). The echo-quarks $r_1^{u,d,s}$, $r_2^{c,s,b}$, $r_3^{c,b}$ stay massless. So G is explicitly broken down to G' :

$$G' = U(3)_L^{\tau_1^{u,d,s}} \times U(3)_R^{\tau_1^{u,d,s}} \times U(3)_L^{\tau_2^{c,s,b}} \times U(3)_R^{\tau_2^{c,s,b}} \times U(2)_L^{\tau_3^{t,b}} \times U(2)_R^{\tau_3^{t,b}} \quad (4.10)$$

Next the spontaneous symmetry breaking of G' produces 22 massless Goldstone bosons. The mass of the remaining pseudo Goldstone bosons can be estimated by using PCAC:

$$m_{\pi_r}^2 \cdot F_{\pi_r}^2 = 2 m(r) \mu_r^3 \quad (4.11)$$

This leads to

$$m_{\pi_\tau}^2 \approx 12 m(\tau) \cdot \mu_\tau \quad (4.12)$$

and using eq. (3.4) we obtain

$$m_{\pi_{\tau_1}} \approx (30, 65, 180) \text{ GeV} \quad (4.13)$$

depending if it contains r_1^c , r_1^b or r_1^t and similarly

$$m_{\pi_{\tau_2}} \approx 220 \text{ GeV}$$

if it contains r_2^t .

Among the 22 massless Goldstone bosons there are 10 charged particles. Therefore they obtain masses from electromagnetic interactions (the massive pseudo Goldstone bosons obtain additional mass from electromagnetic interactions) since G is broken explicitly to G'' by electromagnetic effects:

$$\begin{aligned} G'' = & U(3)_L^{\tau_1^{u,c,t}} \times U(3)_R^{\tau_1^{u,c,t}} \times U(3)_L^{\tau_1^{d,s,b}} \times U(3)_R^{\tau_1^{d,s,b}} \\ & \times U(2)_L^{\tau_2^{c,t}} \times U(2)_R^{\tau_2^{c,t}} \times U(2)_L^{\tau_2^{s,b}} \times U(2)_R^{\tau_2^{s,b}} \\ & \times U(1)_L^{\tau_3^t} \times U(1)_R^{\tau_3^t} \times U(1)_L^{\tau_3^b} \times U(1)_R^{\tau_3^b} \end{aligned} \quad (4.14)$$

The electromagnetic mass shift can be estimated in analogy to the $\pi^\pm$ - mass

$$m_{\pi_\tau^\pm}^2 = \frac{3 \ln 2}{2\pi} \alpha \cdot m_{\rho_\tau}^2 \quad (4.15)$$

Therefore

$$m_{\pi_{\tau_{1,2,3}}^\pm} \approx (11, 16, 22) \text{ GeV} \quad (4.16)$$

So eventually one is left with 12 neutral, true Goldstone bosons, from which one linear combination is eaten by the Z^0 boson. They are explicitly

$$\begin{aligned}
 \pi_{\tau_1} &\sim \bar{\tau}_1^d \tau_1^s, \quad \pi_{\tau_1} \sim \bar{\tau}_1^s \tau_1^d, \quad \pi_{\tau_1} \sim \bar{\tau}_1^d \tau_1^d, \quad \pi_{\tau_1} \sim \bar{\tau}_1^s \tau_1^s, \\
 \pi_{\tau_1} &\sim \bar{\tau}_1^u \tau_1^u, \quad \pi_{\tau_2} \sim \bar{\tau}_2^c \tau_2^c, \quad \pi_{\tau_2} \sim \bar{\tau}_2^s \tau_2^s, \quad \pi_{\tau_2} \sim \bar{\tau}_2^b \tau_2^b, \\
 \pi_{\tau_2} &\sim \bar{\tau}_2^b \tau_2^s, \quad \pi_{\tau_2} \sim \bar{\tau}_2^s \tau_2^b, \quad \pi_{\tau_3} \sim \bar{\tau}_3^t \tau_3^t, \quad \pi_{\tau_3} \sim \bar{\tau}_3^b \tau_3^b.
 \end{aligned} \tag{4.17}$$

Similarly the (pseudo) Goldstone bosons can be determined in the leptonic sector. In eq. (3.2) every left-handed lepton doublet was accompanied by echo-leptons $\tilde{\tau}_i$. In addition there are also $\tilde{\tau}_i$ accompanying the right handed charged leptons. Because $\tilde{\tau}_i$ is a real representation one simply counts the number of $\tilde{\tau}$'s in order to find the global symmetry of this sector. This is:

$$G = SU(9)^{\tilde{\tau}_1} \times SU(6)^{\tilde{\tau}_2} \times SU(3)^{\tilde{\tau}_3} \tag{4.18}$$

The condensates break G to H:

$$H = SO(9)^{\tilde{\tau}_1} \times SO(6)^{\tilde{\tau}_2} \times SO(3)^{\tilde{\tau}_3} \tag{4.19}$$

The spontaneous breaking produces 69 Goldstone bosons. Again the broken generators of $G/SU(3)$ and the electromagnetic interactions generate masses for some of them. At this stage it is useful to note that there exist no pions composed out of one echo-quark and one echo-lepton.

Finally, let us discuss the decay properties of the massive pseudo Goldstone bosons. They are very similar to the decays of the pseudo Goldstone bosons in the E.T.C. models.⁽²¹⁾ The decay of a neutral pseudo Goldstone boson π_r^0 into a pair of gauge bosons B_1, B_2 arises from the triangle (anomaly) graph in analogy to the decay of $\pi^0 \rightarrow 2\gamma$. The amplitude for $\pi_r^0 B_1 B_2$ coupling is:

$$A_{\pi_r^0 B_1 B_2} = \frac{S_{\pi_r^0 B_1 B_2}}{8\pi^2 f_2 F_{\pi_r}} \varepsilon_{\mu\nu\lambda\rho} \varepsilon_1^\mu \varepsilon_2^\nu p_1^\lambda p_2^\rho \tag{4.20}$$

where $S_{\pi_r^0 B_1 B_2}$ is the triangle anomaly factor, which is equal to $-e^2 \dim(r)$ for the $\pi_r^0 \gamma\gamma$ coupling and $g_s^2 1(r)$ for the π_r^0 coupling to two gluons. ($\dim(r)$ is the dimension and $1(r)$ is the index of the r high colour representation. We normalize the index to 1 ($3 = 1$.)

Therefore the dominant decay of π_r^0 is into two gluons:

$$\Gamma(\pi_r^0 \rightarrow gg) = \frac{\alpha_s^2 \ell^2(\sigma) m_{\pi_r}^3}{16 \pi^3 F_{\pi_r}^2} \quad (4.21)$$

For $m_{\pi_{r1}} = (30, 65, 180)$ GeV we obtain

$$\Gamma(\pi_{r1}^0 \rightarrow gg) \approx (8, 80, 1600) \text{ MeV} \quad (r_1 = 6) \quad (4.22)$$

and for $m_{\pi_{r2}} \sim 220$ GeV

$$\Gamma(\pi_{r2}^0 \rightarrow gg) \approx 3.4 \text{ GeV} \quad (r_2 = 15) \quad (4.23)$$

The rate into two photons is smaller by a factor $10^{-3} - 10^{-4}$.

Comparable or even bigger are the decay rates of π_r into fermion-antifermion pair. The coupling to fermions is Higgs like:

$$g_{\pi_r f f'} \approx \frac{m_f}{F_{\pi_r}} \quad (4.24)$$

Then the decay rate is:

$$\begin{aligned} \Gamma(\pi_r \rightarrow f f') &= (3) g_{\pi_r f f'}^2 \cdot m_{\pi_r} = \\ &= (3) 0.1 \frac{m_f^2}{F_{\pi_r}^2} m_{\pi_r} \end{aligned} \quad (4.25)$$

The factor 3 holds for the decay into quarks.

So the pions ($\pi_{r1}^c, \pi_{r1}^b, \pi_{r1}^t$) with mass of order (30, 65, 180) GeV decay mainly into ($\bar{c}c, \bar{b}b, \bar{t}t$):

$$\Gamma\left((\pi_{r1}^c, \pi_{r1}^b, \pi_{r1}^t) \rightarrow (\bar{c}c, \bar{b}b, \bar{t}t)\right) \approx (5, 160, 28000) \text{ MeV} \quad (4.26)$$

The pion π_{r2}^t with mass of order 220 GeV decays mainly to $\bar{t}t$:

$$\Gamma(\pi_{r2}^t \rightarrow \bar{t}t) \approx 17 \text{ GeV} \quad (4.27)$$

5. Low Energy Phenomenology

5.1 Flavour Changing Neutral Currents (F.C.N.C.) from Heavy Gauge Boson Exchanges

5.1.1 Single Gauge Boson Exchange

Let us first examine if in the pure hadronic or leptonic processes any F.C.N.C. can be induced due to single exchange of massive $G_S/SU(3)_C$ gauge bosons.

Contrary to E.T.C. models where the E.T.C. gauge bosons induce horizontal interactions, in the present scheme the heavy vector bosons couple only flavour diagonally. The reason is that every representation of G_S contains only one quark or lepton family. Therefore the heavy vector boson is flavour blind just like the ordinary gluon. So in all of the present schemes, there are no induced transitions contributing to K_L-K_S mass difference etc. by single gauge boson exchange which was the major difficulty of the E.T.C. models.

However in the Pati-Salam models, semileptonic flavour changing neutral interactions are possible due to exchange of the vector boson V_3 . The most stringent limits are provided by the decay of $K_L \rightarrow e\mu$ (see Fig. 4). The branching ratio for this process is given by

$$\frac{\Gamma(K_L^0 \rightarrow e^- \mu^+)}{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)} = \frac{2\pi^2 \alpha_s^2 M_{V_3}^{-4}}{\sin^2 \theta_c \left(\frac{g_1^2}{2M_W^2} \right)^2} \quad (5.1)$$

Demanding this ratio to be less than the upper limit of 0.63×10^{-9} we obtain

$$M_{V_3} \gtrsim 50 \text{ TeV} \quad (5.2)$$

We should in particular emphasize that in $[SU(3)]^n$ -type models of the present scheme even the above decays are absent. This is an additional reason that makes the $[SU(3)]^n$ -type model more attractive than the Pati-Salam.

5.1.2 High Order Gauge Boson Exchanges

Although the couplings of the heavy vector bosons to quarks and leptons are flavour diagonal, F.C.N.C. can be induced by high order diagrams involving loops of high colour fermions.

As we have already discussed in chapter 3, there exist flavour violating condensates which do not disappear after the diagonalization of the quark mass matrix. Therefore, they cause F.C.N.C. when they participate in high order diagrams. Let us consider some of the most dangerous processes in analogy with ref.. (3).

$K_L^0-K_S^0$ mass difference: The contributing diagram is shown in Fig. 5. Using the propagator of the high colour fermion r_1 given in (2.4) for the calculation of this diagram we find:

$$\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}} \approx \frac{54}{4} \alpha_s^2 \theta^2 C_2^2(r_1) \frac{\mu_{r_1}^2}{M_{V_8}^4} \quad (5.3)$$

θ is the angle which describes the flavour violating mixing of the echo-quarks for which we use two values of $0(1)$ and of $0(1/20)$. To respect the limit on the K_L-K_S mass difference, $\mathcal{L}_{(\bar{s}d)^2}^{\text{eff}}$ must be smaller than $0.5 \times 10^{-12}(\text{GeV})^{-2}$, and with $r_1 = 6$, $\mu_{r_1} = 70 \text{ GeV}$ we find,

$$\begin{aligned} M_{V_8} &\gtrsim 10 \text{ TeV} \quad (\theta^2 \sim 1) \\ M_{V_8} &\gtrsim 5 \text{ TeV} \quad (\theta^2 \sim \frac{1}{20}) \end{aligned} \quad (5.4)$$

These bounds are satisfied in our model (see eq. (3.8)) when V_8 are identified with V_{8_1} which are the vector bosons that couple $Q_{r_1} d, s$ to d, s .

$\bar{s}dZ^0$ -vertex: There exist vertex correction diagrams which give in general flavour changing contributions to the couplings of the Z^0 vector boson. The leading contribution to flavour-changing Z^0 vertices involves two helicity flips of the high colour fermions shown in fig. 6 (diagrams without helicity flip do not contribute because

of the flavour conservation of the gauge couplings). We estimate the flavour changing Z^0 coupling to be:

$$(\bar{s} \gamma_\mu d)_{L,R} Z^0_\mu \left(\frac{g_2}{2 \cos \theta_w} \right) \frac{\alpha_s \cdot \Theta \cdot (3 C_2(r_1))^2 \mu_{r_1}^2}{2\pi M_{\nu_8}^2} \quad (5.5)$$

In order these contributions to be less than the typical G.I.M. suppressed neutral currents one needs:

$$\frac{\Theta \cdot \alpha_s (3 C_2(r_1))^2 \mu_{r_1}^2}{M_{\nu_8}^2} \lesssim \frac{m_c^2}{M_w^2} \quad (5.6)$$

With $r_1 = 6$ and $\mu_6 \approx 70$ GeV we obtain

$$\begin{aligned} M_{\nu_8} &\geq 13 \text{ TeV} & (\theta^2 \sim 1) \\ M_{\nu_8} &\geq 6 \text{ TeV} & (\theta^2 \sim \frac{1}{20}) \end{aligned} \quad (5.7)$$

Also these bounds are satisfied in our model (see eq. (3.8)).

$\mu \rightarrow e \gamma$: Because the high colour fermions which accompany the ordinary leptons transform according to real $SU(3)_C$ representations there are no flavour violating condensates. This is due to the fact that to make a symmetric singlet from $\tilde{r}_i \times \tilde{r}_j$ one has to choose $i = j$. Therefore the process $\mu \rightarrow e \gamma$ and any other leptonic process cannot be induced in one loop.

5.2 The problems with true Goldstone Bosons

As we have discussed in chapter 4.3 there exist 11 neutral massless Goldstone bosons in the quark sector of our model. They behave like axions. Therefore, one can derive limits on the corresponding decay constant F_{π_R} using the bounds obtained in the searches for axions²²⁾. The massless Goldstone bosons contribute also to F.C.N.C. because of the existence of flavour violating condensates.

Let us discuss the most dangerous processes:

$\psi \rightarrow \gamma + \pi_r^0$: This process is shown in Fig. 7. The Goldstone boson involved in this process is the $\pi_{r_2} \sim \bar{r}_2^c r_2^c$ (see eq. (4.17)). It couples to the $\bar{c}c$ -state in a Higgs-like way, namely proportionally to the c -mass.

Then the decay rate for this process is given by⁽²³⁾:

$$\frac{\Gamma(\psi \rightarrow \gamma + \pi_{r_2}^0)}{\Gamma(\psi \rightarrow \mu^+ \mu^-)} \simeq \frac{m_c^2}{8 F_{\pi_{r_2}}^2 \pi \alpha} \quad (5.8)$$

Demanding this ratio to be less than 2×10^{-4} (24) one obtains

$$\mu_{r_2} \gtrsim 1 \text{ TeV} \quad (5.9)$$

This bound is already one order of magnitude bigger than the typical values given in eq. (3.4).

$K_L^0 - K_S^0$ mass differences: The process is shown in Fig. 8. It is induced by the exchange of the $\pi_{r_1} \sim \bar{r}_1^d r_1^s$ (see eq. (4.17)). This Goldstone boson couples proportionally to the d -quark mass. We obtain

$$\mathcal{L}_{(5d)}^{\text{eff}} \simeq \frac{m_d^2}{F_{\pi_{r_1}}^2} \cdot \frac{1}{m_K^2} \leq 10^{-12} (\text{GeV})^{-2} \quad (5.10)$$

This leads to a limit for μ_{r_1} ,

$$\mu_{r_1} \gtrsim 50 \text{ TeV} \quad (5.11)$$

which is far above from the value that was needed in eq. (3.4).

$K^0 \rightarrow \pi^0 \pi_r^0$: The last process we consider is the decay of K^0 into an ordinary pion π^0 and a high colour pion $\pi_{r_1}^0 \sim \bar{r}_1^d r_1^s$ (see Fig. 9).

The decay rate is given by⁽²²⁾:

$$\frac{\Gamma(K^0 \rightarrow \pi^0 \pi_r^0)}{\Gamma(K^+ \rightarrow \pi^0 e^+ \nu)} \simeq \frac{m_d^2 m_K}{16\pi F_{\pi_{r_1}}^2} \left(\frac{G_F^2 m_K^5}{192 \pi^3} \right)^{-1} \leq \quad (5.12)$$

This leads to

$$\leq 8 \times 10^{-4}$$

$$\mu_r \gtrsim 10^5 \text{ TeV} \quad (5.13)$$

in complete disagreement with any of the values of eq. (3.4).

In conclusion there is an apparent serious difficulty related to the existence of truly massless high colour Goldstone bosons in the present scheme. The problem exists in all the models which have been constructed so far and it is not a peculiarity of the particular model of section 3.

Are there possibilities to avoid this problem? The first but not very appealing way out is to make all the massless Goldstone bosons heavy by explicit breaking of the global symmetries. For example this could be due to some explicit masses for the high colour fermions blaming some non renormalizable interactions for their appearance as was suggested in ref. (25). Nevertheless this possibility is not very attractive in the context of dynamical fermion mass generation via the radiative mechanism.

Another possibility is to introduce additional horizontal gauge interactions which are spontaneously broken and therefore could absorb some of the unwanted Goldstone bosons. The last of the obvious ways out is to introduce as few as possible high colour representations. Then the fermion hierarchies are generated by the mass differences of the heavy vector bosons V .

But so far we have not succeeded in incorporating any of the above suggestions in a realistic model.

As we have already mentioned there could be also massless Goldstone bosons in the leptonic sector. But these are coupled as we have emphasized only flavour diagonally so they behave like not dangerous Majorons.⁽²⁶⁾ On the other hand in contrary to E.T.C. the massive pseudo Goldstone bosons do not cause any problems in our models.

6. Production of High Colour Fermions in High Energy Accelerators

In this final chapter, we shall discuss the production of the high colour baryons and pions in present and future high energy machines.

6.1 Production of High Colour Baryons/Mesons

Let us consider first the production of the high colour baryons/mesons containing only one echo-quark Q_r . This is equivalent to the production of single equo-quark Q_r .

$p\bar{p} (pp) \rightarrow Q_r + \bar{Q}_r + X$: The high colour baryons/mesons like the echo-quarks are expected to have masses in the range of 100 to 200 GeV (see eq. (4.2)), and should therefore show up at hadron-hadron colliders in the TeV $\sqrt{s}$ range. At high s they are dominantly produced via gluon fusion (see ref. (27)). In Fig. 10 the production cross sections for colour sextet echo-quarks at collider energies $\sqrt{s} = 0.6, 2, 20, 40$ TeV are shown, employing the quark and gluon distribution function (set 1) of Duke and Owens⁽²⁸⁾. For $\sqrt{s} = 600$ GeV (2 TeV) quark-antiquark annihilation dominates over gluon fusion for $M_{Q_6} > 60$ GeV (160 GeV).

Our results are consistent with those of Abolins et al.⁽²⁹⁾. The corresponding production cross sections for colour octet pair production are practically identical. It is interesting that for $M_{Q_r} < 80$ GeV ($\sigma > 10$ pb), echo-quarks should already be copiously produced at the CERN collider. Contrary to composite models where colour octet exotics decay into a high energetic lepton and a gluon jet, the echo-quarks of the present scheme do not have such spectacular signals. The decay signatures of the high colour fermions of the present scheme will depend on the class of models they belong to (see chapter 4.1). In the Pati-Salam-type models, there was no stable high colour baryon/meson, they decay (see Fig. 3 and eq. (4.4)) always to a high energetic lepton plus one or several quark jets. In the $[SU(3)]^n$ option however, the lowest high colour baryon/meson is stable. Thus in this case we expect as a signature the track of a heavy, strong interacting particle.

At hadron-hadron colliders, within the Pati-Salam type models, the V_8 exchange in (du) or (dd) collisions leads to the production of e^- and an echo-quark (see Fig. 11). In Table 1 we present cross sections in pb for $M_{Q_6} \approx 100$ GeV in colliders with energy from 2 to 40 TeV. The values are tiny if $M_{V_8} > 10$ TeV. However, the signatures could be very impressive since the isolated electron is extremely energetic and has a large p_T .

Lepton-Hadron Collider: At lepton hadron colliders the exchange of a V_8 between the incoming e^- and a quark or antiquark leads to the production of an echoquark Q_8 and an echo-lepton Q_6 (see Fig 12a) within the $[SU(3)]^n$ models.

The cross section is given by:

$$\sigma(e^- p \rightarrow Q_8 Q_6 + X) = \frac{2\alpha_s^2}{s} \cdot \frac{8}{3} \int_{x_0}^1 \frac{dx}{x} \left(\sum_{i=q, \bar{q}} D^i(x, Q^2) \right) \int d\text{Phs} \frac{Z}{N} \quad (6.1)$$

where

$$Z = 2 \left(\hat{s}^2 + \hat{u}^2 - \hat{s} (M_{Q_6}^2 + M_{Q_8}^2) + \hat{u} (M_{Q_6}^2 - M_{Q_8}^2) \right) + \\ + \frac{M_{Q_8}^2 M_{Q_6}^2}{M_V^4} \left[\hat{t}^2 - \hat{t} (M_{Q_6}^2 - M_{Q_8}^2) + 4 \hat{s} M_V^2 \right]$$

$$N = \left(\hat{t} + M_{Q_8}^2 - M_V^2 \right)^2$$

$$\hat{s} = 2 k_e p_q, \quad \hat{t} = -2 k_e p_{Q_8}, \quad \hat{u} = -2 k_e p_{Q_6}$$

$$d\text{Phs} = \frac{d^3 p_{Q_8}}{2 E_{Q_8}} \cdot \frac{d^3 p_{Q_6}}{2 E_{Q_6}} \delta^4(k_e + p_q - p_{Q_8} - p_{Q_6})$$

$$x_0 = \frac{(M_{Q_6} + M_{Q_8})^2}{s}$$

and $8/3$ is a colour factor.

For numerical studies, we take $M_{Q_8} = M_{Q_6} = M_Q$. In Fig. 13a we present (with dashed lines) the production cross sections for the HERA collider

($\sqrt{s} = 314$ GeV) as a function of the V_8 mass. In the case of $M_Q = 100$ GeV, the cross section is larger than $1/10$ pb for $M_{V_8} < 1.1$ TeV. In view of our bounds from F.C.N.C. induced by high order exchange of gauge boson V_8 (see eqs. (5.4, 5.7)) such a low value seems to be excluded within our $[SU(3)]^n$ models. In Fig. 13b we show (with dashed lines) the corresponding limits for M_{V_8} when $\sqrt{s} = 2$ TeV. In the case that $M_Q = (100, 200)$ GeV we obtain the limits $M_{V_8} < 7,6$ TeV in order the cross section to be larger than $\frac{1}{10}$ pb. Comparing these masses with the bounds from F.C.N.C. eq. (5.4, 5.7) we see that with a mixing $\theta^2 < \frac{1}{10}$ (like the usual Cabbibo mixing) the bounds can be satisfied. Therefore, taking seriously our $[SU(3)]^n$ -models, one could expect some events in these machines.

In the Pati-Salam type models, the exchange of a V_8 between the incoming e^- and u or $\bar{d}$ of the proton sea leads to a d-quark jet and a anti-echo-quark (see Fig. 12b). The cross section can be extracted from eq. (6.1) by setting $M_{Q_8} = 0$ and replacing the colour factor by 6. The predictions are included in Fig. 13 (with solid curves). The cross sections are larger than $1/10$ pb at HERA, if $M_{V_8} < 4.5$ TeV for $M_Q = 100$ GeV. Comparing the values of M_{V_8} with the bounds from $K \rightarrow \mu e$ (eq. (5.2)) they seem to be excluded.

6.2 Production of Pseudo Goldstone Boson

Since the masses of pseudo Goldstone bosons lie in the range of 30 - 220 GeV (see eq. (4.13)), we expect them to be produced quite often in hadron-hadron collisions. The production of single Pseudo Goldstone bosons is dominated by two-gluon fusion. The cross section is given by:

$$\sigma(\bar{p}p \rightarrow \pi_r + X) = \frac{\pi^2 \Gamma(\pi_r \rightarrow gg)}{8 m_{\pi_r}^3} \int_0^1 \frac{dx}{x} G(x) G\left(\frac{\tau}{x}\right); \quad (6.2)$$

$$\tau = \frac{m_{\pi_r}^2}{s}$$

The corresponding decay rate of π_r^0 into two gluons was given in eq. (4.21). In Fig. 14 the cross section is shown for $\sqrt{s} = 0.6, 2, 20$ and 40 TeV, choosing $r = 6$ and $\mu_6 = 70$ GeV. The main decay of the pseudo Goldstone bosons

proceeds via 2 gluons or a quark-antiquark pair which lead to a two jet decay signature. However the calculated cross section⁽³⁰⁾ for the CERN collider exceeds the two jet background calculations of usual Q.C.D. (dashed curve in Fig. 14) only for: $m_{\pi_r} < 28 \text{ GeV}$.

Therefore, comparing this bound with the calculated mass values of eq. 4.13 ($m_{\pi_r} > 30 \text{ GeV}$) it seems rather unlikely for the present CERN collider to detect a π_r^0 signal in the two jet cross section except possibly in very high statistics searches. But also other non-gluonic decay modes like into $\gamma\gamma$, $Z\gamma$ are very suppressed and also strongly constrained by general unitarity arguments.⁽³¹⁾ The branching ratio of the decay into these particles is of order of 10^{-3} . If we require about 5 such events at least 5000 π_r^0 have to be produced. This requires, at an integrated luminosity of about 310 nb^{-1} , a production cross section

$$\sigma(p p \rightarrow \pi_r^0 + X) \gtrsim 16 \text{ nb} \quad (6.3)$$

Then the curve of Fig. 14 implies that

$$m_{\pi_r^0} \lesssim 32 \text{ GeV} \quad (6.4)$$

in order to obtain a detectable signal at the present CERN collider.

7. Summary

We have discussed the phenomenological consequences of models involving high colour fermions. High colour fermions were introduced in an attempt to understand dynamically the electroweak symmetry breaking, the fermion mass generation mechanism and the flavour mixing. Such an attempt is supported by recent lattice calculations.

Our model building efforts can be summarized in two classes of models: models with quark and lepton unification a la Pati-Salam and $[\text{SU}(3)]^n$ -type models with quark and leptons in distinct representations. A common feature of these models is the enlargement of the strong gauge group to G_s and its subsequent breaking to $\text{SU}(3)_c$. In this way, normal fermions are coupled to high colour fermions via gauge interactions and as a result fermion

masses and mixing angles are generated. The advantage of these models, compared to technicolour models, is the lack of tree order FCNC by heavy gauge boson exchange. However the heavy vector bosons do induce flavour changing neutral currents in higher orders which puts limits on their masses. These limits are easily satisfied in our models when they are confronted with the values which are needed in order to reproduce the fermionic spectrum.

Models that can be constructed within the presented scheme suffer from the presence of true massless Goldstone bosons which induce rare processes in unacceptably high rates. Therefore better models are needed in which the dangerous Goldstone bosons are either absorbed by vector bosons or become massive in some way.

In this paper we have also considered the signals that could be seen in high energy accelerators. We expect very clear signals of high colour baryons/mesons to be detected in future hadron-hadron colliders like the tevatron with $\sqrt{s} = 2$ TeV. They could already be copiously produced at the present CERN collider if their mass is lower than about 80 GeV which however seems not very likely within the models that have been constructed. At lepton hadron colliders the exchange of the heavy vector bosons leads to the production of high colour fermions. However at HERA it will be hard to obtain a detectable signal according to our models, while in the $\sqrt{s} = 2$ TeV collider we expect some events. For the production of pseudo Goldstone bosons in hadron-hadron collisions the same is true. In future experiments they should be detected but in the present CERN collider they are just at the boarder.

Although our specific models should not be taken as totally realistic, they have a number of attractive points and definite predictions. Therefore one should search for the signals of high colour fermions and keep in mind that the existence of these particles with the described or similar properties will be a big step towards the understanding of the origin of the $W^\pm$, Z^0 , quark and lepton masses.

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Figure Captions:

- Fig. 1: Scale dependence of α_s in the presence of dynamical echo-quark mass μ_r and cut off Λ .
- Fig. 2: Radiative diagrams generating ordinary fermion masses.
- Fig. 3: Decay diagram of an echo-quark in the Pati-Salam type models.
- Fig. 4: Diagram for the decay of K_L into μ^+e^- in the Pati-Salam type models.
- Fig. 5: Diagram contributing to the $K_L^0-K_S^0$ mass difference involving flavour violating echo-quark condensates.
- Fig. 6: Diagram for the induced $\bar{s}dZ^0$ vertex.
- Fig. 7: Diagram for the decay of Ψ into a massless Goldstone boson π_{I_2} and a photon.
- Fig. 8: Diagram contributing to the $K_L^0-K_S^0$ mass difference via the exchange of a massless Goldstone boson.
- Fig. 9: Diagram for the decay of K^0 into a massless Goldstone boson and an ordinary pion.
- Fig. 10: Pair production cross section for a colour sextet echo-quark Q_6 as a function of M_{Q_6} for $\sqrt{s} = 0.6, 2, 20, 40$ TeV in proton - anti proton colliders.
- Fig. 11: Diagram for the production of an electron and an echo-quark for the Pati-Salam type models in hadron-hadron colliders.
- Fig. 12: Diagrams for echo-quark production in lepton-hadron colliders.
 (a) for the $[SU(3)]^n$ -type models
 (b) for the Pati-Salam type models
- Fig. 13: Lepto production cross section for a single Q_6 via V_3 exchange in Pati-Salam type models (solid line) and for a Q_6, Q_8 -pair via V_8 exchange in $[SU(3)]^n$ type models (dashed line) as a function of the mass of the exchanged vector boson
 a) for HERA ($\sqrt{s} = 314$ GeV)
 b) for a $\sqrt{s} = 2$ TeV lepton-hadron collider.

Fig. 14: Production cross sections for Goldstone boson π_6 as a function at m_{π_6} for $\sqrt{s} = 0.6, 2, 20, 60$ TeV in proton - antiproton colliders. The dashed curve represents the cross section of the two jet background of usual QCD at $\sqrt{s} = 600$ GeV.

Table 1:

Cross section, average electron energy and average transversal electron momentum for Q_6 production in hadron-hadron colliders via V , exchange.

	G [pb]				$\langle E_e \rangle$ [GeV]	$\langle P_{Te} \rangle$ [GeV]
	$M_{V_3} =$					
$\sqrt{s} =$	5 TeV	10 TeV	20 TeV	30 TeV		
2 TeV	$0.44 \cdot 10^{-2}$	$0.28 \cdot 10^{-3}$	$0.18 \cdot 10^{-4}$	$0.34 \cdot 10^{-5}$	190	132
10 TeV	0.11	$0.74 \cdot 10^{-2}$	$0.47 \cdot 10^{-3}$	10^{-4}	765 - 804	463 - 496
20 TeV	0.36	$0.26 \cdot 10^{-1}$	$0.17 \cdot 10^{-2}$	$0.34 \cdot 10^{-3}$	1330 - 1510	748 - 909
40 TeV	1.0	$0.82 \cdot 10^{-1}$	$0.58 \cdot 10^{-2}$	$0.12 \cdot 10^{-2}$	2250 - 2890	1090 - 1700

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