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SPONTANEOUSLY BROKEN $N = 3$ SUPERGRAVITY

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A B S T R A C T

We discuss partial breaking of $N = 3$ extended supergravity coupled to matter, and exploit conditions which enable us to find stationary points with one unbroken supersymmetry, thus avoiding the study of the scalar potential. We consider gauge groups $SO(3) \times K_n$, where K_n is the gauge group of $N = 3$ matter. We find $N = 1$ extrema in anti-de Sitter space for those gauge groups K_n which act as isometries of quaternionic coset spaces, surfaces of $SU(3,n)/SU(3) \times SU(n) \times U(1)$. Explicit solutions for the quaternionic surfaces $SO(n)/SO(n-4) \times SO(4)$, $n \geq 8$, and $E_6/SU(2) \times SU(6)$, $n = 78$ are exhibited.

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1. INTRODUCTION

Extended supergravities provide an interesting framework for model field theories of particle interactions, including gravity, in which symmetries between particles of different spin and statistics are present.

Four-dimensional extended supergravities can be separated into three classes depending on the number of spinorial charges Q_α^i ($i = 1, \dots, N$).

For $N \leq 2$, supergravity theories admit two different kinds of matter multiplets, one with spin-0 and spin-1/2 particles, the other with spin-1 gauge particles and lower spin states [1, 2].

For $N = 3$ and 4 the matter multiplets [3, 4] are unique and contain particles with spin-0 and up to spin-1. For $N > 4$, no matter multiplets are possible, and all particles, including the spin-1 gauge fields, belong to the gravitational multiplet, i.e. the multiplet which includes the spin-2 graviton and the spin-3/2 gravitinos.

In view of physical applications, one of the most outstanding problems is the question of supersymmetry breaking. Supersymmetry is such a constrained symmetry that its breaking can only occur in special circumstances.

In extended supersymmetry one may consider situations in which N supersymmetries are broken down to a lower N' -extended supersymmetry.

A particularly interesting situation is $N' = 1$, which would correspond to an extended supersymmetry broken to simple supersymmetry.

It is known that for a sequential breaking of supersymmetry to be possible, one needs local rather than global supersymmetry. This is due to a theorem of Witten [5] which states that in any theory without ghosts, global N -extended supersymmetry is either unbroken or completely broken, with all supersymmetry charges broken at the same scale. In supergravity theories the situation is different owing to the fact that the positivity properties of the scalar potentials are no longer true.

This is already the case in $N = 1$ supergravity [1] where the structure of the supergravity potential allows supersymmetry breaking with vanishing cosmological constant.

In extended supergravities the lack of positive-definiteness for the potential allows the possibility of sequential breaking of supersymmetries.

The reason why such partial breaking can occur is best expressed by a matrix Ward identity which relates the fermionic shifts of spin-1/2 and spin-3/2 particles (functions of the scalar fields of the theory) and the overall scalar potential [6],

$$-6 S_{AC}(\phi) S^{CB}(\phi) + \bar{Z}_A^i(\phi) \bar{Z}_i^B(\phi) = \delta_A^B V(\phi) \quad (1.1)$$

in which

$$\delta \chi^i = \bar{Z}_A^i(\phi) \varepsilon^A + \dots \quad (1.2)$$

$$\delta \psi_\mu^A = D_\mu \varepsilon^A + S^{AB}(\phi) \gamma_\mu \varepsilon_B + \dots \quad (1.3)$$

In Eqs. (1.1) and (1.2) the spin-1/2 shift $\bar{Z}_A^i(\phi)$ is related to the spin-1/2 and spin-3/2 interference term in the Lagrangian,

$$\bar{\chi}^i \bar{Z}_i^A(\phi) \gamma^\mu \psi_{\mu A} \quad (1.4)$$

In Eqs. (1.1) and (1.3) the symmetric matrix $S^{AB}(\phi) = S^{BA}(\phi)$ is the gravitino mass matrix

$$\bar{\psi}_{A\mu} \sigma^{\mu\nu} \psi_{\nu B} S^{AB}(\phi) \quad (1.5)$$

and $V(\phi)$ in Eq. (1.1) is the scalar potential.

The extended supergravity Ward identities yield a further equation if we demand invariance up to terms that are linear in the Fermi fields (but to all orders in non-derivative terms in the scalar fields) [7]:

$$v_{ij}(\phi) \bar{\Sigma}_A^i(\phi) - 4 S_{AB}(\phi) \bar{\Sigma}_i^B(\phi) + \frac{\partial W}{\partial \phi_\alpha} L_{\alpha Ai}(\phi) = 0 \quad (1.6)$$

In Eq. (1.6) v_{ij} is the spin-1/2 mass matrix and $L_{\alpha Ai}$ is related to the scalar field supergravity variation,

$$\delta \phi_\alpha = \bar{\chi}^i \varepsilon^A L_{\alpha Ai} \quad (1.7)$$

(ϕ_α means those combinations of scalar fields which transform with respect to ε^A).

For extended supergravities with $N > 2$, the functions Γ_A^i , S^{AB} , and v_{ij} are non-vanishing only if some subgroup K of the full isometry group G of the scalar manifold is gauged [3]. For $N = 1, 2$ the functions under consideration are determined by the tensor calculus [1, 2], and in these exceptional cases they can be determined even for scalar manifolds which are not coset spaces.

Examples of the breaking of $N = 2$ supersymmetry down to $N = 1$ in anti-de Sitter space have been given [8], whilst a residual unbroken $N = 1$ Poincaré supersymmetry is impossible [9]. For $N = 2$ supergravity a partial breaking of $N = 2$ supersymmetry down to $N = 1$ needs the simultaneous occurrence of scalar fields belonging to different kinds of multiplets: vector multiplets and hypermultiplets, the latter containing only spin-0 and spin-1/2 states [8]. This is required from the structure of massive $N = 2$ vector multiplets [10] and from the fact that scalars belonging to vector multiplets are inert under the $SO(2)$ gauge symmetry which acts on the doublet of gravitinos. Therefore $SO(2)$ breaking, needed to split the gravitino masses, can only occur because of hypermultiplets taking a non-vanishing vacuum expectation value [8].

In higher N supergravity, the super-Higgs sector becomes irreducible. This is so because scalar fields either belong to matter or to gravitational multiplets.

For $N > 4$, partial breaking of supersymmetry down to $N = 2$ or $N = 0$ in anti-de Sitter space has been found [11]. Extrema of the $N > 4$ supergravity potentials with a residual $N = 1$ supersymmetry have not been found.

The $N = 3$ and $N = 4$ supergravities coupled to matter offer possible theoretical frameworks in which one could have the simultaneous occurrence of a residual supersymmetry and gauge symmetry. This is so because the scalars of the Higgs sector transform both under the gauge group K , subgroup of the isometry group G of the scalar manifold, and with respect to the global $SU(N)$ symmetry which rotates the N -spinorial generators.

The scalar manifolds that are compatible with $N = 3$ and $N = 4$ supergravity are the following coset spaces

$$\begin{aligned} N=3 & : \quad SU(3, n) / SU(3) \otimes SU(n) \otimes U(1) \\ N=4 & : \quad (SO(6, n) / SO(6) \otimes SO(n)) \otimes (SU(4, 1) / U(1)) \end{aligned} \quad (1.8)$$

where n denotes the number of matter multiplets and is therefore related to the dimension D of the Lie algebra K to be gauged ($D = n + 3$ for $N = 1$ and $D = n + 6$ for $N = 4$).

In extended supergravity with $N \geq 3$, scalar fields are always coordinates of coset spaces and the spin-1/2 and spin-3/2 fermionic shift, that is, the functions $\Gamma_A^i(\varphi)$ and $S^{AB}(\varphi)$ can only be non-vanishing if a suitable subgroup K of the full isometry group G has been gauged. In particular, since the group G acts on the scalars as well as on the vector fields through duality transformations [12], the group K must be a subgroup $\tilde{G} \subset G$ which leaves the

Lagrangian invariant. For $N = 3$, $\tilde{G} = SO(3, n)$ and K can be either $SO(3) \otimes K_n$ or $SO(3, 1) \otimes K_{n-3}$ in which K_n is a subgroup of $SO(n)$ of dimension n such that the vector representation of $SO(n)$, reduced to K_n , becomes the adjoint representation.

In coset spaces the fermionic shifts are entirely determined in terms of boosted structure constants of K , defined in terms of coset representatives $L_{\Lambda}^{\Delta}(\phi)$, i.e. of representatives of G on G/H through the formula:

$$\zeta_{\Lambda}^{\cdot \vec{z} \Delta}(\phi) = \left(L^{-1}(\phi) \right)_{\Lambda}^{\Lambda'} L_{\vec{z}'}^{\vec{z}}(\phi) L_{\Delta'}^{\Delta}(\phi) f_{\Lambda'}^{\cdot \vec{z}' \Delta'} \quad (1.9)$$

where $f_{\Lambda'}^{\cdot \vec{z}' \Delta'}$ are the structure constants of K . The coset representative is a matrix in the G -representations of the duality transformations of vector fields. The particular fermionic shifts defined through Eqs. (1.2) and (1.3) in terms of the quantities (1.9) are given in Section 2 for the case $N = 3$.

Also, the function $L_{\alpha Ai}$ appearing in Eq. (1.6) can be expressed through the inverse vierbein of the manifold G/H , suitably contracted with Clebsch-Gordan coefficients relative to the decomposition of the G representation of duality transformations into the H representation of spin-1/2 fermions.

The presence of at least one unbroken supersymmetry to which corresponds a killing spinor η^A ensures that the potential remains stationary [6]. It turns out that in order to find extrema of N -extended supergravity potentials with at least an unbroken supersymmetry, it is necessary (and sufficient) to look for algebraic solutions of the equations

$$\vec{z}_A^{\cdot i}(\phi) \eta^A = 0 \quad (1.10)$$

In fact, by contraction of Eq. (1.1) with η^A , one finds

$$-6 S_{AC} S^{CB} \eta^A = \eta^B W \quad (1.11)$$

and by multiplication with Γ_B^i one gets

$$\eta^A (SS^*)_A{}^B \sum_B^i = 0 \quad (1.12)$$

Equation (1.12) implies that Γ_B^i is orthogonal to the vector $\eta^A (SS^*)_A{}^B$ whenever Eq. (1.10) is satisfied. Since S^{AB} can be chosen to be real, Eq. (1.12) implies the stronger condition

$$\eta^A S_{AB} \sum^{Bi} = 0 \quad (1.13)$$

Equations (1.10) and (1.13) therefore imply

$$\frac{\partial W}{\partial \phi_\alpha} = 0 \quad (1.14)$$

because of Eq. (1.6).

The solutions of Eq. (1.10) for a class of gauge groups K of $N = 3$ extended supergravity will be given in Section 3.

These are the first examples of $N \geq 3$ theories in $D = 4$ dimension with one residual unbroken supersymmetry.

In the present paper we look for extrema of $N = 3$ supergravity coupled to matter in which one supersymmetry is unbroken. Our criterion is to have surfaces in the scalar manifold with some special symmetry properties.

The scalar manifold G/H has $SU(3) \times U(N)$ as the isotropy group, and we confine ourselves to compact gaugings $SO(3) \times K$ in which K is a suitable subgroup of $SU(N)$. We consider surfaces whose coordinates are singlets under the action of some subgroup S of H , under which at least one spin-3/2 fermion is a singlet. The largest group with this property is a group that is isomorphic to $SU(2)$. Different choices are possible. If we take $SU(2)$ to be a subgroup

of $SU(3)$ so that it commutes with K , then, as will be shown in Section 2, the gravitino mass matrix is proportional to the identity matrix, and one gets $N = 3$ unbroken or completely broken supersymmetry in anti-de Sitter space.

This situation is similar to the extrema one found in Ref. [11] in the context of $N = 5$ supergravity.

A more interesting situation occurs if the $SU(2)$ group does not commute with K . This is possible if $SU(2)$ is taken to be the diagonal subgroup of $SU(2)_1 \times SU(2)_2$ in which $SU(2)_1 \subset SU(3)$ and $SU(2)_2 \subset K$. Since the coordinates are doublets under $SU(2)_1$, the group K must have the property that doublets under its subgroup $SU(2)_2$ are contained in the adjoint representation [13]. The latter condition implies that the surface we are considering consists of scalar fields which belong to the quaternionic manifold $K/SU(2) \times N_K$ and to the adjoint representation of N_K in which N_K is the normalizer of $SU(2)$ in K ^{*)}. The coordinates of the quaternionic manifolds can be regarded as scalars belonging to would-be $N = 2$ hypermultiplets, whilst the coordinates of N_K are scalars belonging to would-be $N = 2$ vector multiplets. Some general features emerge from the present analysis, and they can be summarized as follows:

- i) The surfaces with maximal symmetry described above admit $N = 1$ supersymmetric extrema provided that both groups $SO(3)$ and K have non-vanishing coupling constants.
- ii) For different classes of quaternionic spaces, the ratio between the $SO(3)$ and K coupling constants, the masses of the two gravitinos, and the 'apparent' mass of the gravitino corresponding to unbroken supersymmetry in anti-de Sitter space are the same.

^{*)} This technique is similar in spirit to the method used in Ref. 14 to make a consistent truncation of $D = 10$ supergravity to $D = 4$ $N = 1$ supergravity.

iii) The quaternionic manifolds considered in this paper are the series $SO(v+4)/SO(v) \times SO(4)$, with $v > 4$, and the exceptional manifold $E_6/SU(2) \times SU(6)$.

All other quaternionic manifolds are supposed to share the same properties.

In analogy with $N = 8$ supergravity, we expect that supersymmetry-breaking with zero cosmological constant (Minkowski background) can only occur if the gauge group K is non-semisimple. [This must be a non-semisimple subgroup of $SO(3,n)$ whose adjoint representation has dimension $n + 3$. For instance for $n = 3$, can K be $IO(3)$?]

2. FEATURES OF THE $N = 3$ THEORY AND OF ITS POTENTIAL

In $N = 3$ matter-coupled supergravity [3] the scalar manifold is $SU(3,n)/SU(3) \times SU(n) \times U(1)$, and the gauge group can be either

$$G = SO(3,1) \otimes K_{n-3} \quad (2.1)$$

or

$$G = SO(3) \otimes K_n \quad (2.2)$$

where K_n is compact and semisimple. Calling $f_{\Lambda\Gamma}^{\Pi}$ the structure constants of G , the fundamental objects we have to consider are the 'boosted' structure constants

$$C_{\Lambda\bar{\Sigma}}^{\Pi} = (L^{-1})_{\Lambda}^{\Lambda'} (L^{-1})_{\bar{\Sigma}}^{\bar{\Sigma}'} L_{\Pi'}^{\Pi} f_{\Lambda'\bar{\Sigma}'}^{\Pi'} \quad (2.3)$$

where $L = L(X)$ is the coset representative, which, for convenience, can be chosen to be of the following type:

$$L = L_{\Lambda} \xi = \begin{pmatrix} (1+XX^\dagger)^{1/2} & X \\ X^\dagger & (1+X^\dagger X)^{1/2} \end{pmatrix} \quad (2.4)$$

X being a $3 \times n$ complex matrix representing the scalar fields.

Besides the gravitino ψ_A , we have in this theory three types of spin-1/2 fermions ($\chi_\otimes$, λ_i , λ_{iA}), and their respective supersymmetry shifts are given by^{*)}

$$\delta_{AB} = -\frac{i}{8} (C_A^{\cdot PQ} \varepsilon_{BPQ} + C_B^{\cdot PQ} \varepsilon_{APQ}) \Rightarrow \delta \psi_A \quad (2.5a)$$

$$\mathcal{U}_A = -\frac{1}{4} C_{AM}^{\cdot\cdot H} \Rightarrow \delta \chi^\otimes \quad (2.5b)$$

$$N_{iA} = -\frac{1}{2} C_i^{\cdot PQ} \varepsilon_{APQ} \Rightarrow \delta \lambda_i \quad (2.5c)$$

$$M_{iA}^{\cdot\cdot B} = -C_{iA}^{\cdot\cdot B} + \frac{1}{2} \delta_A^B C_{iH}^{\cdot\cdot H} \Rightarrow \delta \lambda_{iA} \quad (2.5d)$$

The explicit form taken by the general relation between the shifts [6] and the potential is, in our case,

$$\begin{aligned} -W \delta_A^B &= 2 S_{AM} S^{MB} - \frac{2}{3} \mathcal{U}_A \mathcal{U}^B - \frac{1}{6} N_{iA} N^{iB} \\ &\quad - \frac{1}{6} M_{iA}^{iH} M_{iH}^B \end{aligned} \quad (2.6)$$

from which one obtains the following formula for W:

$$12W = \frac{1}{2} |C_{iA}^B|^2 + \frac{1}{2} |C_i^{PQ}|^2 - \left(|C_A^{PQ}|^2 - |C_{AH}^H|^2 \right) \quad (2.7)$$

In the next section we shall show that this potential admits extrema that break $N = 3$ supersymmetry down to $N = 1$. We shall find these extrema by looking at

^{*)} Please note that from now on we use the normalizations and conventions of Ref. 3, adequate to the $N = 3$ theory. They differ from those utilized in the general discussion of Section 1. (For instance, S_{AB} is now pure imaginary rather than real.)

the fermion shifts on a certain submanifold $\tilde{m}$ of the scalar manifold whose coordinates are invariant under a specially chosen $SU(2)$. In this section we show, instead, that on the hypersurface spanned by those scalar fields which are singlet under the $SU(2)$ subgroup of the isotropy group $SU(3)$, the potential (2.7) is calculable in closed form and admits extrema that break all supersymmetries. Introducing a new variable

$$Z = X(1 + X^\dagger X)^{-1/2} \quad (2.8)$$

one can set

$$Z = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \omega^I \end{pmatrix} \quad (2.9)$$

where $\omega^I = Z_3^I$ is the singlet under $SU(2)$, the index I running in the adjoint representation of G .

The matrix L corresponding to this special kind of Z has the following structure:

$$L = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & E_1 & 0 & 0 & E_1 \omega^I \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & E_1 \bar{\omega}_J & 0 & 0 & 1 - E_2 \bar{\omega}_J \omega^I \end{array} \right) \quad (2.10)$$

where

$$E_1 = (1 - |\omega|^2)^{-1/2} ; \quad E_2 |\omega|^2 = 1 - E_1 \quad (2.11)$$

and the potential is easily calculated. Setting

$$\hat{f} \cdot \Sigma \pi = (e \epsilon_{ABC}, g h_{IJK}) \quad (2.12)$$

where e and g are the $SO(3)$ and K coupling constants, respectively, we get

$$12W = \frac{1}{1-|\omega|^2} \left\{ e(|\omega|^2 - 6) + \frac{g}{2} \frac{1}{1-|\omega|^2} \mathcal{U}(\omega) \right\} \quad (2.13)$$

$$\mathcal{U}(\omega) = h_{IKM} h_{IK'M'} \bar{\omega}_M \bar{\omega}_{M'} \omega^K \omega^{K'} \quad (2.14)$$

This potential has exactly the same form as the potential found in certain $N = 2$ theories [15] and admits the same extrema. They break the supersymmetry completely. Any possibility of partial breaking is in any case ruled out *a priori* by the fact that the gravitino mass-matrix S_{AB} is, in the situation of (2.9), completely degenerate. Indeed

$$S_{AB} = -\frac{i}{4} E_1(\omega) \delta_{AB} \quad (2.15)$$

3. $N = 1$ SUPERSYMMETRIC EXTREMA OF THE $N = 3$ THEORY

In this section we apply the technique described in the Introduction to the search for the $N = 1$ supersymmetric extrema of the $N = 3$ theory. We consider the case where the gauge group G is compact:

$$G = SO(3) \otimes K \quad (3.1)$$

and we further restrict our hunting-ground by taking K to be simple. Correspondingly, we have a theory with two coupling constants e and g , associated with the groups $SO(3)$ and K , respectively.

The result we have obtained and whose derivation we shall now present can be described in the following way.

Let the group K admit a subgroup $SU(2) \times N_K$ so that the decomposition of the adjoint representation $\text{adj } K$ is:

$$\text{adj } K \xrightarrow{SU_2 \otimes N_K} (\underline{3}, \underline{1}) \oplus (\underline{1}, \text{adj } N_K) \oplus (\underline{2}, \underline{D}) \quad (3.2)$$

where $\underline{D}$ is a pseudoreal representation of the normalizer N_K of $SU(2)$ in K .

Equation (3.2) is the statement that

$$M_K = \frac{K}{SU_2 \otimes N_K} \quad (3.3)$$

is a 'quaternionic coset manifold' [13]. The list of such cosets is known in mathematical literature; it consists of three infinite families plus five exceptional cases. They are given here for the reader's convenience:

$$\begin{aligned} HP(\nu) &= \frac{Sp(2\nu+2)}{Sp(2\nu) \otimes Sp(2)} ; \quad X(\nu) = \frac{SU(\nu+2)}{SU(\nu) \otimes SU(2) \otimes U(1)} \\ Y(\nu) &= \frac{SO(\nu+4)}{SO(\nu) \otimes SO(4)} ; \quad \frac{G_2}{SU_2 \otimes SU_2} \\ &\quad \frac{F_4}{SU_2 \otimes Sp(6)} ; \quad \frac{E_6}{SU_2 \otimes SU_6} ; \quad \frac{E_7}{SU_2 \otimes SO_{12}} ; \quad \frac{E_8}{SU_2 \otimes E_7} \end{aligned} \quad (3.4)$$

Assuming K to have this structure, we can single out, in the scalar manifold of the $N = 3$ theory

$$M_{\text{scalar}} = \frac{SU(3, n)}{SU(3) \otimes SU(n) \otimes U(1)} \quad (3.5)$$

(where $n = \dim K$), a submanifold $\bar{m} \subset m_{\text{scalar}}$ composed of those points whose coordinates are singlets under the following $SU(2)$ -diagonal subgroup of the isotropy group $H = SU(3) \otimes SU(n) \otimes U(1)$:

$$H = SU(3) \otimes SU(n) \otimes U(1) \supset SU(2)^{\text{diag}} = SU(2)^I \oplus SU(2)^{II} \quad (3.6)$$

where $SU(2)^I \subset SU(3)$ is the 'isospin' subgroup of $SU(3)$, and $SU(2)^{II} \subset K \subset SU(n)$ is the $SU(2)$ subgroup of K singled out by the quaternionic decomposition (3.2). The $SU(2)^{II}$ is also a subgroup of $SU(n)$ since the whole K is a subgroup. Indeed the adjoint representation of the compact group K is unitary in n -dimensions. The submanifold $\bar{m} \subset m$ is complex and has the following real dimension:

$$\dim \bar{m} = 2(\mu + 2\nu) \quad (3.7a)$$

$$\mu = \dim N_K \quad (3.7b)$$

$$2\nu = \dim \underline{D} \text{ of } N_K \quad (3.7c)$$

It is within the submanifold $\bar{m} \subset m$ that we find the $N = 1$ extrema of the $N = 3$ potential. Actually by computing the 'shifts' of the spin-1/2 fields and imposing that they are zero along one of the three supersymmetries, we obtain equations which admit solutions only if the ratio between $SO(3)$ and K coupling constants has a certain value. We have not checked explicitly all the cases listed in Eq. (3.4), confining ourselves (for no better reason than the criterion of choosing one infinite family and one exceptional case) to the choices

$K = SO(v+4)$ and $K = E_6$. However, from working out these two examples we have learned that there is a general pattern, independent of the choice of the group K within the class (3.4).

Calling φ^I the μ -complex coordinates associated with the adjoint representation of N_K , and (a^P, b^P) the $2v$ -complex coordinates associated with the pseudo-real representation D , we shall find that at the $N = 1$ extremum their norms squared satisfy the following conditions independently of the choice of K :

$$|\varphi|^2 = 2(|a|^2 + |b|^2) = t^2 \quad (3.8a)$$

$$t^2 = \sqrt{1 + t^2} \implies t^2 = \frac{1 + \sqrt{5}}{2} \quad (3.8b)$$

Moreover, the ratio of the $SO(3)$ and K coupling constants is

$$\frac{e}{g} = -\sqrt{\frac{\sqrt{5} - 1}{2}} \quad (3.9)$$

and the unbroken gravitino mass is

$$\mathcal{S}_{33} = -\frac{i}{4} (1 + \sqrt{5}) e \quad (3.10)$$

which, through the relation (1.6), implies the following value for the cosmological constant

$$W = -2 \left| \mathcal{S}_{33} \right|^2 = -\frac{e^2}{4} (3 + \sqrt{5}) < 0 \quad (3.11)$$

The universal value of W , being negative, guarantees that we are in anti-de Sitter space.

Before going to the derivation of these results, let us try to justify why we looked for $N = 1$ extrema in the submanifold $\bar{m}$. The reason is simple. Since the three gravitinos form an $SU(3)$ triplet, an extremum $z = z_0$ which breaks $N = 3$ down to $N = 1$ must have an isotropy group

$$\mathcal{H}_{z_0} \subset H = SU(3) \otimes SU(n) \otimes U(1) \quad (3.12)$$

so that the $(\underline{3}, \underline{1})$ of $SU(3) \otimes SU(n) \otimes U(1)$ decomposes under $\mathcal{H}_{z_0}$ into a singlet plus something else:

$$(\underline{3}, \underline{1}) \xrightarrow{\mathcal{H}_{z_0}} (\underline{1}, \underline{1}) \oplus \text{more} \quad (3.13)$$

The simplest possibility is to choose $\mathcal{H}_{z_0} = SU(2)^{\mathcal{H}} \times \bar{\mathcal{H}}$, where $\bar{\mathcal{H}}$ is some subgroup of $SU(n)$. In this way we obtain

$$(\underline{3}, \underline{1}) \xrightarrow{SU(2)^{\mathcal{H}} \otimes \bar{\mathcal{H}}} (\underline{1}, \underline{1}) \oplus (\underline{2}, \underline{1}) \quad (3.14)$$

The subtle point, however, is that the $SU(2)^{\mathcal{H}}$ factor of $\mathcal{H}_{z_0}$ does not need to be all contained in $SU(3)$, but it may be a combination of the isospin subgroup of $SU(3)$ with some $SU(2)$ subgroup of $SU(n)$. The reason why we turned to the second possibility is that if $SU(2)^{\mathcal{H}} \subset SU(3)$, then, as we showed in the previous section, the gravitino mass-matrix S_{AB} is completely degenerate:

$$S_{AB} = i m(z) \delta_{AB} \quad (3.15)$$

This would rule out any partial supersymmetry breaking.

Let us now come to the derivation of the announced results. First of all we want to discuss the general form of the structure constants of a Lie algebra K

which admits the decomposition (3.2). To this effect we decompose the generators T_i of K according to the following scheme:

$$T_i = (T_{\dot{A}}, T_I, T_{\mu p}) \quad (3.16)$$

where $T_{\dot{A}}$ ($\dot{A} = \dot{1}, \dot{2}, \dot{3}$) are the generators of $SU(2)^{II}$, T_I ($I = 1, \dots, \mu$) are the generators of N_K , and $T_{\mu p}$ are the coset generators which transform in the pseudoreal representation D of N_K and in the $\underline{2}$ of $SU(2)^{II}$. This is why $T_{\mu p}$ has a pair of indices.

The index μ takes 4-values which we characterize as follows:

$$\mu = 1\uparrow, 1\downarrow, 2\uparrow, 2\downarrow \quad (3.17)$$

On the other hand, p runs on v values, v being [according to Eq. (3.7c)] one half of the dimension of the pseudoreal representation D .

We can think of the index μ as spanning the four real dimensions of the $\underline{2}$ (complex) $SU(2)^{II}$ representation. On this basis the $SU(2)^{II}$ generators are given by the following 4×4 matrices $D_{\dot{A}\mu\nu}$:

$$D_{\dot{1}} = \begin{pmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & -1/2 & 0 \\ 0 & 1/2 & 0 & 0 \\ -1/2 & 0 & 0 & 0 \end{pmatrix}; D_{\dot{2}} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ -1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 1/2 & 0 \end{pmatrix}; D_{\dot{3}} = \begin{pmatrix} 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \\ -1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \end{pmatrix} \quad (3.18)$$

Calling h_{IJK} the completely antisymmetric structure constants of the compact N_K group, the completely antisymmetric structure constants f_{ijk} of K can be written as follows:

$$f_{\dot{A}\dot{B}\dot{C}} = \varepsilon_{\dot{A}\dot{B}\dot{C}} \quad (3.19a)$$

$$f_{IJK} = h_{IJK} \quad (3.19b)$$

$$f_{\dot{A}(\mu p)(\nu q)} = D_{\dot{A}\mu\nu} \delta_{pq} \quad (3.19c)$$

$$f_I(\mu p)(\nu q) = \begin{pmatrix} f_I & \tau_I & g_I & s_I \\ -\tau_I & f_I & s_I & -g_I \\ -g_I & -s_I & f_I & \tau_I \\ -s_I & g_I & -\tau_I & f_I \end{pmatrix} \quad (3.19d)$$

In Eq. (3.19d) the (4×4) matrix with $\mu\nu$ indices has entries which are $v \times v$ matrices with the following symmetries:

$$\begin{aligned} f_I^T &= -f_I & ; & & g_I^T &= g_I \\ \tau_I^T &= -\tau_I & ; & & s_I^T &= s_I \end{aligned} \quad (3.20)$$

The structure of Eqs. (3.19) can be explained in the following way. A pseudo-real representation of N_K is generated by complex matrices t_I satisfying the condition

$$\Omega t_I \Omega = -t_I^* \quad (3.21)$$

where $\Omega^T = -\Omega$ ($\Omega^2 = -1$) is a matrix representation of the imaginary unity. We can choose Ω to be of the form

$$\Omega = \left(\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \right) \quad (3.22)$$

According to the block structure (3.22) the index spanning the $2v$ -dimensional representation D can be written as a pair $(p\uparrow)$ or $(p\downarrow)$, where p runs on v -values

and the arrow tells you whether you are in the first or second half of the column vector. Utilizing this notation we can write

$$\Omega_{(p\uparrow)(q\downarrow)} = 0 ; \Omega_{(p\uparrow)(q\downarrow)} = -\Omega_{(p\downarrow)(q\uparrow)} = \delta_{pq} ; \Omega_{(p\downarrow)(q\downarrow)} = 0 \quad (3.23)$$

Let $(\text{Re}_A, \text{Im}_A)$ be the real and imaginary parts of the $SU(2)$ generators in the doublet representation [namely the real and imaginary parts of $(i/2)\sigma_A$] and let $(\text{Re}_I, \text{Im}_I)$ be the real and imaginary parts of the N_K generators t_I in the pseudo-real representation D . We can write the action of $SU(2) \times N_K$ on a real object $T_{\hat{a}\alpha}$ which has an index $(\hat{a} = \hat{1}, \hat{2})$ transforming under $SU(2)$ and an index $\alpha = (p\uparrow, p\downarrow)$ transforming under N_K as follows

$$\delta_A T_{\hat{a}\alpha} = f_{A\hat{a}\alpha}{}^{\hat{b}\beta} T_{\hat{b}\beta} = (\text{Re}_A \delta_{\hat{a}\hat{b}} \delta_{\alpha\beta} + \Omega_{\alpha\beta} \text{Im}_A \delta_{\hat{a}\hat{b}}) T_{\hat{b}\beta} \quad (3.24a)$$

$$\delta_I T_{\hat{a}\alpha} = f_{I\hat{a}\alpha}{}^{\hat{b}\beta} T_{\hat{b}\beta} = (\text{Re}_I \delta_{\hat{a}\hat{b}} \delta_{\alpha\beta} + \varepsilon_{\hat{a}\hat{b}} \text{Im}_I \delta_{\alpha\beta}) T_{\hat{b}\beta} \quad (3.24b)$$

The idea is that the imaginary unit has been replaced by a matrix operation with square equal to minus one acting on the complementary set of indices. $T_{\hat{a}\alpha}$ are to be identified with the coset generators in the decomposition (3.2). Using Eq. (3.17) and $\alpha = (p\uparrow, p\downarrow)$ we see that indeed

$$T_{\hat{a}\alpha} = T_{\mu p} \quad (3.25)$$

Moreover, identifying

$$f_{Ipq} = \text{Re}_{I(p\uparrow)(q\downarrow)} ; g_{Ipq} = \text{Im}_{I(p\uparrow)(q\uparrow)} , \quad (3.26a)$$

$$g_{Ipq} = \text{Re}_{I(p\uparrow)(q\uparrow)} ; f_{Ipq} = \text{Im}_{I(p\uparrow)(q\downarrow)} \quad (3.26b)$$

we arrive at Eqs. (3.19c) and (3.19d) for the structure constants $f_{\dot{A}\mu\nu q}$ and $f_{I(\mu p)(\nu q)}$.

Having characterized our K Lie algebra, we now look at the general form of an $SU(3,n)/SU(3) \times SU(n) \times U(1)$ coset element whose coordinates are invariant under $SU(2)^{\text{diag}}$ defined in Eq. (3.6). We utilize the coset parametrization of Eq. (2.4), and we impose that the action of $SU(2)^{\text{diag}}$ on the scalar fields X_A^i yields zero:

$$\delta_{(SU_2^{\text{diag}})} X_A^i = \frac{i}{2} \varepsilon_A^{\dot{A}} \lambda_{\dot{A}B}^{\dot{A}} X_B^i - \varepsilon_A^{\dot{A}} f^{\dot{A}ij} X_A^j = 0 \quad (3.27)$$

In Eq. (3.27) $\lambda_{\dot{A}B}^{\dot{A}}$ are the first three Gell-Mann λ matrices. Utilizing their explicit form and equations (3.19) we conclude that an X which is invariant under $SU(2)^{\text{diag}}$ has the form

$$X^{\dagger} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ a^p & b^p & 0 \\ ia^p & ib^p & 0 \\ b^p & a^p & 0 \\ -ib^p & a^p & 0 \\ 0 & 0 & \varphi^I \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \hline [a] & 0 \\ \hline 0 & \varphi \end{pmatrix} \quad (3.28)$$

where a^p , b^p are two v -dimensional complex vectors, and φ^I is μ -dimensional and belongs to the adjoint representation of N_K . This result justifies Eq. (3.7).

The next problem to solve is now to compute the explicit form of the spin-1/2 'shifts' in the direction of the third supersymmetry which is a singlet under $SU(2)^{\text{diag}}$. Splitting the $SU(3)$ -triplet index as follows: $A = (a, 3)$, and introducing the following auxiliary quantities,

$$\begin{aligned} \Theta_I &= f_{I(\mu p)(\nu q)} \begin{bmatrix} 1 \\ 4 \end{bmatrix}_{\mu p} \begin{bmatrix} 2 \\ 4 \end{bmatrix}_{\nu q} = \\ &= 2 \left(a^T \mathcal{G}_I a + b^T \mathcal{G}_I b + i a^T \mathcal{F}_I a - i b^T \mathcal{F}_I b + 2 i a^T \mathcal{Z}_I b - 2 a^T \mathcal{J}_I b \right) \end{aligned} \quad (3.29a)$$

$$\begin{aligned} \bar{Z}_I &= f_{I(\mu p)(\nu q)} \left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}_{\mu p} \right)^* \begin{bmatrix} 1 \\ 4 \end{bmatrix}_{\nu q} = \\ &= 2 \left(a^+ \mathcal{F}_I a + b^+ \mathcal{F}_I b + i a^+ \mathcal{Z}_I a - i b^+ \mathcal{Z}_I b + a^+ \mathcal{G}_I b \right. \\ &\quad \left. - b^+ \mathcal{G}_I a - i a^+ \mathcal{J}_I b - i b^+ \mathcal{J}_I a \right) \end{aligned} \quad (3.29b)$$

$$\mathcal{D}_I = h_{IJK} \varphi^J \bar{\varphi}^K \quad (3.29c)$$

after some lengthy manipulations we find that the spin-1/2 shifts defined by Eqs. (2.5b, c, d) take, on the submanifold $\bar{m}$, the following form:

$$\mathcal{U}^3 = \frac{1}{2} g \varphi^I \bar{Z}_I, \quad (3.30a)$$

$$N_{\dot{A}3} = N_{(\mu p)3} = 0, \quad (3.30b)$$

$$\begin{aligned} N_{I3} &= e \varphi_I \left[1 + 2(|a|^2 + |b|^2) \right] \\ &\quad - g \left[\Theta_I - \left(1 - \sqrt{1 + |\varphi|^2} \right) \frac{\varphi_I \bar{\varphi}_J \Theta_J}{|\varphi|^2} \right] \end{aligned} \quad (3.30c)$$

$$M_{\dot{A}3}^3 = M_{\dot{A}a}^3 = M_{Ia}^3 = M_{(\mu p)3}^3 = 0 \quad (3.30d)$$

$$M_{I3}^3 = g \left\{ \frac{1}{2} \mathcal{D}_I + \bar{Z}_I - \left(1 - \sqrt{1 + |\varphi|^2} \right) \frac{\varphi_I \bar{\varphi}_J \bar{Z}_J}{|\varphi|^2} \right\} \quad (3.30e)$$

$$M_{(\mu p)a}^3 = \varepsilon_{ab} \bar{\zeta}_{(\mu p)}^3 \left(e + g \frac{1 - \sqrt{1 + 2(|a|^2 + |b|^2)}}{2(|a|^2 + |b|^2)} \varphi_I \bar{\Theta}_I \right) - g \varphi^I \bar{f}_I(\mu p)(\nu q) \bar{\zeta}_{\nu q}^a \quad (3.30f)$$

Requiring that

$$\mathcal{U}^3 = N_{I3} = M_{I3}^3 = M_{(\mu p)a}^3 = 0 \quad (3.31)$$

we obtain a set of equations on φ^I , a^p , b^q whose solutions are the $N = 1$ extrema of the $N = 3$ potential. The equations read,

$$\varphi^I (\mathcal{F}_I a + i \mathcal{G}_I a + \mathcal{G}_I b - i \mathcal{F}_I b) = 0 \quad (3.32a)$$

$$\varphi^I (-\mathcal{F}_I b + i \mathcal{G}_I b + \mathcal{G}_I a + i \mathcal{F}_I a) = 0 \quad (3.32b)$$

$$e + g \frac{1 - \sqrt{1 + 2(|a|^2 + |b|^2)}}{2(|a|^2 + |b|^2)} \varphi_I \bar{\Theta}^I = 0 \quad (3.32c)$$

$$\varphi^I \bar{z}_I = 0 \quad (3.32d)$$

$$-\frac{1}{2} \mathcal{Q}_I - \bar{z}_I + \frac{1 - \sqrt{1 + |\varphi|^2}}{|\varphi|^2} \varphi_I \bar{\varphi}^J \bar{z}_J = 0 \quad (3.32e)$$

$$e \varphi_I \left[1 + 2(|a|^2 + |b|^2) \right] - g \Theta_I + g \frac{1 - \sqrt{1 + |\varphi|^2}}{|\varphi|^2} \varphi_I \bar{\varphi}_J \Theta_J = 0 \quad (3.32f)$$

The reason why the solutions of Eqs. (3.31) are automatically extrema of the potential is that in the submanifold $\bar{m}$ the gravitino mass matrix S_{AB} is diagonal and has the following form:

$$S_{AB} = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A + \Delta A \end{pmatrix} \quad (3.33a)$$

$$A = -\frac{i}{2} e^{\sqrt{1+|\varphi|^2}} \left(\sqrt{1+2(|a|^2+|b|^2)} \right)^2 \quad (3.33b)$$

$$\Delta A = \frac{i}{2} g \bar{\varphi}^I \Theta_I \quad (3.33c)$$

This ensures that the condition (1.10) is satisfied when Eqs. (3.31) are satisfied and guarantees that, solving Eqs. (3.32), we obtain $N = 1$ extrema of the potential.

In the next section we consider two explicit examples.

4. THE $N = 1$ EXTREMUM IN THE $K = SO(v+4)$ AND $K = E_6$ CASES

Let us begin with the $K = SO(v+4)$ example. According to Eq. (3.4) we must divide K by its subgroup $SO(m) \times SO(4)$, where $SO(4)$ is to be regarded as the product of two $SU(2)$. Hence

$$N_K = SU(2)^{\text{III}} \otimes SO(\nu) \quad (4.1)$$

The index i of the adjoint representation of $SO(v+4)$ is a pair of antisymmetric indices $i = (\alpha\beta) = -(\beta\alpha)$, where α and β take $(4+v)$ -values. The structure constants of $SO(v+4)$ are written in standard notation as:

$$f_{(\alpha\beta)(\gamma\delta)}^{(\eta\theta)} = -\delta_{\alpha\gamma} \delta_{\beta\delta}^{(\eta\theta)} - \delta_{\beta\delta} \delta_{\alpha\gamma}^{(\eta\theta)} + \delta_{\alpha\delta} \delta_{\beta\gamma}^{(\eta\theta)} + \delta_{\beta\gamma} \delta_{\alpha\delta}^{(\eta\theta)} \quad (4.2)$$

We subdivide the range of α in the following way:

$$\alpha = 1, 2, 3, 4, p \quad ; \quad p = 5, 6, \dots, 4+\nu \quad (4.3)$$

and we identify the first four values of α with the index μ of Eq. (3.17) according to the following scheme:

$$1\uparrow = 1 \quad ; \quad 1\downarrow = 3 \quad ; \quad 2\uparrow = 2 \quad ; \quad 2\downarrow = 4 \quad (4.4)$$

Moreover, we introduce the following new generators:

$$T_{\dot{A}} = (T_{\dot{1}}, T_{\dot{2}}, T_{\dot{3}}) \quad ; \quad T_{\ddot{A}} = (T_{\ddot{1}}, T_{\ddot{2}}, T_{\ddot{3}}) \quad (4.5)$$

where

$$T_{\dot{1}} = -\frac{1}{2} (T_{14} + T_{23}) \quad ; \quad T_{\ddot{1}} = \frac{1}{2} (T_{14} - T_{23}) \quad (4.6a)$$

$$T_{\dot{2}} = -\frac{1}{2} (T_{34} + T_{12}) \quad ; \quad T_{\ddot{2}} = \frac{1}{2} (T_{34} - T_{12}) \quad (4.6b)$$

$$T_{\dot{3}} = \frac{1}{2} (T_{24} + T_{31}) \quad ; \quad T_{\ddot{3}} = -\frac{1}{2} (T_{24} - T_{31}) \quad (4.6c)$$

Both sets of generators span an $SU(2)$ subalgebra which we identify respectively with the $SU(2)^{II}$ to be linearly combined with $SU(2)^I$ to form $SU(2)^{\text{diag}}$, and with $SU(2)^{III}$ sitting in N_K [see Eq. (4.1)].

Using this basis the Lie algebra of $SO(v+4)$ is in the desired form (3.19).

Since N_K is a direct product, we have two kinds of T_I generators, namely

$$T_I = \begin{cases} T_{\ddot{A}} \\ T_{pq} = -T_{qp} \end{cases} \quad (4.7)$$

The explicit form of the $\mathcal{V}_I, \mathcal{J}_I, \mathcal{G}_I, \mathcal{F}_I$ ($v \times v$) matrices is, in this case,

$$\mathcal{V}_{\dot{i}} = \mathcal{G}_{\dot{i}} = \mathcal{F}_{\dot{i}} = 0 ; \quad \mathcal{J}_{\dot{i}} = -\frac{1}{2} \delta_{pq} \quad (4.8a)$$

$$\mathcal{V}_{\dot{2}} = \mathcal{J}_{\dot{2}} = \mathcal{F}_{\dot{2}} = 0 ; \quad \mathcal{G}_{\dot{2}} = \frac{1}{2} \delta_{pq} \quad (4.8b)$$

$$\mathcal{J}_{\dot{3}} = \mathcal{G}_{\dot{3}} = \mathcal{F}_{\dot{3}} = 0 ; \quad \mathcal{V}_{\dot{3}} = \frac{1}{2} \delta_{pq} \quad (4.8c)$$

$$\mathcal{V}_{pq} = \mathcal{G}_{pq} = \mathcal{J}_{pq} = 0 ; \quad \mathcal{F}_{(pq)} = -\delta_{(pq)}^{(rs)} \quad (4.8d)$$

and the information of Eqs. (4.8) suffices to make Eqs. (3.32) explicit. We solve them with an ansatz. We assume that $v \geq 4$ and we introduce the v orthonormal vectors in v -space $\vec{e}_p$:

$$\vec{e}_p \cdot \vec{e}_q = \delta_{pq} \quad (4.9)$$

Then we set

$$a^p = z (e_1^p + i e_2^p) \quad (4.10a)$$

$$b^p = z (e_3^p + i e_4^p) \quad (4.10b)$$

where z is some complex number whose square we denote by m :

$$m = |z|^2 > 0 \quad (4.11)$$

The ansatz (4.10) guarantees that the following relations are true:

$$a^p b^p = a^p a^p = b^p b^p = \bar{a}^p b^p = 0 \quad (4.12a)$$

$$\bar{a}^P a^P = \bar{b}^P b^P = m \quad (4.12b)$$

Moreover, we set

$$\varphi^{\ddot{A}} = 0 \quad \varphi^{Pq} = k a^P b^q \quad (4.13)$$

where k is another number to be determined. With this ansatz, Eqs. (3.32a) and (3.32b) are automatically satisfied, and we get

$$\odot_{\ddot{A}} = 0 \quad \odot_{Pq} = 4 a_{[P} b_{q]} \quad (4.14a)$$

$$\vec{\Sigma}_{\ddot{A}} = 0 \quad \vec{\Sigma}_{Pq} = -2 \left(\bar{a}_{[P} a_{q]} + \bar{b}_{[P} b_{q]} \right) \quad (4.14b)$$

$$\mathcal{D}_{\ddot{A}} = 0 \quad \mathcal{D}_{Pq} = \frac{k^2 m}{4} \left(\bar{a}_{[P} a_{q]} + \bar{b}_{[P} b_{q]} \right) \quad (4.14c)$$

which yield

$$\varphi^I \vec{\Sigma}_I = \bar{\varphi}^I \vec{\Sigma}_I, \quad \bar{\varphi}^I \odot_I = 2 k m^2 \quad (4.15)$$

Inserting these results in Eqs. (3.32c), (3.32e), and (3.32f), one obtains the following system of equations:

$$4e = -g \left(1 - \sqrt{1 + 4m} \right) k m \quad (4.16a)$$

$$e k (1 + 4m) + 4g \sqrt{1 + 4m} = 0 \quad (4.16b)$$

$$k^2 m = 16 \quad (4.16c)$$

which can only be solved if e and g are in the ratio (3.9). The solution is:

$$m = \frac{1}{4} t^2 = \frac{1}{8} (1 + \sqrt{5}) ; k = 4 \sqrt{2(\sqrt{5} - 1)} \quad (4.17)$$

Inserting Eqs. (4.17) in Eqs. (3.33) and using Eq. (3.9), one obtains the results (3.10) and (3.11) for the unbroken gravitino apparent mass and for the cosmological term, respectively. The anti-de Sitter mass of the broken gravitinos is then easily calculated:

$$\begin{aligned} |m_{3/2}^{(broken)}|^2 &= |\Delta A|^2 = \frac{e^2}{16} (1 + \sqrt{5})^2 = \\ &= -\frac{1}{2} W \end{aligned} \quad (4.18)$$

Let us now discuss the example $K = E_6$. In this case

$$N_K = SU(6) \quad (4.19)$$

and the pseudoreal representation D is the 20 of $SU(6)$. The structure constants of E_6 are very efficiently read off from its Maurer-Cartan equations. In line with the decomposition (3.2), we write E_6 left-invariant 1-forms as follows:

$$\omega^\alpha_\beta = -\bar{\omega}_\beta^\alpha ; \quad \omega^\alpha_\alpha = 0, \quad \alpha = 1, 2 \quad (4.20a)$$

$$\Omega_i^j = -\bar{\Omega}_j^i ; \quad \Omega_i^i = 0, \quad i = 1, 2, \dots, 6 \quad (4.20b)$$

$$K_{ijk}^\alpha = \frac{1}{3} \varepsilon^{\alpha\beta} \varepsilon_{ijkpqz} \bar{K}_\beta^{pqz} \quad (4.20c)$$

where ω_β^α and Ω_i^j generate $SU(2)$ and $SU(6)$, respectively, and K_{ijk}^α are the missing $2 \times 20 = 40$ generators,

$$\underline{78} \xrightarrow{SU(2) \otimes SU(6)} (\underline{3}, \underline{1}) \oplus (\underline{1}, \underline{35}) \oplus (\underline{2}, \underline{20}) \quad (4.21)$$

The Maurer-Cartan equations read,

$$dK_{ijk}^\alpha + \omega^\alpha_\beta \wedge K_{ijk}^\beta + 3\Omega_{[i}^m \wedge K_{jk]m} = 0 \quad (4.22a)$$

$$d\omega^\alpha_\beta + \omega^\alpha_\gamma \wedge \omega^\gamma_\beta - \frac{1}{48} K_{pqz}^\alpha \wedge \overline{K}_\beta^{pqz} = 0 \quad (4.22b)$$

$$d\Omega_i^j + \Omega_i^k \wedge \Omega_k^j - \frac{1}{16} K_{imn}^\alpha \wedge \overline{K}_\alpha^{jmn} = 0 \quad (4.22c)$$

In order to obtain the structure constants in the form (3.19) we decompose the indices i, j , and k , running from 1 to 6 in the following way:

$$i = (\alpha, 6) \quad (4.23)$$

and we identify the real generators $T_{\mu[xy]}$ of the $(\underline{2}, \underline{20})$ with the real and imaginary parts of K_{xy6}^α according to the rule:

$$K_{xy6}^1 = T_{(1\downarrow)[xy]} + i T_{(1\uparrow)[xy]} \quad (4.24a)$$

$$K_{xy6}^2 = T_{(2\downarrow)[xy]} + i T_{(2\uparrow)[xy]} \quad (4.24b)$$

We also decompose the adjoint representation of $SU_6 \subset E_6$ into $SO(5)$ tensors,

$$\begin{aligned} \Omega_x^\gamma &= T^{[xy]} + i T^{\{xy\}} + i \delta^{xy} T^\otimes \\ \Omega_i^j &\begin{cases} \nearrow \Omega_x^\gamma = T^{[xy]} + i T^{\{xy\}} + i \delta^{xy} T^\otimes \\ \rightarrow \Omega_6^x = T^x + i T^{x'} \\ \searrow \Omega_6^6 = -5i T^\otimes \end{cases} \end{aligned} \quad (4.25)$$

and we set

$$\omega^\alpha{}_\beta = \frac{i}{2} (\sigma_{\dot{A}})^\alpha{}_\beta T^{\dot{A}} \quad (4.26)$$

The list of E_6 generators is, in this notation,

$$\begin{array}{ll} T_{\dot{A}} \Rightarrow \dot{A} & 3 \} \text{SU}(2) \text{ subgroup} \\ T_{[xy]} \Rightarrow [xy] & 10 \\ T_{\{xy\}} \Rightarrow \{xy\} & 14 \\ T_x \Rightarrow x & 5 \\ T_{x'} \Rightarrow x' & 5 \\ T_\otimes \Rightarrow \otimes & 1 \\ T_{\mu[xy]} \Rightarrow \mu[xy] & 40 \} \text{SU}(6) \text{ subgroup} \\ & E_6/\text{SU}(2) \otimes \text{SU}(6) \text{ coset} \end{array} \quad (4.27)$$

and from the Maurer-Cartan equations (4.22) we can read off the structure constants h_{IJK} and $\mathfrak{F}_I$, $\mathfrak{G}_I$, $\mathfrak{T}_I$, and $\mathfrak{V}_I$ listed in Table 1.

Using Table 1 we can solve Eqs. (3.32) with an ansatz similar to that utilized in the $SO(v+4)$ case.

In terms of the five orthonormal unit vectors $\vec{e}_x$ we introduce three complex vectors:

$$\begin{aligned} \vec{v} &= \sqrt{\frac{\mu}{2}} (\vec{e}_1 + i\vec{e}_2) ; & \vec{w} &= \sqrt{\frac{\mu}{2}} (\vec{e}_3 + i\vec{e}_4) \\ \vec{k} &= \sqrt{\nu} \vec{e}_5 ; \end{aligned} \quad (4.28)$$

which satisfy the relations

$$\left. \begin{aligned}
 v^x v^x &= v^x w^x = w^x w^x = v^x k^x = w^x k^x = \\
 &= v^x \bar{w}^x = v^x \bar{k}^x = w^x \bar{k}^x = 0 \\
 k^x k^x &\neq 0 \quad |v|^2 = |w|^2 = \mu \quad |k|^2 = \nu
 \end{aligned} \right\} \quad (4.29)$$

The ansatz for a^{xy} , b^{xy} , and φ^I is then

$$a^{xy} = v [x k^y] \quad ; \quad b^{xy} = w [x k^y] \quad (4.30a)$$

$$\varphi^x = \varphi^{x'} = \varphi^\otimes = 0 \quad (4.30b)$$

$$\varphi^{[xy]} = \alpha v [x w^y] \quad ; \quad \varphi^{\{xy\}} = \beta v \{x w^y\} \quad (4.30c)$$

Lengthy but straightforward manipulations lead to the evaluations of the auxiliary quantities Θ_I , Γ_I , and $\mathcal{D}_I$, which read:

$$\Theta_x = \Theta_{x'} = \Theta_\otimes = 0 \quad (4.31a)$$

$$\Theta_{[xy]} = 2 k_m k_m v [x w_y] \quad (4.31b)$$

$$\Theta_{\{xy\}} = 2i k_m k_m v [x w_y] \quad (4.31c)$$

$$\bar{\Sigma}_x = \bar{\Sigma}_{x'} = \bar{\Sigma}_\otimes = 0 \quad (4.31d)$$

$$\bar{\Sigma}_{[xy]} = \left(\bar{v}_x v_y + \bar{w}_x w_y \right) \nu \quad (4.31e)$$

$$\vec{Z}_{\{xy\}} = i \left(\bar{v}_{\{x} v_{y\}} - \bar{w}_{\{x} w_{y\}} \right) \nu \quad (4.31f)$$

$$\mathcal{D}_x = \mathcal{D}_{x'} = \mathcal{D}_\otimes = 0 \quad (4.31g)$$

$$\mathcal{D}_{[xy]} = 2 \left(|\alpha|^2 + |\beta|^2 \right) \mu \left(v_{[x} \bar{v}_{y]} + w_{[x} \bar{w}_{y]} \right) \quad (4.31h)$$

$$\mathcal{D}_{\{xy\}} = -2 \left(\alpha \bar{\beta} - \bar{\alpha} \beta \right) \mu \left(v_{\{x} \bar{v}_{y\}} - w_{\{x} \bar{w}_{y\}} \right) \quad (4.31i)$$

Inserting these results into Eqs. (3.32), one finds that Eqs. (3.32a, b, and d) are identically satisfied. The remaining equations are satisfied if

$$\beta = i\alpha \quad \alpha = \alpha^* \quad (4.32a)$$

$$\nu = \frac{\alpha^2 \mu}{2} \quad (4.32b)$$

$$\mu^2 \alpha^2 = \frac{1 + \sqrt{5}}{2} \quad (4.32c)$$

Moreover, the ratio e/g has to be as in Eq. (3.9). Computing the quantity $\varphi_I \bar{\theta}_I$ we get

$$\varphi_I \bar{\theta}_I = \mu^3 \alpha^3 = \left(\frac{1 + \sqrt{5}}{2} \right)^{3/2} \quad (4.33)$$

which again yields the values (3.10) and (3.11) for the gravitino apparent mass and the cosmological term.

Therefore, no difference exists between the two examples checked by us, and this encourages us to conjecture that for all quaternionic decomposable groups (3.4) there is a spontaneous breaking of $N = 3$ supersymmetry down to $N = 1$, with the universal condition (3.9) on the coupling constants, and the universal values (3.11) of the cosmological constant and (4.18) of the broken gravitino masses.

Table 1

 E_6 structure constants

$$h_{[xy][zw][st]} = 2 \left(\delta_{wt} \delta_{zs}^{xy} + \delta_{zs} \delta_{wt}^{xy} - \delta_{ws} \delta_{zt}^{xy} - \delta_{zt} \delta_{ws}^{xy} \right)$$

$$h_{[xy]wt} = -2 \delta_{wt}^{xy}$$

$$h_{[xy]\{zw\}\{st\}} = 2 \left(\delta_{wt} \delta_{zs}^{xy} + \delta_{zs} \delta_{wt}^{xy} + \delta_{zt} \delta_{ws}^{xy} + \delta_{ws} \delta_{zt}^{xy} \right)$$

$$h_{w\{xy\}t'} = -\frac{1}{2} \left(\delta_{wx} \delta_{yt} + \delta_{wy} \delta_{xt} \right)$$

$$h_{[xy]w't'} = -2 \delta_{xy}^{wt}$$

$$h_{\otimes xy'} = \frac{2}{5} \delta_{xy}$$

$$f_{\otimes} = g_{\otimes} = s_{\otimes} = 0 \quad \tau_{\otimes} = -\frac{2}{5} \delta_{xy}^{wt}$$

$$g_{[xy]} = \tau_{[xy]} = s_{[xy]} = 0 \quad f_{[xy][zw][st]} = \frac{1}{4} h_{[xy][zw][st]}$$

$$f_{\{xy\}} = g_{\{xy\}} = s_{\{xy\}} = 0$$

$$f_{\{xy\}[zw][st]} = \frac{1}{2} \left(\delta_{zw}^{xt} \delta_{ys} + \delta_{zw}^{yt} \delta_{xs} - \delta_{zw}^{ys} \delta_{xt} - \delta_{zw}^{xs} \delta_{yt} \right) - \frac{2}{5} \delta_{xy}^{st} \delta_{zw}$$

$$f_x = \tau_x = s_x = 0 \quad g_{x[zw][st]} = -\frac{1}{4} \varepsilon_{xzwst}$$

$$f_{x'} = \tau_{x'} = g_{x'} = 0 \quad s_{x'[zw][st]} = -\frac{1}{4} \varepsilon_{x'zwst}$$

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