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STOCHASTIC REGULARIZATION OF QED

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ABSTRACT

Stochastic quantization and stochastic regularization of QED is studied using gauge and chiral covariant Langevin equations. It is shown that all naive quadratic divergences of the photon vacuum polarization tensor cancel at the one-loop level, as they should. The leading logarithmically divergent contribution is then transverse, and the masslessness of the photon is easily checked. If Zwanziger's stochastic gauge fixing is used, the transversality of the vacuum polarization tensor is lost, thus showing its incompatibility with stochastic regularization. Furthermore, a simple method for obtaining the chiral anomaly in a massive theory at large fictitious time is developed.

1. - INTRODUCTION

The stochastic quantization method of Parisi and Wu¹⁾ provides an interesting alternative to the standard quantization of field theories. Moreover, it allows for a new regularization method²⁾⁻⁵⁾, resulting from a specific non-Markovian generalization of the Parisi-Wu scheme. This stochastic regularization method is a priori very promising insofar as it preserves the four-dimensional symmetries of the unregularized theory.

This paper discusses the consistency and usefulness of the stochastic regularization scheme when applied to QED. More precisely, the questions of gauge and chiral invariance are tackled. The main conclusion we can draw concerning gauge invariance is that it is imperative to use gauge covariantly-transforming Langevin equations both for the fermions and for the gauge fields, thus excluding Zwanziger's stochastic gauge-fixing procedure⁶⁾ within the framework of the stochastic regularization scheme.

The problems with chiral invariance have been previously discussed⁷⁾ regarding the impossibility of implementing a chiral theory at finite fictitious time. We develop here a perturbative method which allows us to get the anomaly in the $t \rightarrow \infty$ limit of the fictitious time for a theory with massive fermions.

In Section 2, we discuss our covariant Langevin equations and generalize methods^{8),9)} known so far for the unregularized case, in order to calculate quickly the $t \rightarrow \infty$ limit of stochastically regularized stochastic diagrams.

In Section 3, we calculate the divergent part of the photon vacuum polarization tensor at the one-loop level. Transversality is proved and the masslessness of the photon is checked to this order, as at zero external momentum the photon vacuum polarization is shown to vanish. The same calculation performed using Zwanziger's stochastic gauge fixing results in a loss of transversality for the vacuum polarization tensor (however, the photon would still remain massless). In consequence, the incompatibility of the stochastic regularization and stochastic gauge fixing is shown at the one-loop level. An elegant argument¹⁰⁾ has recently remarked the same incompatibility at two or more loops.

In Section 4, we develop a simple method for calculating the chiral anomaly at infinite fictitious time for a theory with massive fermions. The calculation is done perturbatively, with the gauge field considered as an external background field.

2. - STOCHASTIC QUANTIZATION AND REGULARIZATION OF QED

Let us consider the Euclidean action for QED:

$$S = \int d^4x \left[\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\Psi} i (\not{D} - ie \not{A} + im) \Psi \right] \quad (2.1)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2.2)$$

We take ψ and $\bar{\psi}$ as independent Grassmann variables with

$$\{\chi_\mu, \chi_\nu\} = -2 \delta_{\mu\nu} \quad (2.3)$$

Quantization of QED within the stochastic framework of Parisi and Wu requires special care due to the positive and negative eigenmodes of the Dirac operator, which would lead to exponentially increasing correlation functions in the fictitious time. One is therefore led^{2),11)} to introduce non-trivial kernel operators $K(x,y)$ which effectively bosonize the drift terms in the fermionic Langevin equations and guarantee relaxation to equilibrium for large t . The introduction of the kernel operators has to be accompanied by a corresponding modification of the noise structure of the fermionic Langevin equations and, preferring to double the number of fermionic noise terms^{12),13),7)}, we arrive generally at the following set of coupled Langevin equations describing QED:

$$\begin{aligned} \dot{\Psi} &= + \int d^4y \, K(x,y) \left[i (\not{D} + im) \Psi + \eta_1 \right] (y,t) + \eta_2(x,t) \\ \dot{\bar{\Psi}} &= + \int d^4y \, \bar{K}(x,y)^T \left[i (\not{D}' + im) \bar{\Psi} + \bar{\eta}_2 \right] (y,t) + \bar{\eta}_1(x,t) \\ \dot{A}_\mu &= \partial_\nu F_{\nu\mu} + e \bar{\Psi} \gamma_\mu \Psi + \eta_\mu \end{aligned} \quad (2.4)$$

Let us recall for completeness that t denotes the fictitious time co-ordinate and that $x = (x_0, x_1, x_2, x_3)$; T means matrix transposition. In the above equations, we introduced

$$\begin{aligned} D &\equiv \partial - ie A \\ D' &\equiv -\partial - ie A \end{aligned} \quad (2.5)$$

$\eta_1, \bar{\eta}_1, \eta_2, \bar{\eta}_2$ represent Gaussian, Grassmann noises and stochastic regularization is implemented by

$$\langle \eta_i(x, t) \rangle = \langle \bar{\eta}_i(x, t) \rangle = 0; i=1, 2$$

$$\langle \eta_i(x, t) \bar{\eta}_k(x', t') \rangle = 2 \delta_{ik} \delta^4(x-x') \alpha_\Lambda(t-t') \quad (2.6)$$

where we choose as stochastic regulator^{2), 5)}

$$\alpha_\Lambda(t) = -\Lambda^4 \frac{\partial}{\partial \Lambda^2} \left(\frac{\alpha_\Lambda^s(t)}{\Lambda^2} \right) \quad (2.7)$$

$$\alpha_\Lambda^s(t) = \frac{\Lambda^2}{2} e^{-\Lambda^2 |t|} \quad (2.8)$$

which obeys

$$\lim_{\Lambda \rightarrow \infty} \alpha_\Lambda(t) = \delta(t) \quad (2.9)$$

Similarly we could modify the bosonic white noise correlations of the gauge field

$$\langle \eta_\mu(x, t) \rangle = 0$$

$$\langle \eta_\mu(x, t) \eta_\nu(x', t') \rangle = 2 \delta_{\mu\nu} \delta^4(x-x') \delta(t-t') \quad (2.10)$$

but we refrain from doing so, as we will not need it for the calculations in this paper.

There is an apparent freedom in the choice of the kernel operator: the simplest one is given by¹¹⁾

$$\begin{aligned} K(x, y) &= i (\not{\partial}_x - i m) \delta(x-y) \\ \bar{K}(x, y) &= i (-\not{\partial}_x - i m) \delta(x-y) \end{aligned} \quad (2.11)$$

However, gauge covariance of the fermionic Langevin equations is lost in this case. One could naively argue that this would not matter in the equilibrium limit $t \rightarrow \infty$ (where in the unregularized case the usual field theory with all its symmetries is recovered), but this is not so in the framework of stochastic regularization; calculating the photon vacuum polarization tensor to one loop, one indeed finds a quadratic divergence in Δ , which may be compared with the similar result obtained when using a naive momentum cut-off as regulator. Quadratic divergences in Δ and further inconsistencies of the choice (2.11) for QED have also been reported in Ref. 14). We conclude that the choice (2.11) is not acceptable for the purpose of stochastic regularization.

It seems immediately favourable to generalize (2.11) to the covariant choice^{12),7)}

$$\begin{aligned} K(x, y) &= i (\not{D}_x - im) \delta(x-y) \\ \bar{K}(x, y) &= i (\not{D}_x' - im) \delta(x-y) \end{aligned} \quad (2.12)$$

so that QED is described by the following gauge and chiral (for $m = 0$) covariant Langevin equations

$$\begin{aligned} \dot{\Psi} &= -(\not{D} - im)(\not{D} + im)\Psi + i(\not{D} - im)\eta_1 + \eta_2 \\ \dot{\bar{\Psi}} &= -(\not{D}' - im)^T (\not{D}' + im)^T \bar{\Psi} + \bar{\eta}_1 + i(\not{D}' - im)^T \bar{\eta}_2 \\ \dot{A}_\mu &= 2\nu F_{\nu\mu} + e \bar{\Psi} \gamma_\mu \Psi + \eta_\mu \end{aligned} \quad (2.13)$$

In the rest of this paper, we will use this above set of coupled equations and discuss its usefulness and consistency.

After standard manipulations¹⁾, we find immediately the following set of integral equations, corresponding to (2.13) in momentum space, as:

$$\left\{ \begin{aligned} \Psi(k,t) = & \int_0^t d\tau e^{-(k^2+m^2)(t-\tau)} \left\{ (K+m) \eta_1(k,\tau) + \eta_2(k,\tau) \right. \\ & + \frac{e}{(2\pi)^4} \int dp dq \delta(k-p-q) \left[(\not{p} \not{A}(p,\tau) - 2 q_\nu A_\nu(p,\tau)) \Psi(q,\tau) + \not{A}(p,\tau) \eta_1(q,\tau) \right] \\ & \left. - \frac{e^2}{(2\pi)^8} \int dp dq dr \delta(k-p-q-r) A_\nu(p,\tau) A_\nu(q,\tau) \Psi(r,\tau) \right\} \end{aligned} \right. \quad (2.14)$$

$$\left\{ \begin{aligned} \bar{\Psi}(k,t) = & \int_0^t d\tau e^{-(k^2+m^2)(t-\tau)} \left\{ \bar{\eta}_1(k,\tau) + \bar{\eta}_2(k,\tau) (K+m) \right. \\ & + \frac{e}{(2\pi)^4} \int dp dq \delta(-k-p+q) \left[\bar{\Psi}(q,\tau) (-\not{A}(p,\tau) \not{p} - 2 q_\nu A_\nu(p,\tau)) + \bar{\eta}_2(q,\tau) \not{A}(p,\tau) \right] \\ & \left. - \frac{e^2}{(2\pi)^8} \int dp dq dr \delta(k-p-q+r) A_\nu(p,\tau) A_\nu(q,\tau) \bar{\Psi}(r,\tau) \right\} \end{aligned} \right. \quad (2.15)$$

$$\left\{ \begin{aligned} A_\mu(k,t) = & \int_0^t d\tau \left[e^{-k^2(t-\tau)} T_{\mu\nu} + L_{\mu\nu} \right] \left\{ \eta_\nu(k,\tau) + \right. \\ & \left. + \frac{e}{(2\pi)^4} \int dp dq \delta(k+p-q) \bar{\Psi}(p,\tau) \gamma_\mu \Psi(p,\tau) \right\} \end{aligned} \right. \quad (2.16)$$

In (2.16) we used the projection operators

$$T_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$L_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} \quad (2.17)$$

With $\alpha_\Lambda^S(t)$ from (2.8), we then easily find to lowest order

$$\langle \psi(k, t) \bar{\psi}(k', t') \rangle^S = (2\pi)^4 \delta(k - k') (k + m) \cdot$$

$$\frac{\Lambda^2}{(k^2 + m^2)^2 - \Lambda^4} \left[-\frac{\Lambda^2}{k^2 + m^2} e^{-(k^2 + m^2)|t - t'|} + e^{-\Lambda^2|t - t'|} \right] + \dots \quad (2.18)$$

$$\langle \eta_1(k, t) \bar{\psi}(k', t') \rangle^S = \langle \psi(k, t) \bar{\eta}_2(k', t') \rangle^S =$$

$$= \frac{(2\pi)^4}{2} \delta(k - k') \frac{\Lambda^2}{k^2 + m^2 - \Lambda^2} \left[-\frac{2\Lambda^2}{k^2 + m^2 + \Lambda^2} e^{-(k^2 + m^2)|t - t'|} + e^{-\Lambda^2|t - t'|} \right] + \dots \quad (2.19)$$

where the dots indicate exponentially suppressed contributions. Diagrammatically we represent (2.18) and (2.19) by lines with a cross. For convenience of notation we substitute $k^2 + m^2 \rightarrow k^2$ for all momenta in the following. As already remarked in the Introduction, the most appealing feature of stochastic regularization lies in the fact²⁾ that no symmetries of the theory under consideration are broken by the regulator $\alpha_\Lambda(t)$ as the δ -function in the x -dependent part of the noise-noise correlation guarantees their conservation. Unfortunately, the price one has to pay is a non-Markovian stochastic process, of which the equilibrium limit (in sharp contrast to the unregularized case) is not explicitly known, since simple Focker-Planck formulations are lost^{2), 10)}. Moreover, perturbative calculations become rather involved due to the non-Markovian structure (2.8).

In the following, we generalize methods^{8), 9)} known from the unregularized case in order to find easily the large time limit (of sums) of stochastically regularized stochastic diagrams: we observe that $\langle \phi \bar{\phi} \rangle$ as well as the $\langle \eta_1 \bar{\psi} \rangle$ and $\langle \phi \bar{\eta}_2 \rangle$ correlations (2.18) and (2.19) each contain two terms, having different fictitious time dependence in the exponents. Let us therefore split a given stochastic diagram where (2.18) and (2.19) are involved in all parts, such that in crossed (fermionic) lines only single terms of (2.18) and (2.19) appear. Now we may introduce fictitious time orderings and construct the large time limit of the stochastically regulated diagrams as in Ref. 8): the only difference emerging is that now there are contributions in the denominators which do not depend on (momenta)² alone, but also on Λ^2 , corresponding to a part of (2.18) or (2.19) which involved $e^{-\Lambda^2 t}$.

As an example, let us consider a specific contribution to the one-loop correction to $\langle A_\mu A_\nu \rangle^S$ as depicted in Fig. 1 [here again, the index s means regularization with $\alpha_\Lambda^S(t)$]. Let us further assume that we are interested just in the transverse part of the photon contribution (2.16), which is in fact the case when calculating a gauge-invariant quantity in QED (see, for instance, Section 3 for the example of the vacuum polarization tensor).

The stochastic diagram of Fig. 1 contains one regulated (fermion) line and is split according to (2.18) into two parts. The fictitious time integrations then immediately give for $t \rightarrow \infty$ ⁸⁾ in the first case

$$\frac{1}{k^2 + p^2 + q^2} \cdot \frac{1}{2k^2} \quad (2.20a)$$

whereas in the second case

$$\frac{1}{k^2 + \Lambda^2 + q^2} \cdot \frac{1}{2k^2} \quad (2.20b)$$

Thus the large time limit of the stochastic diagram of Fig. 1 follows by using (2.14), (2.15), (2.16) and (2.18) as

$$\int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \delta(k-p+q) T_{\mu\rho} T_{\nu\sigma} \text{tr} \{ \not{p} \gamma_\rho (\not{k} \gamma_\sigma - 2\not{p}) \} \cdot \frac{1}{k^2} \left(\frac{1}{k^2 + p^2 + q^2} \frac{1}{2k^2} \frac{(-\Lambda^4)}{(p^4 - \Lambda^4)p^2} + \frac{1}{k^2 + \Lambda^2 + q^2} \frac{1}{2k^2} \frac{\Lambda^2}{p^4 - \Lambda^4} \right) \quad (2.21)$$

Another simplification which one should make use of is to use a recursion relation⁹⁾ to calculate the contribution of sums of stochastic diagrams, and not sum them up separately. In combination with the above-mentioned method, the large t limit of sums of stochastically regularized stochastic diagrams can be found fairly easily (see, e.g., the next section). We remark finally that one may apply the above simplifications in a similar way if the photon lines are regularized as well.

3. - THE PHOTON VACUUM POLARIZATION

Let us recall at the beginning of this section an open problem in stochastic quantization, namely that due to the lack of an action principle, no direct Noether procedure to construct conserved currents has been found²⁾. Checking, for example, gauge invariance of the stochastic regularization scheme at the one-loop level, one relies rather on indirect methods, such as proving the transversality of the photon vacuum polarization tensor $\Pi_{\mu\nu}(k)$, which has to be extracted from the one-loop corrections to the gauge field propagator [in a similar way as for standard QED or as in Ref. 5)].

To be more specific, we will calculate in the following the divergent part of $\Pi_{\mu\nu}$ at the one-loop level. For this purpose, it is consistent to omit the longitudinal contribution in the gauge field integral equation (2.16) and, moreover, substitute $T_{\mu\nu} \rightarrow \delta_{\mu\nu}$. In fact, we will prove transversality of the divergent part of $\Pi_{\mu\nu}$ by using stochastic regularization, so that all longitudinal contributions to the one-loop correction of the gauge field propagator are projected away, leaving just the Landau gauge result. In fact, Bern's calculation⁵⁾ of $\Pi_{\mu\nu}$ in scalar QED effectively relied on the same features, though he actually used the standard gauge-fixing procedure, choosing the Feynman gauge.

As mentioned in the last section, calculations in the framework of stochastic quantization can conveniently be shortened by using a recursion method of summing stochastic subdiagrams. In our case, the application of this method requires a calculation of the "effective" stochastic vertex

$$V_{\mu}^s(p_1, p_2, p_3) = \lim_{t \rightarrow \infty} \langle \Psi(p_1, t) A_{\mu}(p_2, t) \bar{\Psi}(p_3, t) \rangle^s \quad (3.1)$$

where the index s again means regularization with respect to $\alpha_{\Lambda}^s(t)$ (2.8). The diagrams contributing are depicted in Fig. 2. Let us introduce

$$\begin{aligned} \Sigma &\equiv p_1^2 + p_2^2 + p_3^2 & \Sigma_3 &\equiv p_1^2 + p_2^2 + \Lambda_3^2 \\ \Sigma_1 &\equiv \Lambda_1^2 + p_2^2 + p_3^2 & \Sigma_{13} &\equiv \Lambda_1^2 + p_2^2 + \Lambda_3^2 \end{aligned} \quad (3.2)$$

With the methods of Section 2, we easily calculate the $t \rightarrow \infty$ limits of these diagrams and have, by omitting the overall factor $(2\pi)^4 \delta(p_1+p_2-p_3)$

$$(a) = \cancel{p}_1 \frac{\Lambda_1^2}{p_1^4 - \Lambda_1^4} \frac{\Lambda_3^2}{p_3^4 - \Lambda_3^4} \left[\frac{\Lambda_1^2 \Lambda_3^2}{p_1^2 p_3^2} \frac{1}{\Sigma} - \frac{\Lambda_1^2}{p_1^2} \frac{1}{\Sigma_3} - \frac{\Lambda_3^2}{p_3^2} \frac{1}{\Sigma_1} + \frac{1}{\Sigma_{13}} \right] \gamma_\mu \cancel{p}_3 \quad (3.3)$$

$$(b) = \frac{1}{p_2^2} (-\cancel{p}_2 \gamma_\mu - 2 p_{3\mu}) \cancel{p}_3 \frac{\Lambda_3^2}{p_3^4 - \Lambda_3^4} \left(-\frac{\Lambda_3^2}{p_3^2} \frac{1}{\Sigma} + \frac{1}{\Sigma_3} \right) \quad (3.4)$$

$$(c) = \frac{1}{p_2^2} \cancel{p}_1 \frac{\Lambda_1^2}{p_1^4 - \Lambda_1^4} \left(-\frac{\Lambda_1^2}{p_1^2} \frac{1}{\Sigma} + \frac{1}{\Sigma_1} \right) (\gamma_\mu \cancel{p}_2 - 2 p_{1\mu}) \quad (3.5)$$

$$(d) = \frac{\gamma_\mu}{p_2^2} \frac{1}{2} \frac{\Lambda_3^2}{p_3^2 - \Lambda_3^2} \left(-\frac{2\Lambda_3^2}{p_3^2 + \Lambda_3^2} \frac{1}{\Sigma} + \frac{1}{\Sigma_3} \right) \quad (3.6)$$

$$(e) = \frac{\gamma_\mu}{p_2^2} \frac{1}{2} \frac{\Lambda_1^2}{p_1^2 - \Lambda_1^2} \left(-\frac{2\Lambda_1^2}{p_1^2 + \Lambda_1^2} \frac{1}{\Sigma} + \frac{1}{\Sigma_1} \right) \quad (3.7)$$

Having calculated the effective stochastic vertex V_μ^S , we can immediately find [see Ref. 9] for a similar application] the $t \rightarrow \infty$ limit of all stochastically regularized stochastic diagrams contributing to the one-loop correction of the photon propagator (having omitted the tensor structure of the gauge field) as

$$\langle A_\mu A_\nu \rangle = \int \frac{d^4 p_1 d^4 p_3}{(2\pi)^4} \delta(p_1 + p_2 - p_3) \frac{1}{p_2^2} \cdot \Lambda_1^4 \Lambda_3^4 \frac{\partial}{\partial \Lambda_1^2} \frac{\partial}{\partial \Lambda_3^2} \text{tr} \left\{ \gamma_\nu V_\mu^S(p_1, p_2, p_3) \frac{1}{\Lambda_1^2 \Lambda_3^2} \right\} \quad (3.8)$$

Keeping just potentially (i.e., for $\Lambda \rightarrow \infty$) diverging contributions, we then identify $\Pi_{\mu\nu}$ as

$$\begin{aligned} \Pi_{\mu\nu}(k) = & \int \frac{d^4 p}{(2\pi)^4} \left\{ \text{tr} [\gamma_\nu \not{p} \gamma_\mu (\not{p} + k)] \cdot k^2 \Lambda^8 \cdot \right. \\ & \cdot \frac{\partial}{\partial \Lambda_1^2} \frac{\partial}{\partial \Lambda_3^2} \frac{1}{p^2 + \Lambda_1^2} \frac{1}{p^2 + \Lambda_3^2} \frac{1}{p^2} \frac{1}{(p+k)^2} \frac{1}{\Sigma} \Big|_{\Lambda_1 = \Lambda_3 = \Lambda} \\ & - 2 \text{tr} [\gamma_\nu \not{p} (\gamma_\mu k - 2p_\mu)] \Lambda^4 \frac{\partial}{\partial \Lambda^2} \frac{2p^2 + \Lambda^2}{\Sigma p^2 (p^2 + \Lambda^2)^2} \\ & \left. - 2 \text{tr} (\gamma_\mu \gamma_\nu) \Lambda^4 \frac{\partial}{\partial \Lambda^2} \frac{1}{\Sigma (p^2 + \Lambda^2)} \right\} \quad (3.9) \end{aligned}$$

where we partly performed a shift $p_1 \rightarrow -p_1 - p_2$ for simplification and finally put $p_1 = p$, $p_2 = k$. Evaluating the traces, differentiating and reinserting the mass terms explicitly (at places of relevance only), we get

$$\Pi_{\mu\nu}(k) = A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} \quad (3.10)$$

with

$$\begin{aligned} A_{\mu\nu} = & \int \frac{d^4 p}{(2\pi)^4} \frac{-[p^2 + (pk)] \delta_{\mu\nu} + 2p_\mu p_\nu + p_\mu k_\nu + p_\nu k_\mu}{\Sigma (p^2 + m^2) [(p+k)^2 + m^2]} \frac{4\Lambda^8 k^2}{(p^2 + \Lambda^2)^4} \\ B_{\mu\nu} = & \int \frac{d^4 p}{(2\pi)^4} \frac{2p_\mu p_\nu - (pk) \delta_{\mu\nu} + p_\mu k_\nu + p_\nu k_\mu}{(p^2 + m^2) \Sigma} \cdot \frac{8\Lambda^4 (3p^2 + \Lambda^2)}{(p^2 + \Lambda^2)^3} \\ C_{\mu\nu} = & - \int \frac{d^4 p}{(2\pi)^4} \frac{\delta_{\mu\nu} 8\Lambda^4}{\Sigma (p^2 + \Lambda^2)^2} \end{aligned} \quad (3.11)$$

By power counting, we expect the following types of divergences:

$$\delta_{\mu\nu} k^2 \log \Lambda^2, \quad k_\mu k_\nu \log \Lambda^2 \quad (3.12)$$

and

$$\delta_{\mu\nu} \Lambda^2, \quad \delta_{\mu\nu} m^2 \log \Lambda^2 \quad (3.13)$$

Let us first study the k -dependent, logarithmically-divergent part of $\Pi_{\mu\nu}$. Since it is a polynomial in k of degree 2, we may distinguish three cases:

- a) there is in (3.11) an explicit $\delta_{\mu\nu} k^2$ or $k_\mu k_\nu$ factor, so that we may set $k = 0$ in the rest of the corresponding loop integration, as all k -dependence has already been found;
- b) there is an explicit k_μ or k_ν factor in (3.11); denoting the diverging part of $\Pi_{\mu\nu}$, of which we omitted the above k_μ or k_ν factor, by $\Pi_{\mu\nu}^{1,div}$, it then holds that

$$\Pi_{\mu\nu}^{1,div} = k^\sigma \frac{\partial}{\partial k^\sigma} \Pi_{\mu\nu}^{1,div} \quad (3.14)$$

or effectively

$$\frac{1}{\Sigma} \rightarrow - \frac{(pk)}{2(p^2+m^2)^2} \quad (3.15)$$

- c) there is no explicit k -factor in (3.11); in this case the diverging part of $\Pi_{\mu\nu}$ fulfils

$$\Pi_{\mu\nu}^{2,div} = \frac{1}{2} k^\sigma \frac{\partial}{\partial k^\sigma} \Pi_{\mu\nu}^{2,div} \quad (3.16)$$

or effectively

$$\frac{1}{\Sigma} \rightarrow \frac{1}{2} \left[\frac{(pk)^2}{(p^2+m^2)^3} - \frac{k^2}{(p^2+m^2)^2} \right] \quad (3.17)$$

Using furthermore

$$\int d^4 p f(p^2) p_\mu p_\nu (pk)(pk) = \frac{1}{24} \int d^4 p f(p^2) p^4 [\delta_{\mu\nu} k^2 + 2 k_\mu k_\nu] \quad (3.18)$$

we easily find to leading order, with

$$I = \frac{1}{(2\pi)^4} \int d^4 p \frac{\Lambda^2}{p^2 (p^2 + m^2) (p^2 + \Lambda^2)} = \frac{1}{(2\pi)^4} \log \Lambda^2 + \dots \quad (3.19)$$

that

$$\begin{aligned} A_{\mu\nu} &= -k^2 \delta_{\mu\nu} I \\ B_{\mu\nu} &= -\frac{2}{3} (\delta_{\mu\nu} k^2 + 2 k_\mu k_\nu) I \\ C_{\mu\nu} &= 3 k^2 \delta_{\mu\nu} I \end{aligned} \quad (3.20)$$

Thus the k -dependent, divergent part of $\Pi_{\mu\nu}$ adds up to the transverse result

$$T_{\mu\nu}^{\text{div}}(k) = \frac{4}{3} (k^2 \delta_{\mu\nu} - k_\mu k_\nu) I \quad (3.21)$$

On the contrary, with Zwanziger's gauge-fixing procedure⁶⁾, taking care of the additional terms in the fermionic Langevin equations [see, e.g., Ref. 10) for a discussion of scalar QED], we would get an additional contribution to $B_{\mu\nu}$, namely

$$\int \frac{d^4 p}{(2\pi)^4} \frac{\text{tr}\{\gamma_\nu \not{p}\} k_\mu 2 \Lambda^4 (3p^2 + \Lambda^2)}{(p^2 + \Lambda^2)^3 \sum (p^2 + m^2)} \quad (3.22)$$

which clearly violates transversality.

What still remains to be shown is that the photon does not acquire a mass at the one-loop level, which is guaranteed if the vacuum polarization tensor is vanishing for external zero momentum. This can indeed be seen from

$$\begin{aligned} \Pi_{\mu\nu}(0) = & \int \frac{d^4 p}{(2\pi)^4} \frac{4 \delta_{\mu\nu} \Lambda^4}{(p^2 + \Lambda^2 + m^2)^2 (p^2 + m^2)} \cdot \\ & \cdot \left[\frac{p^2 (3p^2 + \Lambda^2 + m^2)}{(p^2 + m^2)(p^2 + \Lambda^2 + m^2)} - 2 \right] \end{aligned} \quad (3.23)$$

and a few straightforward manipulations [compare also with the similar result in Ref. 5)].

4. - THE CHIRAL ANOMALY

In this section, we apply stochastic regularization to calculate easily the chiral anomaly of QED. The calculation is done perturbatively, with the gauge field considered as an external background field (which does not depend on the fictitious time and has no own Langevin equation). We consider massive fermions and first perform the $t \rightarrow \infty$ limit in our calculations before finally allowing $m = 0$ [see Ref. 7) for a critical discussion].

In the light of our remarks at the beginning of Section 3, we have nothing new to say on how to define the axial vector current properly within the stochastic framework. We rather study the standard axial vector current derivative [thus doing no better than Refs. 7), 15), 14) and 16)] and show how to obtain the correct anomalous contribution. Further investigation along these lines clearly seems desirable. To be specific, we have

$$\dot{j}_\mu^5(x, t) = \bar{\Psi}(x, t) \gamma_\mu \gamma_5 \dot{\Psi}(x, t) \quad (4.1)$$

and

$$\langle \partial_\mu \dot{j}_\mu^5 \rangle = \langle (\not{\partial}^T \bar{\Psi}) \gamma_5 \dot{\Psi} \rangle - \langle \bar{\Psi} \gamma_5 \not{\partial} \dot{\Psi} \rangle \quad (4.2)$$

We could now iterate straightforwardly our Langevin equations (2.14) and (2.15) and calculate the $t \rightarrow \infty$ limit of the (several) corresponding, stochastically regularized stochastic diagrams. We prefer, however, to use these Langevin equations to replace $\not{\partial} \psi$ and $\not{\partial}^T \bar{\psi}$ by

$$\not{D}\psi = -(\not{D}-im)^{-1}\dot{\psi} - i\gamma_1 - (\not{D}-im)^{-1}\not{h}_2 - im\psi + ie\not{A}\psi \quad (4.3)$$

and similarly for $\not{D}^T\bar{\psi}$, to get

$$\begin{aligned} \langle \partial_\mu j_\mu^5 \rangle = & \langle [(\not{D}'-im)^T{}^{-1}\dot{\bar{\psi}}] \gamma_5 \psi \rangle + \\ & + \langle \bar{\psi} \gamma_5 (\not{D}-im)^{-1}\dot{\psi} \rangle + 2im \langle \bar{\psi} \gamma_5 \psi \rangle + \\ & + \langle \{i\bar{h}_1 + [(\not{D}'-im)^T{}^{-1}\bar{h}_2]\} \gamma_5 \psi \rangle + \\ & + \langle \bar{\psi} \gamma_5 (i\gamma_4 + (\not{D}-im)^{-1}h_2) \rangle \end{aligned} \quad (4.4)$$

The first two terms on the right-hand side of the equation above vanish for $t \rightarrow \infty$ due to the appearance of fictitious time derivatives¹²⁾, so we get as the anomalous contribution

$$A = 2 \langle \{i\bar{h}_1 + [(\not{D}'-im)^T{}^{-1}\bar{h}_2]\} \gamma_5 \psi \rangle \quad (4.5)$$

Now it is interesting to remark that in a perturbative expansion of $\psi, \bar{\psi}$ there is just one term contributing to A: if we do not iterate $\psi, \bar{\psi}$ often enough, the trace of γ -matrices, involving γ_5 , gives zero; if we iterate too often, then the stochastic regulator leads to a vanishing result for $\Lambda \rightarrow \infty$.

Let us remind the reader that this phenomenon is very similar to what happens when using Schwinger's point-splitting method¹⁷⁾ as a regularization scheme, and where also only one diagram out of infinitely many contributed to the anomaly.

Iterating thus $\psi, \bar{\psi}$ to the second order in e , using

$$\int (\not{D}'_x - im)^T{}^{-1} (\not{D}'_{x'} - im) \delta(x-x') dx = 1 \quad (4.6)$$

everything which contributes to A is given by

$$A = \frac{e^2}{(2\pi)^{12}} i \, 2 \int d^4y \, d^4y' \, \text{tr} \{ \gamma_5 \not{\partial}_y A(y) \not{\partial}_{y'} A(y') \} \cdot B(\Lambda^2, x, y, y') \quad (4.7)$$

where

$$B(\Lambda^2, x, y, y') = - \int d^4k \, d^4k' \, d^4k'' \cdot \frac{\Lambda^2}{(k^2 + \Lambda^2)(k'^2 + \Lambda^2)(k''^2 + \Lambda^2)} \cdot e^{\{ i k(x-y) + i k'(y-y') + i k''(y'-x) \}} \quad (4.8)$$

Finally, it is easy to prove by Fourier transformation that

$$\lim_{\Lambda^2 \rightarrow \infty} B(\Lambda^2, x, y, y') = \frac{-\pi^2}{2} (2\pi)^8 \delta(x-y) \delta(x-y') \quad (4.9)$$

so that we obtain the well-known result

$$A = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} \quad (4.10)$$

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REFERENCES

- 1) G. Parisi and Wu Yongshi, *Scientia Sinica* 24 (1981) 483.
- 2) J.D. Breit, S. Gupta and A. Zaks, *Nucl. Phys.* B233 (1984) 61.
- 3) M. Namiki and Y. Yamanaka, *Hadronic Journ.* 7 (1984) 594.
- 4) J. Alfaro, *Nucl. Phys.* B253 (1985) 464.
- 5) Z. Bern, *Nucl. Phys.* B251 (1985) 633.
- 6) D. Zwanziger, *Nucl. Phys.* B192 (1981) 259.
- 7) J. Alfaro and M.B. Gavela, *Phys. Lett.* 158B (1985) 473.
- 8) W. Grimus and H. Hüffel, *Z. Phys.* C18 (1983) 129.
- 9) H. Hüffel and P.V. Landshoff, *Nucl. Phys.* B260 (1985) 545.
- 10) Z. Bern and M.B. Halpern, LBL preprint 19714 (1985).
- 11) P.H. Damgaard and K. Tsokos, *Nucl. Phys.* B235 (1984) 75.
- 12) K. Ishikawa, *Nucl. Phys.* B241 (1984) 589.
- 13) B. Sakita, Lectures at Kanpur, India (December 1984).
- 14) U.B. Kaulfuss and U.G. Meissner, Ruhr Universität Bochum preprint (1985).
- 15) E.S. Egorian, E.R. Nissimov and S.J. Pacheva, INRNE preprints (1984, 1985).
- 16) R. Tzani, City University of New York preprint (1985).
- 17) J. Schwinger, *Phys. Rev.* 82 (1951) 664.

FIGURE CAPTIONS

Fig. 1 : One example of a stochastic diagram contributing to the one-loop correction of the photon propagator.

Fig. 2 : The stochastic diagrams contributing to the effective vertex V_μ .

Fig. 1

Fig. 2